Math 164 - Assignment #2, Fall 2024

- **Due Date:** 11:59 pm on October 18th, 2024

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1. Let $f(x) = x^2 + 4\cos x, x \in \mathbb{R}$. We wish to find the minimizer x^* of f over the interval $[1, 2]$. (Calculator users: Note that in $\cos x$, the argument x is in radians.)

   (a) Plot $f(x)$ versus x over the interval $[1, 2]$.

   (b) Use the golden section method to locate x^* to within an uncertainty of 0.2. Display all intermediate steps using a table:

Iteration k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	?	?	?	?	$[?, ?]$
2	?	?	?	?	$[?, ?]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

   (c) Repeat part b using the bisection method, with $\varepsilon = 0.05$. Display all intermediate steps using a table.

   (d) Apply Newton's method, using the same number of iterations as in part b, with $x^{(0)} = 1$.

2. Suppose that $\rho_1, \ldots, \rho_N$ are the values used in the Fibonacci search method. Show that for each $k = 1, \ldots, N, 0 \leq \rho_k \leq 1/2$, and for each $k = 1, \ldots, N-1$,

$$\rho_{k+1} = 1 - \frac{\rho_k}{1 - \rho_k}$$

3. Suppose that we have an efficient way of calculating exponentials. Based on this, use Newton's method to devise a method to approximate $\log(2)$ [where "log" is the natural logarithm function]. Use an initial point of $x^{(0)} = 1$, and perform two iterations.

4. Consider the problem of finding the zero of $g(x) = (e^x - 1)/(e^x + 1), x \in \mathbb{R}$, where e^x is the exponential of x. (Note that 0 is the unique zero of g.)

   (a) Write down the algorithm for Newton's method of tangents applied to this problem. Simplify using the identity $\sinh x = (e^x - e^{-x})/2$.

   (b) Find an initial condition $x^{(0)}$ such that the algorithm cycles [i.e. $x^{(0)} = x^{(2)} = x^{(4)} = \cdots$]. You need not explicitly calculate the initial condition; it suffices to provide an equation that the initial condition must satisfy. *Hint*: Draw a graph of g.

   (c) For what values of the initial condition does the algorithm converge?

5. Consider using a gradient algorithm to minimize the function

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^\top \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x}$$

   with the initial guess $\boldsymbol{x}^{(0)} = [0.8, -0.25]^\top$.

   (a) To initialize the line search, apply the bracketing procedure in Figure 7.11 (in the book) along the line starting at $\boldsymbol{x}^{(0)}$ in the direction of the negative gradient. Use $\varepsilon = 0.075$.

   (b) Apply the golden section, Fibonacci or bisection method to reduce the width of the uncertainty region to 0.01 . Organize the results of your computation in a table format similar to that of Exercise 2.