HW4 - SHILPA MUKHOPADHYAY
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Given Problem

Initial State: $(x_s, y_s, \theta_s) = (S_x, S_y, 0)$

Final state : $(x_g, y_g, \theta_g) = (10\text{ cm}, 10\text{ cm}, \pi/2)$

Velocity $\in [-2, 2]$ cm/s.

To output :- Sequence of actions $(v_l, v_r, \Delta t)$

Inverse Kinematics for Differential Drive

For forward drive we have

A $$x_t = \frac{1}{2}\int_0^t [v_r(t) + v_l(t)] \cos[\theta(t)]\, dt$$

$$y_t = \frac{1}{2}\int_0^t [v_r(t) + v_l(t)] \sin[\theta(t)]\, dt$$

$$\theta_t = \frac{1}{l}\int_0^t [v_r(t) - v_l(t)]\, dt.$$

(1) If we consider $v_r \neq v_l$, we get

$$x(t) = \frac{l}{2}\frac{v_r + v_l}{v_r - v_l} \sin\left(\frac{t}{l}(v_r - v_l)\right) = R \sin \omega t.$$

$$y(t) = -\frac{l}{2}\frac{v_r + v_l}{v_r - v_l} \cos\left(\frac{t}{l}(v_r - v_l)\right) = -R \cos \omega t.$$

$$\theta(t) = \frac{t}{l}(v_r - v_l) = \omega t.$$

(2) If $v_r = v_l = v$, from section A, we can conclude

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x + v \cos\theta . \Delta t \\ y + v \sin\theta\, \Delta t \\ \theta \end{bmatrix} \rightarrow \text{moving in straight line}$$

(3) Similarly, if $v_r = -v_l = v$, then section A gives

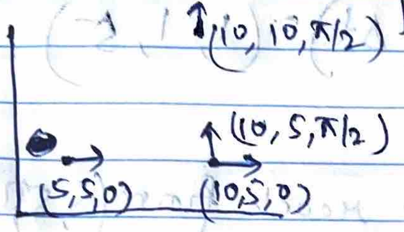
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta + 2v\dfrac{\Delta t}{l} \end{bmatrix} \rightarrow \text{rotating at same place.}$$

Now to solve the given problem, let me assume 3 actio

Action 1:- move from $(5, 5, 0)$ to $(10, 5, 0)$

Action 2 $\rightarrow$ Rotate from $(10, 5, 0)$ to $(10, 5, \pi/2)$

Action 3 $\rightarrow$ move from $(10, 5, \pi/2)$ to $(10, 10, \pi/2)$.



~~let~~

Using eqn (2) to perform action 1, let my $\Delta t = 5$ secs

$$\begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 + v\cos 0^\circ . 5 \\ 5 + v\sin 0^\circ . 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 + 5v \\ 5 \\ 0 \end{bmatrix}$$

$\therefore v = \dfrac{10-5}{5} = 1$ cm/s. $\quad \therefore v_r = v_l = v$.

$\therefore$ Action 1 $= (v_l, v_r, \Delta t) = (1, 1, 5)$

Using eqn 3 to perform action 2, [let my Δt be 5 secs]

$$\begin{bmatrix} 10 \\ 5 \\ \pi/2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 + 2v \times \dfrac{5}{4} \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ \dfrac{5v}{2} \end{bmatrix}$$

$\therefore \dfrac{\pi}{2} = \dfrac{5v}{2} \Rightarrow v = \dfrac{\pi}{5} = 0.63$ cm/s. $\because v_r = -v_l = v$.

$\therefore$ Action 2 $:= (v_l, v_r, \Delta t) = (-0.63, 0.63, 5)$

Using eqn 2 to solve action 3 (let $\Delta t = 5$ sec)

$$\begin{bmatrix} 10 \\ 10 \\ \pi/2 \end{bmatrix} = \begin{bmatrix} 10 + v \cos \pi/2 \cdot 5 \\ 5 + v \sin \pi/2 \cdot 5 \\ \pi/2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 + 5v \\ \pi/2 \end{bmatrix}$$

$v = 1$ cm/s

$\therefore v_l = v_r = 1$ cm/s

$\therefore$ Action 3 :- $(v_l, v_r, \Delta t) = (1, 1, 5)$

$\therefore$ Final actions to reach from $(5, 5, 0)$ to $(10, 10, \pi/2)$

A1: $(1, 1, 5)$

A2: $(-0.63, 0.63, 5)$

A3: $(1, 1, 5)$