Foundations of data science, summer 2020

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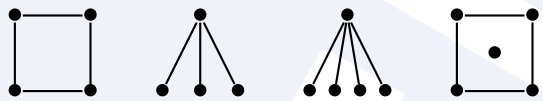
5. Exercise sheet
Hand in solutions until Thursday, 21 May 2020, 12:00

Exercise 5.1 (Eigenvectors with different eigenvalues). (4 points)

Show that if A is a symmetric matrix and α and α' are distinct eigenvalues then their corresponding eigenvectors $|x\rangle$ and $|x'\rangle$ are orthogonal. $\boxed{4}$

Exercise 5.2 (Eigenvalues of graphs). (10+2 points)

(i) What are the eigenvalues of (the adjacency matrices of) the graphs shown below? What does this say about using eigenvalues to determine whether two graphs are isomorphic? $\boxed{8}$



(ii) Let A be the adjacency matrix of an undirected, k-regular graph G, ie. each vertex hase degree k. Prove that the largest eigenvalue α_0 of A is equal to k. $\boxed{2+2}$

Hint: It is easy to name an eigenvector for eigenvalue k.

Exercise 5.3 (Separating balls). (0 points)

Consider a unit ball A centered at the origin and a unit ball B whose center is at distance s from A. Suppose that a random point x is drawn from the mixture distribution: "with probability $\frac{1}{2}$, draw at random from A; with probability $\frac{1}{2}$, draw at random from B". Show that a separation $s \gg \frac{1}{\sqrt{d-1}}$ is sufficient so that prob$\,(x \in A \cap B) = o\,(1)$, ie. for any $\varepsilon > 0$ there exists c such that if $s \geq \frac{c}{\sqrt{d-1}}$ then prob$\,(x \in A \cap B) < \varepsilon$. In other words, this extent of separation means that nearly all of the mixture distribution is identifiable.

Hint: Use the theorem about the tropics.