Foundations of data science, summer 2020

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9. Exercise sheet
Hand in solutions until Thursday, 18 June 2020, 12:00

Exercise 9.1 (Approximating $A\,|x\rangle$). (0 points)

In the course we have considered approximating $A\,|x\rangle$ by using the SVD. Well, this task asks you to set up and run some experiments in order to check out whether this really works and how well. Feel free to use python or whichever implementation you prefer.

Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(i) Compute its 2-truncated SVD $A_2 = U_2 D_2 V_2$.

(ii) Compute $A^T\,|y\rangle$ and $A_2^T\,|y\rangle$ with $|y\rangle = \left|(0, 0, 0, \frac{1}{2}, 0, \frac{1}{2}, 0, 0)\right\rangle$.

(iii) How large is the error? Compare it to the upper bound from the course.

(iv) Pick some suitable rank-2 matrices B and compute $\|A - B\|_F$ and $\|A - B\|_2$. Compare to $\|A - A_2\|_F$ and $\|A - A_2\|_2$, respectively.

(v) Plot the rows of $V_2 D_2$. Which points are closest to $\langle y|\,U_2 D_2$?

(vi) Repeat thelike with some other matrices (and vectors) of your choice. Report your findings.

Solution. See `09-programming-task.ipnyb` for most of the above.

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