Foundations of data science, summer 2020

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8. Exercise sheet
Hand in solutions until Thursday, 11 June 2020, 12:00

Exercise 8.1 (The greedy algorithm works). (14 points)

We would like to check that the greedy algorithm works in a further sense: compare the definition to the singular value decomposition, ie. numerically the result of the numpy command `numpy.linalg.svd()`.

You may use for your implementation that if we have a singular value decomposition $A = \sum_{i<k} \sigma_i |u_i\rangle \langle v_i|$ we have that $|v_0\rangle$ is exactly the first singular vector.

(i) Prove that the second singular vector $|v_1\rangle$ is exactly the first singular vector of the matrix [3]

$$A \cdot (1 - |v_0\rangle \langle v_0|).$$

(ii) Write a Python routine `first` that for any matrix A returns the first singular vector. You may use `numpy.linalg.svd()` for this. [4]

(iii) Randomly generate a matrix for at least dimension $d = 4$ and sample size $n = 8$ (to get an $n \times d$-matrix). Apply this routine to the matrix to obtain the first singular vector and apply (i) to also obtain the second. [3]

(iv) Compare numerically the second singular vector to the vector $|v_1\rangle$ from the SVD given by `numpy.linalg.svd()`. [1]

(v) With a method similar to (i), compute also the third singular vector and compare it to the SVD. [3]