Foundations of data science, summer 2020

JONATHAN LENNARTZ, MICHAEL NÜSKEN, ANNIKA TARNOWSKI

6. Exercise sheet
Hand in solutions until Thursday, 28 May 2020, 12:00

Exercise 6.1 (Eigenvalues and -vectors). (10 points)

Take the matrix $A = \begin{bmatrix} 2 & & 4 \\ & -1 & \\ 4 & & 3 \end{bmatrix}$.

(i) Compute eigenvalues and eigenvectors of A. $\boxed{6}$

(ii) How does A stretch vectors?

- Describe. $\boxed{2}$
- Plot how A maps the unit circle in the (x, z)-plane and the eigenvectors therein. $\boxed{2}$

Exercise 6.2 (To norm or not to norm?). (6 points)

For the following expressions, decide (and prove) if the following are norms or not.

(i) The "0-norm (?)": For a vector $|x\rangle$ we define $\boxed{2}$

$$\||x\rangle\|_0 := \#\{i \,|\, x_i \neq 0\}.$$

(ii) The "scaled 1-norm (?)": For a vector $|x\rangle$ we define $\boxed{2}$

$$\||x\rangle\|_s = n\,|x_0| + (n-1)\,|x_1| + (n-2)\,|x_2| + \cdots + 1\,|x_{n-1}|.$$

(iii) The "last-coordinate-norm (?)": For a vector $|x\rangle$ we define $\boxed{2}$

$$\||x\rangle\|_c = x_{n-1}.$$

Exercise 6.3 (Some lemmata). (0 points)

Prove the following lemmata.

(i) **Lemma** (Relations). *If A is symmetric and rank k then*

$$\|A\|_2^2 \leq \|A\|_F^2 \leq k \, \|A\|_2^2 \, .$$

(ii) **Lemma.** *Let A be symmetric. Then $\|A\|_2 = \max_{\||x\rangle\|_2=1} |\langle x| \, A \, |x\rangle|$.*

(iii) **Lemma.** *For each column $|A_{\cdot,j}\rangle$ of A we have $\||A_{\cdot,j}\rangle\|_2 \leq \|A\|_2$.*