## **Foundations of Data science**

Foundations of data science, summer 2020 Jonathan Lennartz, Michael Nüsken, Annika Tarnowski

## 4. Exercise sheet Hand in solutions until Thursday, 14 May 2020, 12:00

(8+3 points)

Exercise 4.1 (Random point in a ball).

Use your notion with $a=b$ to pick 100 points and plot them, so that we can search by the corrections of the algorithm.  Can you also plot a sample with $d=37$ Exercise 4.2 (Random ext).  Consider an experiment for which some general indicator variable $X$ has provided the continuation of the continuati		Write a Python routine (eg. in a jupyter notebook) to choose a uniformly random point in a unit ball of dimension $d$ .	5
Exercise 4.2 (Random exit). (10 points)  Consider an experiment for which some general indicator variable $X$ has problem of the problem of			3
Consider an experiment for which some general indicator variable $X$ has $\operatorname{prob}(X_i = 1) = p \neq 0$ . We create new random variables as following: Repeat the experiment to obtain $X_i$ for all $i \in \mathbb{N}$ . That is, (fill int). Then we set the exit important to obtain $X_i$ for all $i \in \mathbb{N}$ . That is, (fill int). Then we set the exit important in the important into		Can you also plot a sample with $d=3$ ?	+3
We create new random variables as following: Repeat the experiment to obtain $X_i$ for all $i \in \mathbb{N}^n$ . That is,(fill int)  Then we set the exit time $T = \min\{i \mid X_i = 1\}.$ We simply may assume that this minimum always exists.  (i) Prove that $T = i \iff X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_1 = 0.$ (ii) Compute() prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .  (iii) Prove a formula for prob $(T = 1)$ .  (iv) Express E(T) as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} i x^{i-1}$ .  (vi) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ runtime?  i) Given the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ runtime?  i) Given $(X_i) = 0 \land \cdots \land X_i = 0$ Since we know by definition of indicators variables; $X \longleftarrow Q_0, 1 \circlearrowleft_3, P(1) \qquad \text{is the probability}$ of the indicator variables and the first of the probability of the indicator $(X_i) = 0$ .  Finally $(X_i) = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i = 0 \land \cdots \land X_i = 0$ $(X_i) = 0 \land \cdots \land X_i $		Exercise 4.2 (Random exit). (10 points)	
We create new random variables as following: Repeat the experiment to obtain $X_i$ for all $i \in \mathbb{N}^r$ . That is, (fill in!)  Then we set the set it time $T = \min\{i \mid X_i = 1\}.$ We simply may assume that this minimum always exists.  (i) Prove that $T = i \iff X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_i = 0.$ (ii) Compute(f) prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .  (iii) Prove a formula for prob $(T = 1)$ .  (iv) Express $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{n} i x^{i-1}$ .  (vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $[T]$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $[T]$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $[T]$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $[T]$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $[T]$ runtime?  (vii) Consider the method 0 algorithm from class. What is its expected $[T]$ in the probability of the probability $[T]$ is the probability $[T]$ in the probability $[T]$ is the probability $[T]$ in the probability $[T]$ in the probability $[T]$ is the probability $[T]$ in the probability $[T]$ in the probability $[T]$ is the probability $[T]$ in		•	
Then we set the exit time $T = \min\{i \mid X_i = 1\}$ . We simply may assume that this minimum always exists.  (i) Prove that $T = i \iff X_i = 1 \land X_{i-1} = 0 \land \dots \land X_i = 0$ .  (ii) Compute(!) prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .  (iii) Prove a formula for prob $(T = i)$ .  (iv) Express $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{n} ix^{i-1}$ .  (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  (viii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?		We create new random variables as following: Repeat the experiment to	
We simply may assume that this minimum always exists.  (a) Prove that $T = i \iff X_i = 1 \land X_{i-1} = 0 \land \dots \land X_1 = 0$ .  (ii) Compute(1) prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .  (iii) Prove a formula for prob $(T = i)$ .  (iv) Espress $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} i e^{i-1}$ .  (vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  i) Given: $T = i \implies X_i = 1 \land X_{i-1} = 0 \land \dots \land X_1 = 0$ Since we know by definition of indicators variable $i$ : $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability  of the indicator worksholds: $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability $X \longleftarrow Q_0 \land 13$ , $P(i)$ is true probability.			
(i) Prove that $T = i \iff X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_i = 0.$ (ii) Compute(!) prob $(T = 1)$ , prob $(T = 2)$ .    (iii) Prove a formula for prob $(T = i)$ .    (iv) Express $E(T)$ as an infinite sum.    (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} ix^{i-1}$ .    (vi) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtime?  i) Cfirst : $T = i \implies X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_1 = 0$ Since we know by definition of indicators variable: $X \leftarrow 20, 13,  P(i)  \text{is the probability}$ of the indicator wardship $X_i = 0 \land \cdots \land X_1 = 0$ $X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_1 = 0$ $X_i = 0 \land \cdots \land X_1 = 0$ $X_$			
(ii) Compute(l) prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .  (iii) Prove a formula for prob $(T = 1)$ .  (iv) Express $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} ix^{i-1}$ .  (vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ runtine?  i) Given: $T = i \implies X_i = 1 \land X_{i-1} = 0 \land \cdots \land X_1 = 0$ Since we know by definition of indicators variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ of the indicator variable; $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, 13, P(i) \text{ is the probability}$ $X \leftarrow 20, P(i) \text{ is the probability}$ $X \leftarrow 20, P(i)  is the pro$			Б
(iii) Prove a formula for prob $(T=i)$ .  (iv) Express $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} ix^{i-1}$ .  (vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ runtime?  i) Given: $T=i \Rightarrow \chi_i=1  \chi_{i-1}=0  \chi_{i-1}=0$ Since we know by definition of indicators variables: $\chi \leftarrow \boxed{20, 13},  \rho(i) \text{ is the probability}$ of the indicator variables: $\chi \leftarrow \boxed{20, 13},  \rho(i) \text{ is the probability}$ of the indicator variables: $\chi \leftarrow \boxed{20, 13},  \rho(i) \text{ is the probability}$ of the indicator $\chi_{i-1}=0  \chi_{i-2}=0$ $\chi_{i-1}=0  \chi_{i-1}=0  \chi_{i-2}=0$ $\gamma_{i-1}=1  \chi_{i-1}=0  \chi_{i-2}=0$ $\gamma_{i-1}=1  \chi_{i-2}=0  \chi_{i-2}=0$ $\gamma_{i-1}=1  \chi_{i-2}=0  \chi_{i-2}=0$ $\gamma_{i-2}=0  \chi_{i-2}=0  \chi_{i-2}=0$ $\gamma_{i-2}=0  \chi_{i-2}=0  \chi_{i-2}=0$ $\gamma_{i-2}=0  \chi_{i-2}=0  \chi_{i-2}=0  \chi_{i-2}=0$ $\gamma_{i-2}=0  \chi_{i-2}=0  \chi_{$		· ·	[2]
(iv) Express $E(T)$ as an infinite sum.  (v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} ix^{i-1}$ .  (vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ runtime?  i) Given: $T=i \Rightarrow \chi_i = 1 \land \chi_{i-1} = 0 \land \cdots \land \chi_1 = 0$ Since we know by definition of indicators variable; $\chi \leftarrow 20, 13, \rho(1)$ is true probabilistry  of the indicator wards $A = 2 \times 13$ .  Ti = $\chi_i = 1 \land \chi_{i-1} = 0 \land \cdots \land \chi_{i=0} \land \gamma_{i=0} \land \cdots \land \chi_{i=0} \land \cdots \land \chi_{i=0}$		(ii) Compute(!) prob $(T = 1)$ , prob $(T = 2)$ , prob $(T = 3)$ .	1
(v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} ix^{i-1}$ . $\boxed{1}$ (vi) Compute the expected exit time $E(T)$ . $\boxed{1}$ (vii) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ runtime?  i) Given: $T=i \Rightarrow \chi_i=1 \land \chi_{i-1}=0 \land \cdots \land \chi_1=0$ Since we know by definition of indication variable: $\chi \leftarrow 20, 13, P(1)$ is the probability  of the indicated want. $A=\underbrace{2} x=13$ .  Ti = $\chi_i=1 \land \chi_{i-1}=0 \land \cdots \land \chi_1=0$ $\chi_i=0 \land \cdots \land \chi_1=0$ $\chi_1=0 \land \cdots \land \chi_1=0$ $\chi_1=$		(iii) Prove a formula for prob $(T = i)$ .	2
(vi) Compute the expected exit time $E(T)$ .  (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ (vii) Consider the method 0 algorithm from class. What is its expected $\mathbb{Z}$ (i) $T_i = X_i =   X$		(iv) Express $E(T)$ as an infinite sum.	1
(vii) Consider the method 0 algorithm from class. What is its expected $\boxed{2}$ i) Given: $T=i \Rightarrow x_i=1 \land x_{i-1}=0 \land \cdots \land x_1=0$ Since we know by definition of indicators variable;  of the Indicated out $A=2 \land x=13$ .  Ii) $T_i=x_i=1 \land x_{i-1}=0 \land \cdots \land x_{i=0}$ prob $(x_i=1)$ Prob $(x_{i-1}=0)$ $\cdots \land x_{i=0}$			
i) Given: $T=i \Rightarrow xi=1 \land x_{i-1}=0 \land \cdots \land x_{i}=0$ Since we know by definition of indicator verticals: $x \leftarrow 20, 13, P(i)$ is the probability of the indicated event. $A=2 \times =13$ . (ii) $T_i = x_i = 1 \land x_{i-1} = 0 \land \cdots \land x_{i=0} \land x_{i=0} \land \cdots \land x_{i=0} \land x_{i=0} \land \cdots \land x_{i=0} \land x_{i=$			
Since we know by definition of indicator variable: $X \leftarrow 20, 13, P(1) \text{ is the probability}$ of the indicated event. $A = 2 \times = 13$ . $T_i = X_i = 1 \land X_{i-1} = 0 \land - \cdots \land X_{1} = 0$ $P \text{ reab}(T_i) = P \text{ reab}(X_i = 1 \land X_{i-1} = 0)$ $= \text{prob}(X_{i-1} = 0) \cdot \cdots \cdot P \text{ reab}(X_{1} = 0)$ $= P(1-P) - \cdots \cdot (1-P)$			[2]
Since we know by definition of indicator variable: $X \leftarrow 20, 13, P(1) \text{ is the probability}$ of the indicated event. $A = 2 \times = 13$ . $T_i = X_i = 1 \land X_{i-1} = 0 \land \land X_{i=0} \land \land X$	i) Ginen	: T-1 -3 V = 1 AV = = 0 A	VX1=0
of the indicated event. $A = \{ \{ \{ \{ \{ \} \} \} \} \} \}$ .  Ti = $\{ \{ \{ \{ \} \} \} \} \} \} = \{ \{ \{ \{ \} \} \} \} \} $	., ,	1-2 -7 VC . 1/V/-1 - 2	1. 1
$= P \cdot (1-P) \cdot (1-P) = P(1-P)^{2}$	(11) Ti=	Kested went. A= 2 x= 19 Xi = 1 \ Xi-1 = 0 \ Presb(Ti) = Presb(Xi=1 Xi=1). Presb (Xi-1 = 0) (1-P) (1-P)	N X1=0 N X1=0 - Prop(x1=0) X=1). Prop(x1=0)
	1	- P. (1-P) (1-	$-P) = P(1-P)^2$

## **Foundations of Data science**

iv) By the dylinition we know
$$E(0) = \sum_{X \in D} x |_{Puob} (D = x)$$
Here  $f(T) = \sum_{X=1}^{\infty} x |_{Puob} T = x$ 

$$\sum_{i=1}^{\infty} x |_{Pio} x$$