Foundations of data science, summer 2020

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3. Exercise sheet
Hand in solutions until Thursday, 7 May 2020, 12:00

Exercise 3.1 (Cylinder). (8 points)

Given a d-dimensional circular cylinder of radius r and height h

(i) What is the surface in terms of $\mathrm{vol}(B^{d'})$ and $\mathrm{surface}(B^{d'})$ for appropriate d'? $\boxed{6}$

(ii) What is the volume? $\boxed{2}$

Exercise 3.2 (Annuli). (7 points)

(i) Compute and estimate the volume of the $\frac{1}{100}$-annulus compared to the volume of the d-dimensional ball B^d. $\boxed{1}$

(ii) Compute and estimate the volume of the $\frac{1}{\sqrt{d}}$-annulus compared to the volume of the d-dimensional ball B^d. $\boxed{1}$

(iii) Compute and estimate the volume of the $\frac{1}{d^2}$-annulus compared to the volume of the d-dimensional ball B^d. $\boxed{1}$

(iv) Plot the three functions and the relative volume of the $\frac{1}{d}$-annulus for $d = 1..20$. $\boxed{2}$

(v) For which ε does the ε-annulus have at least 99% of the ball volume? $\boxed{2}$

Exercise 3.3 (Gamma). (0 points)

(i) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. $\boxed{+0}$

Hint: Change the variable and use $\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x = \sqrt{\pi}$.

(ii) Recall the formula for integration by parts (look it up if need be...), $\int_a^b f(x)g'(x)\,\mathrm{d}x = \dots$, where f and g are any suitably nice functions. Use this formula to show that indeed $\Gamma(x+1) = x\Gamma(x)$. $\boxed{+0}$