Foundations of data science, summer 2020

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4. Exercise sheet
Hand in solutions until Thursday, 14 May 2020, 12:00

Exercise 4.1 (Random point in a ball). (8+3 points)

Write a Python routine (eg. in a jupyter notebook) to choose a uniformly random point in a unit ball of dimension d. $\boxed{5}$

Use your routine with $d = 2$ to pick $1\,000$ points and plot them, so that we can visually check the correctness of the algorithm. $\boxed{3}$

Can you also plot a sample with $d = 3$? $\boxed{+3}$

Exercise 4.2 (Random exit). (10 points)

Consider an experiment for which some general indicator variable X has

$$\mathrm{prob}\,(X = 1) = p \neq 0.$$

We create new random variables as following: Repeat the experiment to obtain X_i for all $i \in \mathbb{N}'$. That is, . . . (fill in!)

Then we set the exit time

$$T = \min\,\{i \mid X_i = 1\}\,.$$

We simply may assume that this minimum always exists.

(i) Prove that $\boxed{2}$

$$T = i \iff X_i = 1 \wedge X_{i-1} = 0 \wedge \cdots \wedge X_1 = 0.$$

(ii) Compute(!) $\mathrm{prob}\,(T = 1)$, $\mathrm{prob}\,(T = 2)$, $\mathrm{prob}\,(T = 3)$. $\boxed{1}$

(iii) Prove a formula for $\mathrm{prob}\,(T = i)$. $\boxed{2}$

(iv) Express $\mathrm{E}(T)$ as an infinite sum. $\boxed{1}$

(v) Use the geometric series to derive a formula for $\sum_{i=1}^{\infty} i x^{i-1}$. $\boxed{1}$

(vi) Compute the expected exit time $\mathrm{E}(T)$. $\boxed{1}$

(vii) Consider the method 1 algorithm from class. What is its expected runtime? $\boxed{2}$