**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B

*μ* = 45

*σ* = 8

P (X > 50) = P (X > 50) = 1 – P (X ≤ 50).

Z= 50-45/8 = 0.625

P (Z ≤ 0.625) =73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Mean = 38, SD = 6

Z= (value-mean)/SD

A: More employees at the processing center are older than 44 than between 38 and 44.???

Z score for 44 = 44-38/6 = 1

Rcode🡪Pnorm (1) = 0.8413447

People above 44 age = 1- 0.8413447 = 0.1586553 = 15.87% ≈ 63    out of 400 people.

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence the statement: More employees at the processing center are older than 44 than between 38 and 44 is **FALSE**

B: Z score for 30 = (30 - 38)/6 = -1.33 = 9.15 %   ≈ 36 out of 400

Hence the statement: A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if,

X~N (µ1, σ12), and Y~N (µ2, σ22) are two independent random variables, then:

X+Y ~ N (µ1+ µ2, σ12 +σ22) and X-Y ~ N (µ1 - µ2, σ1^2 + σ2^2).

Similarly, if Z= aX+bY, where X and Y are as defined above, i.e. Z is linear combination of X and Y then,

Z~ N (aµ1+ bµ2, a2 σ12 +b2σ22).

Therefore, in the question,

2X1~ N (2 µ, 4 σ2) and,

X1+X2 ~ N (µ + µ, σ2+ σ2) ~ N (2 µ,2 σ2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D

We have a & b symmetric about mean, and the probability to select a random variable between them is given as 99%. We have to work in reverse order to get the values for a & b i.e. x here.

Probability to take between a & b is 0.99

So, probability for not to select between them is .01

Since it is symmetrical about mean, we can split them as,

Towards the left of a = 0.005

Towards right of b = 0.005

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

P(x<=a) = 0.005

P(x<=b) = 0.995

We have to find the z score for these probabilities, to get X values.

Z (0.005): Rcode🡪qnorm (.005) = -2.575829

Z (0.995): Rcode🡪qnorm (.995) = 2.575829

To find x value, Z \* σ + μ = X

X at a: = -2.575829 \*20 +100 = 48.48342

X at b: = 2.575829\*20+100 = 151.51658

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answers:

A: construct confidence interval with a confidence of 95%

Division 1:

Range or Interval: [51.96\*3] = [-0.88 10.88]

Division 2:

Range or Interval: [7] = [-0.84 14.84]

Company’s total profit range in $: [-1.72 25.72]

Company’s total profit range in Rs contains 95% probability for profit:

[-77.4 1157.4] million rupees.

1. 5th percentile of profit

Z score (0.05)🡪 qnorm (0.05) = -1.644854

= -1.64\* 3+5 = 0.08

= -1.64\*4+7 = 0.44

Total profit = 0.08+0.44 = 0.52 = 23.4 million rupees.

1. Considering the breakeven point for as 0 between loss and profit.

Division 1:

Z= 0-5/3 = -1.67

Percentile of the breakeven point = 4.75

i.e. probability to get the profit of 0 Rs = 0.0475 = 4.75%

Division 2:

Z= 0-7/4 = -1.75

i.e. probability to get the profit of 0 Rs = .0400 = 4%

So, Division 1 has the larger probability of making loss in the given year for a percentage of 4.75.