Lecture on
Fuzzy if then Rules(Unit 4 Lecture 3)

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Outline of Presentation

- Introduction.
- History.
- Fuzzy If-Then Rules.
- References.

Introduction

- fuzzy rule-based systems are rule-based systems, where fuzzy sets and fuzzy logic are used as tools for representing different forms of knowledge about the problem at hand, as well as for modeling the interactions and relationships existing between its variables.
- The act or process of inferring by deduction or induction.
- That which inferred; a truth or proposition drawn from another which is admitted or supposed to be true; a conclusion; a deduction. –Milton.
- Inference is a process of obtaining new knowledge through existing knowledge.

History

In 1973, Lotfi Zadeh published his second most influential paper *. This paper outlined a new approach to analysis of complex systems, in which Zadeh suggested capturing human knowledge in **fuzzy rules**.

A fuzzy rule generally assumes the form

$$\boxed{\textbf{R:} \quad \textbf{If } x \textbf{ is A} \;, \quad \textbf{then } y \textbf{ is B.}}$$

where A and B are linguistic values defined by fuzzy sets on universes of discourse X and Y, respectively. The rule is also called a "**fuzzy implication**" or fuzzy conditional statement. The part "*x* is A" is called the "***antecedent***", while "*y* is B" is called the "***consequence***" or "conclusion". It typically expresses an inference such that if we know a fact or a hypothesis (antecedent,), then we can infer, or derive, another fact called a conclusion (consequent).

Fuzzy If-Then Rules.

- Fuzzy sets and fuzzy sets operations are the subjects and verbs of fuzzy logic. If-Then rule statements are used to formulate the conditional statements that comprise fuzzy logic.
- A single fuzzy If-Then rule assumes the form.
  If x is A1 Then y is B2 ....(Eq 1)
- where A1 and B2 are linguistic variables defined by fuzzy sets on the ranges (i.e. universe of discourse) X and Y respectively. The If-part of the rule 'x is A1' is called the antecedent or premise and the Then-part of the rule 'y is B2' is called the consequent.
- In other words, the conditional statement can be expressed in a mathematical form
  If A1 Then B2 or A1 $\rightarrow$ B2 .... (Eq 2)

Example

Speed and pressure of a steam engine can be expressed with the following linguistic conditional statement

If ***Speed*** is ***Slow*** Then ***Pressure*** should be ***High***.

Graphically, this statement looks like

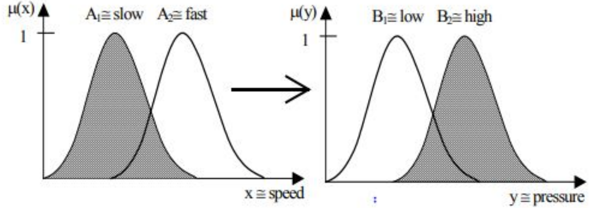

Figure 1: If-Then rule.

Rule Forms

- In general, three forms of rule exist for any linguistic variables.

  1. Assignment statement

  e.g. x is not large AND not very small

  2. Conditional statement

  e.g. IF x is very big THEN y is medium

  3. Unconditional statement

  e.g. set pressure high

Fuzzy If-Then Rules.

Before we employ fuzzy if-then rules to model and analyze a system, first we have to formalize what is meant by the expression:

R: If x is A then y is B

which is sometimes abbreviated as

$$\boxed{R: A \rightarrow B}$$

In essence, the expression describes a relation between two variables x and y. This suggests that a fuzzy rule can be defined as a binary relation R on the product space X x Y.

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6.2. Classical vs. Fuzzy Rules

A **classical** IF-THEN rule uses binary logic, for example,

Rule: 1
IF speed is > 100km/h
THEN stopping distance is 200m

Rule: 2
IF speed is < 40km/h
THEN stopping distance is 60m

The variable speed can have any numerical value e.g. between 0 and 220km/h, and the variable stopping distance can take values between say 10m and 300m. In other words, classical rules are expressed in the black-and-white language of Boolean logic.

Classical vs. Fuzzy Rules (contd.)

We can also represent the stopping distance rules in a **fuzzy** form:

Rule: 1	Rule: 2
IF speed is fast	IF speed is slow
THEN stopping distance is long	THEN stopping distance is short

In fuzzy rules, the linguistic variable speed also has the range (the universe of discourse) between 0 and 220km/h, but this range includes fuzzy sets, such as slow, medium and fast. The universe of discourse of the linguistic variable stopping distance can be between 0 and 300m and may include such fuzzy sets as short, medium and long.

Fuzzy rules relate fuzzy sets.

In a fuzzy system, all rules fire to some extent, or in other words they fire partially.

If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

Fuzzy If-Then Rules.

A fuzzy rule can have multiple antecedents, for example:

IF project_duration is long
AND project_staffing is large
AND project_funding is inadequate
THEN risk is high

IF service is excellent
OR food is delicious
THEN tip is generous

The consequent of a fuzzy rule can also include multiple parts, for instance:

IF temperature is hot
THEN hot_water is reduced;
cold_water is increased

- A fuzzy implication (also known as fuzzy If-Then rule, fuzzy rule, or fuzzy conditional statement) assumes the form :

  If x **is** A **then** y **is** B

  where, A and B are two linguistic variables defined by fuzzy sets A and B on the universe of discourses X and Y, respectively.

- Often, x **is** A is called the antecedent or premise, while y **is** B is called the consequence or conclusion.

Fuzzy If-Then Rules.

- If pressure is High then temperature is Low
- If mango is Yellow then mango is Sweet else mango is Sour
- If road is Good then driving is Smooth else traffic is High
- The fuzzy implication is denoted as $R : A \to B$
- In essence, it represents a binary fuzzy relation R on the (Cartesian) product of $A \times B$

- Suppose, P and T are two universes of discourses representing pressure and temperature, respectively as follows.
- $P = \{ 1,2,3,4\}$ and $T = \{ 10, 15, 20, 25, 30, 35, 40, 45, 50 \}$
- Let the linguistic variable High temperature and Low pressure are given as
- $T_{HIGH} = \{(20, 0.2), (25, 0.4), (30, 0.6), (35, 0.6), (40, 0.7), (45, 0.8), (50, 0.8$
- $P_{LOW} = (1, 0.8), (2, 0.8), (3, 0.6), (4, 0.4)$

Fuzzy If-Then Rules.

- Then the fuzzy implication **If temperature is High then pressu is Low** can be defined as

  $R : T_{HIGH} \rightarrow P_{LOW}$

where, R =

$$\begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.4 \\ 0.7 & 0.7 & 0.6 & 0.4 \\ 0.8 & 0.8 & 0.6 & 0.4 \\ 0.8 & 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

Note : If temperature is 40 then what about low pressure?

Fuzzy If-Then Rules.

$X = \{a, b, c, d\}$

$Y = \{1, 2, 3, 4\}$

Let, $A = \{(a, 0.0), (b, 0.8), (c, 0.6), (d, 1.0)\}$

$B = \{(1, 0.2), (2, 1.0), (3, 0.8), (4, 0.0)\}$

Determine the implication relation :

If x is A then y is B

Here, $A \times B =$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\text{and } \bar{A} \times Y = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R_{mm} = (A \times B) \cup (\bar{A} \times Y) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

This R represents **If** x **is A then** y **is B**

IF x is A THEN y is B ELSE y is C.

The relation R is equivalent to

$$R = (A \times B) \cup (\bar{A} \times C)$$

The membership function of R is given by

$$\mu_R(x, y) = max[min\{\mu_A(x), \mu_B(y)\}, min\{\mu_{\bar{A}}(x), \mu_C(y)]$$

$X = \{a, b, c, d\}$

$Y = \{1, 2, 3, 4\}$

$A = \{(a, 0.0), (b, 0.8), (c, 0.6), (d, 1.0)\}$

$B = \{(1, 0.2), (2, 1.0), (3, 0.8), (4, 0.0)\}$

$C = \{(1, 0), (2, 0.4), (3, 1.0), (4, 0.8)\}$

Determine the implication relation :

If x is A then y is B else y is C

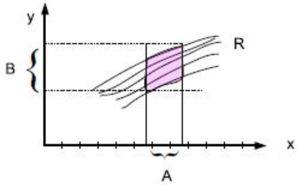
Here, $A \times B =$
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\text{and } \bar{A} \times C = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1.0 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

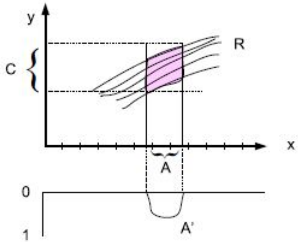
$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1.0 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

Fuzzy If-Then Rules.

If x is A then y is B



If x is A then y is B else y is C



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Queries

Thanking You