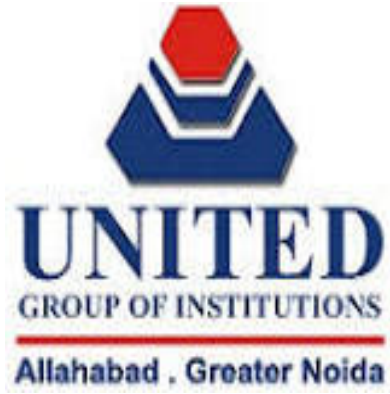
Lecture on
Fuzzy and Crisp conversion(Unit 3 Lecture 6)



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Outline of Presentation

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- Introduction.
- Development of Type-1 Fuzzy Systems.
- Defuzzification Methods
- References.

Introduction

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Why Fuzzy Sets?

- It enables one to work in uncertain and ambiguous situations and solve ill-posed problems or problems with incomplete information.

Two frameworks for Fuzzy Systems

- 1) Development based on Crisp mathematical model and fuzzifying some quantities :
  Model 1 : Fuzzy Mathematical Model
  Example : Fuzzy – K means clustering
  2) Development based on Fuzzy Inference rules:
  Model 2 : Fuzzy Logical Model
  Example : Fuzzy decision Support System

Fuzzy logic system

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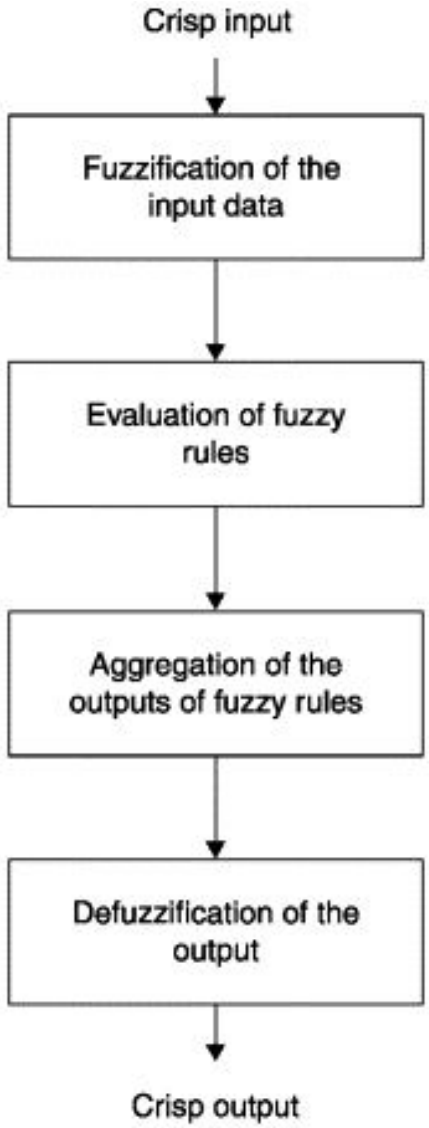
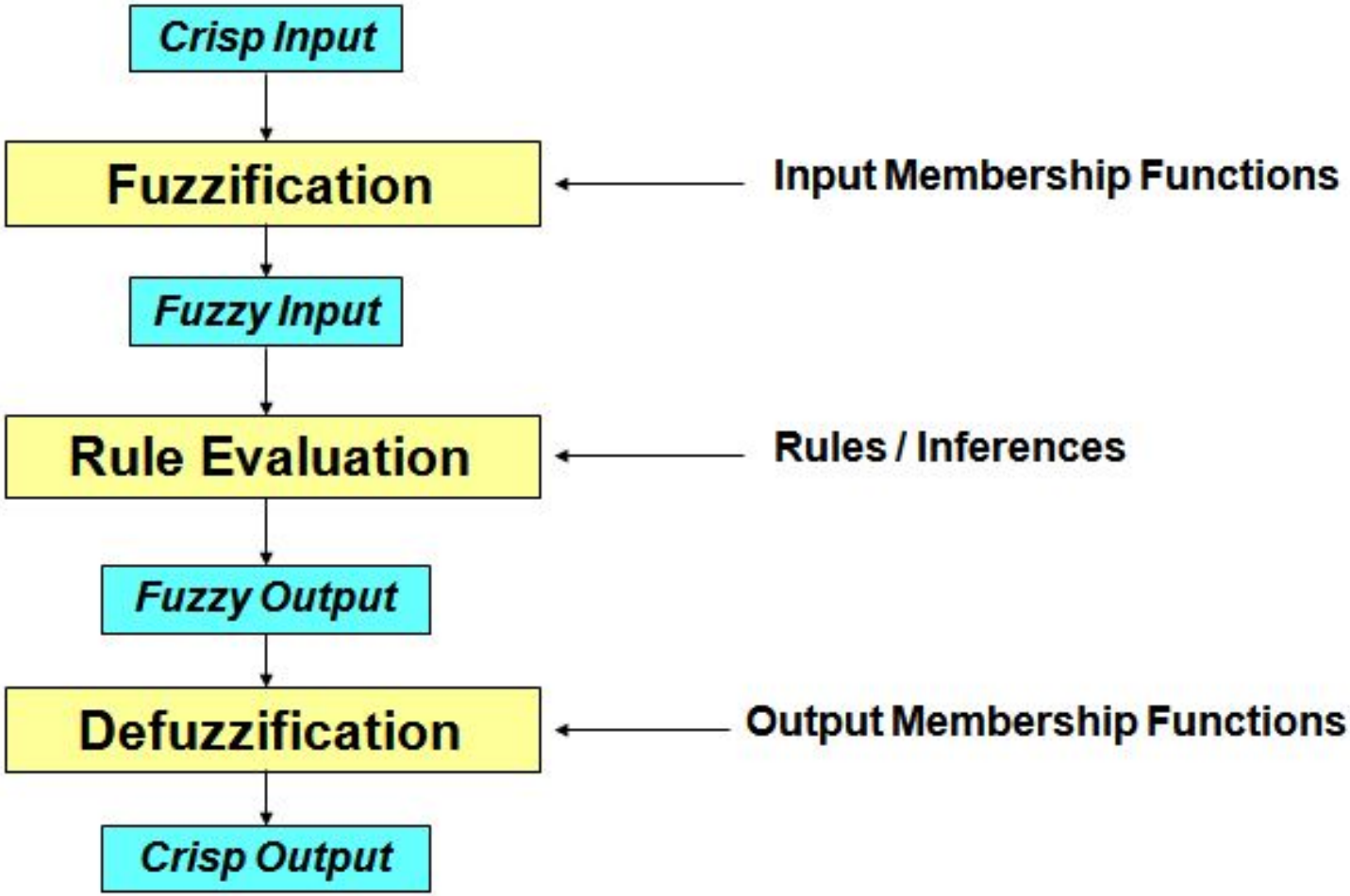
- A fuzzy logic system maps crisp inputs into crisp outputs using the theory of fuzzy sets. In a fuzzy logic system, an inference engine works with fuzzy rules.
- The engine takes inputs, some of which may be fuzzy, and generates outputs, some of which may be fuzzy. The fuzzy core of the inference engine is bracketed by one step that can convert crisp data into fuzzy data, and another step that does the reverse.
- Fuzzy expert system is a collection of membership functions and rules that are used to reason about data.

The general structure of a fuzzy logic system.



The general structure of a fuzzy logic system.



Development of Type-1 Fuzzy Systems.

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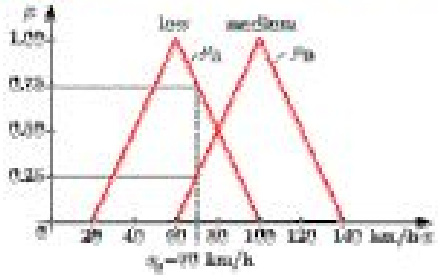
Fuzzy Mathematical Model

- 1) Fuzzification of quantities
  2) Composition of fuzzy sets
  3) Composition of fuzzy relations
  4) Defuzzification of quantities

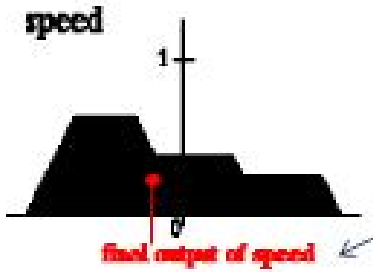
Fuzzy Mathematical Model

Fuzzification: Making a crisp quantity fuzzy.



Assignment of membership functions is the process of fuzzification

Defuzzification: Making a fuzzy quantity crisp.



Calculation of a crisp value from a fuzzy value

Fuzzification of input quantities

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- 1. Discretization and normalization
  2. Fuzzy partition of spaces
  3. Membership function of primary fuzzy set
- Fuzzification is the process of decomposing a system input and/or output into one or more fuzzy sets. Many types of curves and tables can be used, but triangular or trapezoidal-shaped membership functions are the most common, since they are easier to represent in embedded controllers.

Fuzzification

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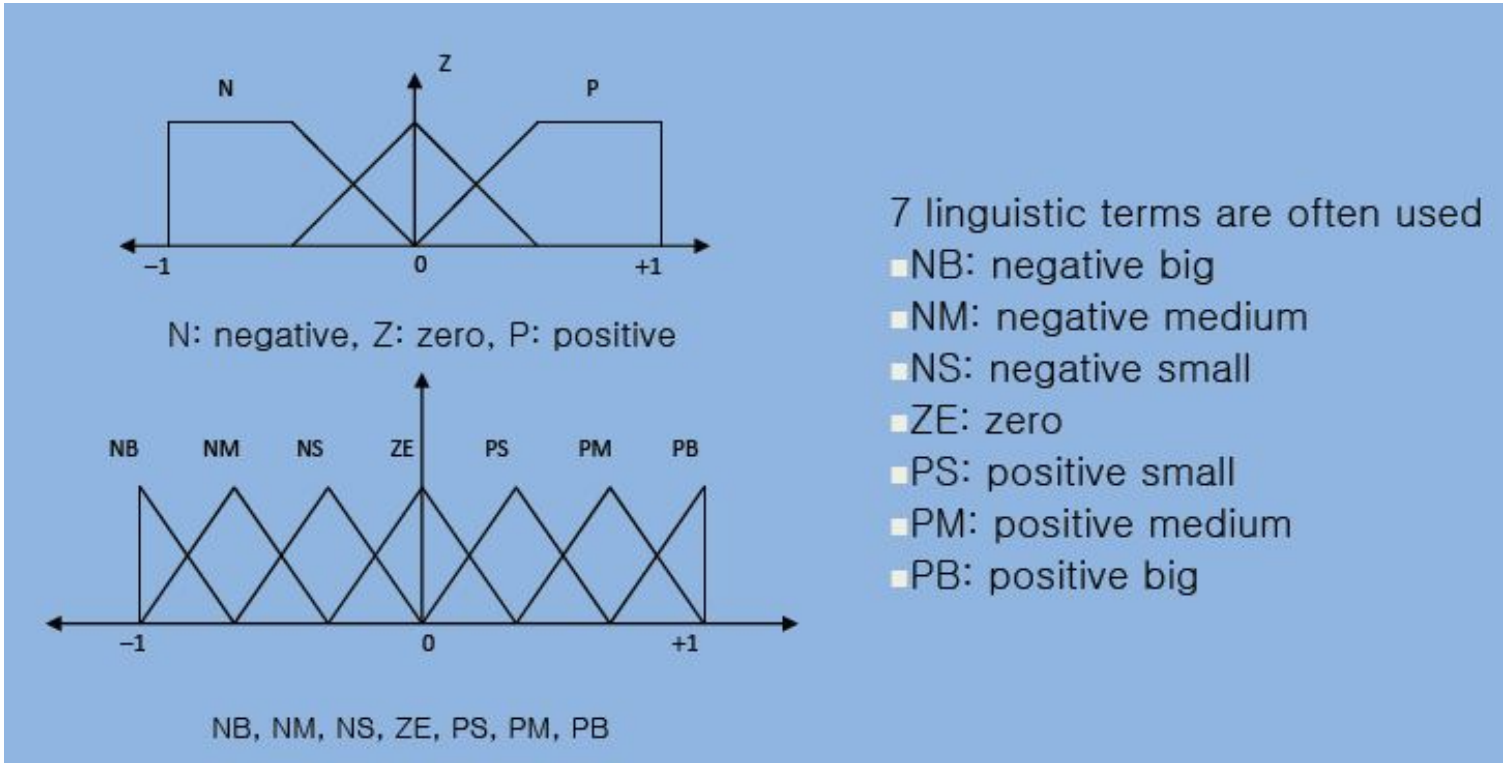
- Fuzzification is the process of converting a crisp input value to a fuzzy value that is performed by the use of the information in the knowledge base.
- Fuzzification may be defined as the process of transforming a crisp set to a fuzzy set or a fuzzy set to fuzzier set. Basically, this operation translates accurate crisp input values into linguistic variables.

Fuzzy Partition : Example

- Example of fuzzy partition with linguistic terms.



Fuzzy Partition : Example

- Fuzzy partition of input and output spaces

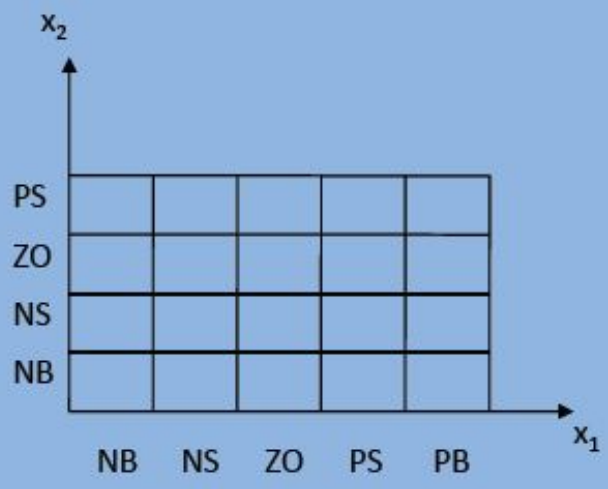


Fig : A fuzzy partition
in 2–dimension input space

the maximum number of control rules = 20 (5x4)

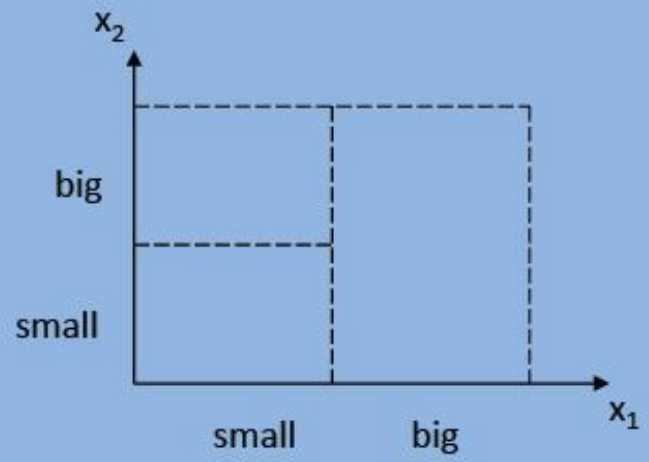


Fig : A fuzzy partition
having three rules

FUZZY-TO-CRISP CONVERSIONS

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$\underset{\sim}{A}$: a fuzzy set A

A_λ: Lambda-Cut set of A

A_λ: $\{x \mid \mu_{\underline{A}}(x) \geq \lambda\}$ where $0 \leq \lambda \leq 1$

The set A_λ is a crisp set.

FUZZY-TO-CRISP CONVERSIONS

Example: $X=\{a,b,c,d,e,f\}$ and $\underset{\sim}{A}=\{\frac{1}{a}+\frac{0.9}{b}+\frac{0.6}{c}+\frac{0.3}{d}+\frac{0.01}{e}+\frac{0}{f}\}$

$\lambda = 1 \quad \rightarrow A_1=\{a\}$;

$\lambda = 0.8 \rightarrow A_{0.8}=\{a,b\}$;

$\lambda = 0.6 \rightarrow A_{0.6}=\{a,b,c\}$;

$\lambda = 0^+ \rightarrow A_{0^+}=\{a,b,c,d,e\}$;

$\lambda = 0 \quad \rightarrow A_0=\{a,b,c,d,e,f\}=X$

Fuzzy notation of λ-Cut Sets:

$\underset{\sim}{A}_{0.6}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{0}{d}+\frac{0}{e}+\frac{0}{f}\}$

$\underset{\sim}{A}_{0.25}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{0}{e}+\frac{0}{f}\}$

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LAMBDA-CUTS FOR FUZZY RELATIONS

$\underset{\sim}{R}$: A fuzzy relation

R_λ: λ-cut relation of $\underset{\sim}{R}$.

$R_\lambda = \{(x,y) \mid \mu_{\underset{\sim}{R}}(x,y) \geq \lambda\}$ for $0 \leq \lambda \leq 1$

FUZZY-TO-CRISP CONVERSIONS

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Example: $R = \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0.4 \\ 0 & 0.4 & 1 \end{bmatrix}$

$\lambda = 1 \rightarrow R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

$\lambda = 0.25 \rightarrow R_{0.25} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$;

$\lambda = 0.5 \rightarrow R_{0.5} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

$\lambda = 0 \rightarrow R_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Defuzzification

- Defuzzification It may be defined as the process of reducing a fuzzy set into a crisp set or to convert a fuzzy member into a crisp member.
- Fuzzy rule based systems evaluate linguistic if-then rules using fuzzification, inference and composition procedures. They produce fuzzy results which usually have to be converted into crisp output. To transform the fuzzy results in to crisp, defuzzification is performed.
- Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set. The defuzzified value in FLC (Fuzzy Logic Controller) represents the action to be taken in controlling the process.

Different Defuzzification Methods

- The following are the known methods of defuzzification.
  1 Center of Sums Method (COS)
  2 Center of gravity (COG) / Centroid of Area (COA) Method
  3 Center of Area / Bisector of Area Method (BOA)
  4 Weighted Average Method
  5 Maxima Methods
  5.1 First of Maxima Method (FOM)
  5.2 Last of Maxima Method (LOM)
  5.3 Mean of Maxima Method (MOM)

Center of Sums (COS) Method

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- This is the most commonly used defuzzification technique. In this method, the overlapping area is counted twice.

Center of gravity (COG) / Centroid of Area (COA) method

- This method provides a crisp value based on the center of gravity of the fuzzy set.
- The total area of the membership function distribution used to represent the combined control action is divided into a number of sub-areas.
- The area and the center of gravity or centroid of each sub-area is calculated and then the summation of all these sub-areas is taken to find the defuzzified value for a discrete fuzzy set.

The important difference between Fuzzification and Defuzzification.

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Sr. No.	Key	Fuzzification	Defuzzification
1	Definition	Fuzzification is the process of transforming a crisp set to a fuzzy set or a fuzzy set to fuzzier set.	Defuzzification is the process of reducing a fuzzy set into a crisp set or converting a fuzzy member into a crisp member.
2	Purpose	Fuzzification converts a precise data into imprecise data.	Defuzzification converts an imprecise data into precise data.
3	Example	Voltmeter.	Stepper motor, D/A converter.
4	Methods used	Inference, Rank ordering, Angular fuzzy sets, Neural network.	Maximum membership principle, Centroid method, Weighted average method, Center of sums.
5	Complexity	Fuzzification is easy.	Defuzzification is quiet complex to implement.
6	Approach	Fuzzification uses if-then rules to fuzzify the crisp value.	Defuzzification uses center of gravity methods to get centroid of sets.

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Queries

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Thanking You