Lecture on Fuzzy Set Theory(Operations and Properties)(Unit-3-Lecture 3)

Presented By

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Outline of Presentation

- Objective of Fuzzy Set theory.
- Introduction.
- Fuzzy set Theory.
- Fuzzy Sets and Membership Functions.
- Fuzzy Set Operations.
- Fuzzy Set properties.
- References.

Objective of Fuzzy Set theory

- 1. To introduce fuzzy sets and how they are used.
  2. To define some types of uncertainty and study what methods are used to with each of the types.
  3. To define fuzzy numbers, fuzzy logic and how they are used.
  4. To study methods of how fuzzy sets can be constructed.
  5. To see how fuzzy set theory is used and applied in cluster analysis.

Introduction

- Fuzzy Set Theory was formalised by Professor Lotfi Zadeh at the University of California in 1965 to generalise classical set theory.
- Since 1992 fuzzy set theory, the theory of neural nets and the area of evolutionary programming have become known under the name of ‘computational intelligence’ or ‘soft computing’.
- Fuzzy set theory formally speaking is one of these theories, which was initially intended to be an extension of dual logic and/or classical set theory.

Fuzzy Set Theory

- The concept of a fuzzy set, on which fuzzy logic (FL) has been built, has been proven to play an important role in
  (1) modeling and representing imprecise and uncertain linguistic human concepts;
  (2) mimicking human thinking; and
  (3) researchers and practitioners developing tools to model behaviors in forms that are easy to understand and implement in computer systems.

Fuzzy Set Theory

- An object has a numeric "degree of membership"
  Normally, between 0 and 1 (inclusive)
  1. 0 membership means the object is not in the set
  2. 1 membership means the object is fully inside the set
  3. In between means the object is partially in the set

Fuzzy Sets and Membership Functions

If U is a collection of objects denoted generically by x, then a *fuzzy set* A in U is defined as a set of ordered pairs:

$$A = \{(x, \underbrace{\mu_A(x)}_{\text{membership function}}) \mid x \in U\}$$

U : universe of discourse.

$$\mu_A : U \rightarrow [0,1]$$

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Characteristic function X, indicating the belongingness of x to the set A

$$X(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

or called **membership**

Hence,

$$A \cup B \to X_{A \cup B}(x)$$
$$= X_A(x) \cup X_B(x)$$
$$= \max(X_A(x), X_B(x))$$

Note: Some books use + for $\cup$, but still it is not ordinary addition!

FUZZY SET OPERATIONS

The fuzzy set operations of union, intersection and complementation are defined in terms of membership functions as follows:

- Union:
  $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
- Intersection:
  $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
- Complement:
  $\mu_{\text{not } A}(x) = 1 - \mu_A(x)$

The other fuzzy set theory constructs that are essential are:

- Fuzzy Set Inclusion:
  $A \subset B$ if and only if $\forall x$ (for all x) $\mu_A(x) \leq \chi_B(x)$
- Fuzzy Set Equality:
  $A = B$ if and only if $\forall x$ (for all x) $\mu_A(x) = \mu_B(x)$.

Fuzzy Set Operation

$$\mu_{A\cup B}(x) = \mu_A(x) \cup \mu_B(x)$$
$$= \max(\mu_A(x), \mu_B(x))$$

$$\mu_{A\cap B}(x) = \mu_A(x) \cap \mu_B(x)$$
$$= \min(\mu_A(x), \mu_B(x))$$

$$\mu_{A'}(x) = 1 - \mu_A(x)$$

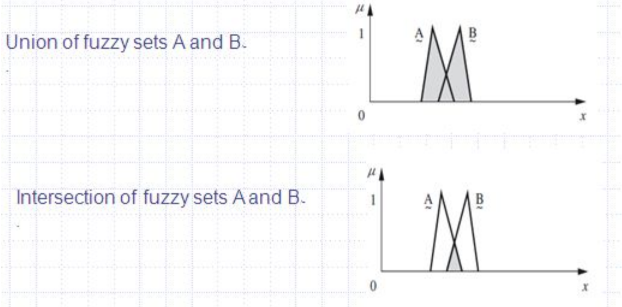
De Morgan's Law also holds:

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

But, in general

$$A \cup A' \neq X$$
$$A \cap A' \neq \phi$$

Fuzzy Set Operation



Fuzzy Set union

Fuzzy union (∪): the union of two fuzzy sets is the maximum (MAX) of each element from two sets.

E.g.

- A = {1.0, 0.20, 0.75}
- B = {0.2, 0.45, 0.50}
- A ∪ B = {MAX(1.0, 0.2), MAX(0.20, 0.45), MAX(0.75, 0.50)} = {1.0, 0.45, 0.75}

Fuzzy Set intersection

- Fuzzy intersection ($\cap$): the intersection of two fuzzy sets is just the MIN of each element from the two sets.
- E.g.
  - $A \cap B$ = {MIN(1.0, 0.2), MIN(0.20, 0.45), MIN(0.75, 0.50)} = {0.2, 0.20, 0.50}

Examples of Fuzzy Set Operations

A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e}

B = {0.6/a, 0.9/b, 0.1/c, 0.3/d, 0.2/e}

Complement:

A' = {0/a, 0.7/b, 0.8/c 0.2/d, 1/e}

Union:

$A \cup B$ = {1/a, 0.9/b, 0.2/c, 0.8/d, 0.2/e}

Intersection:

$A \cap B$ = {0.6/a, 0.3/b, 0.1/c, 0.3/d, 0/e}

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Properties of Fuzzy Sets

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$A \cup A = A$	$A \cap A = A$
$A \cup X = X$	$A \cap X = A$
$A \cup \varnothing = A$	$A \cap \varnothing = \varnothing$

If $A \subseteq B \subseteq C$, then $A \subseteq C$

$$A'' = A$$

Fuzzy Set properties

Operations

- Properties of Standard Fuzzy Operators

1) Involution : $(F^c)^c = F$
2) Commutative : $F \cup G = G \cup F$
   $F \cap G = G \cap F$
3) Associativity : $F \cup (G \cup H) = (F \cup G) \cup H$
   $F \cap (G \cap H) = (F \cap G) \cap H$
4) Distributivity : $F \cup (G \cap H) = (F \cup G) \cap (F \cup H)$
   $F \cap (G \cup H) = (F \cap G) \cup (F \cap H)$
5) Idempotency : $F \cup F = F$
   $F \cap F = F$

Fuzzy Set properties

Operations

6) Absorption :

$$F \cup (F \cap G) = F \quad F \cap (F \cup G) = F$$

7) Absorption by ϕ and U :

$$F \cap \phi = \phi, \qquad F \cup U = U$$

8) Identity :

$$F \cup \phi = F \qquad F \cap U = F$$

9) DeMorgan's Law:

$$(F \cap G)^C = F^C \cup G^C \qquad (F \cup G)^C = F^C \cap G^C$$

10) Equivalence :

$$(F^C \cup G) \cap (F \cup G^C) = (F^C \cap G^C) \cup (F \cap G)$$

11) Symmetrical difference:

$$(F^C \cap G) \cup (F \cap G^C) = (F^C \cup G^C) \cap (F \cup G)$$

Refrences

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Queries

Thanking You