Lecture on
Membership functions in fuzzy logic (Unit 4 Lecture 1)

Presented By
Ashish Tiwari
Assistant Professor

Department of CSE
United College of Engg. and Research,
Prayagraj, India

Outline of Presentation

- Introduction.
- Membership functions.
- Different forms of membership functions.
- Fuzzy Membership Function: Basic Concepts.
- References.

Introduction

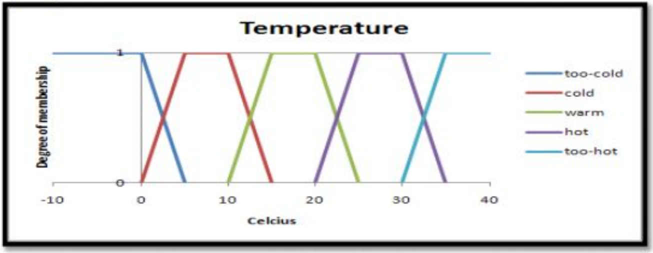
- A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.
- The input space is sometimes referred to as the universe of discourse, a fancy name for a simple concept.
- The output-axis is a number known as the membership value between 0 and 1. The curve is known as a membership function and is often given the designation of μ.

Intro

- Membership function (MF) - A function that specifies the degree to which a given input belongs to a set.
- Degree of membership- The output of a membership function, this value is always limited to between 0 and 1. Also known as a membership value or membership grade.
- Membership functions are used in the fuzzification and defuzzification steps of a FLS (fuzzy logic system), to map the non-fuzzy input values to fuzzy linguistic terms and vice versa.

- Membership functions were first introduced in 1965 by Lofti A. Zadeh in his first research paper "fuzzy sets".
- Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.
- Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.
- Membership functions are represented by graphical forms. Rules for defining fuzziness are fuzzy too.

Example

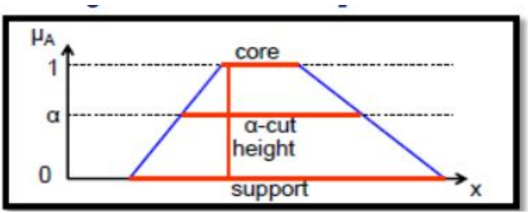


- Membership Functions for T (temperature) = **too-cold, cold, warm, hot, too-hot.**

Fuzzy Membership Function: Basic Concepts

- Support: elements having non-zero degree of membership.
- Core: set with elements having degree of 1.
- α -Cut: set of elements with degree $\geq \alpha$.
- Height: maximum degree of membership.

Example



Fuzzy Membership Function: Basic Concepts

- A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse.
- The only condition a membership function must really satisfy is that it must vary between 0 and 1.

Membership functions in Fuzzy vs. crisp sets

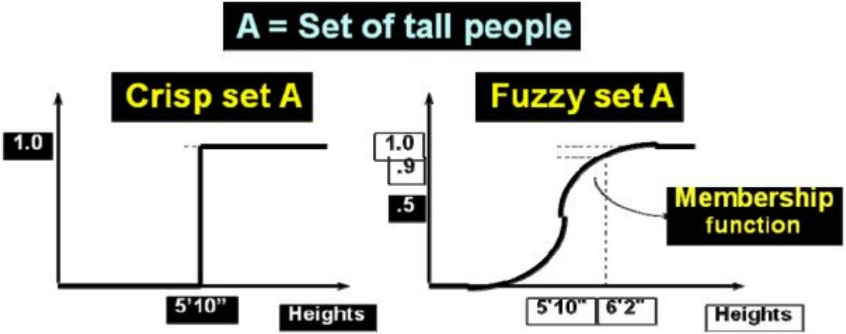
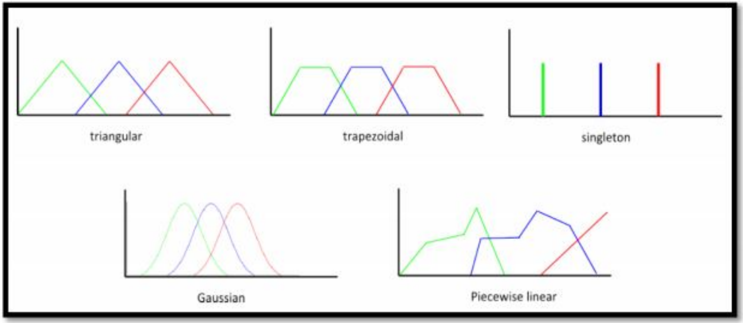


Figure 3.1: Membership functions in Fuzzy vs. crisp sets

Different forms of membership functions

- There are different forms of membership functions such as:
  1 Triangular.
  2 Trapezoidal.
  3 Piecewise linear.
  4 Gaussian.
  5 Singleton.

Example



triangular
trapezoidal
singleton
Gaussian
Piecewise linear

Membership functions: Parameterization and Form

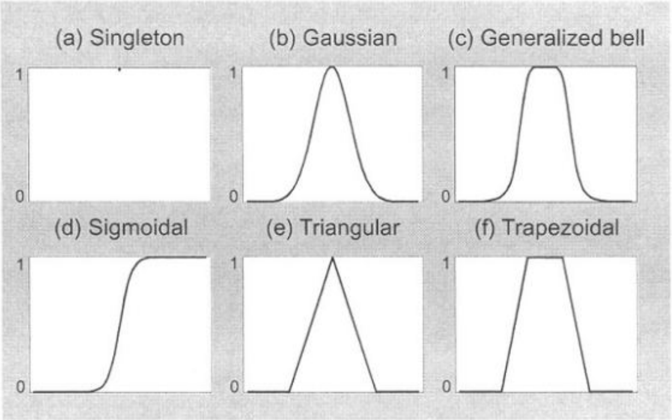
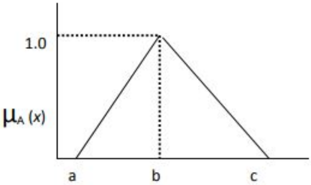


Figure 3.3: Various type of Fuzzy membership functions

Triangular Membership function:

Let a, b and c represent the x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre where membership degree is 1).



$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$

Trapezoidal membership function:

Let a, b, c and d represents the x coordinates of the membership function. then

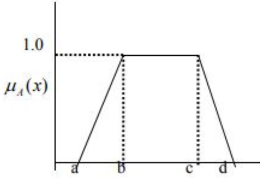
Trapezoid(x; a, b, c, d) = 0 if x ≤ a;

= (x-a)/ (b-a) if a ≤ x ≤ b

= 1 if b ≤ x ≤ c;

= (d-x)/ (d-c) 0 if c ≤ x ≤ d;

= 0, if d ≤ x.



$$\mu_{\text{trapezoid}} = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

Gaussian membership function:

UNITED GROUP OF INSTITUTIONS
Allahabad . Greater Noida

The Gaussian membership function is usually represented as Gaussian(x:c,s) where c, s represents the mean and standard deviation.

$$\mu_A(x,c,s,m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$

Here c represents centre, s represents width and m represents fuzzification factor.

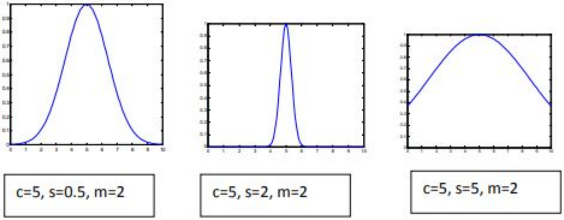
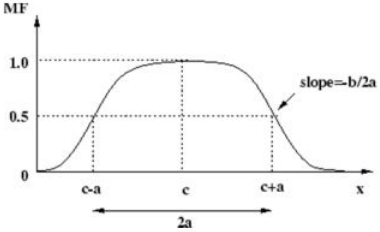


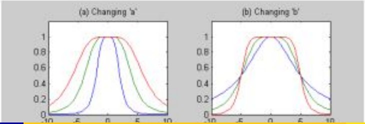
Figure 3.4: Different shapes of Gaussian MFs with different values of s and m.

Generalized Bell membership function:

A generalized bell membership function has three parameters: a –responsible for its width, c – responsible for its center and b –responsible for its slopes. Mathematically,

$$gbellmf(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{b}\right|^{2b}}$$





Sigmoid Membership function:

A sigmoidal membership function has two parameters: a responsible for its slope at the crossover point x = c. The membership function of the sigmoid function can be represented as Sigmf (x:a, c) and it is

$$sigmf(x;a,b,c) = \frac{1}{1+e^{-a(x-c)}}$$

Sigmoid Membership function:

A sigmoidal MF is inherently open right or left and thus, it is appropriate for representing concepts such as "very large" or "very negative". Sigmoidal MF mostly used as activation function of artificial neural networks (NN).A NN should synthesize a close MF in order to simulate the behavior of a fuzzy inference system.

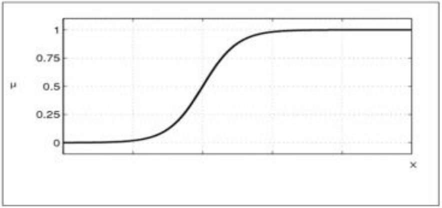


Figure 3.6: A general structures of sigmoid MF.

Fuzzy singleton

- Fuzzy singleton- A fuzzy set with a membership function that is unity at a one particular point and zero everywhere else.
- Singleton output function- An output function that is given by a spike at a single number rather than a continuous curve. In the Fuzzy Logic Toolbox it is only supported as part of a zero-order Sugeno model.

Refrences

- Leszek Rutkowski., Flexible Neuro-Fuzzy Systems: Structures, Learning and Performance Evaluation, Technical University of Czestochowa, Poland, Kluwer Academic Publishers New York.
- Bilgic, Taner and Turksen, Burhan. 1999. Measurement of Membership Function: Theoretical and Experimental Work. In D. Dubois and H. Prade (editors) Handbook of Fuzzy Systems, Vol. 1.
- W. Barada and H. Singh, Generating Optimal Adaptive Fuzzy-Neural Models of Dynamical Systems with Applications to Control, IEEE Transactions on Systems, Man, and Cybernetics, Part C 28 (1998) 371-391. 2001.

Queries

Thanking You