Lecture on
Fuzzy Sets and crisp sets(Unit-3-Lecture 2)

Presented By
Ashish Tiwari
Assistant Professor

Department of CSE
United College of Engg. and Research,
Prayagraj, India

Outline of Presentation

- Introduction.
- Classical set.
- Fuzzy set.
- Why is fuzzy set?
- crisp set.
- What is the difference between crisp set and fuzzy set?
- References.

Introduction

- In mathematics, fuzzy sets are somewhat like sets whose elements have degrees of membership. Fuzzy sets were introduced independently by Lotfi A. Zadeh and Dieter Klaua in 1965 as an extension of the classical notion of set.
- Fuzzy sets can be considered as an extension and gross oversimplification of classical sets. It can be best understood in the context of set membership. Basically it allows partial membership which means that it contain elements that have varying degrees of membership in the set.

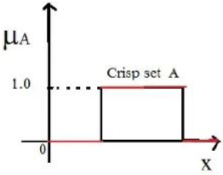
Classical set

- Set: A set is defined as a collection of objects, which share certain characteristics.
  1. Classical set is a collection of distinct objects. For example, a set of students passing grades.
  2. Each individual entity in a set is called a member or an element of the set.
  3. The classical set is defined in such a way that the universe of discourse is spitted into two groups members and non-members. Hence, In case classical sets, no partial membership exists.
  4. Let A is a given set. The membership function can be use to define a set A is given by:

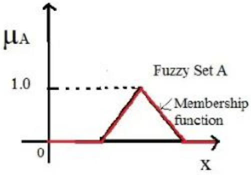
$$\mu A(x)=\{\ 1 \text{ if } x \in A \quad 0 \text{ if } x \notin A\ \}$$

Definition of Crisp Set and Fuzzy Sets

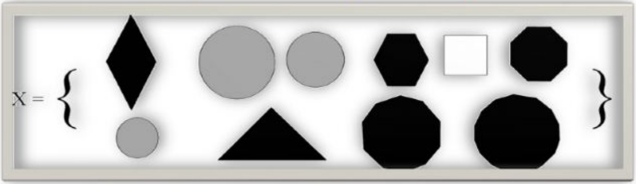
- A ‘crisp’ set, A, can be defined as a set which consists of elements with either full or no membership at all in the set.
- Each item in its universe is either in the set, or not.



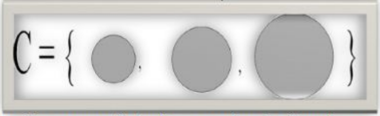
- A “fuzzy set” is defined as a class of objects with a continuum of grades of membership. It is characterized by a “membership function” or “characteristic function” that assigns to each member of the fuzzy set a degree of membership in the unit interval [0,1].



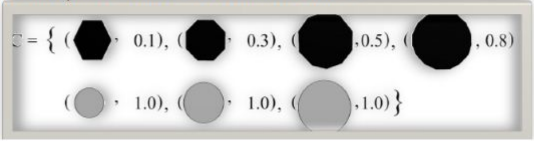
Example of Crisp Set and Fuzzy Sets



One can define the crisp set "circles" as:



The fuzzy set "circles can be defined as:



Fuzzy set

- 1. Fuzzy set is a set having degrees of membership between 1 and 0. Fuzzy sets are represented with tilde character(˜). For example, Number of cars following traffic signals at a particular time out of all cars present will have membership value between [0,1].
  2. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
  3. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.
  4. A fuzzy set A˜in the universe of discourse, U, can be defined as a set of ordered pairs and it is given by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

Fuzzy Set

A fuzzy set is a pair (U, m) where U is a set and $m\colon U \to [0,1]$ a membership function. The reference set U (sometimes denoted by Ω or X) is called **universe of discourse**, and for each $x \in U$, the value $m(x)$ is called the **grade** of membership of x in (U, m). The function $m = \mu_A$ is called the **membership function** of the fuzzy set $A = (U, m)$.

For a finite set $U = \{x_1, \ldots, x_n\}$, the fuzzy set (U, m) is often denoted by $\{m(x_1)/x_1, \ldots, m(x_n)/x_n\}$.

Let $x \in U$. Then x is called

- **not included** in the fuzzy set (U, m) if $m(x) = 0$ (no member),
- **fully included** if $m(x) = 1$ (full member),
- **partially included** if $0 < m(x) < 1$ (fuzzy member).[5]

The (crisp) set of all fuzzy sets on a universe U is denoted with $SF(U)$ (or sometimes just $F(U)$).[

Example Fuzzy set

- Example

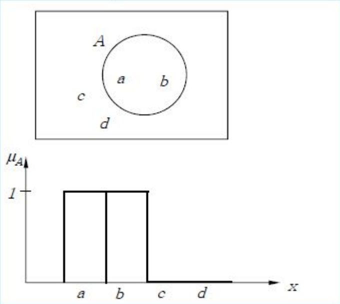


Fig : Graphical representation of crisp set

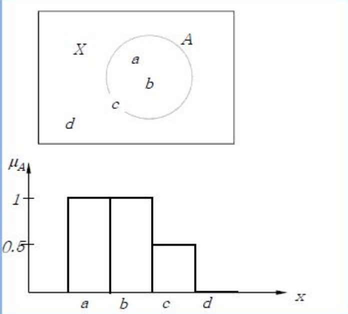


Fig : Graphical representation of fuzzy set

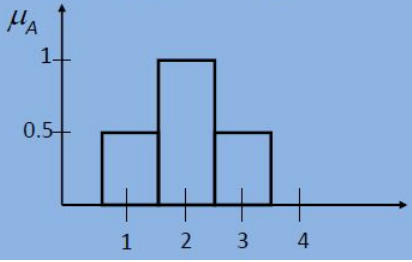
Example Fuzzy set

Consider fuzzy set ‘two or so’. In this instance, universal set X are the positive real numbers.

$X = \{1, 2, 3, 4, 5, 6, \ldots\}$

- Membership function for A =‘two or so’ in this universal set X is given as follows:

$\mu_A(1) = 0.5, \mu_A(2) = 1, \mu_A(3) = 0.5, \mu_A(4) = 0 \ldots$



Why is fuzzy set?

- Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set.
- It enables one to work in uncertain and ambiguous situations and solve ill-posed problems or problems with incomplete information.

crisp set

- Crisp sets are the sets that we have used most of our life. In a crisp set, an element is either a member of the set or not. Fuzzy sets, on the other hand, allow elements to be partially in a set.
- Crisp logic (crisp) is the same as boolean logic(either 0 or 1). Either a statement is true(1) or it is not(0), meanwhile fuzzy logic captures the degree to which something is true.
- Crisp logic is like binary values. That is either statement answer is 0 or 1. In sampler way , It's define as either value is true or false. Only two value it's varying like binary. In short value in between 0 or 1.

Definition of Crisp Set

- The crisp set is a collection of objects (say U) having identical properties such as countability and finiteness.
- A crisp set 'B' can be defined as a group of elements over the universal set U, where a random element can be a part of B or not.
- Which means there are only two possible ways, first is the element could belong to set B or it does not belong to set B. The notation to define the crisp set B containing a group of some elements in U having the same property P, is given below.

$$B = \{x : x \in U \text{ and } x \text{ has same property } P\}$$

- It can perform operations like union, intersection, compliment and difference. The properties exhibited in the crisp set includes commutativity, distributivity, idempotency, associativity, identity, transitivity and involution. Though, fuzzy sets also have the same above given properties.

Crisp Logic

- The traditional approach (crisp logic) of knowledge representation does not provide an appropriate way to interpret the imprecise and non-categorical data. As its functions are based on the first order logic and classical probability theory. In another way, it can not deal with the representation of human intelligence.

crisp Relation

- A Crisp relation represents presence or absence of association, interaction or interconnection between elements of greater than and equal to 2 sets. This concept can be generalized to various degrees or strengths of association or interaction between elements. So, a crisp relation is a restricted case of a fuzzy relation.

Operations on Crisp Relations

Let A and B be two relations defined on X x Y and are represented by relational matrices. The following operations can be performed on these relations A and B

Union

$A \cup B (x,y) = \max [A(x,y) , B(x,y)]$

Intersection

$A \cap B (x,y) = \min [A(x,y) , B(x,y)]$

What is the difference between crisp set and fuzzy

- It can be a partial member of the set. The key difference between a crisp set and a fuzzy set is their membership function. A crisp set has unique membership function, whereas a fuzzy set can have an infinite number of membership functions to represent it.
- Fuzzy set and crisp set are the part of the distinct set theories, where the fuzzy set implements infinite-valued logic while crisp set employs bi-valued logic. Previously, expert system principles were formulated premised on Boolean logic where crisp sets are used.
- But then scientists argued that human thinking does not always follow crisp "yes"/"no" logic, and it could be vague, qualitative, uncertain, imprecise or fuzzy in nature. This gave commencement to the development of the fuzzy set theory to imitate human thinking.

Comparison Chart

BASIS FOR COMPARISON	FUZZY SET	CRISP SET
Basic	Prescribed by vague or ambiguous properties.	Defined by precise and certain characteristics.
Property	Elements are allowed to be partially included in the set.	Element is either the member of a set or not.
Applications	Used in fuzzy controllers	Digital design
Logic	Infinite-valued	bi-valued

Refrences

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Queries

<u>Thanking You</u>