Convolutional Neural Networks

Why convolutions?

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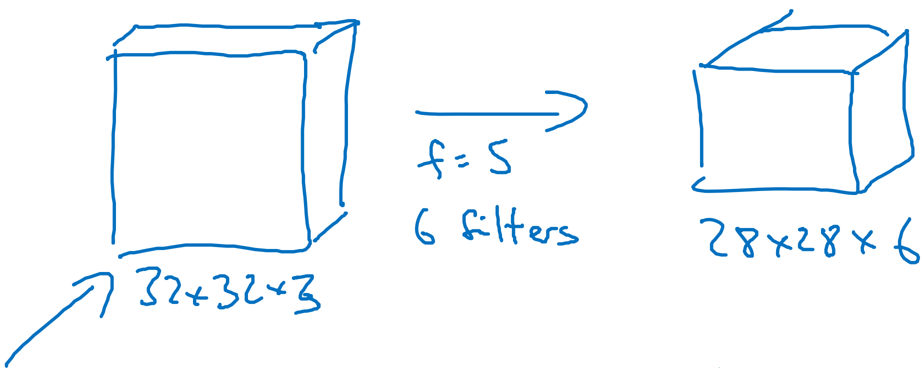
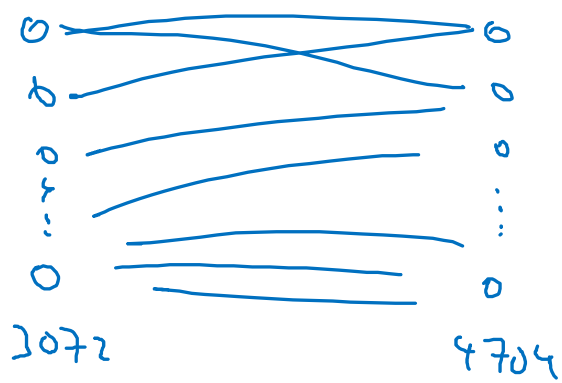
Why convolutions



3,072

4,704



$5 \times 5 - 25$

26

$6 \times 26 = 156$ parameters

$3{,}072 \times 4{,}704 \approx \underline{14M}$

Why convolutions

translation invariance

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

$\underbrace{3 \times 3}$

=

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

→ **Sparsity of connections:** In each layer, each output value depends only on a small number of inputs.

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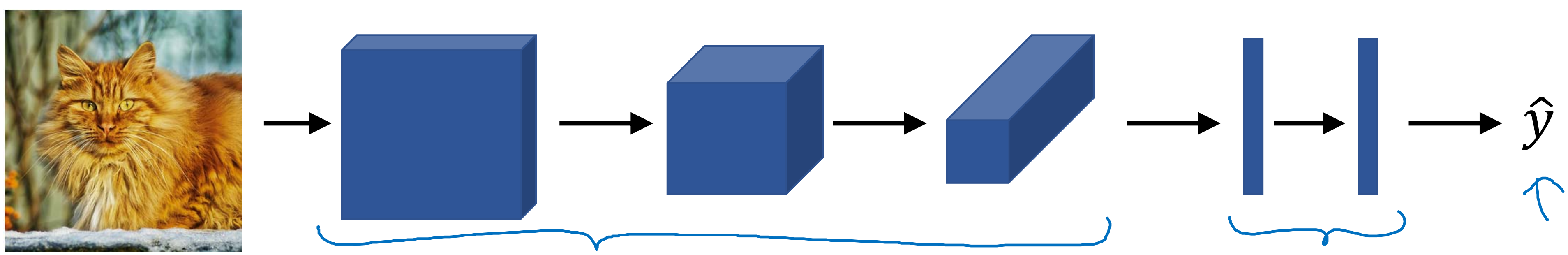
Putting it together

Training set $(x^{(1)}, y^{(1)}) \ldots (x^{(m)}, y^{(m)})$.

$\boxed{w, b}$



Cost $J = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Use gradient descent to optimize parameters to reduce J

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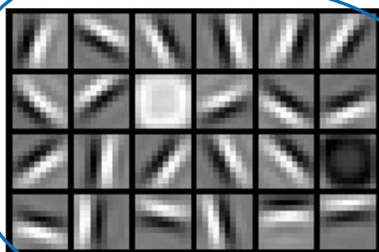
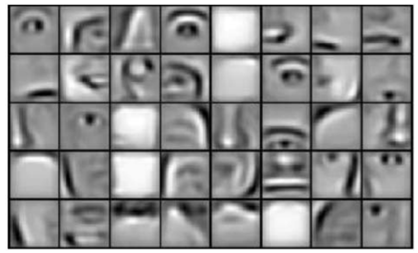
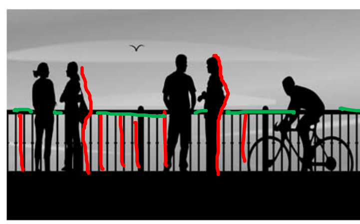
Convolutional Neural Networks

Edge detection example

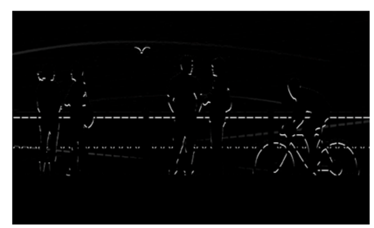
Computer Vision Problem







vertical edges



horizontal edges

Vertical edge detection

$3\times1 + 1\times1 + 2\times1 + 0\times0 + 5\times0 + 7\times0 + 1\times-1 + 8\times-1 + 2\times-1 = -5$

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6×6

"convolution"

$*$

1	0	-1
1	0	-1
1	0	-1

3×3

→ filter
kernel

$=$

-5	-4	0	8
-10	-2	2	3
0	-2	-4	-7
-3	-2	-3	-16

4×4

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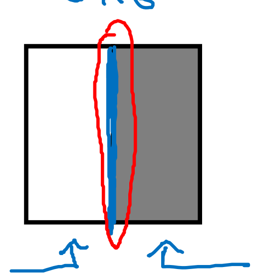
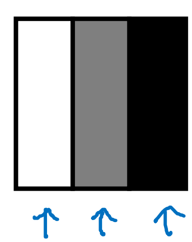
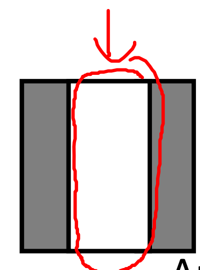
Vertical edge detection

$$
\underbrace{\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix}}_{6 \times 6} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{3 \times 3} = \underbrace{\begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}}_{4 \times 4}
$$



*





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Convolutional Neural Networks

More edge detection

Vertical edge detection examples

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \\ 0 & 0 & 0 & 10 & 10 & 10 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \\ 0 & -30 & -30 & 0 \end{bmatrix}$$

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Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

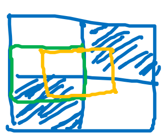
6 x 6

*

1	1	1
0	0	0
-1	-1	-1

=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0



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Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

1	0	-1
2	0	-2
1	0	-1

Sobel filter

3	0	-3
10	0	-10
3	0	-3

Scharr filter

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

convolution

$*$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3x3

$=$

45°
70°
73°

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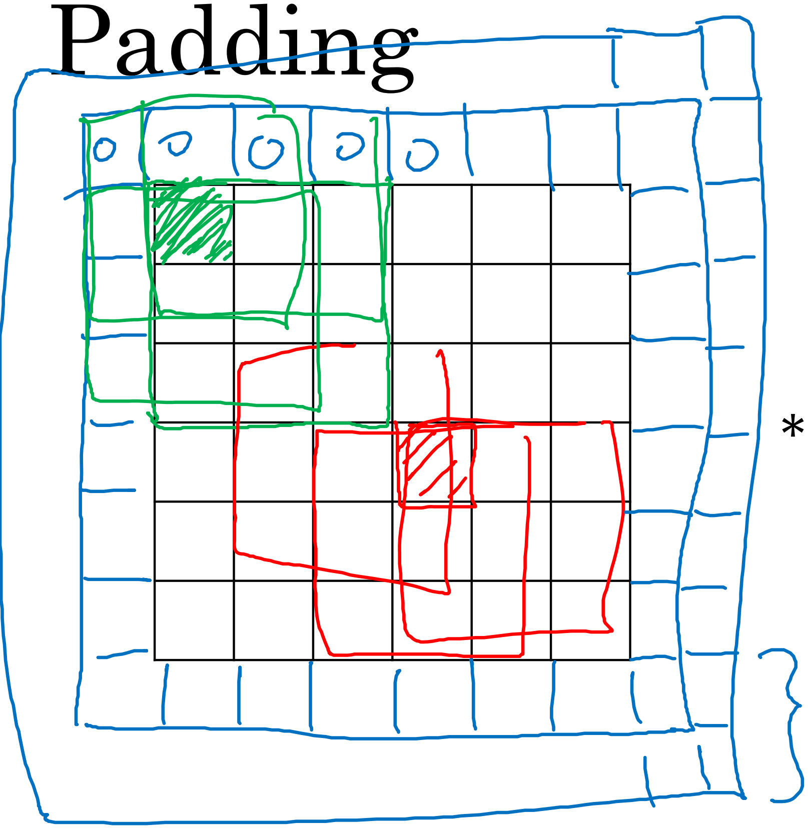
Convolutional Neural Networks

Padding

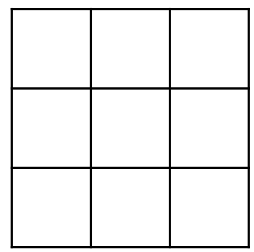
Padding

- shrinky output
- throw away info from edge



p=2

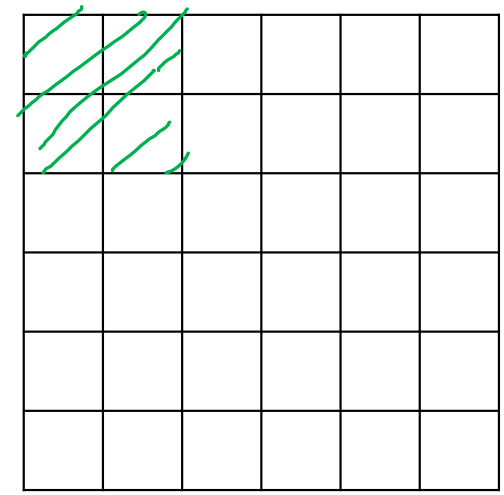
$*$



3×3

$f \times f$

$=$



6×6

~~4×4~~

$6 \times 6 \rightarrow 8 \times 8$

$n \times n$

$n-f+1 \quad \times \quad n-f+1$

$6-3+1=4$

p = padding = $\underline{1}$

$n+2p-f+1 \times n+2p-f+1$

$6+2-3+1 \times \_\_\_ = 6 \times 6$

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Valid and Same convolutions

no padding

"Valid": $n \times n \ * \ f \times f \longrightarrow n-f+1 \times n-f+1$

$6 \times 6 \ * \ 3 \times 3 \longrightarrow 4 \times 4$

"Same": Pad so that output size is the same as the input size.

$n+2p-f+1 \times n+2p-f+1$

$\cancel{n}+2p-f+1 = \cancel{n} \Rightarrow \boxed{p = \frac{f-1}{2}}$

$3 \times 3 \quad p = \frac{3-1}{2} = 1$

$5 \times 5 \quad f=5 \quad p=2$

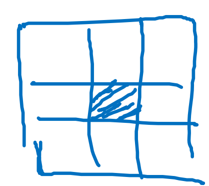
f is usually odd

1×1

3×3

5×5

7×7



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Convolutional Neural Networks

Strided convolutions

Strided convolution

2	3	7	4	6	2	9
6	6	9	8	7	4	3
3	4	8	3	8	9	7
7	8	3	6	6	3	4
4	2	1	8	3	4	6
3	2	4	1	9	8	3
0	1	3	9	2	1	4

7×7

$*$

3	4	4
1	0	2
-1	0	3

3×3

$=$

91	100	83
69	91	127
44	72	74

3×3

stride = 2

$\lfloor z \rfloor = \text{floor}(z)$

$n \times n \quad * \quad f \times f$

padding p $\qquad$ stride s

$s = 2$

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

$$\frac{7+0-3}{2} + 1 = \frac{4}{2} + 1 = 3$$

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Summary of convolutions

$n \times n$ image $\quad f \times f$ filter

padding $p \quad$ stride s

Output size:

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \quad \times \quad \left\lfloor \underbrace{\frac{n+2p-f}{s}} + 1 \right\rfloor$$

Technical note on cross-correlation vs. convolution

Convolution in math textbook:

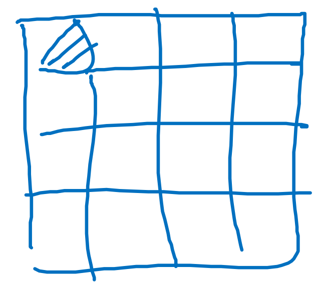
2^{7}	3^{2}	7^{5}	4	6	2
6^{9}	6^{0}	9^{4}	8	7	4
3^{-1}	4^{1}	8^{3}	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

*

3	4	5
1	0	2
-1	9	7

7	2	5
9	0	4
-1	1	3

=



$$(A*B)*C = A*(B*C)$$

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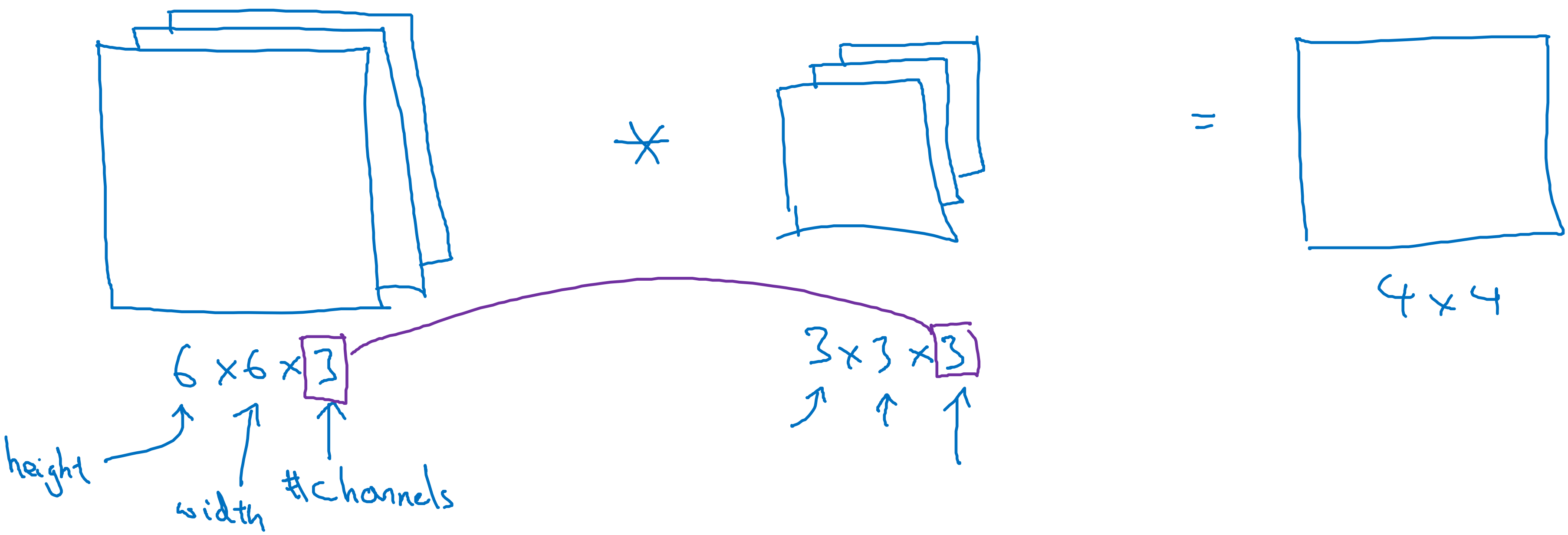
Convolutional Neural Networks

Convolutions over volumes

Convolutions on RGB images



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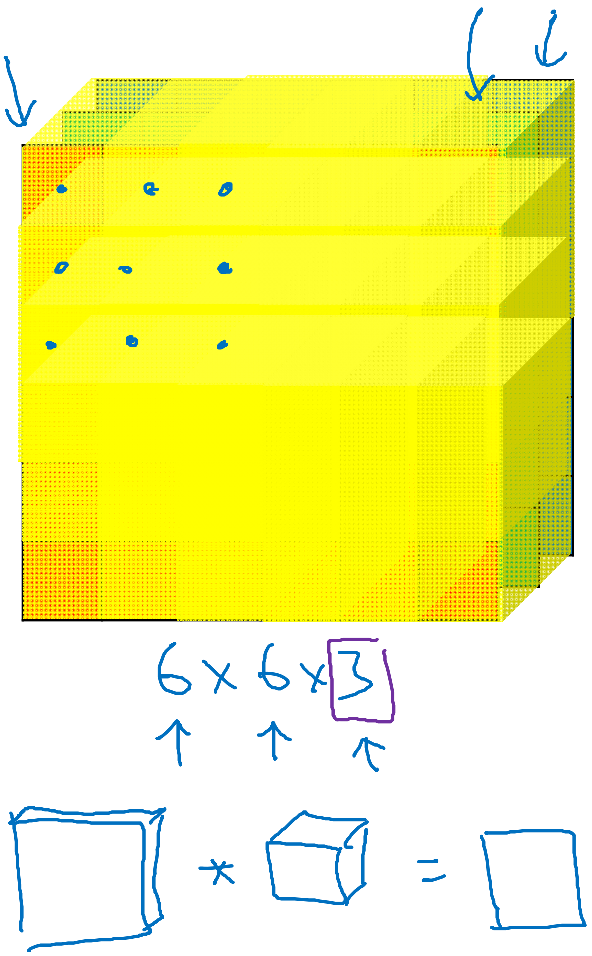
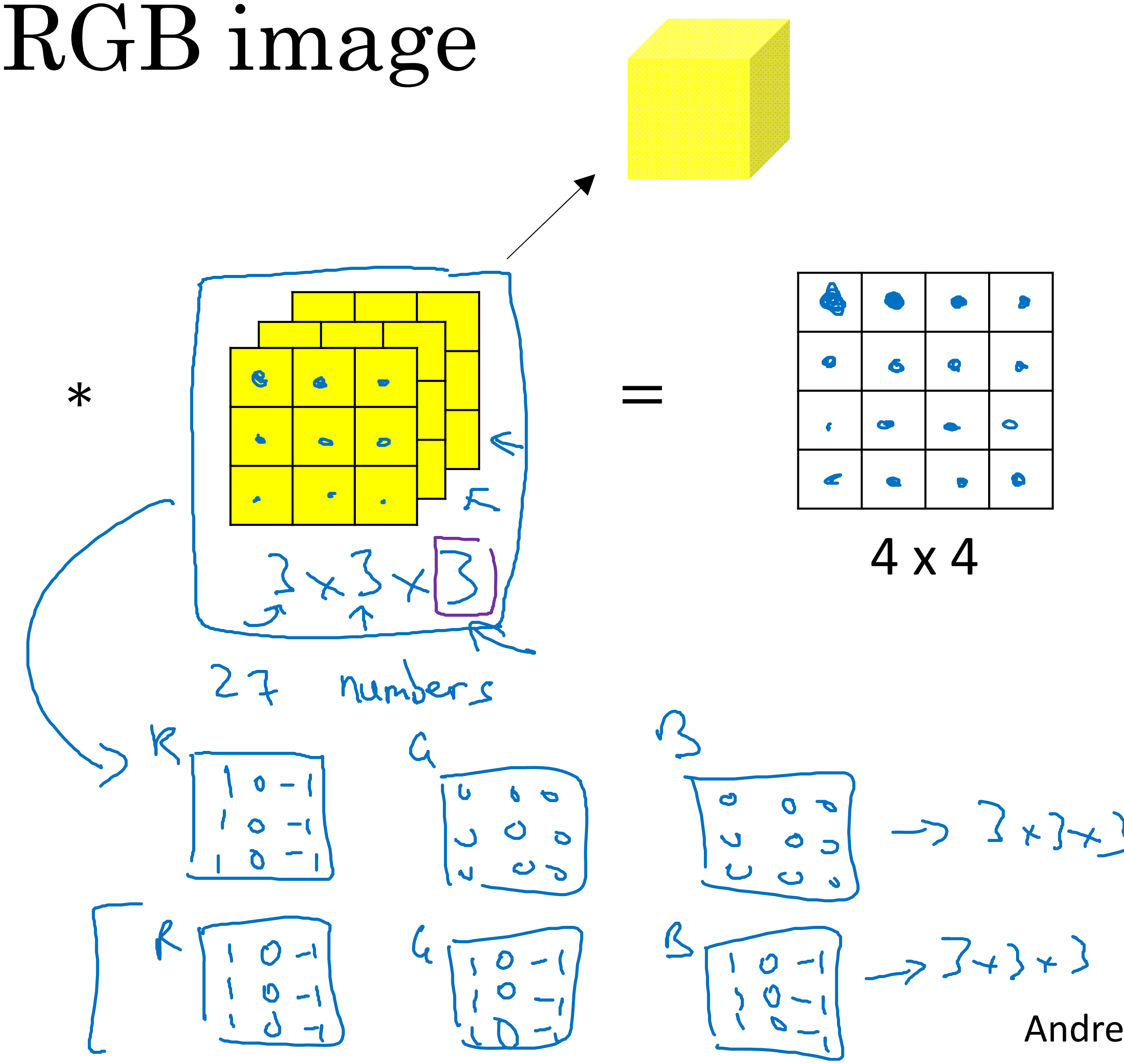
Convolutions on RGB image





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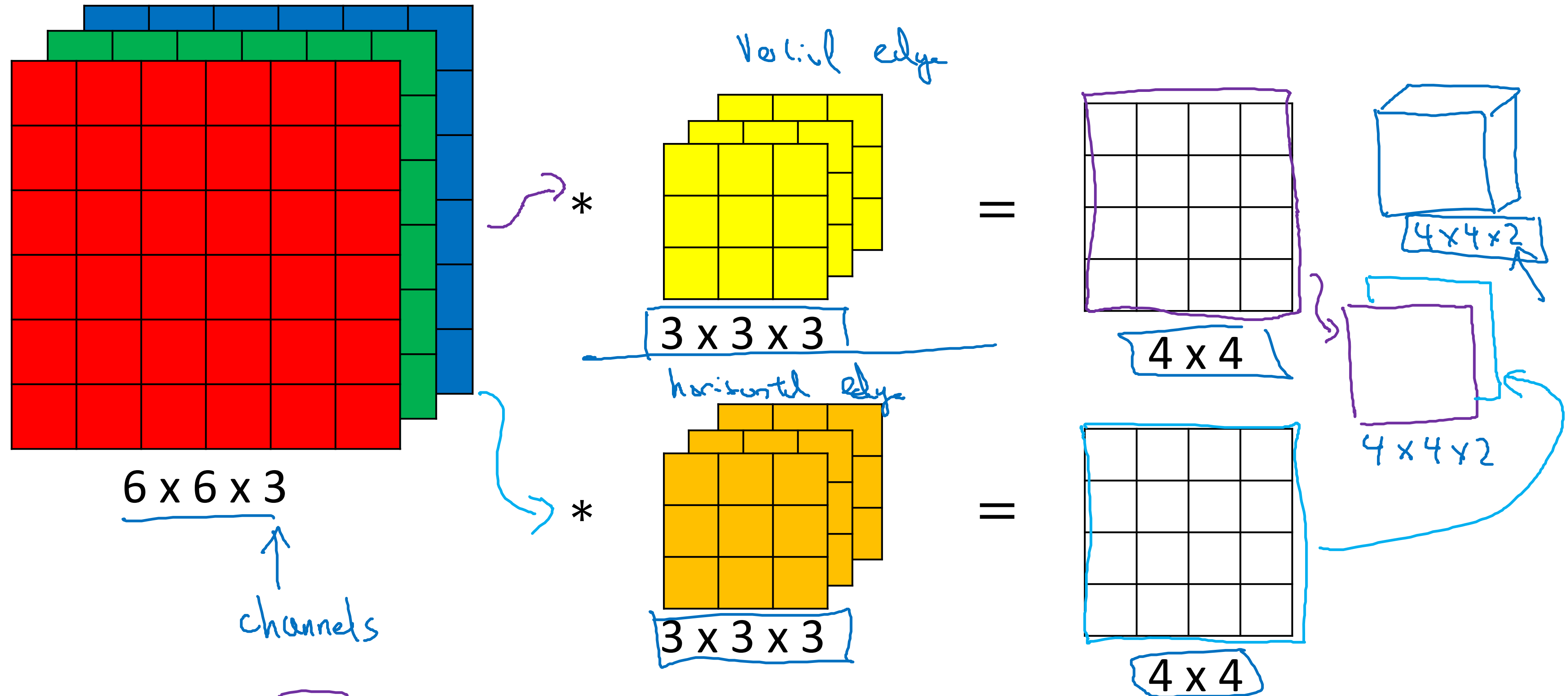
Multiple filters



Summary: $n \times n \times n_c \;\; * \;\; f \times f \times n_c \;\longrightarrow\; \underline{n-f+1} \times \underline{n-f+1} \times n_c'$

$6 \times 6 \times 3 \qquad 3 \times 3 \times 3 \qquad 4 \times 4 \times 2$ ← # filters

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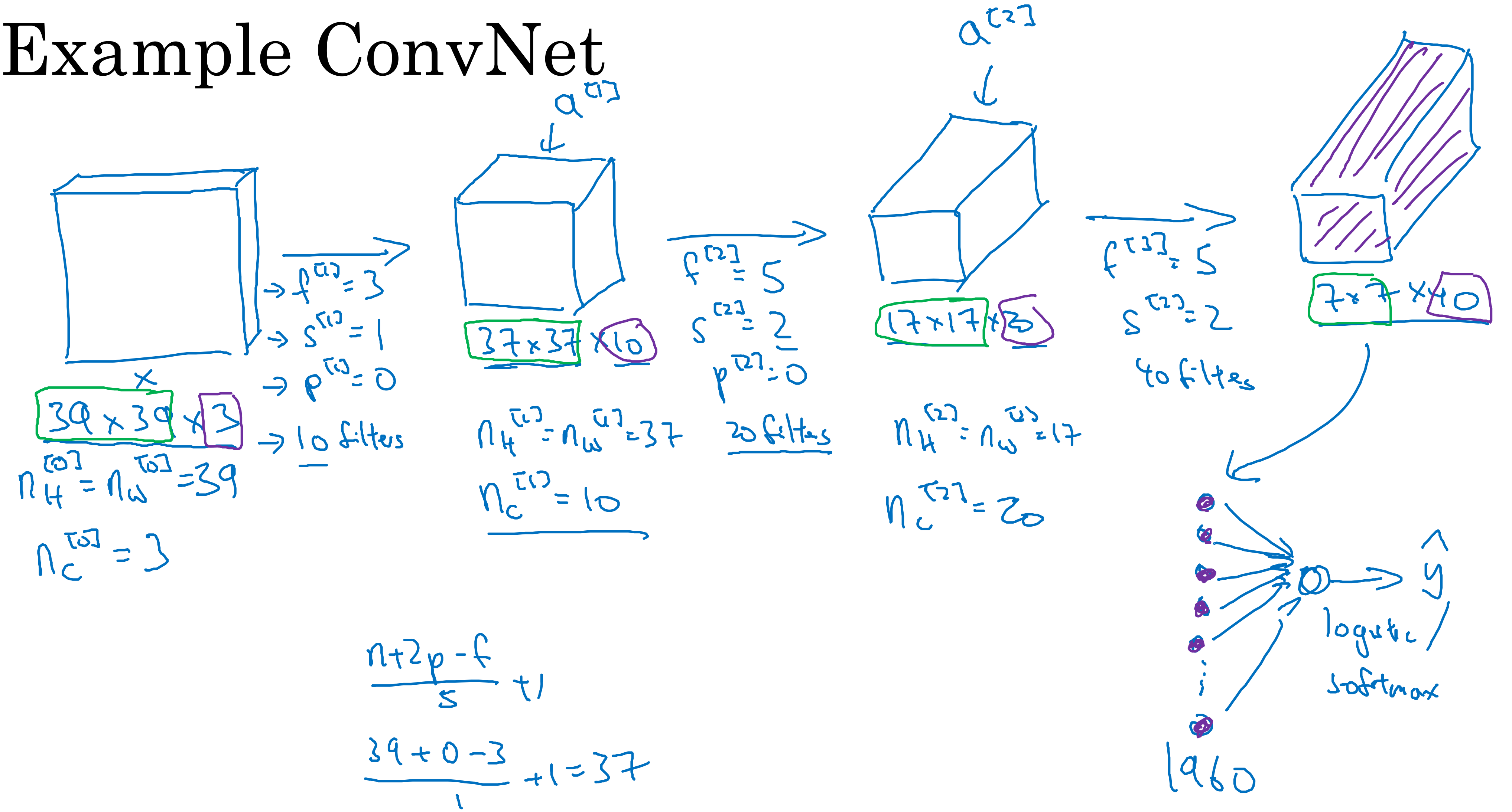
Convolutional Neural Networks

A simple convolution network example

Example ConvNet



Types of layer in a convolutional network:

- Convolution (CONV) ←
- Pooling (POOL) ←
- Fully connected (FC) ←

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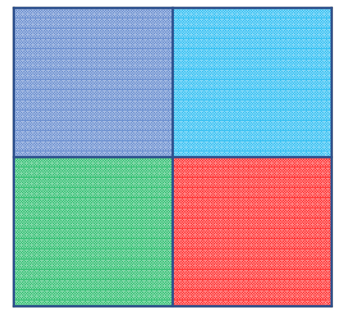
Convolutional Neural Networks

Pooling layers

Pooling layer: Max pooling

1	3	2	1
2	9	1	1
1	3	2	3
5	6	1	2



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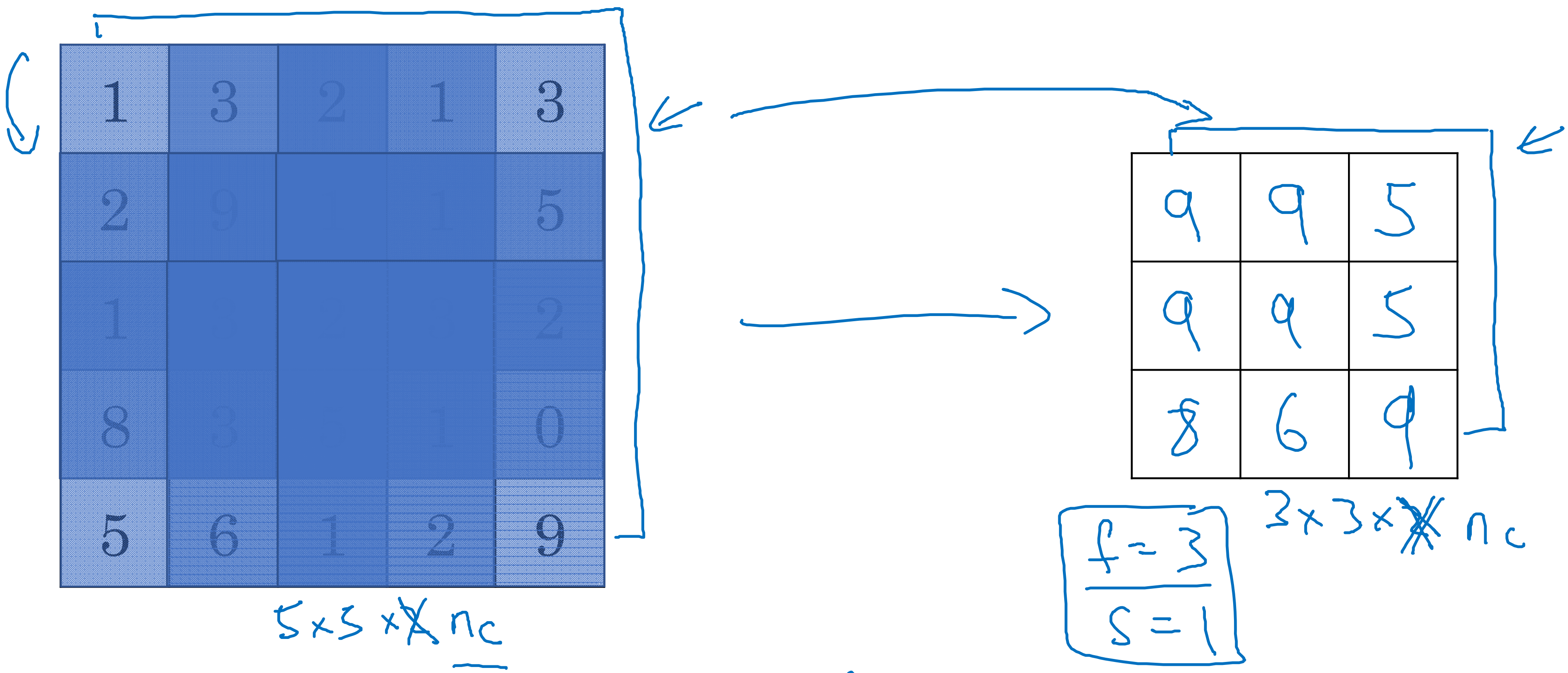
Pooling layer: Max pooling



$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

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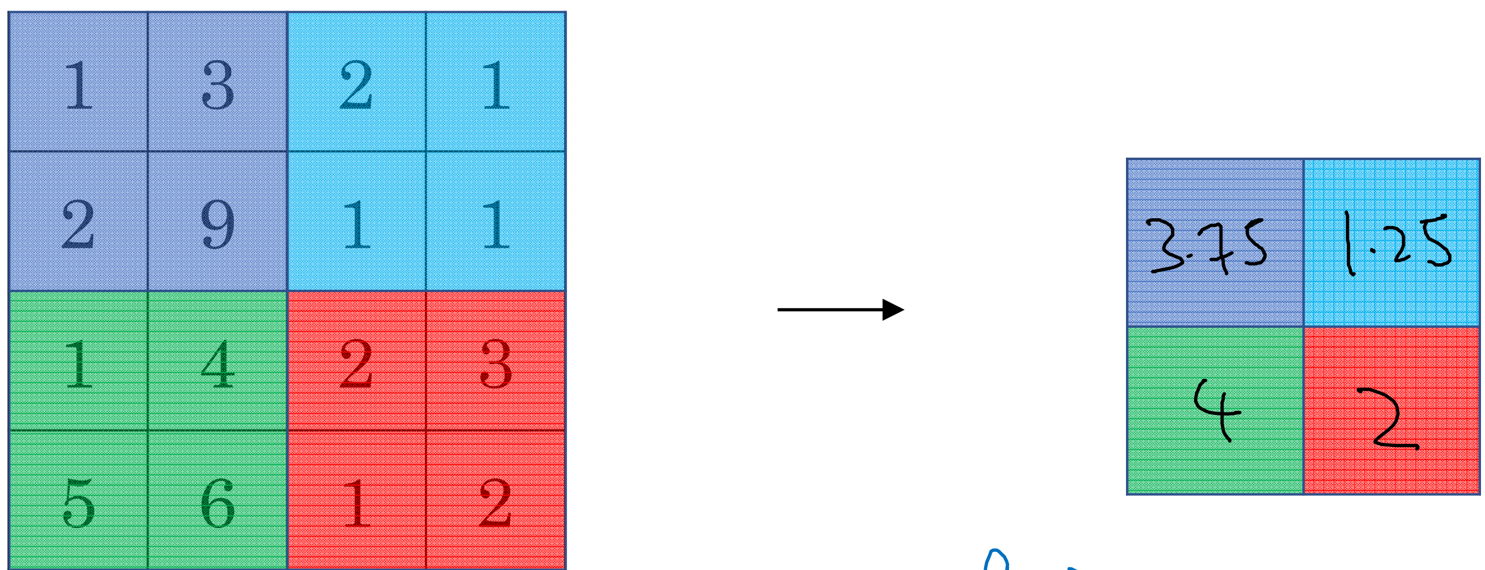
Pooling layer: Average pooling



$f = 2$

$s = 2$

$\underline{7 \times 7} \times 1000 \longrightarrow 1 \times 1 \times 1000$

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Summary of pooling

Hyperparameters:

f : filter size

s : stride

Max or average pooling

$f=2, s=2$

$f=3, s=2$

→ ~~p: padding.~~

No parameters to learn!

$n_H \times n_W \times n_C$

$\downarrow$

$$\left\lfloor \frac{n_H - f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n_W - f}{s} + 1 \right\rfloor$$

$\times \; n_C$

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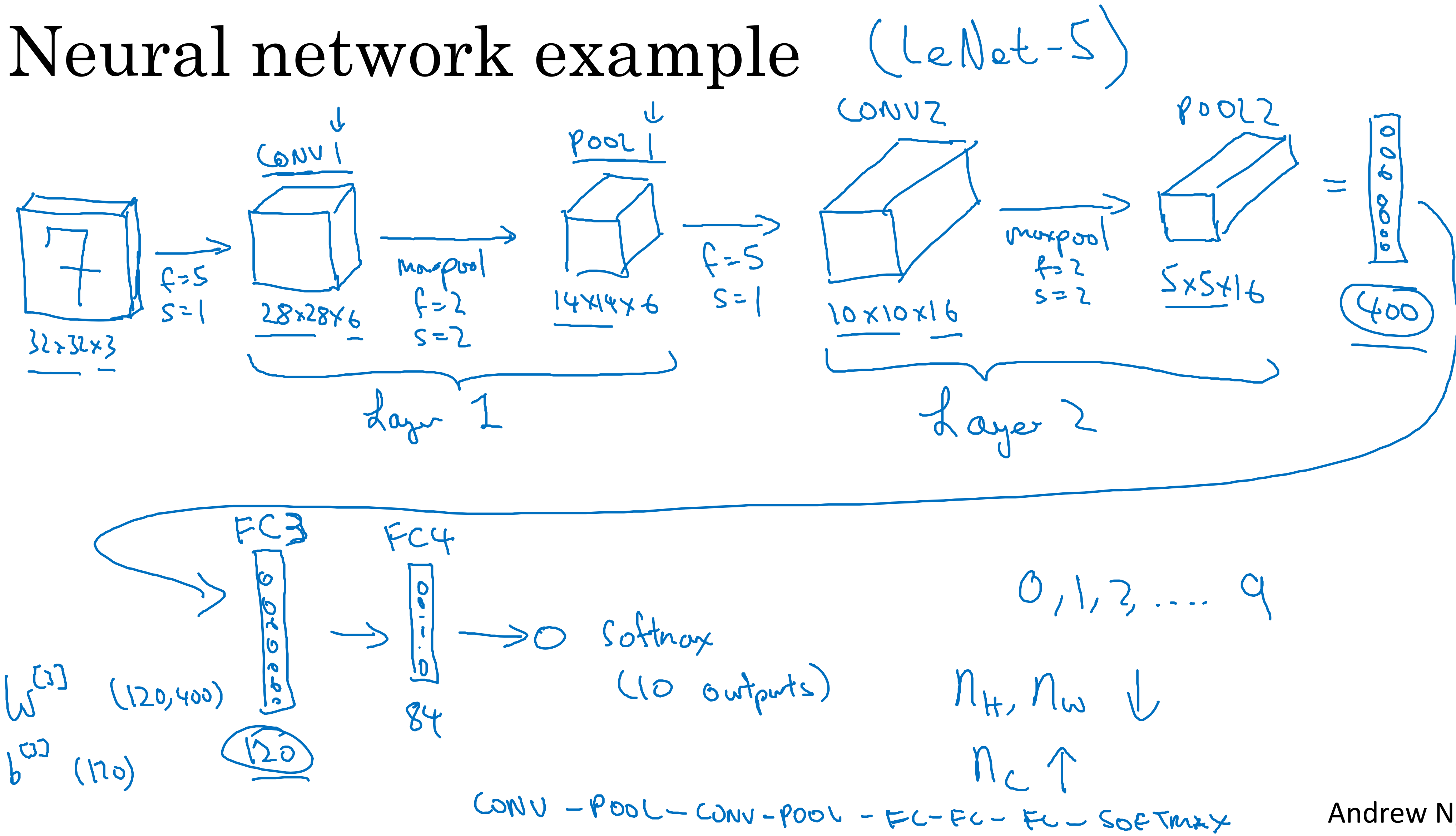
Convolutional Neural Networks

Convolutional neural network example

Neural network example



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Neural network example

	Activation shape	Activation Size	# parameters
Input:	(32,32,3)	3,072 $a^{[0]}$	0

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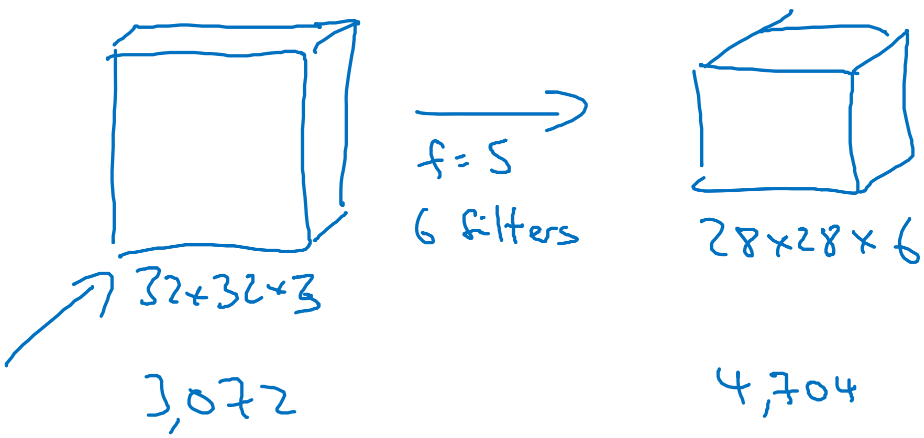
Convolutional Neural Networks

Why convolutions?

Why convolutions



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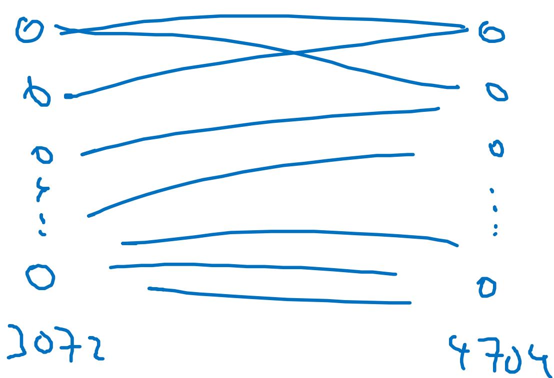

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Why convolutions

translation invariance

$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}}_{3\times 3} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

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Putting it together

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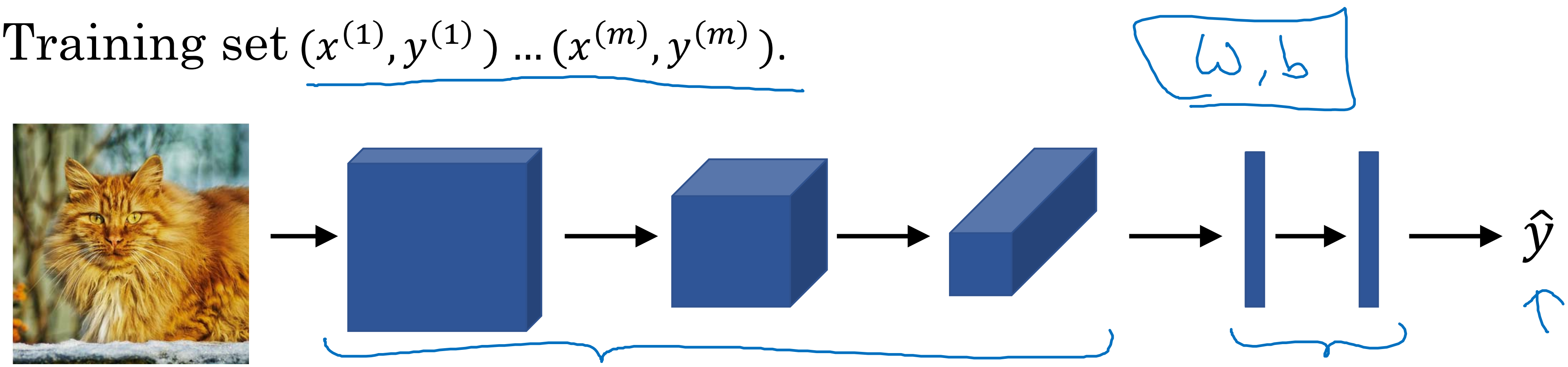

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Use gradient descent to optimize parameters to reduce J