Basics of Neural Network Programming

Binary Classification

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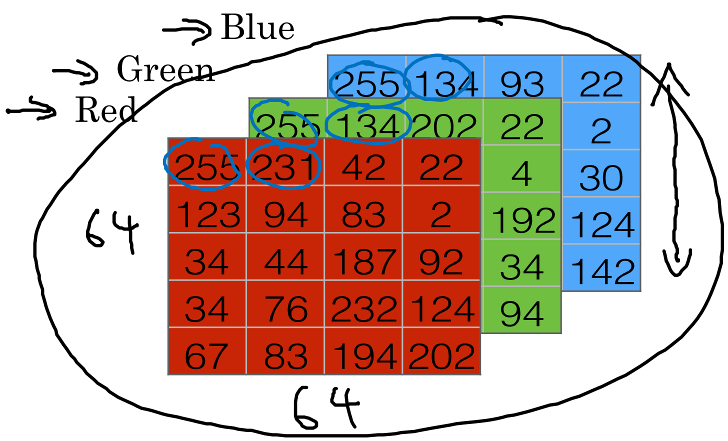
Binary Classification

64

64

1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

$x \longrightarrow y$

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Notation

$(x, y) \qquad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

$M = M_{train} \qquad M_{test} = \#\text{test examples.}$

$$X = \underbrace{\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}}_{m} \left.\vphantom{\begin{bmatrix} | \\ x \\ | \end{bmatrix}}\right\updownarrow n_x \qquad \cancel{\begin{bmatrix} - & x^{(1)T} & - \\ & \vdots & \\ - & x^{(m)T} & - \end{bmatrix}}$$

$X \in \mathbb{R}^{n_x \times m} \qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

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Basics of Neural Network Programming

Logistic Regression

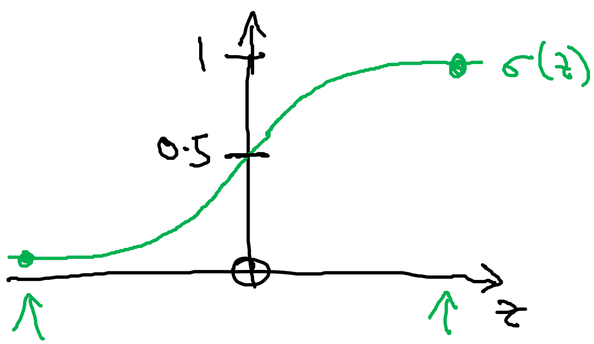
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\qquad 0 \le \hat{y} \le 1$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1$, $\quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{array}{l} \} \, b \leftarrow \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} w \leftarrow \end{array}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Basics of Neural Network Programming

Logistic Regression cost function

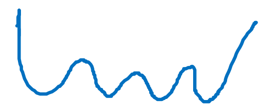
Logistic Regression cost function

$\rightarrow \hat{y}^{(i)} = \underline{\sigma(w^T \underline{x}^{(i)} + b)}$, where $\underline{\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}}$ $z^{(i)} = w^T x^{(i)} + b$

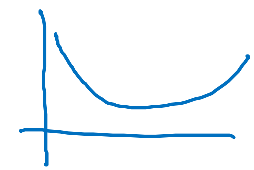
Given $\{(\underline{x^{(1)}, y^{(1)}}), \ldots, (\underline{x^{(m)}, y^{(m)}})\}$, want $\underline{\hat{y}^{(i)}} \approx \underline{y^{(i)}}$.

$x^{(i)}$ $y^{(i)}$ $z^{(i)}$ i-th example.

$\boxed{\text{Loss}}$ (error) function: $\underline{\mathcal{L}}(\hat{y}, y) = \underline{\frac{1}{2}(\hat{y}-y)^2}$



$$\boxed{\mathcal{L}(\hat{y}, y)} = -\left(\boxed{y \log \hat{y}} + \underline{(1-y)\log(1-\hat{y})}\right) \leftarrow$$



If $\underline{y=1}$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y} \leftarrow$ want $\log \hat{y}$ large, $\underline{\text{want } \hat{y} \text{ large.}}$

If $\underline{y=0}$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y}) \leftarrow$ want $\log 1-\hat{y}$ large .... $\underline{\text{want } \hat{y} \text{ small}}$

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)}) \right]$

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Basics of Neural Network Programming

Gradient Descent

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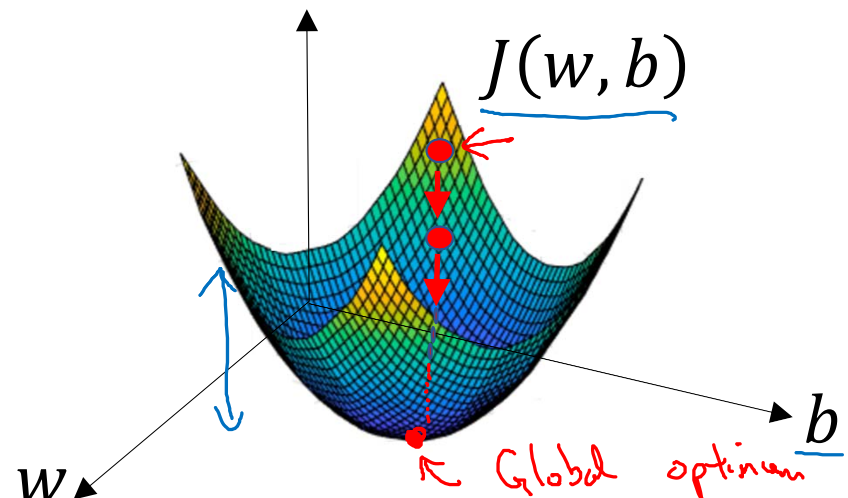
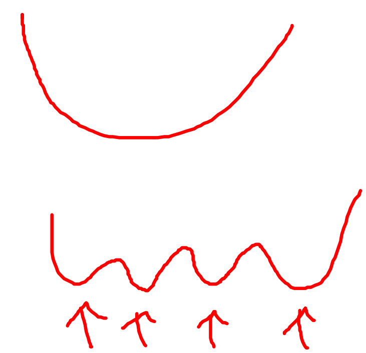
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





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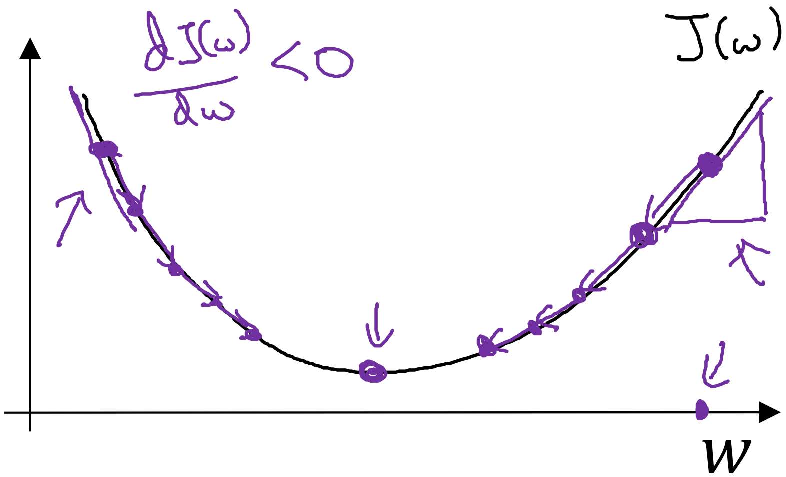
Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

"dw"

}

$$w := w - \alpha \, dw$$

$$\frac{dJ(w)}{dw} = ?$$

$J(w,b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \quad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \quad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d J

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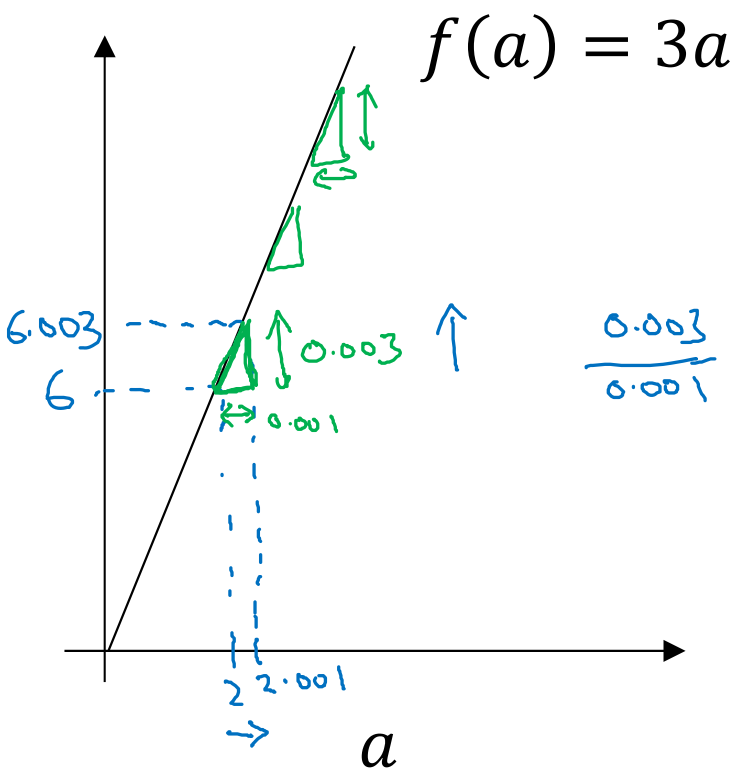
Basics of Neural Network Programming

Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

$\rightarrow$ $a = 2$ $f(a) = 6$

$a = 2.\underline{001}$ $f(a) = 6.\underline{003}$

$\rightarrow$ $\underline{\text{slope}}$ ($\underline{\text{derivative}}$) of $f(a)$ at $a=2$ is 3

$\rightarrow$ $a = 5$ $f(a) = 15$

$a = 5.\underline{001}$ $f(a) = 15.\underline{003}$

slope at $a=5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} \underline{f(a)}$$

0.001

0.00000001

0.000000001

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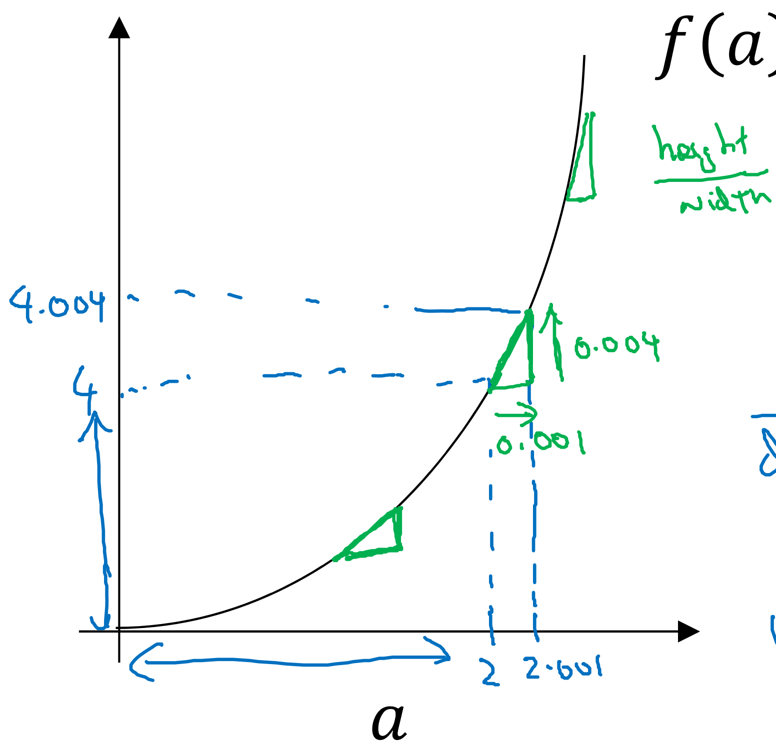
More derivatives examples

Intuition about derivatives



$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

0.001

0.00000.....01

$a = 2$ $f(a) = 4$

$a = 2.001$ $f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of f(a) at $a=2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a=2}$.

$a = 5$ $f(a) = 25$

$a = 5.001$ $f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a=5}$

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = \boxed{2a}$$

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More derivative examples

$$f(a) = a^2 \qquad \frac{d}{da} f(a) = \underbrace{2a}_{4} \qquad a = 2 \quad f(a) = 4$$

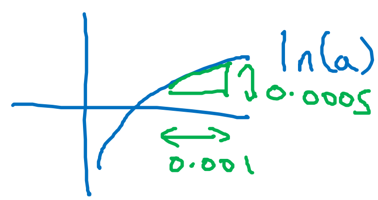
$$a = 2.001 \quad f(a) \approx 4.004$$

$$f(a) = a^3 \qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12} \qquad a = 2 \quad f(a) = 8$$

$$a = 2.\underline{001} \quad f(a) \approx 8.\underline{012}$$

$$f(a) = \log_e(a) \qquad \ln(a) \qquad \frac{d}{da} f(a) = \frac{1}{a} \qquad a = 2 \quad f(a) \approx 0.69315$$

$$a = 2.\underline{001} \quad \underline{f(a)} \approx 0.69365$$



$$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$$

$$0.0005 \qquad 0.0005$$

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Basics of Neural Network Programming

Computation Graph

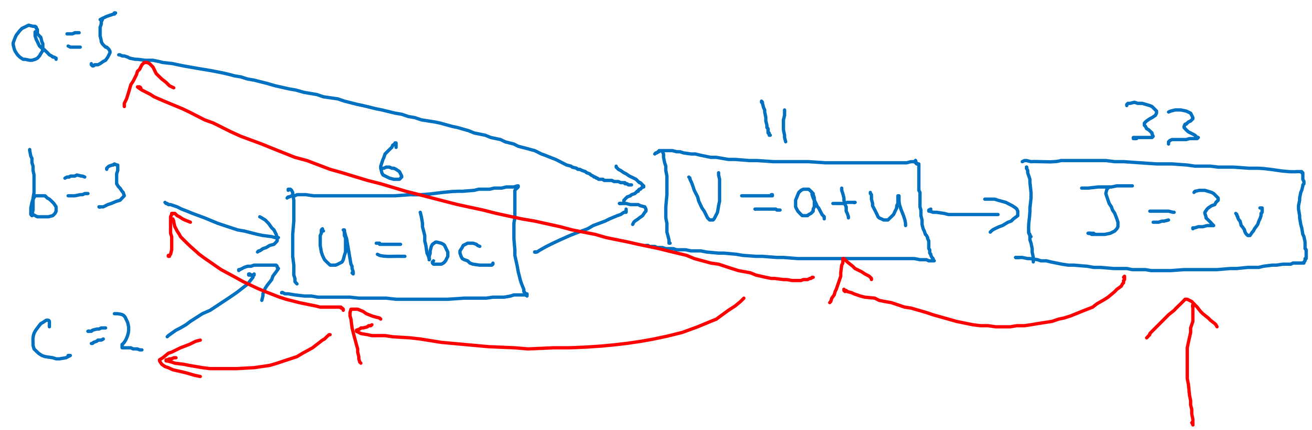
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a + \underbrace{bc}_{u}}_{v})}_{J} = 3(5 + 3\times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$



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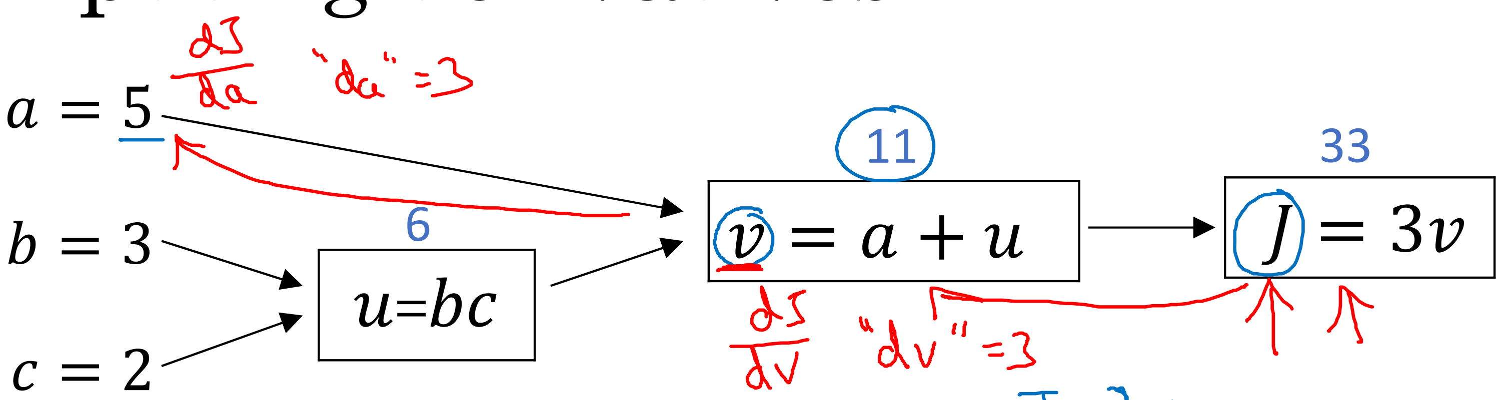
Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives



$\boxed{\frac{dJ}{dv} = ? = 3}$

$a \rightarrow v \rightarrow J$

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{f(a) = 3a}$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$\boxed{J = 3v}$

$\frac{dJ}{dv} = 3$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \; \frac{dv}{da}$

3×1

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{\frac{dv}{da} = 1}$

$\boxed{\frac{d\,\text{FinalOutputVar}}{d\,\text{var}}}$

dJdvar

"dvar"

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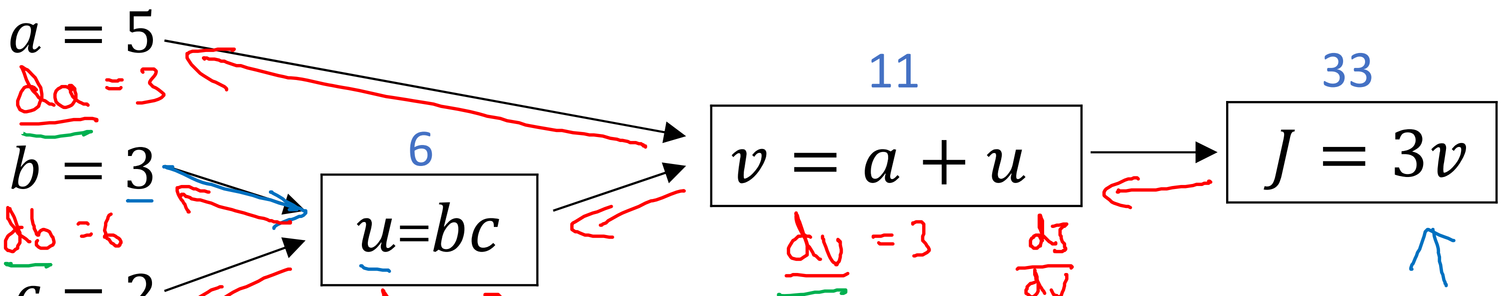
Computing derivatives



$\frac{dJ}{da} \rightarrow da = 3$

$\frac{dJ}{db} = db = 6$

$\rightarrow dc = 9$

$du = 3$

$dv = 3$ $\quad \frac{dJ}{dJ}$

$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\rightarrow 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \underbrace{\boxed{\frac{dJ}{du}}}_{3} \cdot \underbrace{\frac{du}{da}}_{3} = 9 \qquad 3 \times 3$$

$u = 6 \rightarrow 6.001$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$

$u = b \cdot c = 6 \rightarrow 6.002 \qquad c = 2$

$J = 33.006 \qquad .006$

$v = 11.002$

$J = 3v$

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Basics of Neural Network Programming

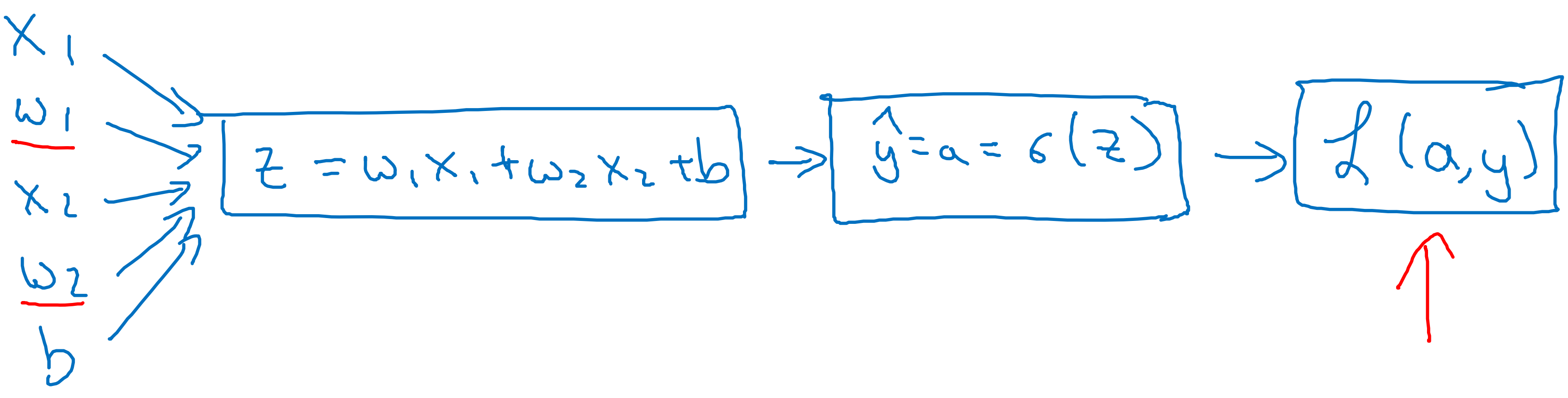
Logistic Regression Gradient descent

Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



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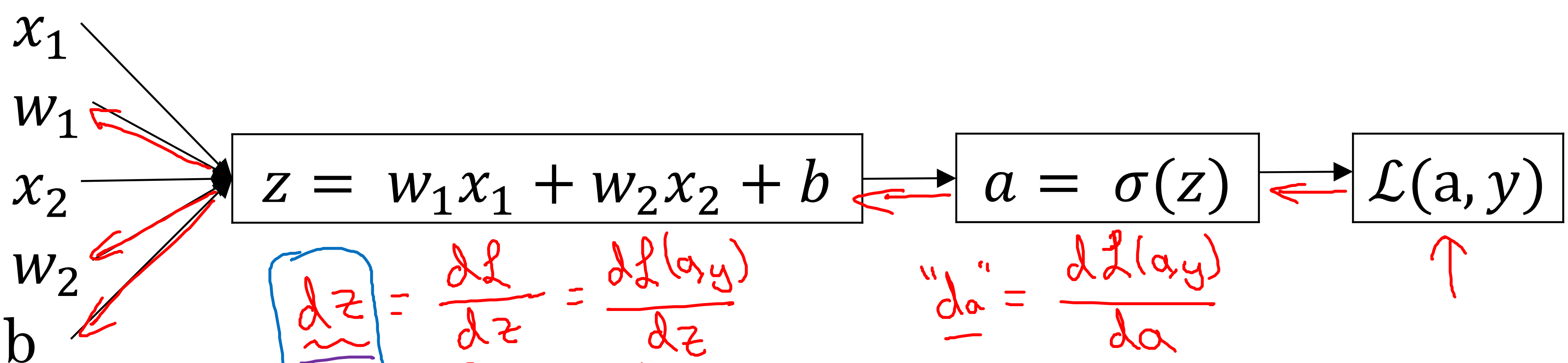
Logistic regression derivatives



$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz} = a - y = \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$$

$$\frac{da}{dz} = a(1-a)$$

$$\text{"}da\text{"} = \frac{d\mathcal{L}(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz \qquad dw_2 = x_2 \cdot dz \qquad db = dz$$

$$w_1 := w_1 - \alpha\, dw_1$$
$$w_2 := w_2 - \alpha\, dw_2$$
$$b := b - \alpha\, db$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$$(x^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$\underline{dw_1^{(i)}},\ \underline{dw_2^{(i)}},\ \underline{db^{(i)}}$$

$$\underline{\frac{\partial}{\partial w_1} J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}}}}$$

$$\underline{dw_1^{(i)}} - (x^{(i)}, y^{(i)})$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 += x_1^{(i)} dz^{(i)}$

$dw_2 += x_2^{(i)} dz^{(i)}$

$db += dz^{(i)}$

$n=2$

dw_3

$\vdots$

dw_n

$J /= m$

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db.$

Vectorization

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Vectorization

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What is vectorization?

$$z = \underbrace{w^T x} + b$$

$w = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $\quad x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $\quad w \in \mathbb{R}^{n_x}$ $\quad x \in \mathbb{R}^{n_x}$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

$$z = \underbrace{np.\underline{dot}(w, x)}_{w^T x} + b$$

→ GPU
→ CPU

} SIMD - single instruction multiple data.

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More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_i \sum_j A_{ij} v_j$

```
u = np.zeros((n,1))
for i ...
    for j ...
        u[i] += A[i][j] * v[j]
```

```
u = np.dot(A,v)
```

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Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v,0)
v**2        1/v
```

Logistic regression derivatives

J = 0, $\boxed{\cancel{dw1 = 0, dw2 = 0}}$, db = 0

$dw = np.zeros((n\_x, 1))$

for i = 1 to n:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$

$dz^{(i)} = a^{(i)}(1 - a^{(i)})$

for $j = 1 \ldots n_x$

$dw_j \mathrel{+}= \ldots$

$n_x = 2$

$dw \mathrel{+}= x^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

J = J/m, $\boxed{\cancel{dw_1 = dw_1/m, \; dw_2 = dw_2/m}}$, db = db/m

$dw \mathrel{/}= m.$

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Basics of Neural Network Programming

Vectorizing Logistic Regression

Vectorizing Logistic Regression

$$\boxed{z^{(1)} = w^T x^{(1)} + b \\ a^{(1)} = \sigma(z^{(1)})} \qquad \boxed{z^{(2)} = w^T x^{(2)} + b \\ a^{(2)} = \sigma(z^{(2)})} \qquad \boxed{z^{(3)} = w^T x^{(3)} + b \\ a^{(3)} = \sigma(z^{(3)})}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \qquad (n_x, m) \qquad \mathbb{R}^{n_x \times m}$$

$$[\; \text{———} \;] \; w^T \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$Z = [z^{(1)} \; z^{(2)} \cdots z^{(m)}] = \underbrace{w^T X + \underbrace{[b \; b \cdots b]}_{1 \times m}} = [\, w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \cdots \quad w^T x^{(m)} + b \,]_{1 \times m}$$

$$Z = \text{np.dot}(w.T, X) + b \qquad (1,1) \quad \mathbb{R}$$

"Broadcasting"

$$A = [a^{(1)} \; a^{(2)} \cdots a^{(m)}] = \sigma(Z)$$

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Basics of Neural Network Programming

Vectorizing Logistic Regression's Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $dz^{(2)} = a^{(2)} - y^{(2)}$ ....

$\boxed{dZ = [dz^{(1)} \; dz^{(2)} \cdots dz^{(m)}]}$ ←
$1 \times m$

$A = [a^{(1)} \cdots a^{(m)}]$. $Y = [y^{(1)} \cdots y^{(m)}]$

→ $dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \cdots]$

→ $dw = 0$
$dw \mathrel{+}= x^{(1)} dz^{(1)}$
$dw \mathrel{+}= x^{(2)} dz^{(2)}$
$\vdots$
$dw \mathrel{/}= m$

$\cancel{dw_1}$
$\cancel{dw_2}$
$\vdots$

$db = 0$
$db \mathrel{+}= dz^{(1)}$
$db \mathrel{+}= dz^{(2)}$
$\vdots$
$db \mathrel{+}= dz^{(m)}$
$db \mathrel{/}= m.$

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$
$$= \frac{1}{m} \, np.sum(dZ)$$

$$\boxed{dw = \frac{1}{m} X \, dZ^T}$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \cdots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[x^{(1)} dz^{(1)} + \cdots + x^{(m)} dz^{(m)} \right]$$
$n \times 1$

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Implementing Logistic Regression

```
J = 0, dw_1 = 0, dw_2 = 0, db = 0
for i = 1 to m:
```

$$z^{(i)} = w^T x^{(i)} + b$$

$$a^{(i)} = \sigma(z^{(i)})$$

$$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$$

$$dz^{(i)} = a^{(i)} - y^{(i)}$$

$$dw \mathrel{+}= x^{(i)} * dz^{(i)}$$

$$db \mathrel{+}= dz^{(i)}$$

$$J = J/m, \quad dw_1 = dw_1/m, \quad dw_2 = dw_2/m$$

$$db = db/m$$

for iter in range(1000):

$$Z = w^T X + b$$

$$= np.dot(w.T, X) + b$$

$$A = \sigma(Z)$$

$$dZ = A - Y$$

$$dw = \frac{1}{m} X dZ^T$$

$$db = \frac{1}{m} np.sum(dZ)$$

$$w := w - \alpha dw$$

$$b := b - \alpha db$$

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Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

$$
\begin{array}{r c}
 & \begin{array}{cccc} \text{Apples} & \text{Beef} & \text{Eggs} & \text{Potatoes} \end{array} \\
\begin{array}{r} \text{Carb} \\ \text{Protein} \\ \text{Fat} \end{array} & \begin{bmatrix} 56.0 & 0.0 & 4.4 & 68.0 \\ 1.2 & 104.0 & 52.0 & 8.0 \\ 1.8 & 135.0 & 99.0 & 0.9 \end{bmatrix}
\end{array} = A \quad (3,4)
$$

0

1

59 cal

$\frac{56}{59} \approx 94.9\%$

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

```
cal = A.sum(axis = 0)
percentage = 100*A/(cal.reshape(1,4))
```

(3,4) / (1,4)

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}100\\100\\100\\100\end{bmatrix} \quad \cancel{100}$$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 200 & 300\\100 & 200 & 300\end{bmatrix}$$

(m,n) $(2,3)$ $(1,n) \rightsquigarrow (m,n)$ $(2,3)$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 100 & 100\\200 & 200 & 200\end{bmatrix} =$$

(m,n) $(m,1) \rightsquigarrow (m,n)$

General Principle

$$(m, n) \quad \begin{matrix} + \\ - \\ * \\ / \end{matrix} \quad \begin{matrix} (1, n) \rightsquigarrow (m, n) \\ (m, 1) \rightsquigarrow (m, n) \end{matrix}$$

matrix

$$(m, 1) \quad + \quad \mathbb{R}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 & 102 & 103 \end{bmatrix}$$

Matlab/Octave: bsxfun

deeplearning.ai

Basics of Neural Network Programming

A note on python/ numpy vectors

Python Demo

Python / numpy vectors

```
import numpy as np

a = np.random.randn(5)

a = np.random.randn((5,1))

a = np.random.randn((1,5))

assert(a.shape = (5,1))
```