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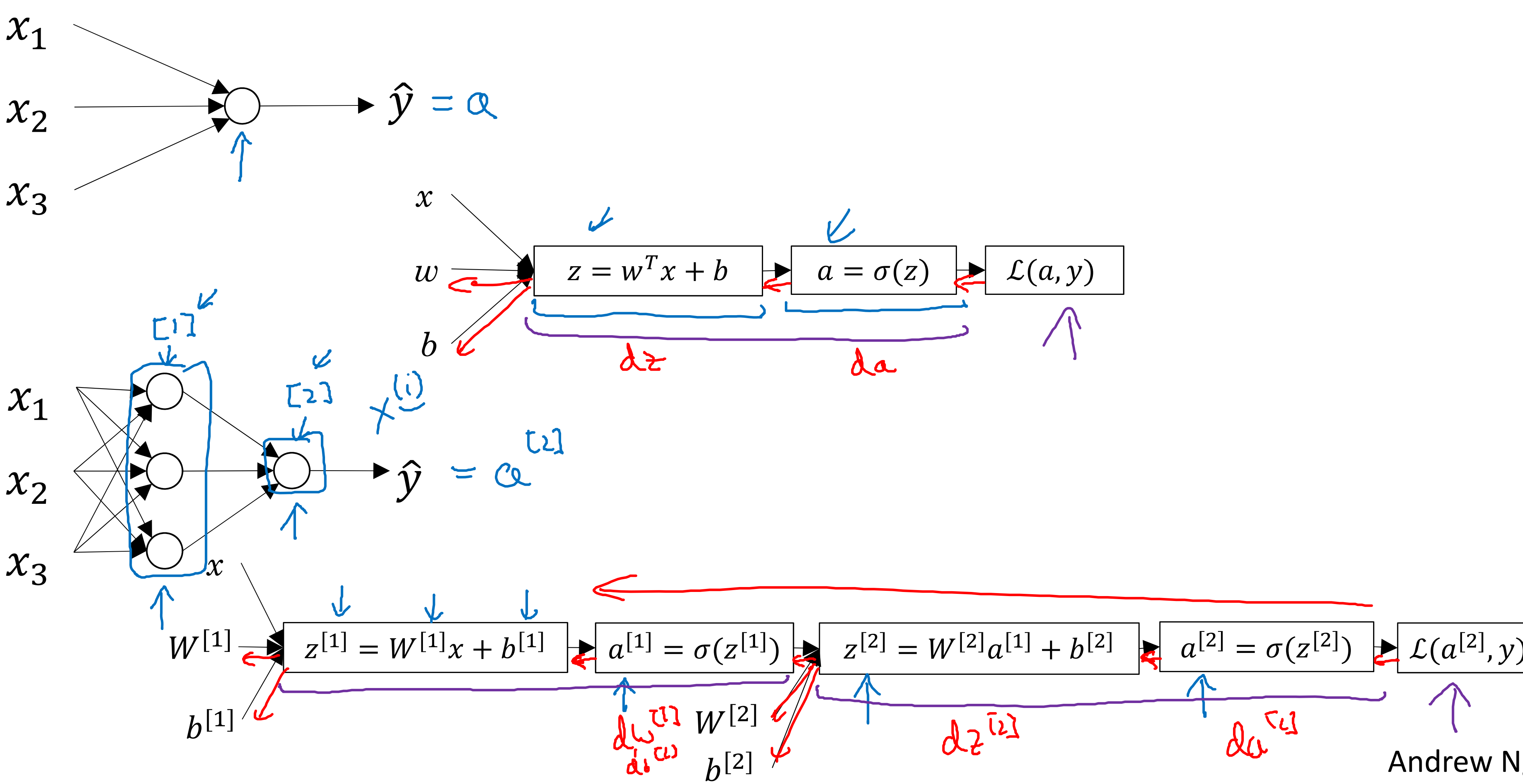
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



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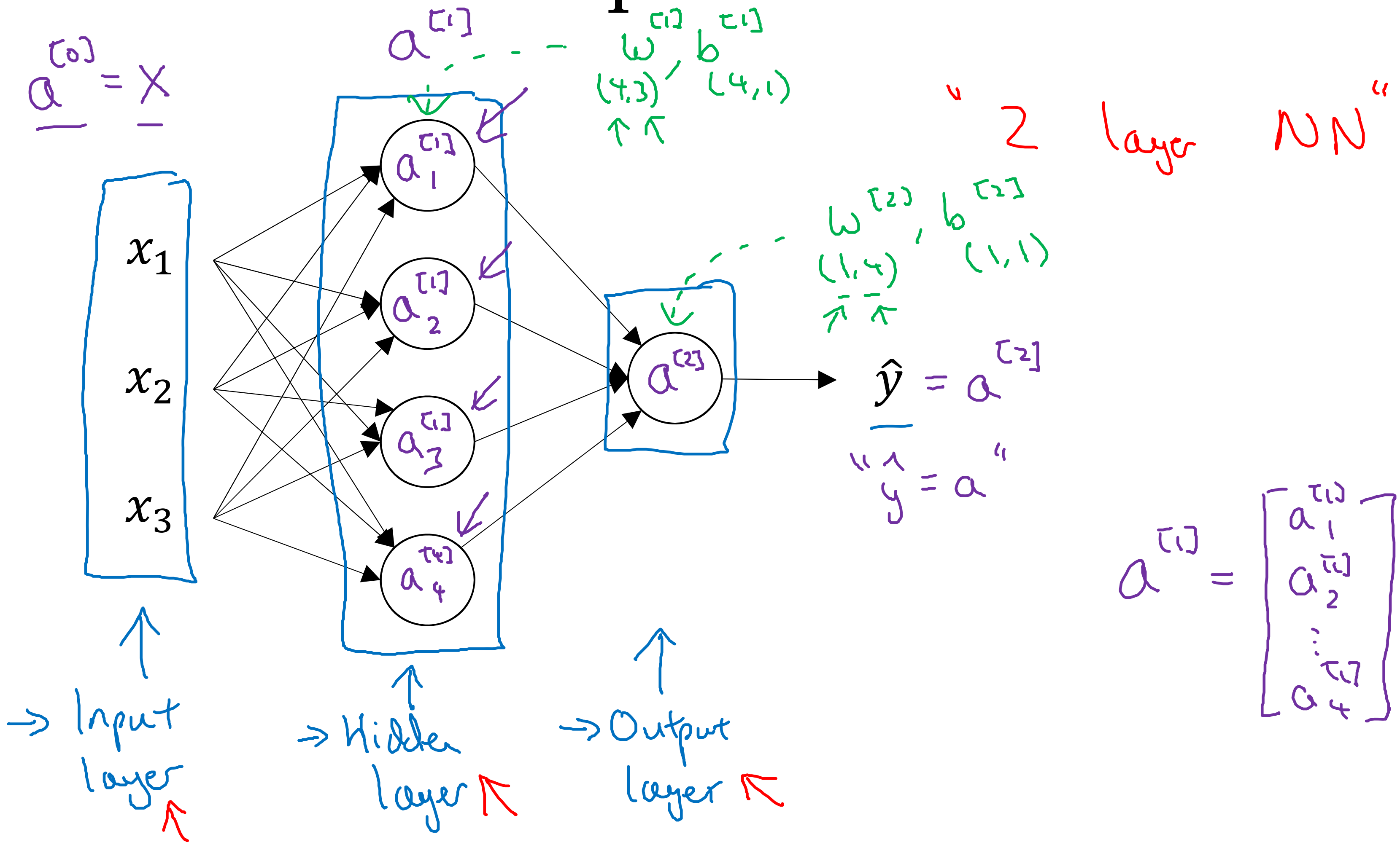
Neural Network Representation

Neural Network Representation



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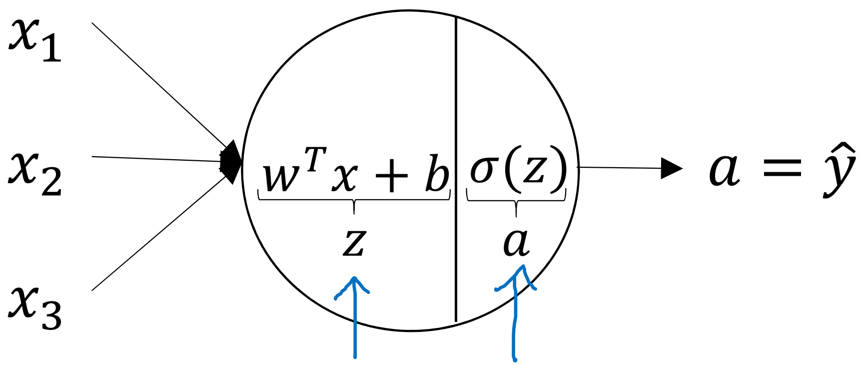
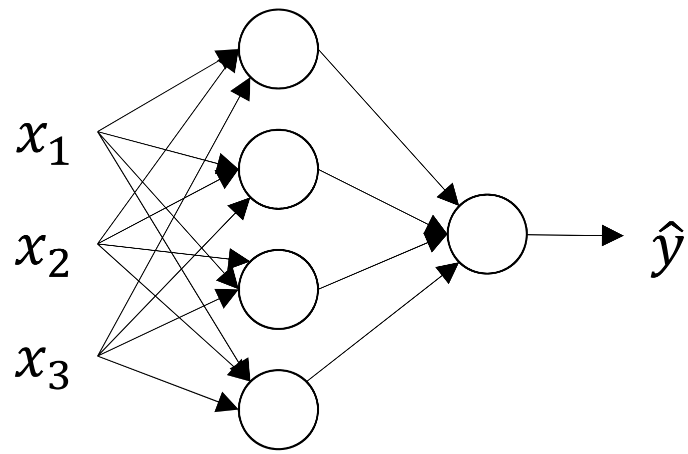
One hidden layer Neural Network

Computing a Neural Network's Output

Neural Network Representation





$$z = w^T x + b$$

$$a = \sigma(z)$$

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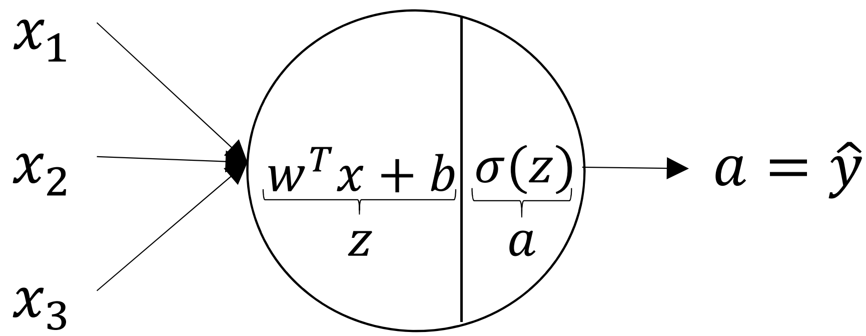
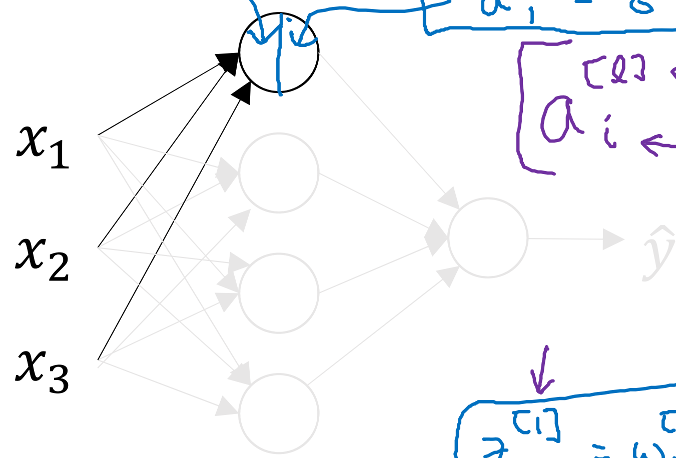
Neural Network Representation



$$z = w^T x + b$$

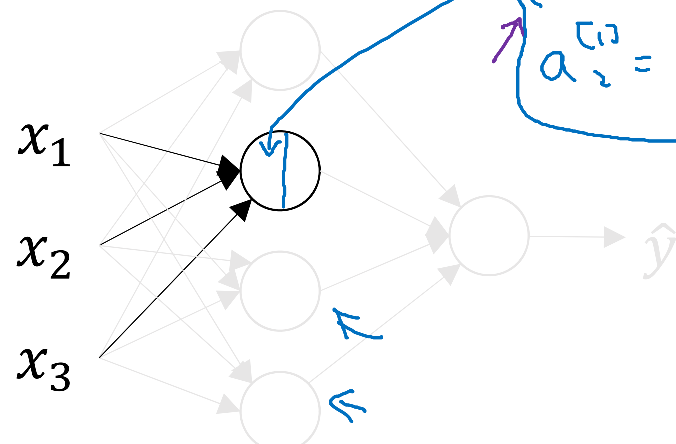
$$a = \sigma(z)$$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$$

$$a_1^{[1]} = \sigma(z_1^{[1]})$$

$a_i^{[l]}$ ← layer
← node in layer.



$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$$

$$a_2^{[1]} = \sigma(z_2^{[1]})$$

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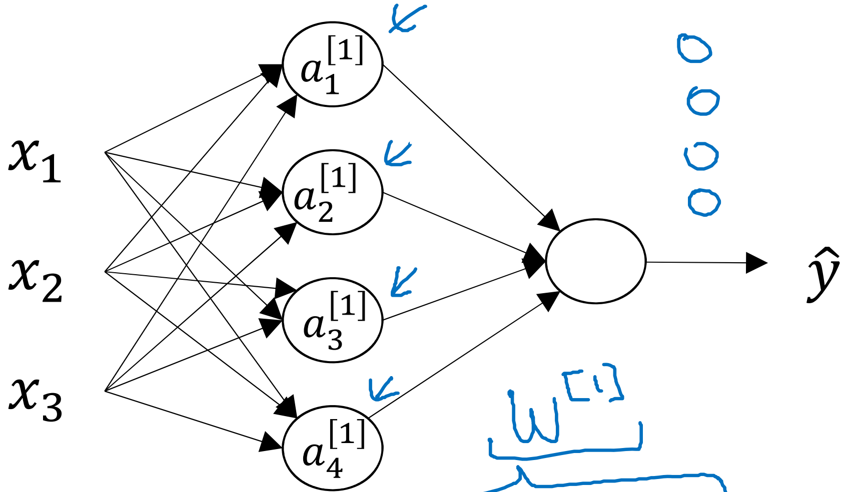
Neural Network Representation



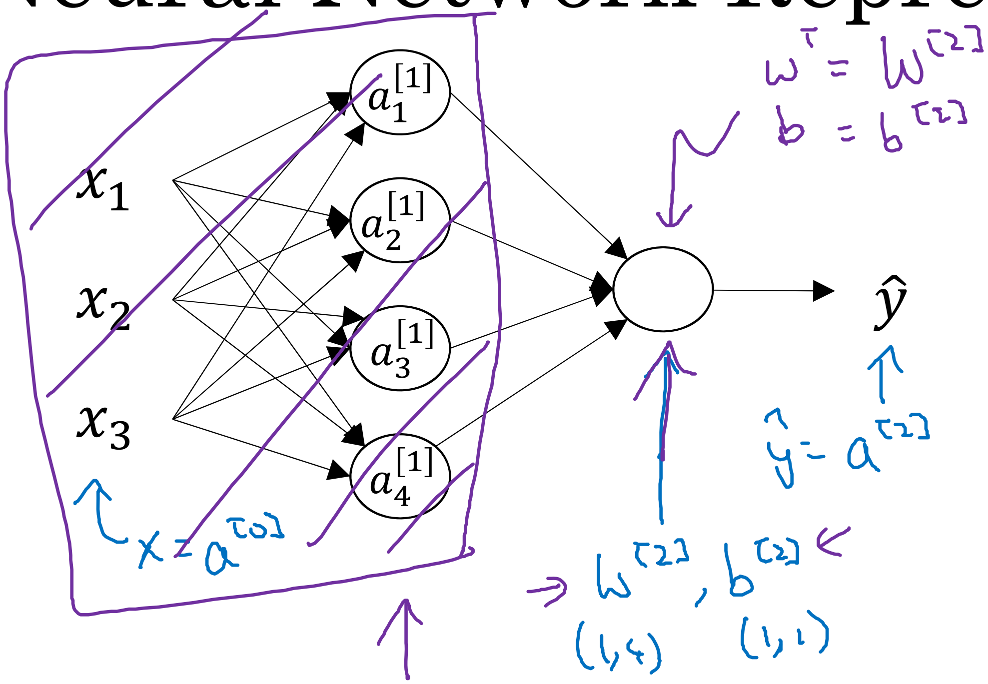
$(w_1^{[1]})^T x$ $\qquad a^{[1]}$

$$z_1^{[1]} = w_1^{[1]T}x + b_1^{[1]},\quad a_1^{[1]} = \sigma(z_1^{[1]})$$
$$z_2^{[1]} = w_2^{[1]T}x + b_2^{[1]},\quad a_2^{[1]} = \sigma(z_2^{[1]})$$
$$z_3^{[1]} = w_3^{[1]T}x + b_3^{[1]},\quad a_3^{[1]} = \sigma(z_3^{[1]})$$
$$z_4^{[1]} = w_4^{[1]T}x + b_4^{[1]},\quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \underbrace{\overbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}^{W^{[1]}}}_{(4,3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}\ (4,1)} = \begin{bmatrix} w_1^{[1]T}x + b_1^{[1]} \\ w_2^{[1]T}x + b_2^{[1]} \\ w_3^{[1]T}x + b_3^{[1]} \\ w_4^{[1]T}x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}}\underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}}\underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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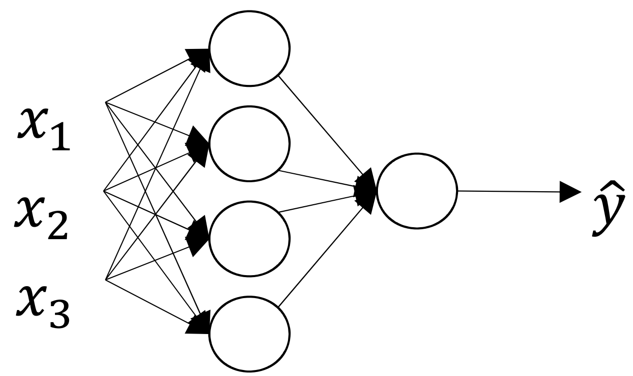
One hidden layer Neural Network

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$$x \longrightarrow a^{[2]} = \hat{y}$$
$$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$$
$$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$$
$$\vdots$$
$$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$$

$a^{[2](i)}$ ← example i

← layer 2

for $i = 1$ to m,

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$

$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$

$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ A^{[1]} &= \sigma(Z^{[1]}) \\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

training examples

hidden units

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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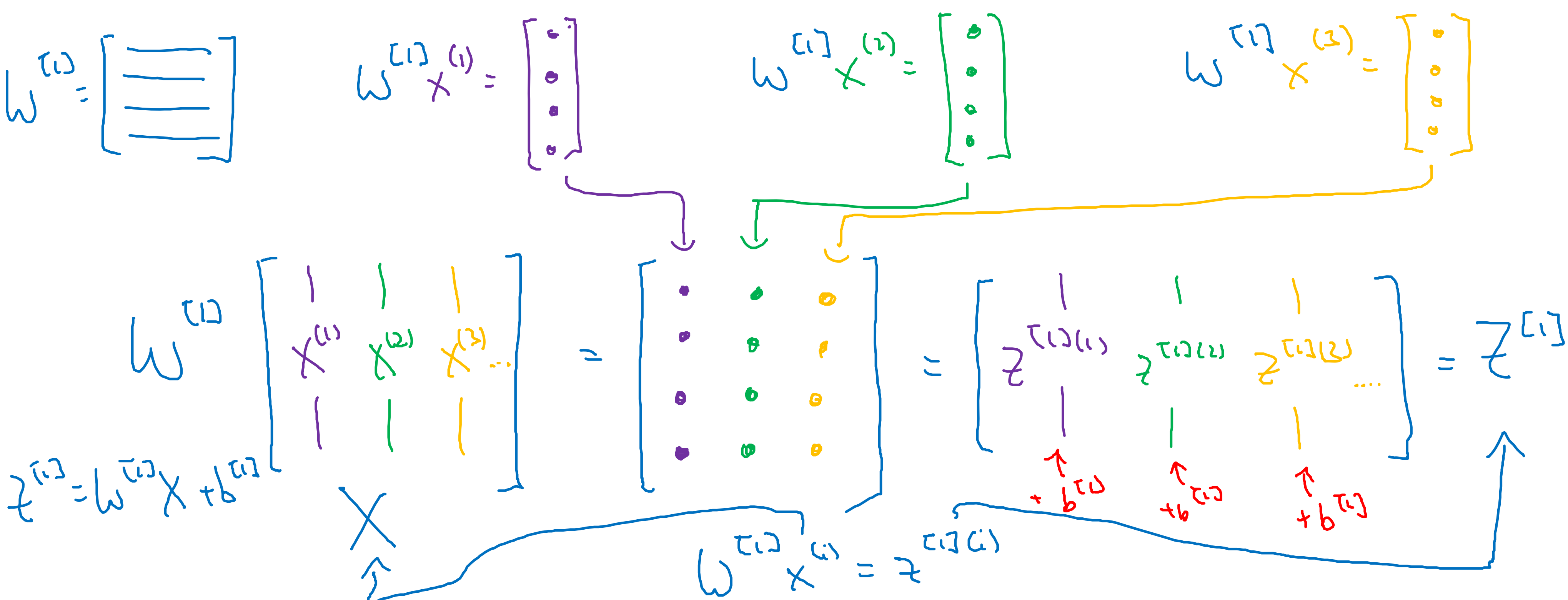
One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$z^{[1](1)} = W^{[1]} x^{(1)} + \cancel{b^{[1]}}$, $\quad z^{[1](2)} = W^{[1]} x^{(2)} + \cancel{b^{[1]}}$, $\quad z^{[1](3)} = W^{[1]} x^{(3)} + \cancel{b^{[1]}}$

$\uparrow 0 \qquad \uparrow 0 \qquad \uparrow 0$



$z^{[1]} = W^{[1]} X + b^{[1]}$

$W^{[1]} x^{(i)} = z^{[1](i)}$

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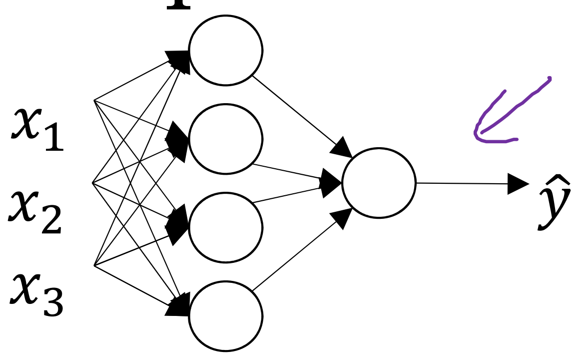
Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$\underline{A^{[1]}} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
```

$$\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$A^{[0]}$ $\quad X = a^{[0]}$ $\quad x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}X + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

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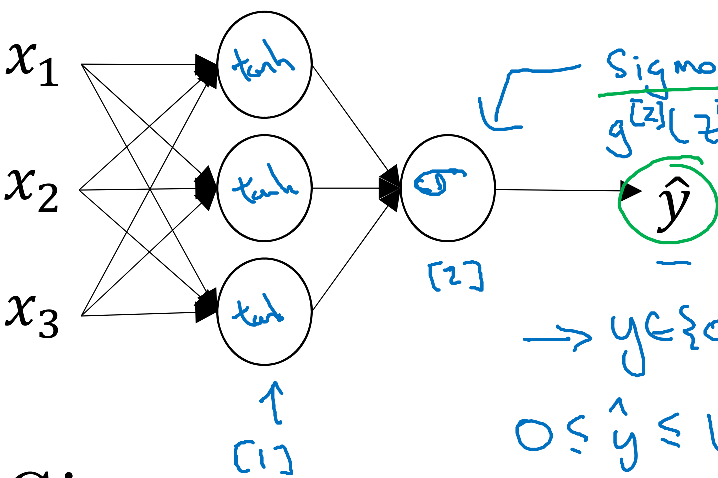
One hidden layer Neural Network

Activation functions

Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$



Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

$\to y \in \{0,1\}$

$0 \le \hat{y} \le 1$

-1 1

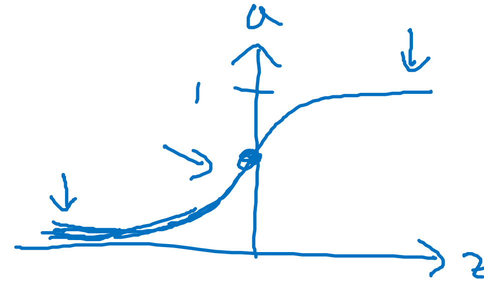
Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\to a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

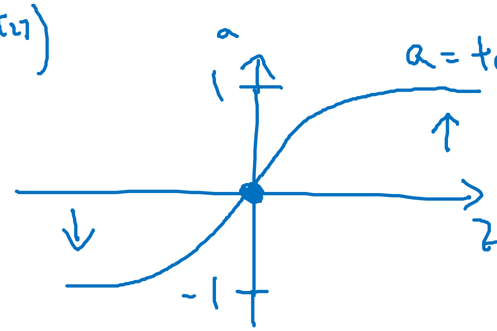
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

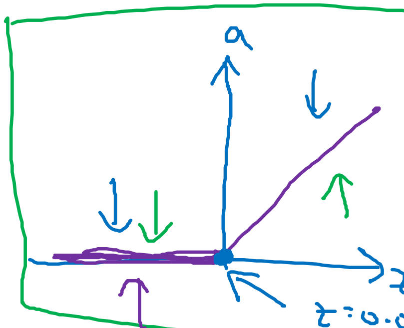
$$\to a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$



$a = \frac{1}{1+e^{-z}}$

$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$





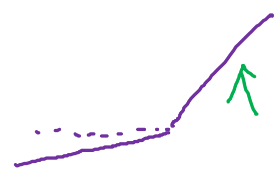
$z > 0$

$a = \max(0, z)$

ReLU

Rectified Linear Unit

$z = 0.000000000000$



Leaky ReLU

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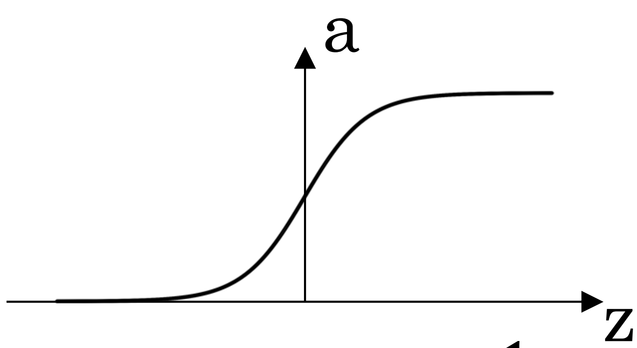
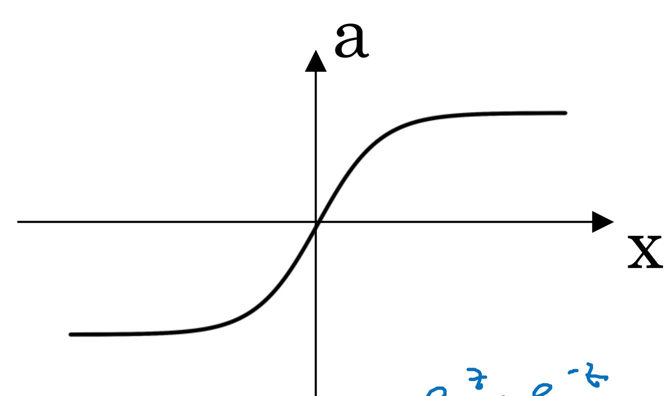
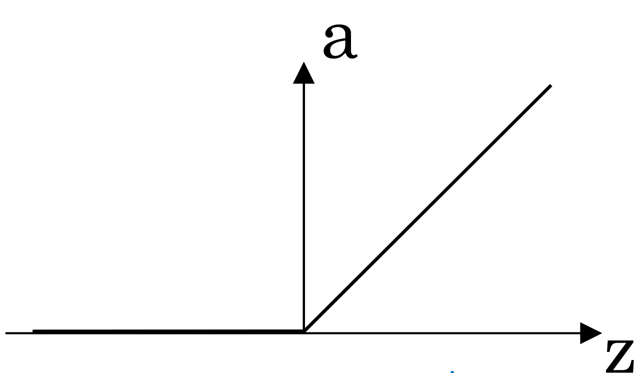
Pros and cons of activation functions



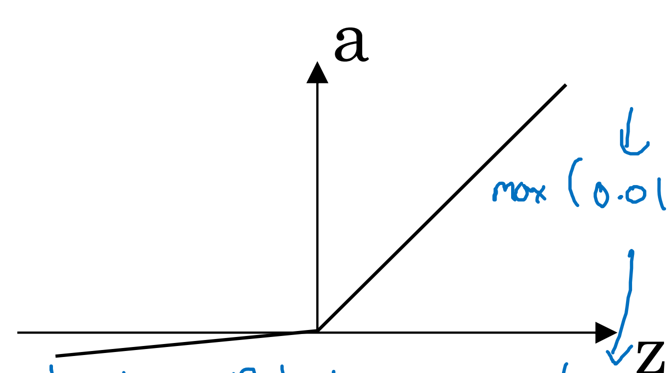
sigmoid: $a = \dfrac{1}{1 + e^{-z}}$



tanh: $a = \dfrac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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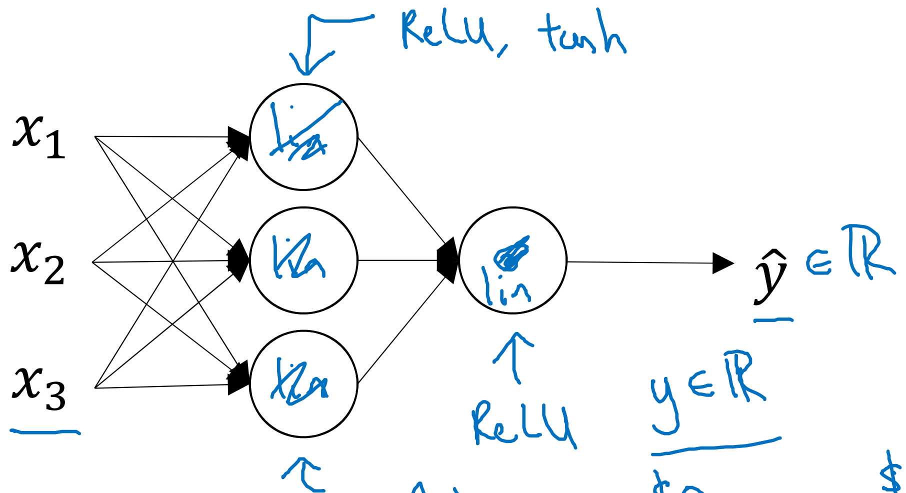
One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000\$

$\hat{y} \geq 0$.

Given x:

$\rightarrow$ $z^{[1]} = W^{[1]}x + b^{[1]}$

$\rightarrow$ $a^{[1]} = \cancel{g^{[1]}(z^{[1]})}$ $z^{[1]}$

$\rightarrow$ $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

$\rightarrow$ $a^{[2]} = \cancel{g^{[2]}(z^{[2]})}$ $z^{[2]}$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}(\underbrace{W^{[1]}x + b^{[1]}}_{a^{[1]}}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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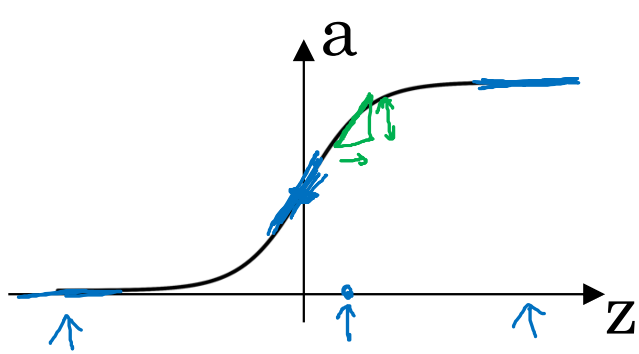
Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\left(1-g(z)\right)}$$

$$= \boxed{a(1-a)}$$

$$g'(z) = a(1-a)$$

$z = 10$. $g(z) \approx 1$

$$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$$

$z = -10$ $g(z) \approx 0$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$z = 0$ $g(z) = \frac{1}{2}$

$$\frac{d}{dz} g(z) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$$

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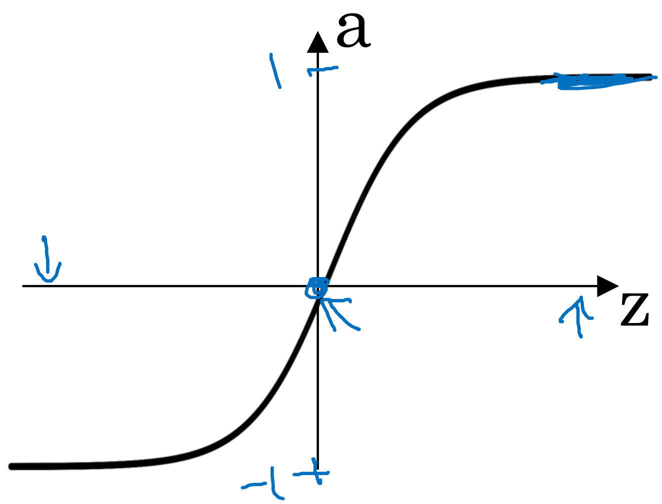
Tanh activation function



$$g(z) = \tanh(z)$$

$$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= 1 - (\tanh(z))^2$$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

$g'(z) = 1$

$$a = g(z), \qquad g'(z) = 1 - a^2$$

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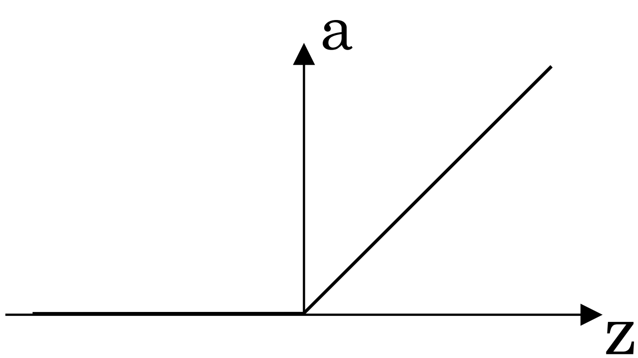
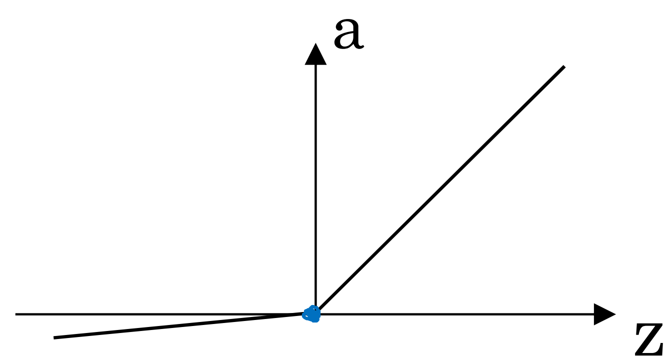
ReLU and Leaky ReLU



ReLU



Leaky ReLU

$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef}} & \cancel{\text{if } z = 0} \end{cases}$$

$$z = 0.0000\ldots\ldots 0$$

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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One hidden layer Neural Network

Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $W^{[1]}$, $b^{[1]}$, $W^{[2]}$, $b^{[2]}$

$(n^{[1]}, n^{[0]})$ $(n^{[1]}, 1)$ $(n^{[2]}, n^{[1]})$ $(n^{[2]}, 1)$

$n_x = n^{[0]}$, $n^{[1]}$, $\underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$

$\hat{y} \leftarrow a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \ldots, m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}$, $\underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}$, ...

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \ldots$ $b^{[2]} := \ldots$

}

Formulas for computing derivatives

Forward propagation:

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]}) \leftarrow$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$$

Back propagation:

$$dZ^{[2]} = A^{[2]} - Y \quad \swarrow \qquad Y = [y^{(1)}\ y^{(2)} \dots, y^{(m)}]$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T} \qquad (n^{[1]},) \leftarrow$$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis}=1, \text{keepdims}=\text{True}) \qquad (n^{[2]}, 1) \leftarrow$$

$$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]}, m)}$$

↑ element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^{T}$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[1]}, 1)$ $\quad (n^{[1]},)$ $\quad$ reshape ↑

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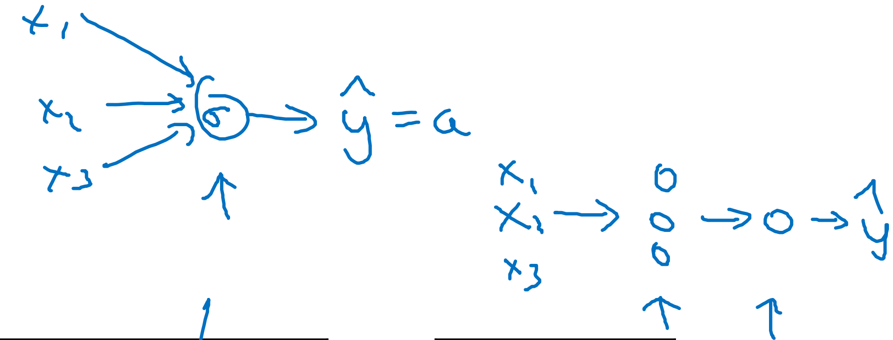
Backpropagation intuition (Optional)

Computing gradients

Logistic regression



$$\left.\begin{array}{l} x \\ w \\ b \end{array}\right\} \rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$$

$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$$\boxed{dz = da \cdot g'(z)}$$

$g(z) = \sigma(z)$

$$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$$

"dz" = "da"

$$\frac{d}{dz} g(z) = g'(z)$$

$$da = \frac{d}{da} \mathcal{L}(a, y) = -y \log a - (1-y) \log(1-a)$$

$$= -\frac{y}{a} + \frac{1-y}{1-a}$$

Neural network gradients

x_1, x_2, x_3 $\rightarrow \hat{y}$

$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]} = 1$

$W^{[2]}$ $dW^{[2]}$

$b^{[2]}$ $db^{[2]}$

x

$W^{[1]}$ $dW^{[1]}$ $\quad b^{[1]}$ $db^{[1]}$

$$z^{[1]} = W^{[1]}x + b^{[1]} \rightarrow a^{[1]} = \sigma(z^{[1]}) \rightarrow z^{[2]} = W^{[2]}x + b^{[2]} \rightarrow a^{[2]} = \sigma(z^{[2]}) \rightarrow \mathcal{L}(a^{[2]}, y)$$

$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$ — element wise product

$da^{[1]} = \dots$

$dz^{[2]} = a^{[2]} - y$

$da^{[2]}$

$dW^{[2]} = dz^{[2]} a^{[1]T}$ $\rightarrow$ "$dw = dz \cdot x$"

$db^{[2]} = dz^{[2]}$

$w^{[1]}: [\;\_\_\_\_\;]$

$W^{[2]}$ $\quad (n^{[2]}, n^{[1]})$

$\rightarrow z^{[2]}, dz^{[2]}$ $\quad (n^{[2]}, 1)$ $\quad - (1,1)$

$\rightarrow z^{[1]}, dz^{[1]}$ $\quad (n^{[1]}, 1)$

foo $\quad W^{[1]}$

dfoo $\quad dW^{[2]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$ $\quad a^{[0]T}$

$db^{[1]} = dz^{[1]}$

$$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]'}(z^{[1]})}$$

$$(n^{[1]}, 1) = (n^{[1]}, n^{[2]})\,(n^{[2]}, 1) * (n^{[1]}, 1)$$

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Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

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Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$(n^{[1]}, 1)$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$dZ^{[2]} = A^{[2]} - Y$$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underset{(n^{[1]}, m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]}, m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

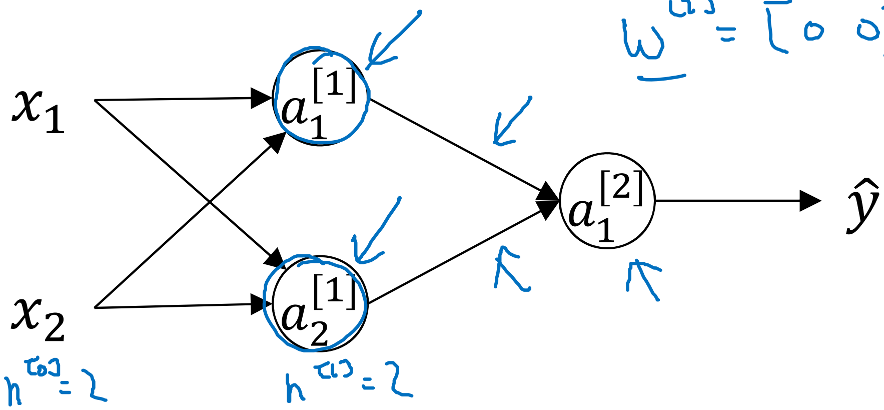
Andrew Ng

deeplearning.ai

One hidden layer Neural Network

Random Initialization

What happens if you initialize weights to zero?



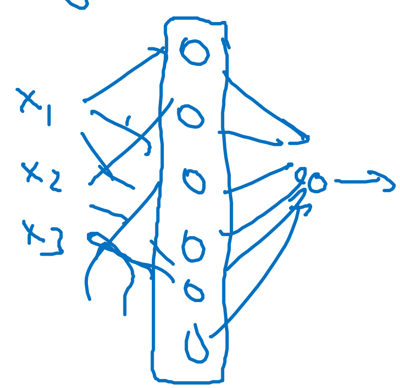
$W^{[2]} = [0 \quad 0]$

$$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]} \qquad dz_1^{[1]} = dz_2^{[1]}$$

$$dw = \begin{bmatrix} u & v \\ u & v \end{bmatrix} \qquad W^{[1]} = W^{[1]} - \alpha dW$$

Symmetric



$$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$$

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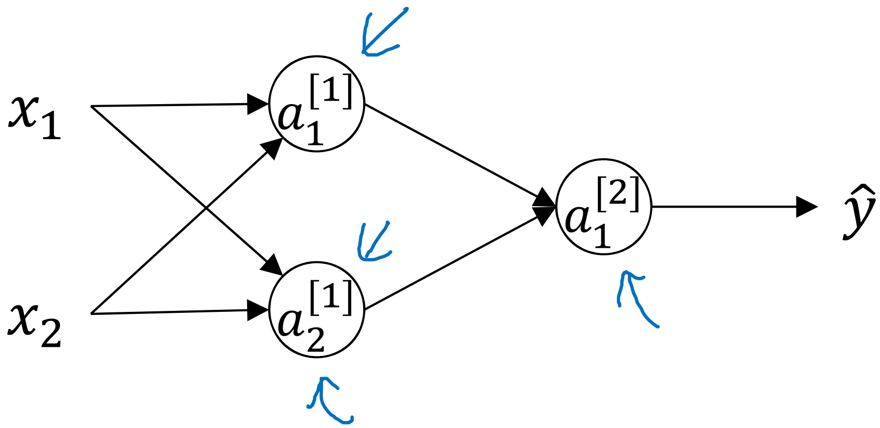
Random initialization



$W^{[1]} =$ np.random.randn((2,2)) * 0.01

100?

$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) * 0.01

$b^{[2]} = 0$

$z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$