The **Ljung–Box test** (named for Greta M. Ljung and George E. P. Box) is a type of statistical test of whether any of a group of autocorrelations of a time series are different from zero. Instead of testing randomness at each distinct lag, it tests the "overall" randomness based on a number of lags, and is therefore a portmanteau test.

Formal definition [edit]

The Ljung–Box test may be defined as:

H_0: The data are independently distributed (i.e. the correlations in the population from which the sample is taken are 0, so that any observed correlations in the data result from randomness of the sampling process).

H_a: The data are not independently distributed; they exhibit serial correlation.

The test statistic is:[2]

$$Q = n\,(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k}$$

where *n* is the sample size, $\hat{\rho}_k$ is the sample autocorrelation at lag *k*, and *h* is the number of lags being tested. Under H_0 the statistic Q follows a $\chi^2_{(h)}$. For significance level α, the critical region for rejection of the hypothesis of randomness is:

$$Q > \chi^2_{1-\alpha,h}$$

where $\chi^2_{1-\alpha,h}$ is the 1-[3]α-quantile of the chi-squared distribution with *h* degrees of freedom.

The Ljung–Box test is uncommonly used in autoregressive integrated moving average (ARIMA) modeling. Note that it is applied to the residuals of a fitted ARIMA model, not the original series, and in such applications the hypothesis actually being tested is that the residuals from the ARIMA model have no autocorrelation. When testing the residuals of an estimated ARIMA model, the degrees of freedom need to be adjusted to reflect the parameter estimation. For example, for an ARIMA(p,0,q) model, the degrees of freedom should be set to $h-p-q$.[4]