Quality tracking

lunedì 28 aprile 2025 11:23

ρ DENSITÀ MISCELA

p PRESSIONE MISCELA

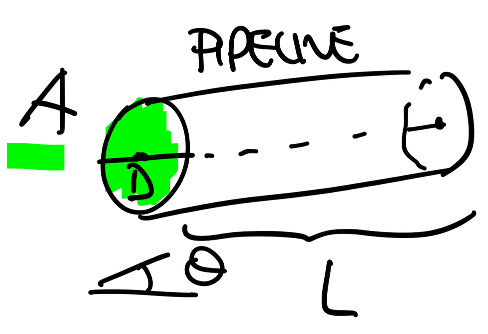
v VELOCITÀ MISCELA

$$\begin{cases} \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho v) = 0 & \text{(mass conservation)} \\ \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\underbrace{\rho v^2}_{\text{NEGLECTED}} + p) + \frac{\lambda \rho v |v|}{2D} + \underbrace{\rho g \sin\theta}_{\text{NEGLECTED}} = 0 & \text{(momentum conservation)} \end{cases}$$

$t \in [0,T]$
$\forall$ pipeline
$x \in [0,L]$

EOS MISCELA $\quad \frac{p}{\rho} = ZRT = c^2$

$A, \lambda, D, \underline{g}, \theta$ note from pipeline



(TOTAL) MASS FLOW RATE MISCELA $\quad \dot{m} = \rho v A = \phi A$

$\phi = \rho v$ MASS FLOW PER AREA MISCELA

CONSIDERING NOW SINGLE COMPONENTS

ρ^α PARTIAL DENSITY OF COMPONENT $\quad \alpha \in \{1, \dots, n\}$ $\quad n = 21$ in EOS GERG

$c^\alpha := \rho^\alpha / \rho$ MASS FRACTION $\quad \sum_{\alpha=1}^{n} \rho^\alpha = \rho \quad \sum_{\alpha=1}^{n} c^\alpha = 1$

$\phi^\alpha = v\rho^\alpha = \gamma^\alpha \phi \quad \dot{m}^\alpha = v\rho^\alpha A = \phi^\alpha A = \gamma^\alpha \dot{m} \quad \sum_{\alpha=1}^{n} \gamma^\alpha = 1$

$\gamma^\alpha := \phi^\alpha / \phi$ VOLUMETRIC FRACTION $\quad \tilde{\rho}^\alpha := \rho^\alpha / \gamma^\alpha$ INDIVIDUAL COMPONENT DENSITY

$p^\alpha / \tilde{\rho}^\alpha = Z_\alpha RT \quad Z_\alpha = 1 + a_\alpha(T) p$

(densità che la componente avrebbe se occupasse tutto il volume)

Now we have:

$$\begin{cases} \frac{\partial}{\partial t}(\rho^\alpha) + \frac{\partial}{\partial x}(\rho^\alpha v) = 0 & \text{MASS} \\ \frac{\partial}{\partial t}(\phi) + \frac{\partial}{\partial x}(p) + \frac{\lambda \phi |v|}{2D} = 0 & \text{MOMENTUM} \end{cases}$$

$\forall \alpha \in \{1, \dots, n\}$
$\forall x \in [0,L]$
$\forall t \in [0,T]$

$\Rightarrow \rho^\alpha = c^\alpha \rho = c^\alpha \left(\frac{p}{c^2}\right) \quad \rho^\alpha v = c^\alpha \rho v = c^\alpha \phi = c^\alpha \frac{\dot{m}}{A}$

$$\left[\frac{\partial}{\partial t}\left(c^\alpha \frac{p}{c^2}\right) + \frac{\partial}{\partial x}\left(c^\alpha \frac{\dot{m}}{A}\right) = 0 \right] \Longleftrightarrow$$

EQUAZIONE DISCRETIZZATA DA MARCO

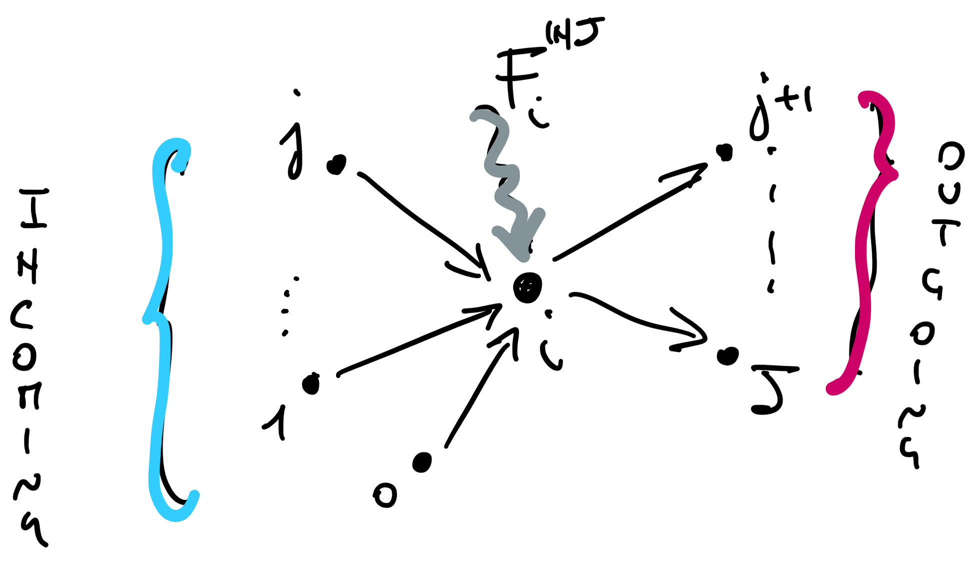
$$\Rightarrow \frac{\partial}{\partial t}(c^\alpha) \frac{p}{c^2} + \frac{\partial}{\partial t}\left(\frac{p}{c^2}\right) c^\alpha + \frac{\partial}{\partial x}(c^\alpha) \frac{\dot{m}}{A} + \frac{\partial}{\partial x}\left(\frac{\dot{m}}{A}\right) c^\alpha = 0$$

$$c^2\left(\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho v)\right) = 0$$

$$\Rightarrow \frac{p}{c^2}\frac{\partial}{\partial t}(c^\alpha) + \frac{\dot{m}}{A}\frac{\partial}{\partial x}(c^\alpha) = 0 \Rightarrow \left[\frac{\partial}{\partial t}(c^\alpha) + v \frac{\partial}{\partial x}(c^\alpha) = 0 \right]$$

TRANSPORT of c^α
$\forall \alpha \in (1, \dots, n)$
$\forall x \in [0,L]$
$\forall t \in [0,T]$

ON NODES



$$\underbrace{\sum_{j \in INC} A_j \bar{c}_j^\alpha \bar{\phi}_j}_{} - \underbrace{\sum_{j \in OUT} A_j \underline{c}_j^\alpha \underline{\phi}_j}_{} = c_i^\alpha F_i - \underbrace{c_{INJ}^\alpha}_{\text{GIVEN}} F_i^{INJ}$$

$\forall i \in \{0, \dots, I\}$ NODES
$\forall \alpha \in (1, \dots, H)$

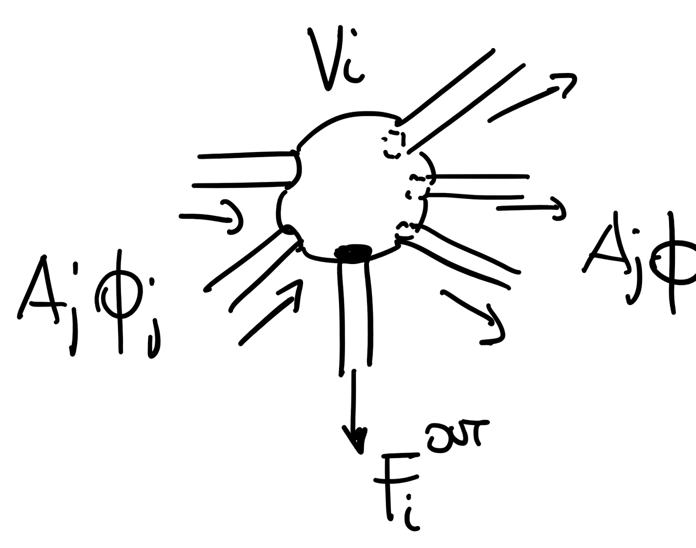
where

$\bar{c}_j^\alpha := c^\alpha(L_j, t) \quad \underline{c}_j^\alpha := c^\alpha(0,t) \quad \bar{\phi}_j := \phi_j(L_j, t) \quad \underline{\phi}_j := \phi_j(0,t)$

$\Rightarrow A_j \bar{\phi}_j = \bar{\dot{m}}_j \quad , A_j \underline{\phi}_j = \underline{\dot{m}}_j$

PERFECT MIXING $\quad \underline{c}_j^\alpha \equiv c_i^\alpha \quad \forall j \in OUT$ (CONTINUITY OF CONCENTRATIONS)

$F_i \equiv$ WITHDRAWAL TOTAL MASS FLOW FROM NODE i



V_i = VOLUME OF NODE i

$$F_i = F_i^{OUT} + \int_{V_i} \frac{\partial \rho_i}{\partial t}$$

HYPOTESIS $\rho_i(x,t) = \rho_i(t) \Rightarrow$ CONSTANT IN SPACE IN THE NODE AS REDUCED TO POINT

$$\Rightarrow F_i = F_i^{OUT} + \frac{\partial \rho_i}{\partial t} V_i = F_i^{OUT} + \frac{V_i}{c^2}\frac{\partial p_i}{\partial t}$$

SUBSTITUTING F_i we obtain the equation of MARCO

$$\sum_{j \in INC} A_j \bar{c}_j^\alpha \bar{\phi}_j - \sum_{j \in OUT} A_j \underline{c}_j^\alpha \underline{\phi}_j = -c_{INJ}^\alpha F_i^{INJ} + c_i^\alpha \left(F_i^{OUT} + \frac{V_i}{c^2}\frac{\partial p_i}{\partial t}\right)$$

$\forall i \in \{0, \dots, I\}$ NODES
$\forall \alpha \in (1, \dots, H)$