Transient Gas Network Model

A closer look:

Coninutiy equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

$G = \rho v A$

$\rho = \frac{p}{ZRT}$

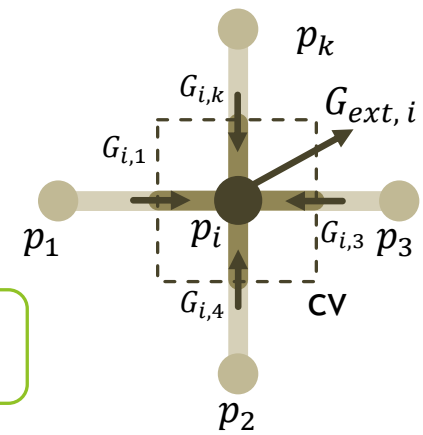
$$\frac{\partial p}{dt} = -\frac{c^2}{A}\frac{\partial G}{\partial x}$$

Control Volume integration

$$\frac{V_i}{c_{i,j}^2}\frac{dp_i}{dt} = \sum_{j=1}^{k} G_{i,j} - G_{ext,i}$$

time discretization

$$\frac{V_i}{c_{i,j}^2 \Delta t}(p_i^{n+1} - p_i^n) = \sum_{j=1}^{k} G_{i,j}^{n+1} - G_{ext,i}^{n+1}$$



$c^2 = ZRT$

$P = p^2$

Flow equation

$$\frac{\partial(\rho v)}{\partial t} + \cancel{\frac{\partial(\rho v^2)}{\partial x}} + \frac{\partial p}{\partial x} + \frac{\lambda}{2D}\rho v|v| + \cancel{\rho g \sin\alpha} = 0$$

creeping motion *horizontal pipelines*

$$\frac{\partial p}{\partial x} = -\frac{\lambda c^2}{2DA^2 p}|G|G - \frac{1}{A}\frac{\partial G}{\partial t}$$

Finite Difference over a pipeline

time discretization

LINEARIZATION NEEDED!

$$\Delta P_j^{n+1} = R_{f_j}|G_j^{n+1}|G_j^{n+1} + R_{I_j}(G_j^{n+1} - G_j^n)$$

with:

Fluid-dynamic Resistance $R_f = \frac{16\,\lambda c^2}{\pi^2 D^5 p}\Delta x$

Inertia Resistance $R_I = \frac{2\,p_m}{A}\frac{\Delta x}{\Delta t}$

Averaged over the pipeline length

- Friction factor λ → Colebrook-White correlation
- Compressibility Factor Z → GERG-2008 wide range equation of state