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myfbcd.py
Automated Feedback Controller Design

https://github.com/shimodatakaki/mypy

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1.1 Overview

Automated Feedback Controller Design:

Exploiting FRF results, the objective is to find a linear FB controller that satisfies desired linear/quadratic, **convex/concave** constraints **for all given FRFs**, i.e.

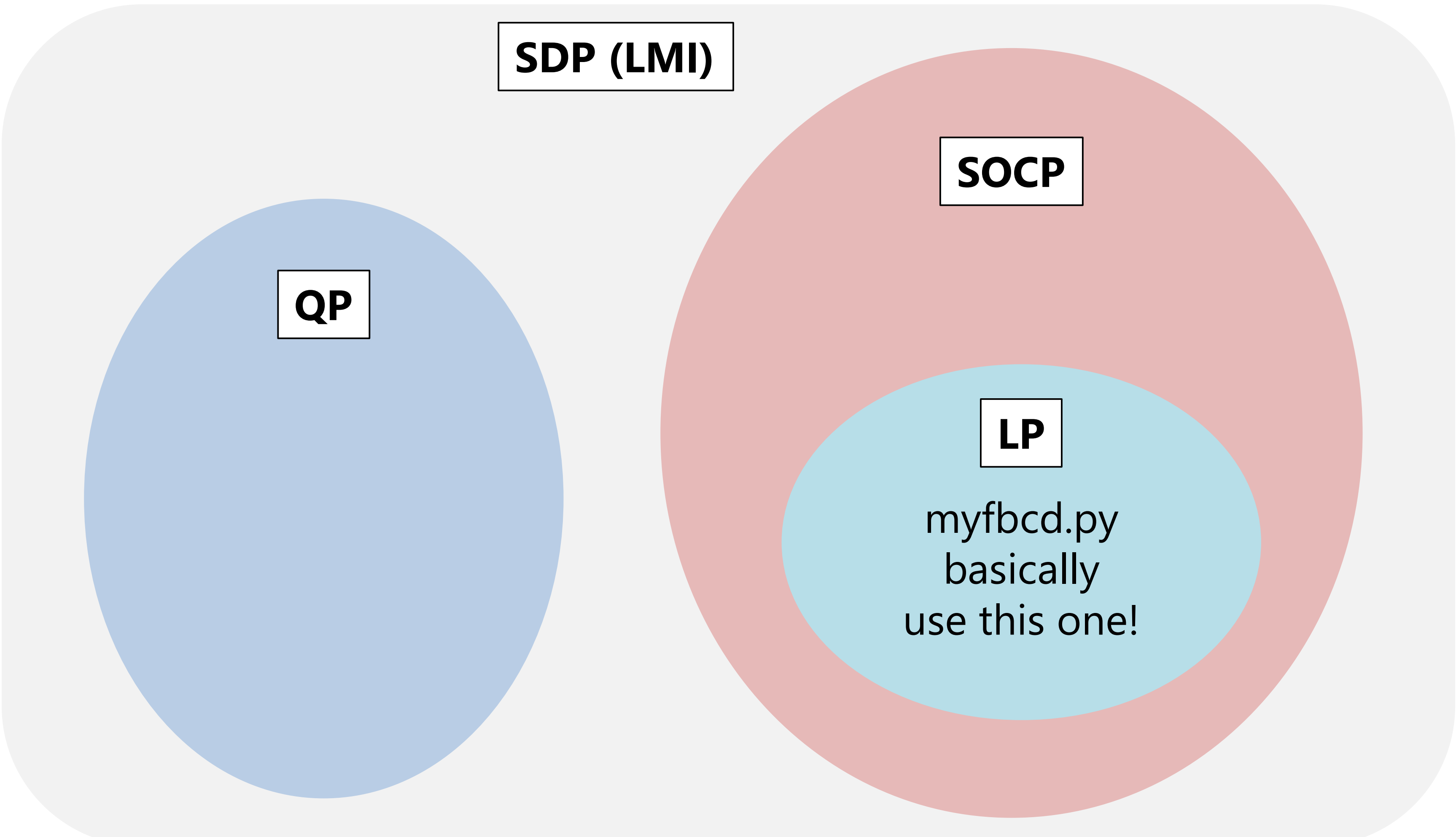
(1) Gain-Crossover Linear Inequalities,
(2) Phase Margin Linear Inequalities,
(3) Gain Margin Linear Inequalities,
(4) Second Phase Margin Linear Inequalities,
(5) Gain Minimum/Maximum Linear Inequalities,
(6) Stability Margin (Disk) Concave Inequalities via CCCP method,
(7) Robust Stability Quadratic Inequalities (using socp or sdp),
(8) Nominal Performance (Disk) Concave Inequalities via CCCP method.

NOTE: (5), (6), and (8) are important constraints.

Default Controller: PIDs + 50 FIRs (3+*50* variables).

1.2 Convex Optimization

Linear/Quadratic Convex Optimization Methods:



1.3: Controllers and FRFs

Given FRF *g*, the open loop transfer function *L* is:

$$L(j\omega_k) = g(j\omega_k)C(j\omega_k)$$

$$C(j\omega_k) = \boldsymbol{\phi}(j\omega_k)^T \boldsymbol{\rho}$$

where C is the controller, ρ is the controller gain vector, and Φ is the basis transfer function vector.

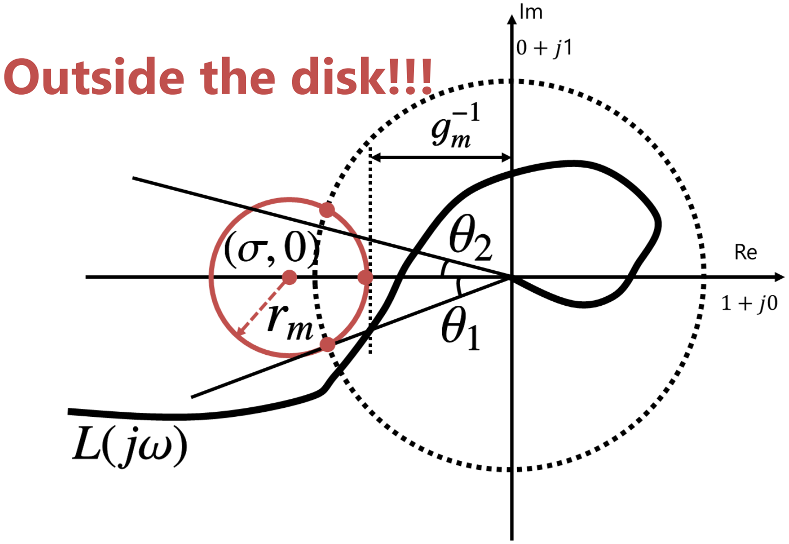
For example,

$$\boldsymbol{\phi}(s_k) = [1, 1/s_k, s_k/(1+0.01s_k)]^T$$

$$\boldsymbol{\rho} = [K_P, K_I, K_D]^T$$

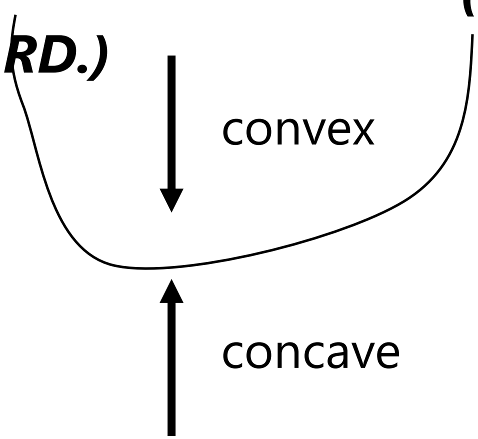
1.4 Concave Optimization

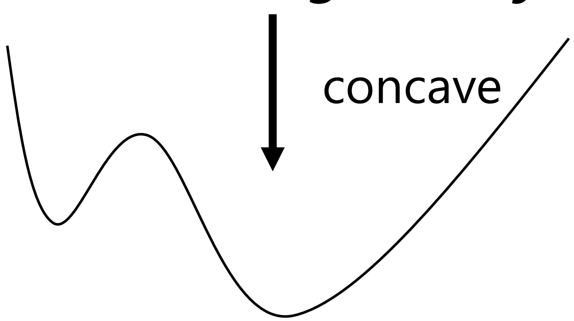
(6) and (8) constraints says
"All of Nyquist curves must be outside the disk given for gain-margin, phase-margin, or nominal-sensitivity".



*But **out-of-disk constraints are typical concave constraints.***

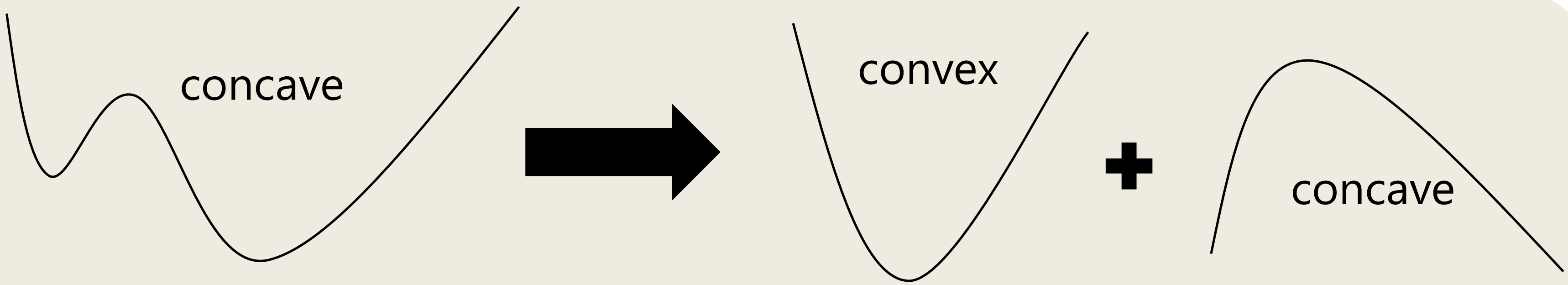
Concave problems cannot be solved by any types of convex solvers in polynomial-time,
i.e. find r s.t. $r^2 > 1 \rightarrow$ the result is ***primal infeasible.***
(Concave optimazation is generally NP−HARD.)





1.5 Concave-Convex Procedure (CCCP)

Concave functions can be split into convex and concave functions

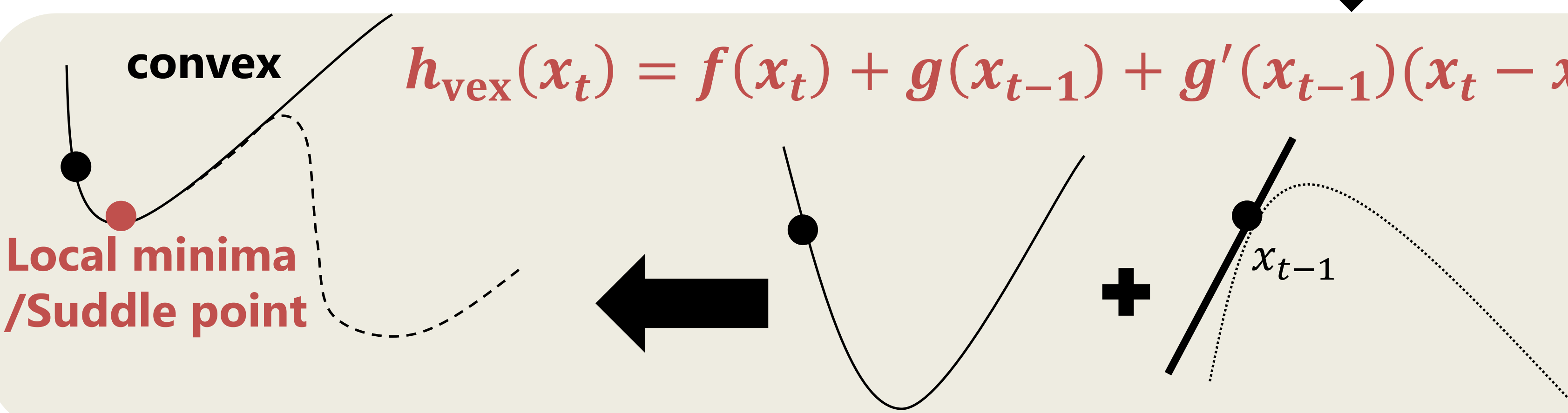


$$h_{\mathbf{cave}}(x) = f(x) + g(x)$$

The concave problem is now reduced into a convex problem!

Taylor Series of g

$$h_{\mathrm{vex}}(x_t) = f(x_t) + g(x_{t-1}) + g'(x_{t-1})(x_t - x_{t-1})$$



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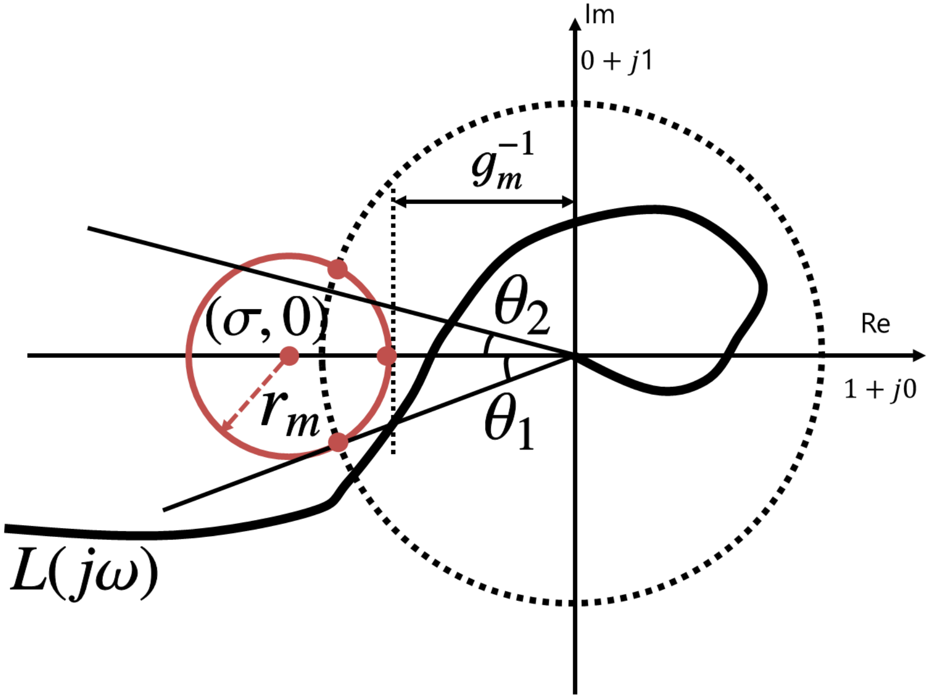
2.1 Stability Margin (Disk) Concave Inequalities via CCCP method



All of $L(j\omega_k)$ should be outside the disk.

$$r_m - |L(j\omega_k) - \sigma| \leq 0$$

for all k

CCCP

like $x_0 x + y_0 y \geq r$

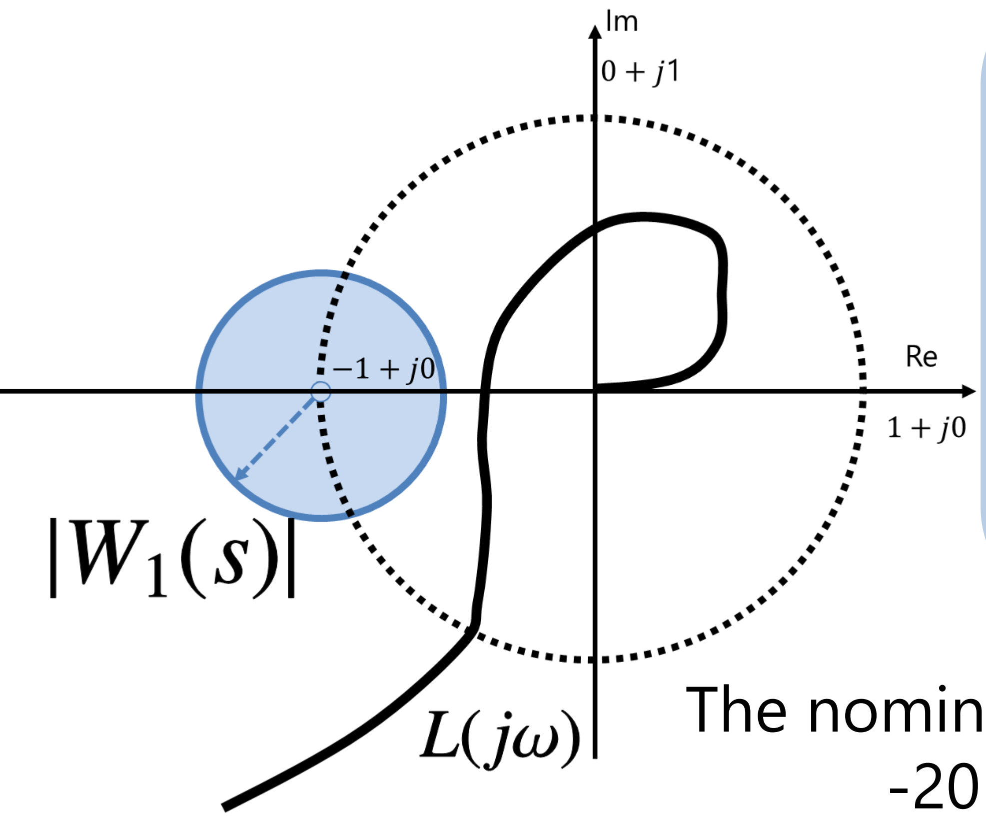
$$\boldsymbol{G}_k \boldsymbol{\rho} \leq \boldsymbol{h}_k$$

for all k

$$G_k = -\Re\left[\frac{L_{t-1}(j\omega_k)^H}{|L_{t-1}(j\omega_k)|}\boldsymbol{\phi}(j\omega_k)^T\right]$$

$$h_k = -r_m - \Re\left[\frac{L_{t-1}(j\omega_k)^H}{|L_{t-1}(j\omega_k)|}\right]\sigma$$

2.2 Nominal Performance (Disk) Concave Inequalities via CCCP method



$$|W_1(j\omega_k)S(j\omega_k)| \leq 1$$

for all k

$$|W_1(j\omega_k)| = \left(\frac{\omega_{ns}}{\omega_k}\right)^m$$

The nominal sensitivity <= 0 dB at ω_{ns}, and
-20 dB/dec when $m = 1$,
-40 dB/dec when $m = 2$,
-60 dB/dec when $m = 3$,
-80 dB/dec when $m = 4$ ...

(The CCCP is omitted since this problem is equivalent to 2.1)

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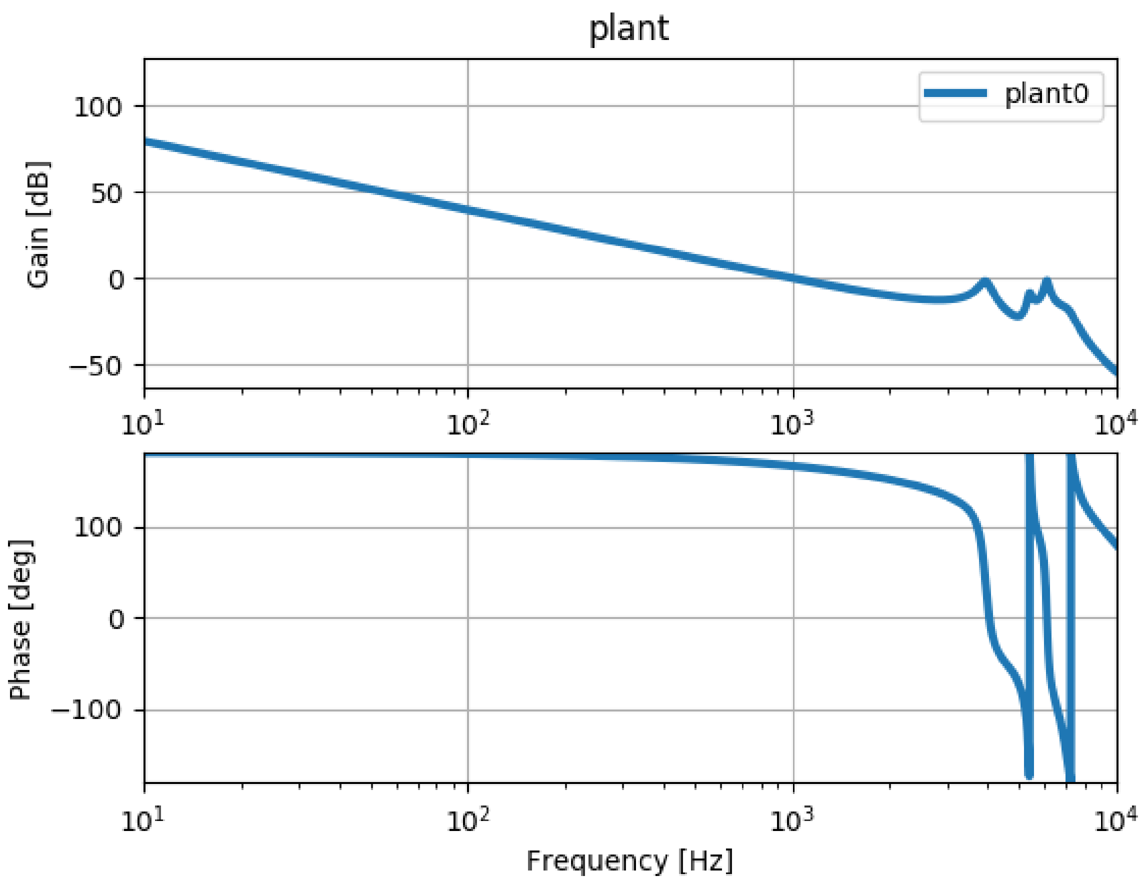
3.1 Plant and Controller

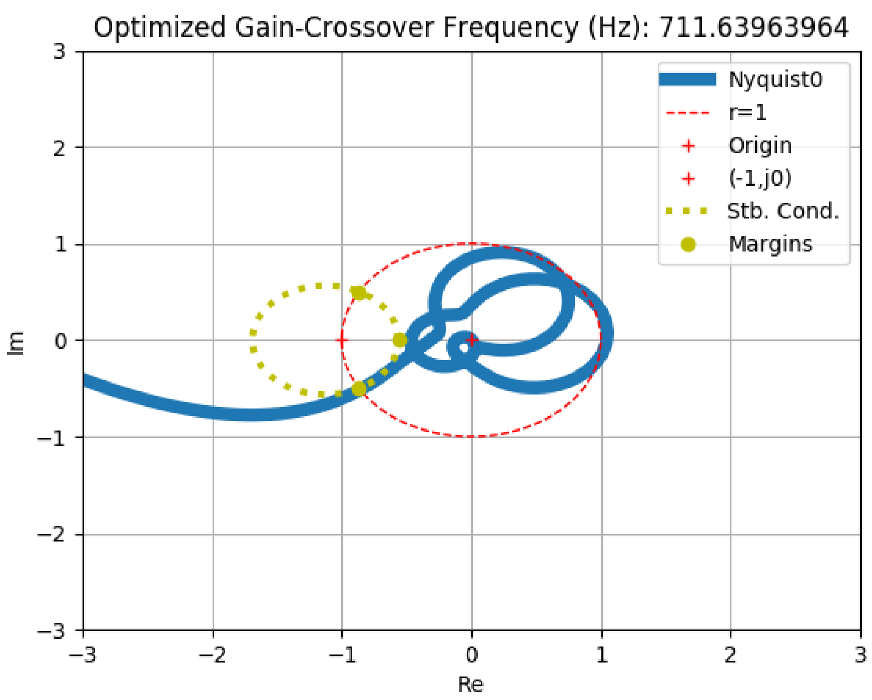


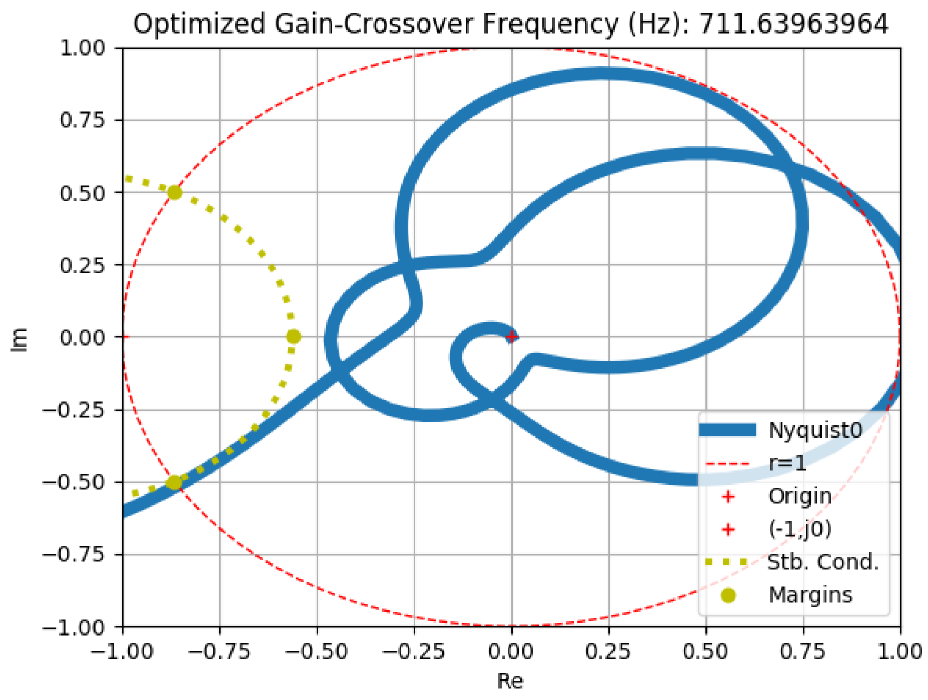
Discretization: 50 us
Delay: 15 us

The controller is P-I-D (pseudo derivative with 100 us cutoff).
The controller is optimized by iteration of bisection search of ω_{ns}.
Gain-Margin: 5 dB, Phase-Margin: 30 degrees, Second Phase-Margin : 30 degrees
Nominal Sensitivity: -60 dB/dec

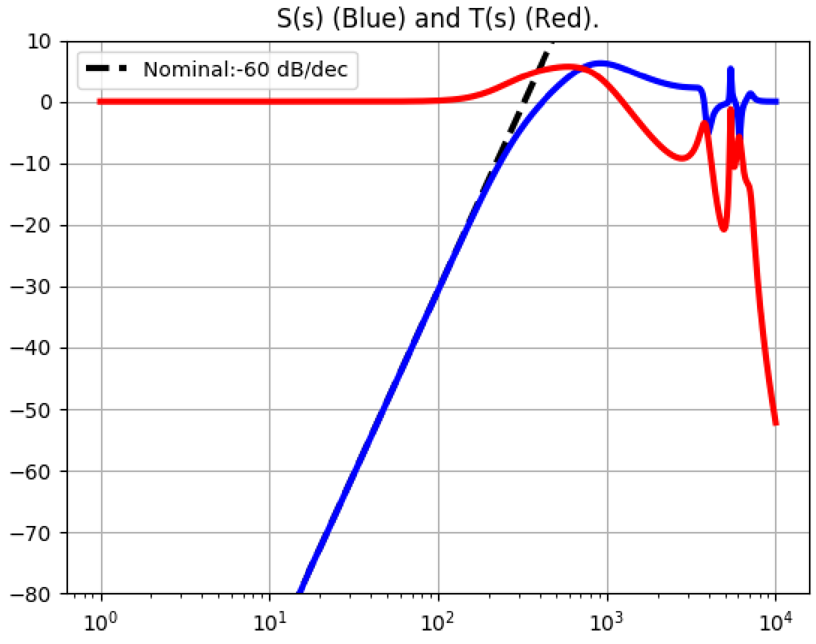
3.2 Nyquist

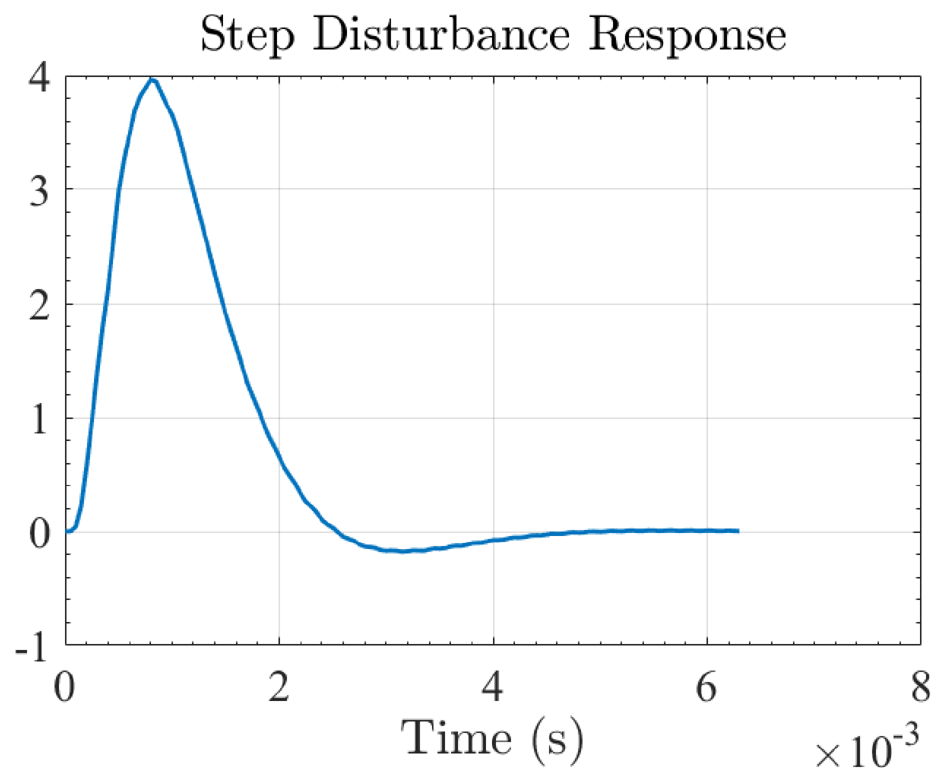
P Gain: 2.16684594e-01, I Gain: 2.29603303e+02, D gain: 1.06905746e-04.





3.3 Sensitivity and Step Disturbance Response





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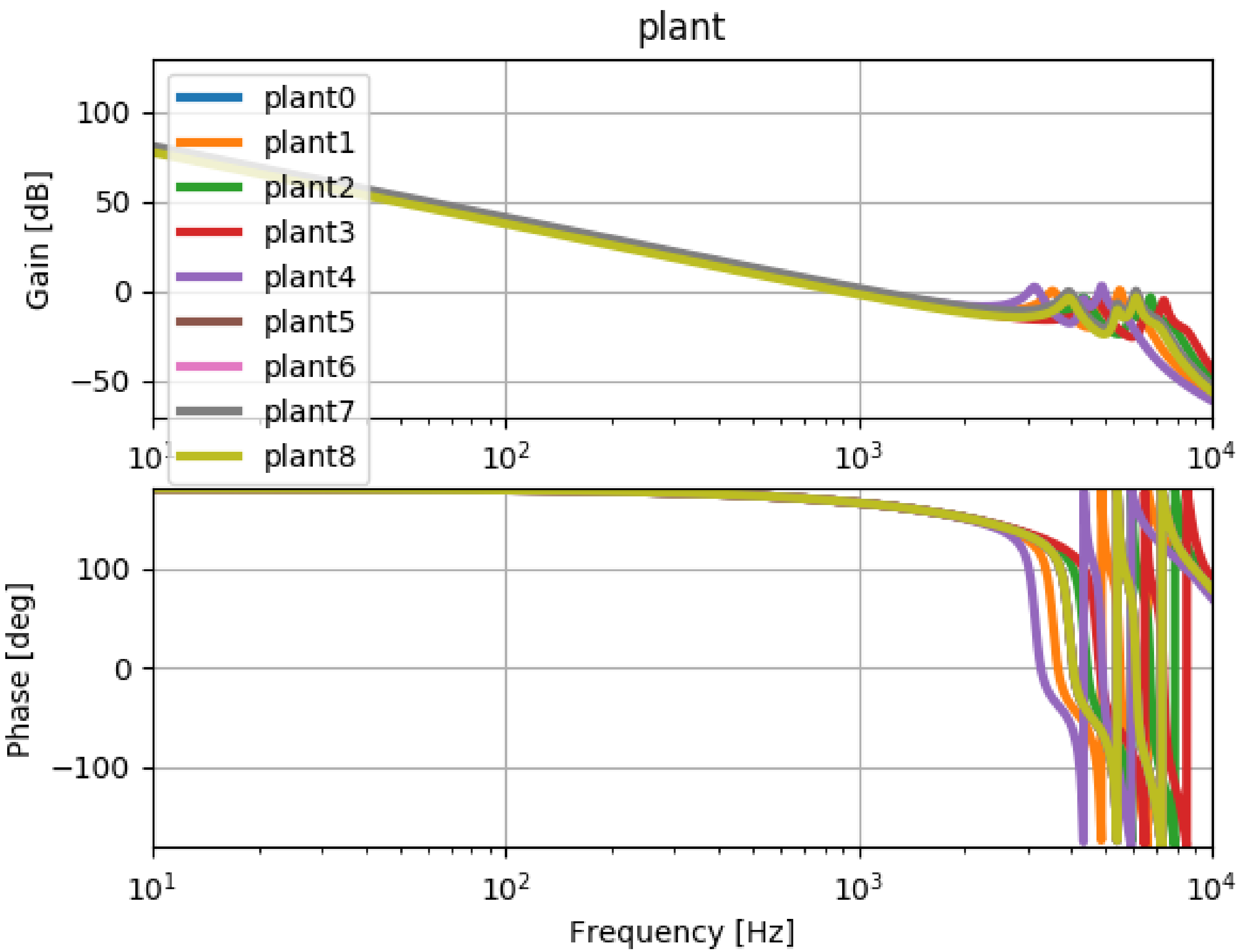
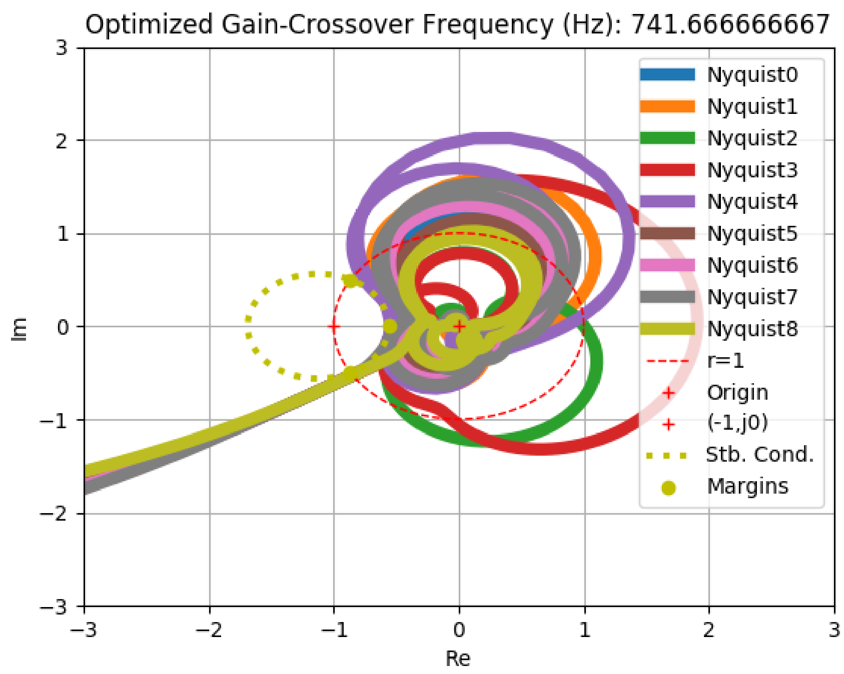
4.1 Plant

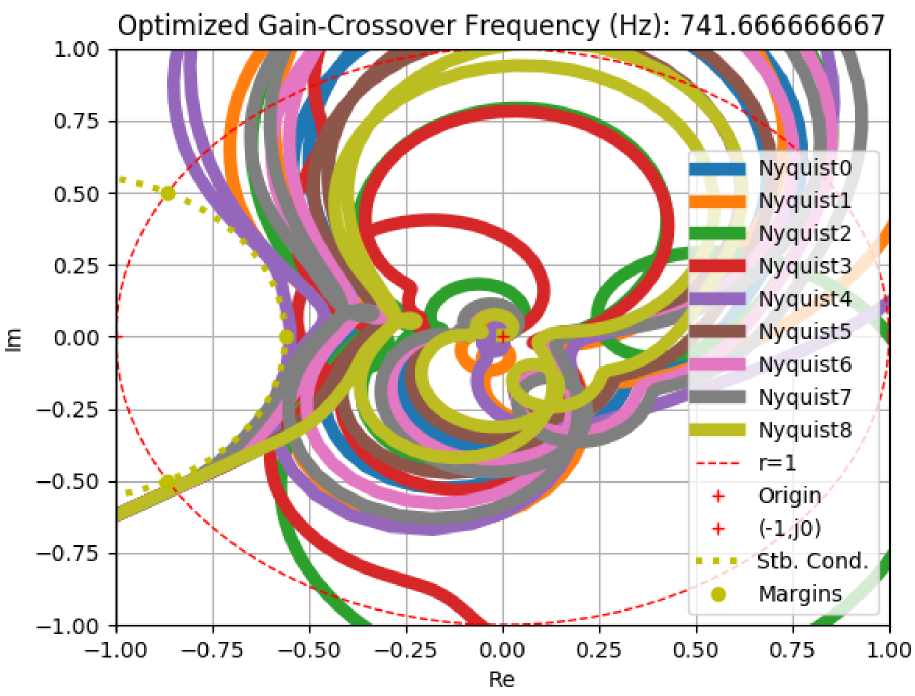


Discretization: 50 us
Delay: 15 us

The controller is PID with 30 parallel FIRs
Gain-Margin: 5 dB, Phase-Margin: 30 degrees, Second Phase-Margin : 30 degrees
Nominal Sensitivity: -40 dB/dec

4.2 Nyquist


Optimized Gain-Crossover Frequency (Hz): 741.666666667
Im
Re
3
2
1
0
−1
−2
−3
Nyquist0
Nyquist1
Nyquist2
Nyquist3
Nyquist4
Nyquist5
Nyquist6
Nyquist7
Nyquist8
r=1
Origin
(-1,j0)
Stb. Cond.
Margins


Optimized Gain-Crossover Frequency (Hz): 741.666666667
Im
Re
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Nyquist0
Nyquist1
Nyquist2
Nyquist3
Nyquist4
Nyquist5
Nyquist6
Nyquist7
Nyquist8
r=1
Origin
(-1,j0)
Stb. Cond.
Margins

4.3 Sensitivity and Openloop