Appendix for

Polarize, Catalyze, Stabilize

How an active minority of norm internalizers amplifies group selection and punishment

by

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Appendix A Model: Full(er) specification

Appendix A.1 Abstract model

The parameters for the abstract model are shown in Table 1.

Category	Variable	Description
Game	b	the benefit produced from cooperating
	c	the cost of cooperating
	$\bar{b}$	the proportion of the public pot that each (unpunished) agent received in the previous round
	f	the base fitness, which all agents receive in every round
	p_{obs}	the probability of being observed
Strategies	ϵ	the probability of acting counter to one's strategy
	β	how much the present value is weighted into a running average
	l	the learning rate
	d	the starting distribution of strategies
	d_{mut}	the distribution on strategies when a mutation occurs
Environment	p_{con}	The probability that a given group will engage in a conflict
	p_{mig}	probability of migration
	p_{mut}	the probability of offspring mutating to another strategy
	σ	the survival rate
	n	number of agents in a group
	g	number of groups
	y	number of rounds to run the model for

Table 1 **Parameters of the abstract model**

Appendix A.1.1 Decision

When deciding whether to cooperate, all agents use the same basic inequality, given in Equation **??**:

$$p\bar{b} \geq (1-\pi)c, \tag{1}$$

though if it is the first round, they cooperate if their propensity to cooperate is greater than $1/2$, and defect otherwise. Like in the abstract model, agents will deviate from their strategy with probability ϵ. Agents learn their parameter π in different ways.

Selfish After each round, they compare their payoff w to their present-biased average payoff $\bar{w}$, and their current propensity to cooperate π to their present-weighted average $\overline{\pi}$ (the present-weighted averages are calculated using β to determine the weight of the present value). They then increment π according to

$$\pi \leftarrow \pi + \frac{(\pi - \overline{\pi})(w - \overline{w})}{\overline{w} + w}. \tag{2}$$

Thus, if their fitness was above average and their π was above average, π will increase, if their fitness was below average and π was above average, π will decrease, and so on. The denominator is present to normalize the increment.

Norm internalizer The norm internalizer is learning strategy is conditional on the number of other cooperators. If the more than a majority are cooperating, the update is

$$\pi \leftarrow (1-l)\pi + l, \tag{3}$$

otherwise, it is

$$\pi \leftarrow (1-l)\pi. \tag{4}$$

These should be interpreted as the weighted average of π and 1 (moving closer to an unconditional cooperator) and the weighted average of π and 0 (moving closer to an unconditional defector). The weight is given by the learning rate, l.

Static - this strategy keeps its π-value constant.

Appendix A.1.2 Intergroup dynamics

In every round, groups are paired up at random, and with probability p_{con} they will engage in a conflict. If groups i and j fight, the probability that i will win is given by

$$\frac{1}{2}(1 + \frac{\overline{w_i} - \overline{w_j}}{\overline{w_i} + \overline{w_j}}), \tag{5}$$

where $\overline{w_i}$ is the average fitness of group i. The victor then replaces the old group with a copy of itself. Thus, higher fitness groups have an advantage in group conflict.

Appendix A.1.3 Selection

Agents act according to their strategy and receive their payoffs according to the rules of the game. Then, in every round, there is a phase of mortality. A survival rate σ determines the proportion of agents in each group that will survive at the end of each round. A set of $n \cdot \sigma$ agents are selected (without replacement) to survive, where an agent is selected to survive with probability

$$\frac{w}{n\overline{w}}, \tag{6}$$

where w is the agent's fitness and $\overline{w}$ is the average fitness of the group.
Each time an agent dies, another agent in the group is chosen to reproduce, where the probability of reproduction is proportional to that agent's fitness. Thus, for every death, any given agent with fitness w is selected to reproduce with probability $\frac{w}{n\overline{w}}$. With probability $(1 - p_{\text{mut}})$, the new agent has the same learning style as their parent. Otherwise, the new agent's strategy is chosen at uniformly at random among the learning styles in the strategy space. When norm internalizers are not present in the strategy space, their probability weight is allocated towards unconditional cooperators.

Appendix A.2 Naturalistic model

The parameters for the naturalistic model are shown in Table 2.

Category	Variable	Description
Game	b	the benefit from cooperation*
	c	the cost of cooperating*
	c_{dist}	the cost of foraging on an adjacent square (rather than one's own)*
	f	the base fitness, a constant proportional to the resources on a square*
	$\bar{b}$	the proportion of the public pot that each (unpunished) agent received in the previous round
	p_{obs}	the probability of being observed
	c_{alive}	the cost of staying alive for another round
	c_{repro}	the cost of reproducing
Strategies	ϵ	the probability of acting counter to one's strategy
	β	how much the present value is weighted into the average
	l	the learning rate
	d	the starting distribution of strategies
	d_{mut}	the distribution on strategies when a mutation occurs
Environment	p_{mut}	the probability of offspring mutating to another strategy
	s	grid side length
	n	starting number of agents in a group
	g	starting number of groups
	y	number of rounds to run the model for

Table 2 **Parameters of the naturalistic model** *strictly speaking, these quantities are not the final costs and benefits; they are first divided by the number of agents foraging on the square

Appendix A.2.1 Decision

For the decision on whether to cooperate, the naturalistic model functions exactly as the abstract model (Appendix A.1.1). As for the decision on where to forage, all agents use the same strategy. First, they calculate expected payoffs of staying on their current square versus going to an adjacent square as follows:

$$\begin{aligned} \pi_{\text{stay}} &= \frac{f}{n_{\text{here}}} \\ \pi_{\text{go}} &= \frac{f - c_{\text{dist}}}{n_{\text{out}}} \end{aligned} \tag{7}$$

where n_{here} is the number of agents on the current square and n_{out} is the number on adjacent squares. Also, π_{go} factors in the cost of foraging on distant lands. Using these payoffs, the agent adopts probabilities for staying or going as follows:

$$\begin{aligned} p_{\text{stay}} &= \frac{\pi_{\text{stay}}}{\pi_{\text{stay}} + \pi_{\text{go}}} \\ p_{\text{go}} &= \frac{\pi_{\text{go}}}{\pi_{\text{stay}} + \pi_{\text{go}}} \end{aligned} \tag{8}$$

Agents decide whether to stay or to go based on these probabilities. If they decide to go, they choose one of the four adjacent squares at random.

Appendix A.2.2 Distribution

The base-fitness payoff (interpreted as the payoff from simply foraging alone) is f/n_{here}, to account for the dwindling of resources as the square gets more crowded. Cooperating will produce a benefit to the group of b/n_{here} because cooperation bears more fruit when fewer agents compete for resources. The cost paid to cooperate is c/n_{here} and the cost paid to forage on distant lands is $c_{\text{dist}}/n_{\text{here}}$ (where n_{here} is the number of agents on the new square). Both of these costs are inversely proportional to the number of agents because, fundamentally, both of these are costs in *time*. If the resources on a particular square are rich (that is, there are few competitors for the resources), then a single unit of time is worth more, because one unit of time foraging will bear more fruit. Thus, if agents pay a constant cost in *time* either to cooperate or to forage on distant lands, the fitness cost will be proportional to the richness of the resources on their square (and inversely proportional to the number of agents on their square).

Thus, an agent generates value shown in Table 3, but because they do not keep the group benefit component that they generate (these get totaled up at distributed to all the agents in the group who were not caught defecting) the final payoff is obtained by subtracting the b wherever it appears and adding back $\bar{b}$ to any agent that was not caught defecting.

	Defect	Cooperate
Here	$\frac{f}{n}$	$\frac{f-c}{n}+b$
Distant	$\frac{f-c_{\text{dist}}}{n}$	$\frac{f-c-c_{\text{dist}}}{n}+b$

Table 3 **Value generated by agents according to their decisions** This is not their final payoff, because it is pre-distribution.

Appendix A.2.3 Intergroup dynamics

Here we make a remark about how the migration rates in the abstract and naturalistic models. They are not commensurate, for two reasons. The first is that lifetimes in the naturalistic model are much longer (20 as opposed to 10/3 rounds), so a certain lifetime migration rate in the naturalistic model would require a lower round-by-round migration rate. But, to complicate matters, agents in the naturalistic model only migrate if they are foraging on another square where another group is located. The migration probability in this very specific situation is the parameter p_{mig}. This would suggest that the naturalistic model would require a *higher* migration rate than the abstract model for the same effect. In the end, for the abstract model, we determined that a lifetime migration rate of 50% would require a $p_{\text{mig}} = 0.2$. For the naturalistic model, we empirically determined that $p_{\text{mig}} = 0.5$ leads to approximately a 50% migration rate.

Appendix A.2.4 Selection

Every round, an agents' fitness score gets incremented by their payoff. Each agent must then pay c_{alive} (the cost of stayin' alive), and if they cannot, they die. Further, every agent has a randomly sampled lifespan, and when they exceed their lifespan, they die.
If an agent can afford to pay the c_{repro}, the reproduction cost, and still have some fitness left over (equal to c_{alive}), they will reproduce. The child is a noisy copy of their parent, with the same learning style and propensity to cooperate, but some noise added to the propensity to cooperate, and probability p_{mut} of a mutation in learning style (in which case one of the three are chosen at random).

Appendix B Results: Parameters

In all tables in this section, we will refer to two values of d and d_{mut}: `non-NI` (no norm internalizers), `NI` (with norm internalizers). These names correspond respectively to distribution (d) values of

```
selfish:  0.49
static:  0.49
norm internalizer:  0.02,

selfish:  0.49
static:  0.49
uncond-coop:  0.02,
```

and mutation distributions (d_{mut}) of

```
selfish:  1/3
static:  1/3
norm internalizer:  1/3,

selfish:  1/3
static:  1/3
uncond-coop:  1/3.
```

Appendix B.1 Abstract Model

Category	Variable	Description
Game	b	`3.0, 3.25, 3.5, 3.75, 4.0, 4.25, 4.5`
	c	`1`
	f	`1`
Strategies	ϵ	`1/20`
	β	`0.2`
	l	`1/2`
	d, d_{mut}	`non-NI`, `NI`
Environment	p_{con}	`1/13`
	p_{mig}	`0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6`
	p_{mut}	`1/100`
	σ	`7/10`
	n	`35`
	g	`60`
	y	`15,000`

Table 4 **Parameters of the abstract model**

Number	b	p_{mig}
6	3	0.2
6	3.25	0.0, ... , 0.6
6	3.5	0.0, ... , 0.6
6	3.75	0.0, ... , 0.6
6	4	0.2
6	4.25	0.2

Table 5 **Runs of the abstract model**

Appendix B.2 Naturalistic Model

Category	Variable	Description
Game	b	55, 60, 65, 70, 75
	c	20
	c_{dist}	5
	f	20
	c_{alive}	2
	c_{repro}	2
Strategies	ϵ	1/20
	β	3/10
	l	1/20
	d, d_{mut}	NI, non-NI
Environment	p_{mut}	1/100
	p_{mig}	0, 0.005, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8
	s	10
	n	20
	g	10
	y	20000

Table 6 **Parameters of the naturalistic model** *strictly speaking, these quantities are not the final costs and benefits; they are first divided by the number of agents foraging on the square

Note that the migration rate is much lower in this model than the abstract one: this is because a generation in this round was around 41 rounds instead of just 10/3 rounds. We selected the migration rate so that lifetime migration rates would be equal between the two models.

Number	b	p_{mig}
8	50	0.5
8	55	0.5
16	60	0.0, 0.1, ... , 0.8
16	65	0.0, 0.1, ... 0.4, 0.6, ... , 0.8
8	65	0.005, 0.05
24	65	0.5
16	70	0.0, 0.1, ... , 0.8
8	75	0.5

Table 7 **Runs of the abstract model**

Appendix C Counterpart figures

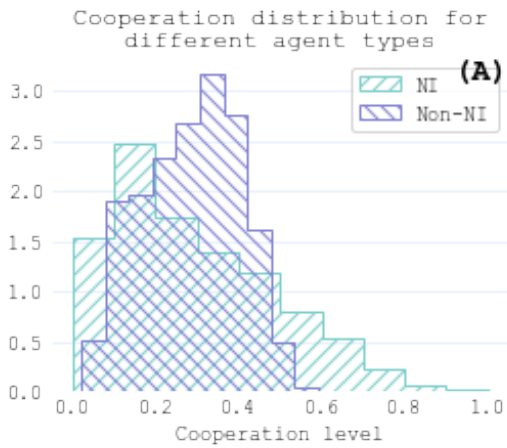

Fig. 1 **Norm internalizer cooperation histogram.** The equivalent of Fig. **??** for the naturalistic model. The norm internalizers have similarly higher variance in their coopereation levels.

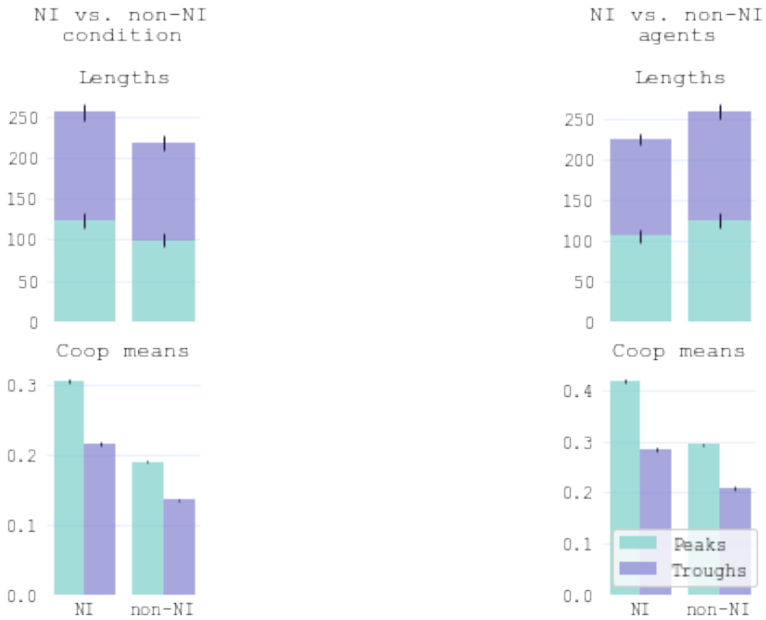

Fig. 2 **Spike-trough comparison for the abstract model.** Equivalent of Fig. **??** for the abstract model. It exhibits a similar pattern, except that the NI troughs are higher than the non-NI troughs (this is not the case in the naturalistic model)

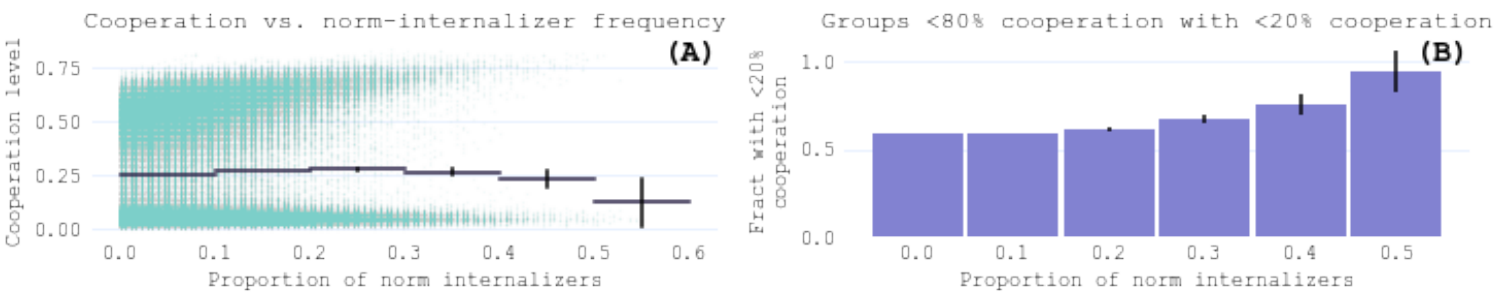

Fig. 3 **Polarization due to norm internalizers in the abstract model.** Equivalent of Fig. **??** for the abstract model, exhibiting a similar pattern

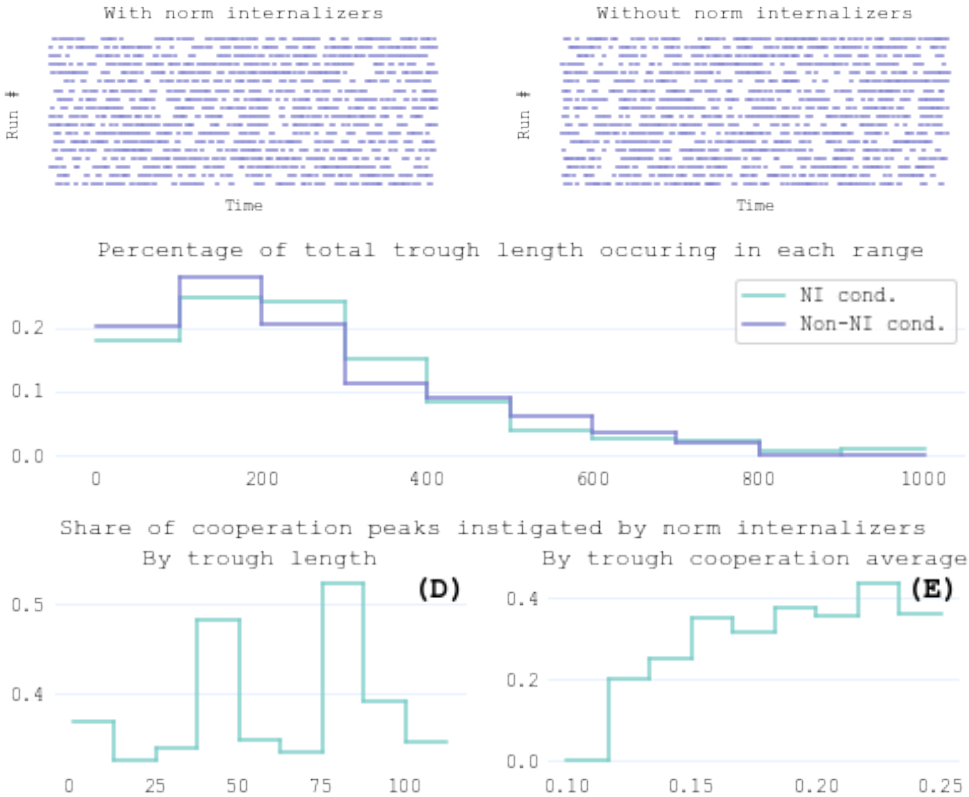

Fig. 4 **Troughs characteristics in the abstract model.** These are the equivalents of Fig. **??** for the abtract model. The abstract model does not exhibit the same pattern where norm internalizers prevent very long troughs

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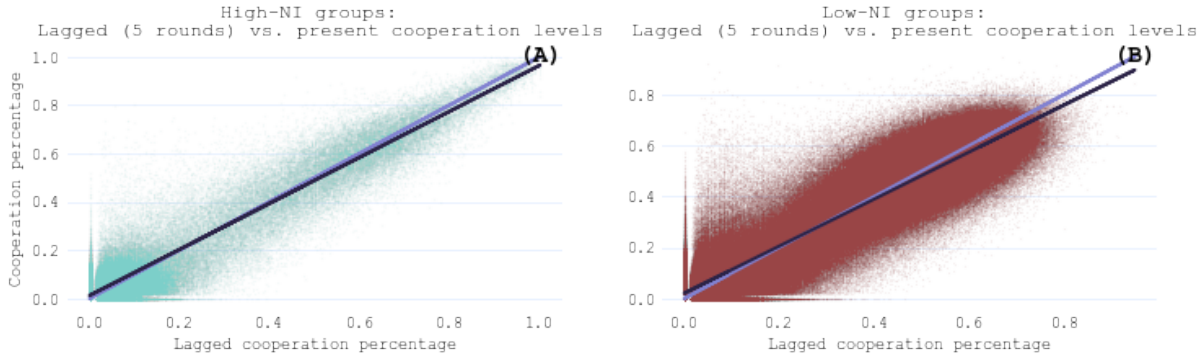

Fig. 5 **Norm internalizers as stabilizers in the naturalistic model.** Equivalent of Fig. **??** for the naturalistic model, a similar pattern exists, with the coefficient for the low-NI state 0.92 (± 0.0005), compared to the high-NI coefficient 0.95 (± 0.0015). Confidence intervals are 95% confidence

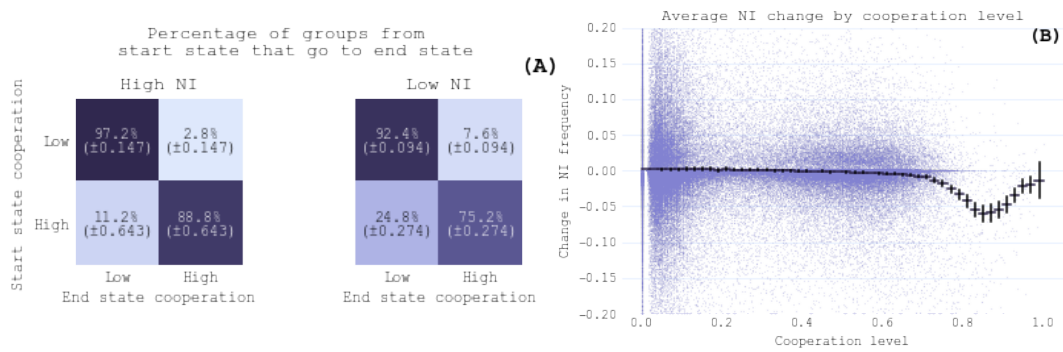

Fig. 6 **Norm internalizers (NIs) as stabilizers, and their flaw - naturalistic model** The equivalent of Fig. **??** for the naturalistic model, exhibiting a very similar pattern. The one difference is the uptick in norm internalizer change for very high cooperation levels — this is because, at this level, there are very few free riders in the group. Groups in the abstract model never attained this level of cooperation