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STAT511 HW 8

1) n=22, $\pi = 0.72$

A) Mean μ_Y = n π = 22*0.72 = 15.84

SD ,$\sigma_Y = \sqrt{n\,\pi(1-\pi)} = \sqrt{22 * 0.72(1-0.72)}$ = 2.105991

B) P(Y ≤ 16) = 0.6100871

C) P(Y < 16) = 0.422552

D) P(16 ≤ Y < 18) = 0.1701999

E) P(Y = 18) = 0.1215714

F) P(Y ≥ 18) = 0.2197129

G) without continuity correction,

P(Y≥18) = 1 – P(Y<18) = 1 - P(Y≤17) = 1-pnorm(17,mean=15.84,sd=2.105991,lower.tail = TRUE)

so, without continuity correction , P(Y≥18) = 0.2908821

H) with continuity correction,

P(Y≥18) = 1 – P(Y<18) = 1 - P(Y≤17.5) = 1-pnorm(17.5,mean=15.84,sd=2.105991,lower.tail = TRUE)

so, without continuity correction , P(Y≥18) = 0.2152818

2) n = #of students = 75 (trial)

y = #of STEM majors = 51 (success)

A) $Sample\ proportion, \hat{\pi} = \frac{y}{n} = \frac{51}{75} = 0.68$ [Estimate of STEM population proportion]

Estimate of standard deviation, SE($\hat{\pi}$) = $\sqrt{\frac{\hat{\pi}\ (1-\hat{\pi}\ )}{n}}$ = = $\sqrt{\frac{(0.68)*(1-0.68)}{75}}$ = 0.05386403

B) 95% confidence interval for true population proportion π = $\hat{\pi} \pm Z_{\alpha/2} * \mathrm{SE}(\ \pi\ ) = 0.68 \pm (1.96 * 0.05386403) = (0.5744265, 0.7855735)$

C) $H_0 : \pi \le 0.5$

$H_A : \pi > 0.5$

$\pi_0 = 0.5$, $\hat{\pi} = 0.68$,

Test statistics z = $\frac{\pi - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.68-0.5}{\sqrt{\frac{0.5*0.5}{75}}} = 3.117691$

Rejection region, z$\geq z_\alpha$, or z z≥ 1.645

In this case, $3.117691 \geq 1.645$, so we can reject the null hypothesis with 95% confidence, which means we can conclude (with 95% certainty) that the population proportion of STEM majors is greater than 0.5.

From running prop.test, p-value = 0.00091114. (Ans)

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        1-sample proportions test without continuity correction

data:  51 out of 75, null probability 0.5
X-squared = 9.72, df = 1, p-value = 0.0009114
alternative hypothesis: true p is greater than 0.5
95 percent confidence interval:
 0.5864651 1.0000000
sample estimates:
   p
0.68
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3) n = #of items = 60 (trial)

y = #of defective items = 10 (success)

A) $Sample\ proportion, \hat{\pi} = \frac{y}{n} = \frac{10}{60} = 0.1666667$ [Estimate of population proportion]

B) α = 0.1, 90% confidence interval = (0.0958273,0.2691848)

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        1-sample proportions test with continuity correction

data:  10 out of 60, null probability 0.5
X-squared = 25.35, df = 1, p-value = 4.782e-07
alternative hypothesis: true p is not equal to 0.5
90 percent confidence interval:
 0.0958273 0.2691848
sample estimates:
        p
0.1666667
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C) 90% confidence interval = (0.09330693 0.26629080)

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Exact binomial test

data:  10 and 60
number of successes = 10, number of trials = 60, p-value =
1.616e-07
alternative hypothesis: true probability of success is not equal to 0.5
90 percent confidence interval:
 0.09330693 0.26629080
sample estimates:
probability of success
             0.1666667
```