STAT511HW2

SHAIKH SHAWON AREFIN SHIMON 830201247

1)

a.

$P(Z \leq 0.57) = 0.7157$

b.

$P(Z \leq -0.32) = 0.3745$

c.

$P(Z > 2.10) = 1 - P(Z \leq 2.10) = 1 - .9821 = 0.0179$

d.

$P(-0.32 \leq Z \leq 1.55) = P(Z \leq 1.55) - P(Z \leq -0.32) = 0.9394 - 0.3745 = 0.5649$

e.

$P(Z \leq z) = 0.3300$; $z = -0.44$

f.

$P(Z > z) = 0.3987$, so $P(Z \leq z) = 1 - 0.3987 = 0.6013$; $z = 0.2567$

2)

Y ~ N (μ = 6, = 0.8)

a.

$P(Y \leq 7) = ?$

Converting normal distribution Y to standard normal distribution Z, we get

$P(Y \leq 7) = P(\frac{Y-\mu}{\sigma} \leq \frac{7-\mu}{\sigma}) = P(Z \leq \frac{7-6}{0.8}) = P(Z \leq 1.25) = 0.8944$

b.

$P(Y > 5.4) = ?$

Converting normal distribution Y to standard normal distribution Z, we get

$P(Y > 5.4) = P(\frac{Y-\mu}{\sigma} > \frac{5.4-\mu}{\sigma}) = P(Z \leq \frac{5.4-6}{0.8}) = P(Z > -0.75) = 1 - P(Z \leq -0.75) = 1 - 0.2266 = 0.7734$

c.

$P(6 \leq Y \leq 7.2) = ?$

Converting normal distribution Y to standard normal distribution Z, we get

$P(6 \leq Y \leq 7.2) = P(\frac{6.0-\mu}{\sigma} \leq \frac{Y-\mu}{\sigma} \leq \frac{7.2-\mu}{\sigma}) = P(\frac{6.0-6.0}{0.8} \leq \frac{Y-\mu}{\sigma} \leq \frac{7.2-6.0}{0.8}) = P(0 \leq Z \leq \frac{1.2}{0.8})$

$= P(0 \leq Z \leq 1.5) = P(Z \leq 1.5) - P(Z \leq 0.0) = 0.9332 - 0.5 = 0.4332$

d.

$P(Y \leq y) = 0.85$

Converting normal distribution Y to standard normal distribution Z, we get

$P(Y \leq y) = P(\frac{Y-\mu}{\sigma} \leq \frac{y-\mu}{\sigma}) = P(Z \leq z) = 0.85$, where $z = \frac{y-\mu}{\sigma}$

From Table 1, z=1.035.

So, $1.035 = \frac{y-6}{0.8}$, making y = 6.828

3)

a.

df=25, $P(T>1.708) = 0.05$

b.

df=9, $P(-t<T<t) = 0.95$.

Two-tailed probability, $\alpha = 0.05$.

So, right tailed probability, $\alpha/2 = 0.025$

$t_{0.025,9}$ = 2.262

c.

df=50, P(T>t) = 0.90.
$t_{0.9,50}$ = -1.299

4)

a.

As the distribution of Y is skewed, empirical rule for normal distribution can't be applied. In this case, Chebyshev's rule should be applied for determining 75% interval for population distribution.

From Chebyshev's rule, for population mean at least 75% of the data lie within ($\mu \pm 2\sigma$) range, or (90±20) = (70,110) range.
So, the interval is (70,110)

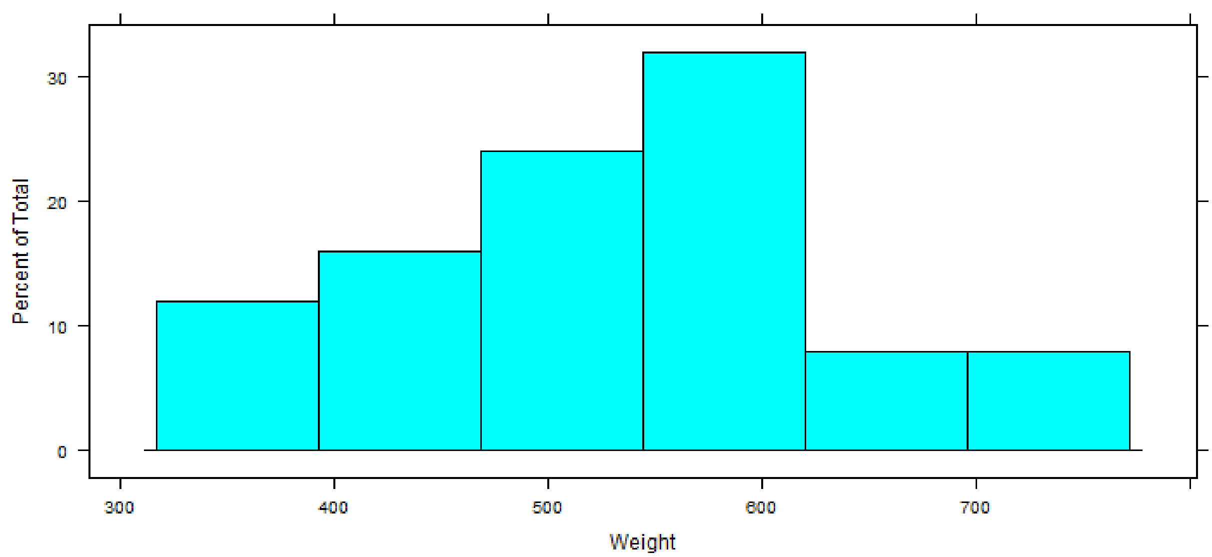
b.

According to the Central Limit Theorem, the distribution of sample mean, $\bar{y}$ is close to normal as n is large (n=100). So the shape of the distribution will follow normal distribution (bell shaped curve) [$\bar{y} \sim (\mu, \frac{\sigma}{\sqrt{n}})$]

Mean = μ = 90
Standard deviation s = $\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{100}} = 1$

5)

a.



Sample mean $\bar{y}$ = 526.12

Sample sd , s= 113.7279

b.

From the histogram, the sample data looks like a normal distribution. So from the empirical rule, 95% (α = 0.05) confidence interval for μ is = ($\bar{y} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$) [Two tailed distribution]
Now, $t_{\alpha/2} = t_{0.05/2} = t_{0.025}$, and df = n-1 = 25-1 = 24.
From Student's t-distribution, $t_{0.025}$ for df , $t_{\alpha/2,\, n-1} = t_{0.025,\, 24} = 2.064$.
So, confidence interval = ($\bar{y} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$) = (526.12 ± (2.064) $\frac{113.7279}{\sqrt{25}}$ = (526.12 ± 46.95) = (479.17 , 573.07)

c.

We can be 95% confident that population mean seed weight is contained in the 95% confidence interval (479.17 , 573.07).

d.

For $H_0 : \mu = 500$
$H_A : \mu \neq 500$
Test Statistic TS, t = $\frac{\bar{y}-500}{s/\sqrt{n}} = \frac{526.12-500}{113.7279/\sqrt{25}} = 1.148.$
Now , $|t| \ngtr t_{\alpha/2,\, n-1}$ as 1.148 < 2.064.
So the test statistics does not fall into the rejection region.

This means, we fail to reject the null hypothesis H_0 with 95% certainty, or we fail to reject that the bean population mean is 500 with 95% certainty.