PHY408 Final Project: Kalman Filter Pairs Trading

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AI Use: None

April 2025

1 Introduction

Financial markets are characterised by complex dynamics along with noisy signals which makes prediction and analysis challenging. This project aims to investigate the dynamic relationship between two semiconductor companies—**NVIDIA (NVDA)** and **AMD**—through a quantitative approach which is grounded in time series analysis. Specifically, *Kalman Filters* are employed, which are a recursive algorithm for estimating the time varying parameters of a linear model, to uncover and model their dynamic hedge ratio for a statistical arbitrage strategy known as **pairs trading**.

Pairs trading is a market neutral strategy that is based on the relative mispricing between two historically correlated assets. Traditional approaches often rely on static regression models to determine the hedge ratio between two securities. However, such methods fail to adapt to structural changes in the market over time. To overcome this limitation, a Kalman Filter based approach is used, which treats the hedge ratio as a latent variable that evolves over time. This method not only adapts to changes in the relationship between the assets but also smooths out noise, providing a more realistic and robust measure of spread.

The Kalman Filter operates within the framework of **State Space Models**, which is a central topic in time series econometrics introduced in this course. In this setting, the observed data which are the asset prices, are linked to unobserved or *hidden states* (e.g., time-varying hedge ratio) through measurement and transition equations. The filter sequentially updates its estimates as new data arrives, making it well-suited for financial applications where data is continuously updated. This approach directly builds on the material covered in class and highlights the theoretical and practical advantages of state space modelling in dynamic environments.

This study not only demonstrates the application of Kalman Filters to real financial data, but also provides insights into the behaviour of asset relationships over time, offering potential for improved risk management and strategy development. The broader objective is to illustrate how classical filtering techniques can be integrated into a modern trading framework to address the limitations of static models.

2 Methodology

This analysis applies a dynamic modelling approach to investigate the statistical relationship between the daily closing prices of NVDA and AMD from a time series perspective. The goal is to estimate and track a time varying hedge ratio suitable for a pairs trading strategy. Our methodology builds upon three key pillars: data preprocessing and normalisation, stationarity verification, and Kalman Filter based regression modelling.

Data Preprocessing and Rebased Normalization

The study began by importing and cleaning the dataset from Bloomberg. Missing values were linearly interpolated which was a reasonable choice given the high frequency and smooth nature of price movements over short intervals along with few missing values. Any remaining NaN values were pruned by dropping those columns. Then both the price series were rebased by dividing each by their initial value, which allowed for a normalised comparison of relative growth and movement over time. This serves to prepare the data for visual inspection and initial correlation analysis.

Alternative methods like min-max scaling or z-score normalization were also considered. However, rebasing was chosen due to its intuitive financial interpretation. It allows the series to be understood in terms of percentage growth from a baseline, which aligns with how traders usually view price dynamics.

Stationarity Checks for Spread Estimation

Pairs trading relies on the assumption that the spread between two assets is mean reverting or stationary. To test this, a spread was constructed based on equal weighted or statically regressed prices. This motivated the shift toward a dynamic modelling framework, where the relationship between the asset prices is treated as time varying rather than fixed. The Kalman Filter emerged as the ideal candidate due to its recursive updating mechanism as well as its optimality under Gaussian noise assumptions.

Kalman Filter as a State Space Regression Model

The core of this method lies in the implementation of a **Kalman Filter** to estimate a dynamic hedge ratio. The hedge ratio β_t was modelled as a latent state evolving over time, expressed within a linear state space model as follows:

$$y_t = \alpha_t + \beta_t x_t + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, R) \quad \text{(Measurement equation)}$$
$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, Q) \quad \text{(State equation)}$$

Here, x_t and y_t are the observed log prices of AMD and NVDA respectively. Log prices were used as they stabilise variance, transform multiplicative growth into additive trends, enables more valid use of linear time series tools like the Kalman Filter, and helps in analysing **relative price movements**, which is crucial in **pairs trading**. The measurement noise ϵ_t accounts for short term deviations from the modelled relationship, while the process noise η_t governs the variability of the hedge ratio itself.

Parameters Q and R (state and observation variances) were selected through a grid search which evaluated combinations that minimised the residual sum of squares (RSS) of the spread. This parameter tuning is crucial since overly small Q values would lead to under fitting (rigid hedge ratio), while large Q values might overreact to noise.

Optimising Parameter Values

In order to optimise the model's performance, a grid analysis was used, varying values for Q (Process noise) and R (observation noise), in order to find the values that yielded the maximum Sharpe ratio. Sharpe ratio is a measure of return which is adjusted for risk. R_a is the return of the asset (the model) and R_f is the risk free rate of return. $E[R_a - R_f]$ is the expected value of excess return of the asset, and σ_a is the standard deviation of the return of the asset. Since this is a simplistic model of pairs trading, the risk free rate is assumed to be 0. The trading period was assumed to be 90 days, which seemed reasonable as the dataset went back to 2021.

$$S_a = \frac{E[R_a - R_f]}{\sigma_a}$$

Model Justification and Rejection of Alternatives

A number of different strategies were examined and disregarded. Despite being straightforward, static ordinary least squares (OLS) regression is unable to account for changes in the assets' connection over time. The Engle-Granger test and other cointegration-based methods were investigated, but they necessitated more robust long-term equilibrium assumptions that the data's stationarity profile could not support.

In contrast, the Kalman Filter offers a probabilistic framework for integrating noise and uncertainty in both observations and model dynamics, in addition to accommodating time-varying parameters. Because of this, it was the best approach for our investigation, both theoretically and empirically.

3 Dataset

For this project, the dataset comprised of the daily closing price data of two leading semiconductor companies, Advanced Micro Devices (AMD) and NVIDIA Corporation (NVDA). This spanned over a period of five years from 2020 to 2025. The data was retrieved from Bloomberg which is a financial terminal. Bloomberg's data is used because it is already adjusted for corporate actions such as stock splits and dividends, which ensures that the time series reflect true market value and simplifies the preprocessing pipeline.

The decision to focus on AMD and NVIDIA was based on both theoretical and empirical considerations. Being direct competitors in the GPU and semiconductor industries and operating in the same industry, these two businesses are subject to comparable macroeconomic factors, technological advancements, and cycles of consumer demand. The likelihood of movement and mean reverting behaviour in their relative pricing is increased by this economic alignment, which is crucial for a pairs trading strategy. Furthermore, both stocks have a high level of liquidity, which makes them appropriate for the practical implementation of any trading strategy derived from this analysis.

Daily data was chosen as the optimal level of granularity. While higher-frequency intraday data could offer more detail, it introduces significant noise, increases computational complexity, and may necessitate more advanced infrastructure for real-time processing. Conversely, lower-frequency data such as weekly or monthly closes would likely miss shorter-term convergence-divergence patterns that are central to effective pairs trading. Daily closing prices strike a balance by providing enough resolution to detect meaningful trends without overfitting the model to short-term volatility.

More than 1,250 data points are available for each asset over the course of five years, which is more than enough for the Kalman filter to generate reliable estimates of time varying hedging ratios. The model is not suited to a single market environment because this time frame encompasses a range of market regimes, such as post pandemic volatility, inflation driven corrections, and several earnings cycles. Dynamic models such as state space filters are generally advised to have a minimum threshold of 500 data points, which our dataset surpasses.

4 Analysis and Results

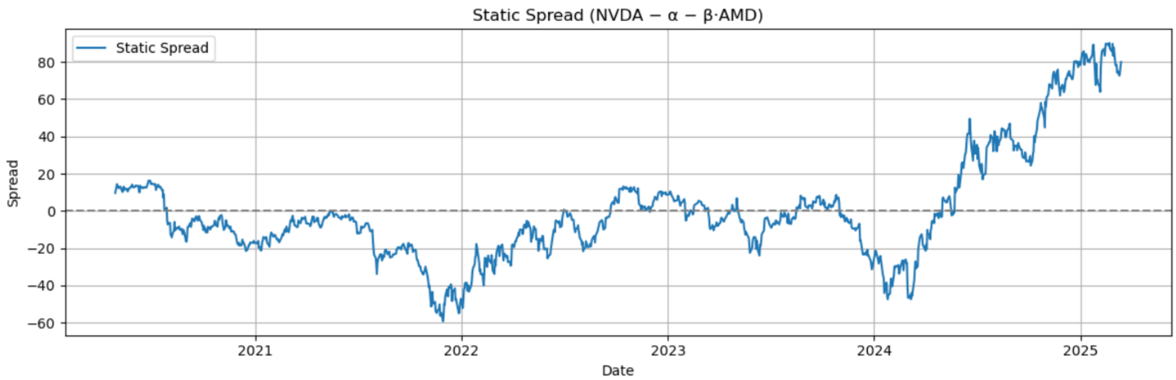


Figure 1: Static spread between AMD and NVDA (Nvidia). This is calculated by $\text{Spread}_t = P_t^{\text{NVDA}} - \beta \cdot P_t^{\text{AMD}} - \alpha$, where α and β are based on a linear regression on data over the last five years. This highlights the deviation of Nvidia's price from its calculated fair value based on AMD's share value. Mean reversion here would highlight opportunities to take advantage of statistical arbitrage, and thereby implement a pairs trading strategy. This plot depicts the spread over a given time period.

The difference between NVIDIA's asset price and AMD's assessed fair value is shown in Figure 1 and Figure 2. While Figure 2 shows the dynamic hedge ratio β_t estimated iteratively using a Kalman Filter, Figure 1 employs a static hedge ratio β derived via ordinary least squares (OLS). The spread in Figure 1 shows a number of noteworthy departures from the mean, especially starting in mid-2023, which corresponds with a time when NVIDIA outperformed the market because of its AI-driven growth. These variations imply that changes in company performance indicators (especially the comparables) along with market conditions are not taken into consideration by the static hedge ratio. Figure 2 presents a far more complex image. When AMD and NVIDIA's relative performance changes, the time-varying hedging ratio β_t adjusts smoothly. Interestingly, β_t shows a comparatively consistent correlation from 2020 to early 2022. But starting in early 2023, β_t rises sharply, hitting levels exceeding 1.2. This suggests that when AMD's price changed, NVIDIA's price started to respond more. One of the main benefits of the Kalman Filter framework is its time-adaptive nature, which enables the model to react to shifting market regimes. β_t incorporates dynamic shifts in co-movement and hedging needs, in contrast to the static β, which assumes a fixed linear relationship throughout the time. This adaptability increases trading signal dependability and spread estimation accuracy.

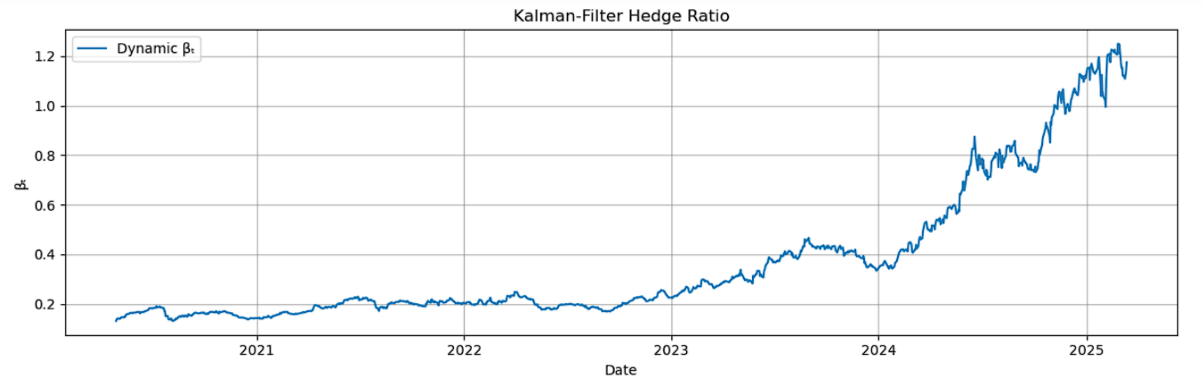


Figure 2: Time-varying hedge ratio. This is found using a Kalman Filter for Nvidia and AMD. This estimate changes over time, highlighting the constantly changing relationship between AMD and Nvidia. The Kalman Filter enables the hedge ratio to move with shifts in market conditions and asset prices; this effectively means that the hedge ratio shifts in accordance with a state space model. β_t oscillates over the period which is especially apparent during volatile market phases

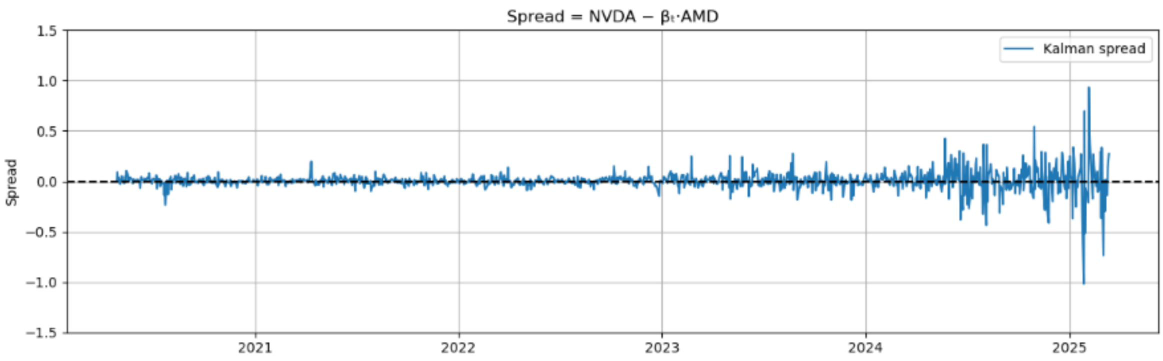


Figure 3: Kalman filtered spread. This is calculated by the equation $\text{Spread}_t = \text{NVDA}_t - \beta_t \cdot \text{AMD}_t$: β_t. This is the Time varying hedge ratio that was calculated above. This spread depicts Nvidia's deviation from its implied value. β is adjusted in accordance with market conditions by the Kalman Filter; one can observe potential opportunities for arbitrage through this. Volatility increases in the time period after 2024; this implies a divergence in correlation between the two assets.

The time-varying hedging ratio β_t, which is dynamically calculated using the Kalman Filter, is used to calculate the spread in Figure 3. In contrast to the static β, which makes the assumption that NVDA and AMD have a constant linear relationship over time, β_t fluctuates in response to market conditions and records structural shifts in the relationship between the assets. This flexibility is especially helpful during times of economic upheaval or industry-specific changes, like the apparent distinction shown in 2024–2025, when NVIDIA's performance surpassed AMD's due to the growth of the AI sector. A significant finding is that, even while the Kalman-filtered spread still shows mean-reverting behavior, its *volatility* sharply rises after 2024. Increased uncertainty or shifting investor sentiment in the semiconductor sector are probably the causes of this variation increase. Wider and more frequent departures from the mean raise the frequency of trading signals, but because of the possible downside, they also call for more stringent risk management. Additionally, at stable times, the spread calculated by β_t fits the dynamic equilibrium line more closely, boosting confidence. This makes β_t more suitable for high-frequency over a medium-horizon, when adaptation to market conditions is crucial. These transitions would be missed by static β models, which could result in incorrect signals or suboptimal trades.

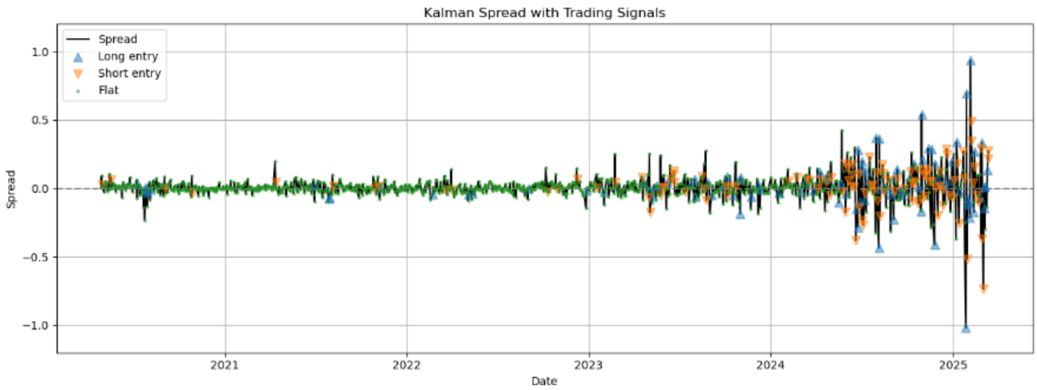
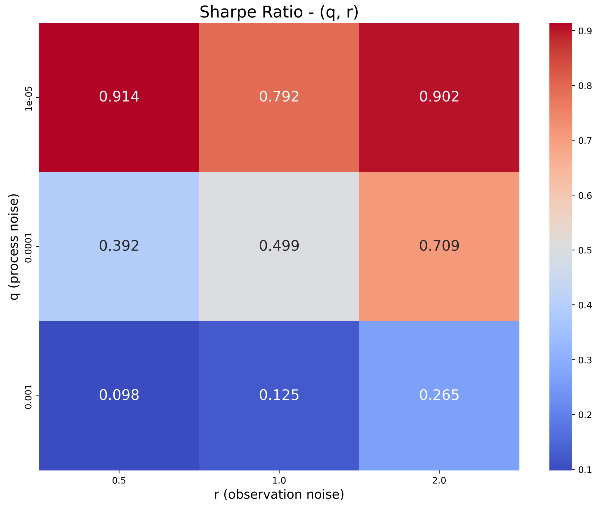


Figure 4: Kalman Filter Spread with Trading Signals. This figure shows the dynamic spread over a time series between Nvidia and AMD: this is calculated with $\text{Spread}_t = \text{NVDA}_t - \beta_t \cdot \text{AMD}_t$, where β_t is dynamic hedge ratio calculated above. In this figure, the black line depicts the spread that changes with time with trading signals superimposed on it. These trading signals are calculated through a Z score threshold. Signals for long entries are illustrated by blue triangles facing upwards, whereas orange triangles facing downwards are short entry signals. A long is entered when the Z-score is below a threshold, whereas a short is entered when the spread is above a threshold. The green dots, which represent flat signals, are when positions are exited.

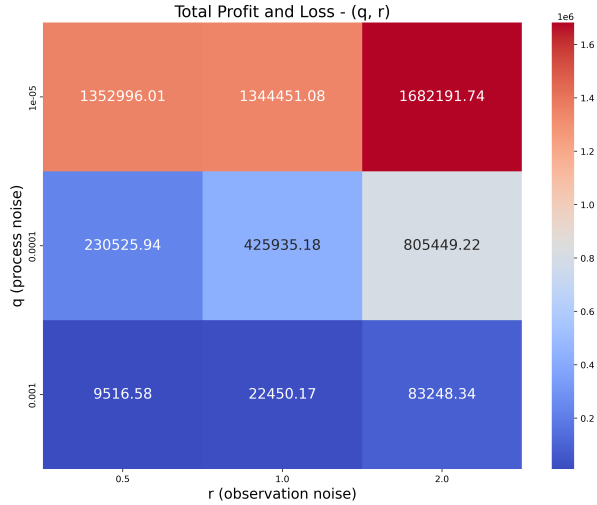
Figure 4 demonstrates how the Kalman-filtered spread enables the generation of dynamic signal based on deviations from the evolving hedge ratio. Trading activity is relatively sparse from the stable periods of 2021–2023 but rises post-2023 as volatility increases, likely due to shifts in market structure and NVIDIA's outperformance. This highlights the model's responsiveness to changing conditions and its value in identifying short-term mispricings.

The sensitivity of signal generation to the Kalman Filter parameters (q, r) is made clear in Figure 5. A higher q results in a more reactive hedge ratio which is useful during volatile periods but also potentially riskier. A lower q yields smoother estimates which is better suited to stable pairs. Similarly, tuning r balances sensitivity to outliers: lower values increase reactivity, while higher values reduce noise.

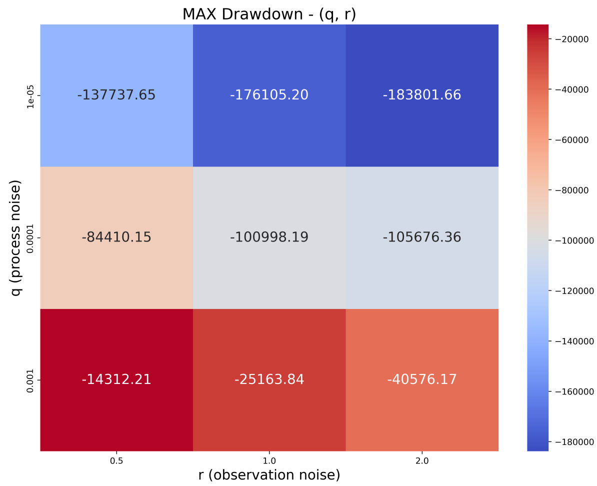
By visualising model results, such drawdown, PnL, and Sharpe ratio, throughout the (q, r) grid, we can evaluate the filter's stability and flexibility. These insights are crucial for understanding how state estimation reacts in various regimes as well as for performance optimisation. The findings of an additional test with a maximum holding period were not conclusive which indicate that the filter's dynamic state estimates, rather than strict constraints, are a better guide to determining parameters.



(a) 5a: Sharpe Ratio over (q, r) grid.



(b) 5b: Total Profit and Loss (PnL) over (q, r) grid.



(c) 5c: Maximum Drawdown over (q, r) grid.

Figure 5: This figure is a series of heatmaps that depict how Sharpe Ratio, Total Profit and Loss, and Maximum Drawdown vary as the Kalman Filter parameters, (q, r), are adjusted. Figure 5a demonstrates that as q decreases, Sharpe ratio increases. The maximum Sharpe ratio occurs when r=0.5 and q=0.0001; the Sharpe ratio itself is 0.914. Meanwhile, the total profit and loss is at its highest at \$$1,682,191$ dollars when q = 0.00001 and r = 2.0. The maximum drawdown occurs when q = 0.00001 and r = 2.0; the maximum drawdown is \$-182801.66

Discussion & Conclusion

The results of the pairs trading method based on the Kalman Filter provide important information about the dynamic pricing relationship between AMD and NVIDIA. The Kalman Filter successfully captured the changing correlation between the two stocks by modelling the hedging ratio as a time varying parameter inside a state space framework, especially during times of market stress and divergence.Figure 4 shows that the dynamic spread is much more responsive to changing market conditions than a static spread calculated using OLS. This flexibility is crucial for spotting profitable arbitrage possibilities instantly.

The gap shown starting in early 2023, when NVIDIA's price growth significantly surpassed AMD's, was of special interest. The time-varying β_t predicted by the Kalman Filter adjusted appropriately, preserving the validity of spread-based trading signals, but the static model found it difficult to capture this changing relationship. Trades that were in line with mean-reversion expectations were produced by these signals, which were controlled by Z-score levels. In order to balance trade frequency and profitability, the entrance and exit thresholds (± 0.8 and ± 0.2, respectively) have to be calibrated.

Additionally, Figure 5's heatmaps demonstrate how sensitive the model is to parameter selections (q, r). Higher q values produced more reactive but riskier behavior, while low q values (low process noise) produced smoother hedge ratio estimates. It's interesting to note that the biggest PnL happened at $q = 0.00001$ and $r = 2.0$, while the highest Sharpe ratio was attained at $q = 0.0001$ and $r = 0.5$. Financial theory supports this trade-off between risk sensitivity and profitability, which emphasizes how crucial it is to adjust these factors to an investor's risk tolerance.

However, it is worthwhile to highlight some of the model's limitations. First, even though the Kalman Filter is good at adapting in real time, it depends on assumptions about Gaussian noise, which could not be true in all market conditions. Additionally, the model makes the assumption that there are no transaction costs or slippage, and a null risk free rate of return which could have a big influence on profitability in a live trading environment. Furthermore, the strategy's dependence on the spread returning to the mean is predicated on stable underlying market dynamics, which could not always hold true in the event of macroeconomic shocks or structural disruptions. Another point of note is that the stock price of Nvidia begins to diverge from AMD's after 2024; this is apparent in Figure 3.

This model could be extended in a number of ways in future research. Using a multivariate Kalman Filter is one intriguing approach that could combine exogenous macroeconomic data or model more than two assets at once. In order to improve performance in volatile or non-linear situations, another approach would be to use unscented or particle filters to loosen Gaussian assumptions. Additionally, the robustness of signal production may be enhanced by using volume or news sentiment data.

In conclusion, our project has shown how well the Kalman Filter models dynamic interactions between correlated assets in a framework for pairs trading. The time-varying technique is more flexible and responsive to market conditions than static models. Although there is room for improvement, the findings support the idea that Kalman Filtering and state space modeling offer a strong basis for creating systematic trading techniques in the financial industry.

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Appendix A: Implementing the model and refining strategy

The pairs trading model implemented, has a number of assumptions that need to be discussed. For one, the trading period was 90 days; this was enforced in order to make sure that the model did not hold till the end of the trading period. Further, entry and exit Z values were defined. These values are usually set by a trader; in this model the entry and exit thresholds were ± 0.8 and ± 0.2, respectively.

The model first started by calculating the dynamic beta, and then carried on to find the spread adjusted for dynamic beta. Then the model was optimized via a grid search for both observation noise r and process noise q

Appendix B: Sample of Dataset Used

Table 1 presents a portion of the dataset used in this study. The data consists of daily closing prices for NVIDIA (NVDA) and AMD, spanning from January to March 2025. This subset illustrates the structure and frequency of the data retrieved from Bloomberg and used for modeling the dynamic hedge ratio with a Kalman Filter.

Table 1: Sample from the Daily Closing Price Dataset (Bloomberg, Jan–Mar 2025)

Date	NVDA- Closing Price	AMD - Closing Price
3/13/25	115.58	98.11
3/12/25	115.74	100.79
3/11/25	108.76	96.76
3/10/25	106.98	96.63
3/7/25	112.69	100.31
3/6/25	110.57	98.85
3/5/25	117.30	101.67
3/4/25	115.99	100.75
3/3/25	114.06	98.23
2/28/25	124.92	99.86
2/27/25	120.15	99.51
2/26/25	131.28	104.74
2/25/25	126.63	103.96
2/24/25	130.28	108.11
2/21/25	134.43	110.84
2/20/25	140.11	114.17
2/19/25	139.23	114.69
2/18/25	139.40	114.28
2/14/25	138.85	113.10
2/13/25	135.29	111.81
2/12/25	131.14	111.72
2/11/25	132.80	111.10
2/10/25	133.57	110.48
2/7/25	129.84	107.56
2/6/25	128.68	110.16
2/5/25	124.83	112.01
2/4/25	118.65	119.50
2/3/25	116.66	114.27
1/31/25	120.07	115.95
1/30/25	124.65	118.86
1/29/25	123.70	117.35
1/28/25	128.99	114.17
1/27/25	118.42	115.01
1/24/25	142.62	122.84
1/23/25	147.22	123.04
1/22/25	147.07	123.75
1/21/25	140.83	122.28
1/17/25	137.71	121.46