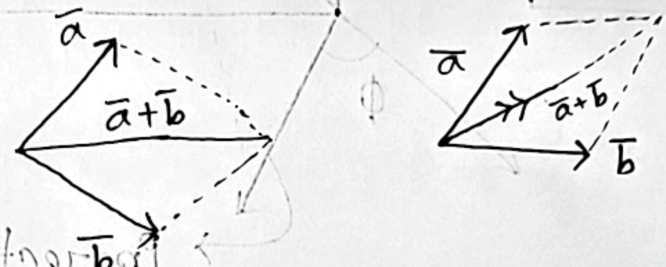
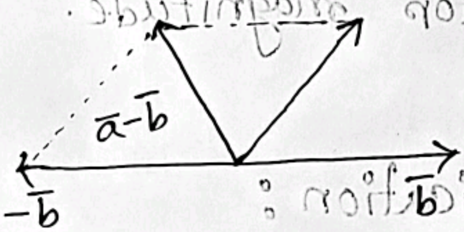
## Vector: A ~~magnitude~~ quantity which have both magnitude and direction.

// Vector Addition: $\vec{a} + \vec{b}$




// Vector substraction: $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$




// Cartesian Co-ordinate: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\hat{k} \rightarrow$ z axis

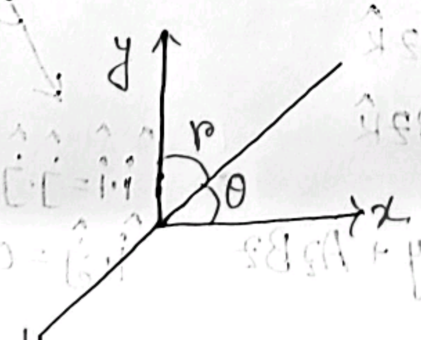
$\hat{j} \rightarrow$ y-axis

$\hat{i} \rightarrow$ Unit vector along x-axis

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

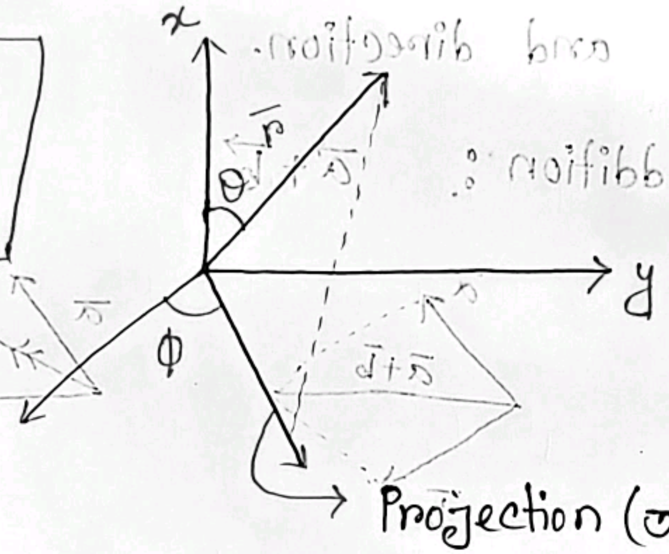
$\therefore \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$

// Polar:



// Spherical Coordinate:

Anticlock = Out
Clock = In




Projection angle with x-axis → $\phi$

r = position vector magnitude.

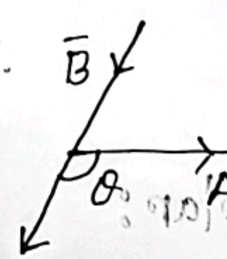
// Vector Multiplication:

→ Dot / Scaler Product

→ Cross / Vector Product

→ $\vec{A}\cdot\vec{B} = (\text{Scaler}) = AB\cos\theta$ → Angle between A & B

→ $\vec{A}\times\vec{B} = (\text{Vector}) = AB\sin\theta$




$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

→ $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0$

$$\rightarrow \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - B_yA_z) - \hat{j}(A_zB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

$$\rightarrow \hat{n} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|}$$

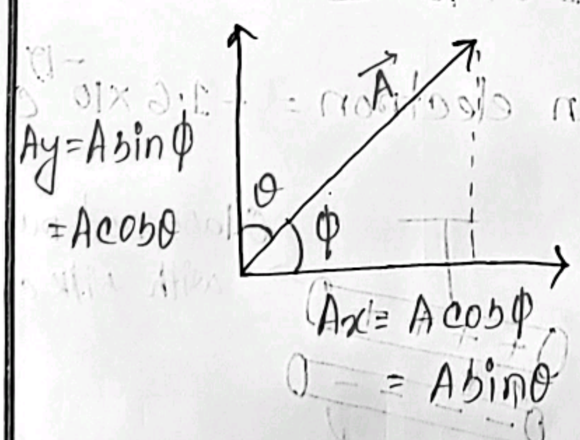
// Gradiant Operator: $\vec{\nabla} = \hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}$ ; $\frac{\delta}{\delta x}f(x)$

$\rightarrow \vec{\nabla}f \rightarrow$ Gradiant

$\frac{d}{dx}(x^n) = nx^{n-1}$ ; $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\frac{d}{dx}(e^{mx}) = me^{mx}$ ; $\int (e^{mx}).dx = \frac{e^{mx}}{m} + c$ //

// Vector Component:




$$\vec{A} = A\cos\phi\,\hat{i} + A\sin\phi\,\hat{j}$$
$$= A\sin\theta\,\hat{i} + A\cos\theta\,\hat{j}$$

angle যার সাথে থাকবে সেটা cos হবে

## Electrostatic:

// History: ⇒ Discovered by greek philosophers. They found that if a piece of Amber is rubbed with for and then brought near straw, it attracks straw.

$$\text{Amber} \longrightarrow \text{electron}.$$

→ They also discovered a stone that attracts iron.

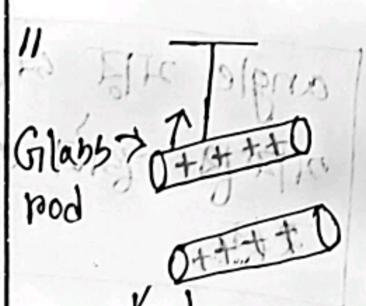
→ Benjamin Franklin's kite and key experiment.

// Electric charge: • it is a fundamental property of a particle.

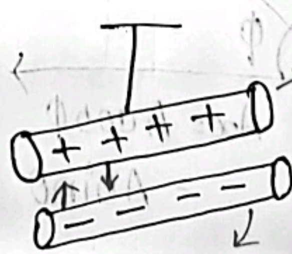
• $e_p = |e| = 1.6 \times 10^{-19}\ C$

• electric charge of an electron $= -1.6 \times 10^{-19}\ C$

//




Glass rod after rubbing silk cloth.

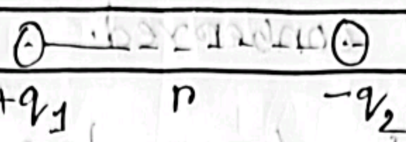


Glass rod rubbing with silk cloth.

G Plastic rubbed by for.

# Chapter-21

## Coulomb's Law :




$$\therefore\ \boxed{F = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}}$$

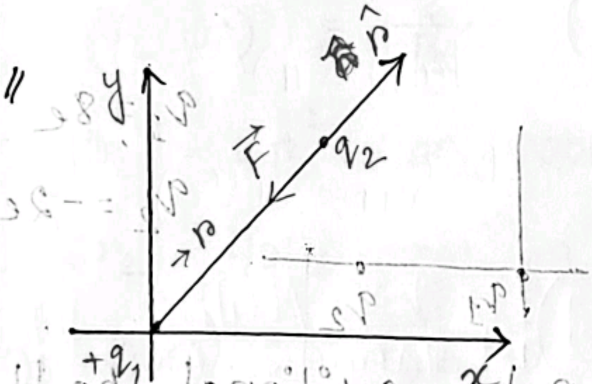
$k = \frac{1}{4\pi\epsilon_0}$

$F = k\cdot\frac{q_1 q_2}{r^2}$

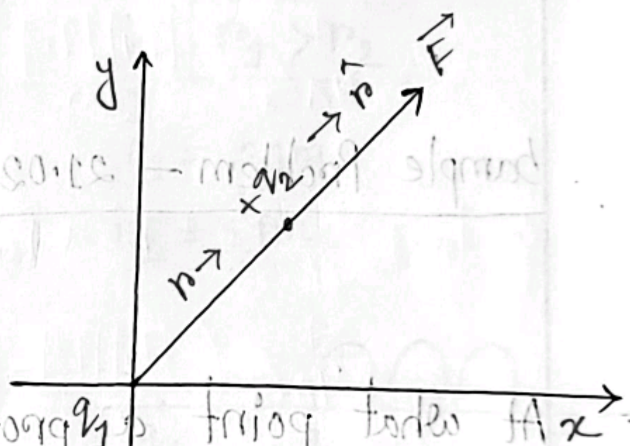
$\epsilon_0$ = ~~Posi~~ Peptivity of free space

$$\epsilon_0 = 8.854\times10^{-12}\ N^{-1}m^{-2}C^{2}$$

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ Nm^{2}C^{-2}$$




$$\vec{F} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}(-\hat{r})$$




$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\ \frac{q_1 q_2}{r^2}\hat{r}$$

($q_1, q_2$ might be positive or negative)

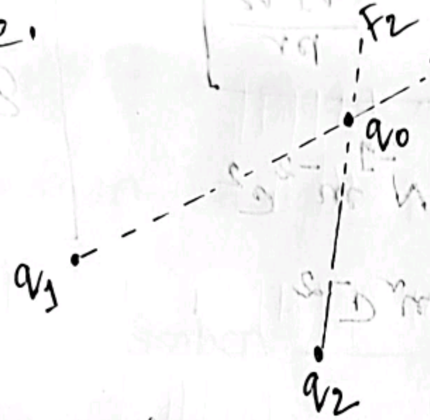
## Charge is quantized → (discrete)

$$\boxed{q = ne} = \pm e, \pm 2e, \pm 3e \ldots$$

$n$ → Integer number.

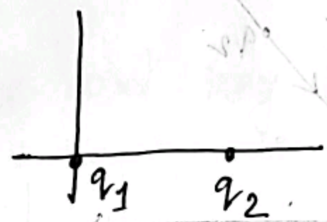
// Charge is conserved.

// Coulomb's ~~fro~~ force law obey superposition principle.




$$\vec{F} = \vec{F_1} + \vec{F_2}$$

Sample Problem — 21.02:



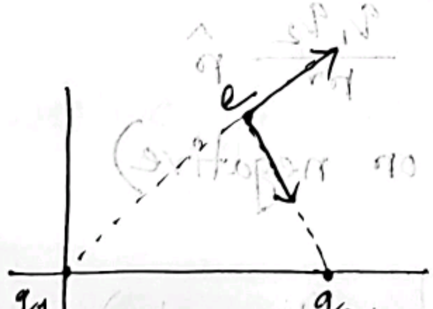

$q_1 = 8e$

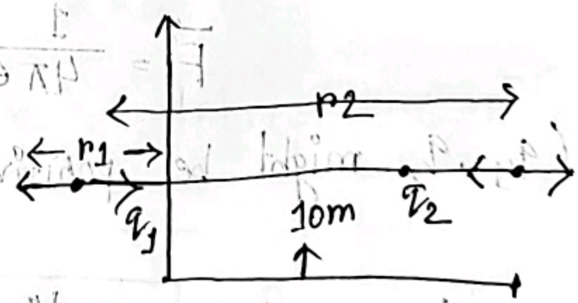
$q_2 = -2e$

At what point a proton can be positioned, so that it remains in the equibilirum? Is the equibilirum stable or non-stable?

Soln:

Case-1

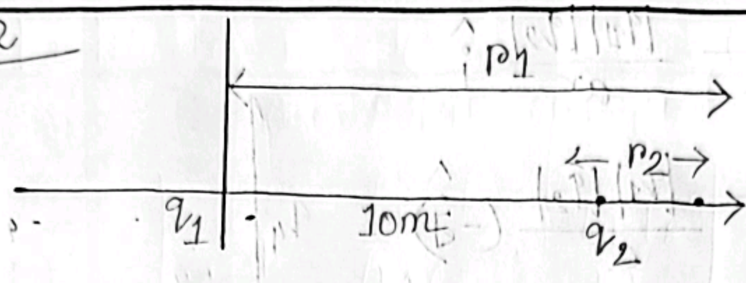



$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{8e^2}{r_1^2}$$ $\rightarrow \boxed{r_2 > r_1}$

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{r_2^2}$$ $\therefore F_1 > F_2$

Case-2




$F_1 = F_2$

$F_1 = \frac{1}{4\pi\epsilon_0}\frac{8e^2}{r_1^2}$

$F_2 = \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2^2}$

$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{8e^2}{r_1^2} = \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2^2}$

$\Rightarrow \frac{8}{(r_2+10)^2} = \frac{2}{r_2^2}$ $\boxed{r_1 > r_2}$

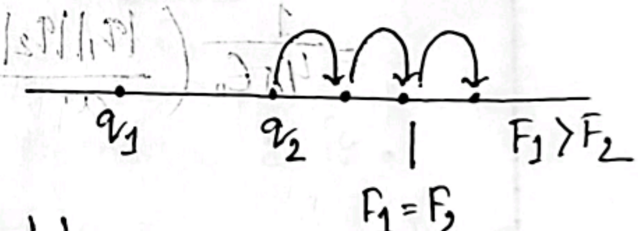
$\Rightarrow 8r_2^2 = 2r_2^2 + 40r_2 + 200$

$r_1 = 21m$

$r_2 = 11m$

$\Rightarrow 6r_2^2 - 40r_2 - 200 = 0$

$r_1 = 20m$ $r_2 = 10$




∴ The equibilirum is non-stable.

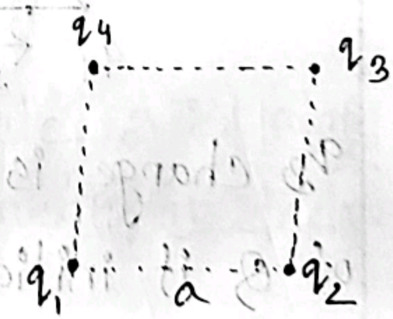
---

## Application of Coulomb's law:

Problem-11: $q_1 = 200\,nc$, $q_2 = -200\,nc$

$q_3 = -100\,nc$, $q_4 = +100\,nc$

$a = 5\,cm$



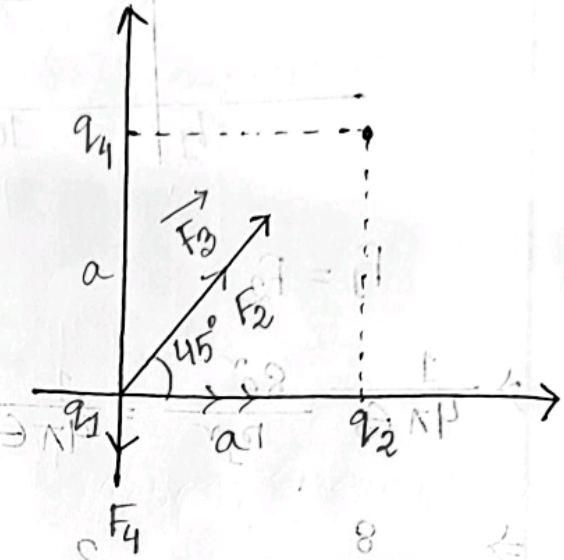

What are the components of net electrostatic force on Particle 1 ?

**Sol<sup></sup>:**

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0}\,\frac{|q_1||q_2|}{a^2}\,\hat{i}$$

$$\vec{F_4} = \frac{1}{4\pi\epsilon_0}\,\frac{|q_1||q_2|}{a^2}\,(-\hat{j})$$

$$|\vec{F_3}| = \frac{1}{4\pi\epsilon_0}\,\frac{|q_1||q_2|}{(\sqrt{2}a)^2}\,\hat{j}$$



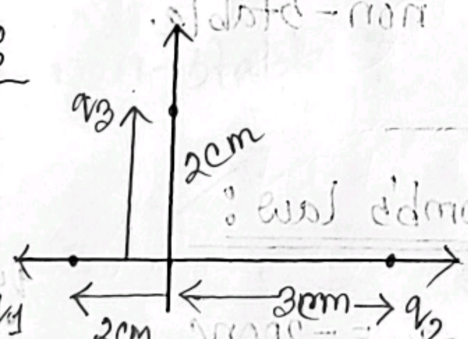

$$\therefore \vec{F_3} = |\vec{F_3}|\cos 45°\,\hat{i} + |\vec{F_3}|\sin 45°\,\hat{j}$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{|q_1||q_3|}{2a^2}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

$$\therefore \vec{F_{net}} = \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{|q_1||q_2|}{a^2} + \frac{|q_1||q_3|}{\sqrt{2}\,2a^2}\right)\hat{i} + \frac{1}{4\pi\epsilon_0}\left(\frac{|q_1||q_3|}{\sqrt{2}\cdot 2a^2} - \frac{|q_1||q_3|}{a^2}\right)\hat{j}$$

**Problem-12:**




$q_1 = 40\,\mu C$

$q_2 = Q$

$q_3 = 20\,\mu C$

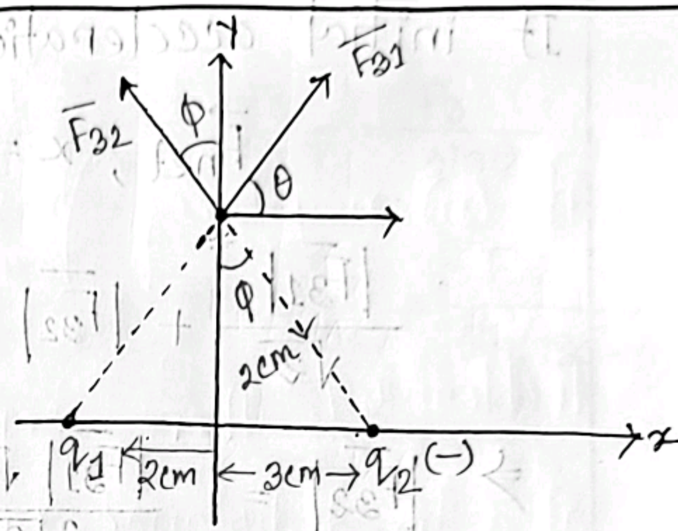
$q_3$ charge is released from rest. What is the value of $Q$ if initial accelaration of particle 3 is in the direction of (i) x-axis ?

(ii) y-axis ?

Soln:

$$|\overline{F_{31}}| = \frac{1}{4\pi\epsilon_0}\frac{|q_3||q_2|}{(2\sqrt{2})^2}$$

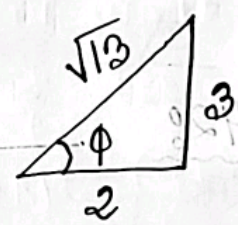
$$|\overline{F_{32}}| = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_4|}{(\sqrt{13})^2}$$




$$\overline{F_{31}} = |\overline{F_{31}}|\cos\theta\,\hat{i} + |\overline{F_{31}}|\sin\hat{j}$$

$$= \frac{|F_{31}|}{\sqrt{2}}(i+j)$$

$$\overline{F_{32}} = |\overline{F_{32}}|\cos\phi\,\hat{i} + |\overline{F_{32}}|\sin\hat{j}$$




$\phi = \tan^{-1} 3/2$

$$= |\overline{F_{32}}|\left(\frac{2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{j}\right)$$

$\therefore$ Total electrostatic force on particle 3 is,

$$F_{net} = \overline{F_{31}} + \overline{F_{32}}$$

$$= \left(\frac{|F_{31}|}{\sqrt{2}} + |F_{32}|\frac{2}{\sqrt{13}}\right)\hat{i} + \left(|\overline{F_{31}}|\frac{1}{\sqrt{2}} + |\overline{F_{32}}|\frac{3}{\sqrt{13}}\right)\hat{j}$$

now, if initial acceleration of particle 3 is along x-axis, there will be no y-components of electrostatic force, so,

$$\frac{|\overrightarrow{F_{31}}|}{\sqrt{2}} + |\overrightarrow{F_{32}}|\frac{3}{\sqrt{13}} = 0 \quad \therefore |\overrightarrow{F_{32}}| = -|\overrightarrow{F_{31}}|\frac{\sqrt{13}}{3\sqrt{2}} = 0$$

$\hookrightarrow q = ?$

ii

If initial acceleration is along y-axis,

$$\vec{F}_{net,x} = 0$$

$$\frac{|\vec{F_{31}}|}{\sqrt{2}} + |\vec{F_{32}}| \frac{2}{\sqrt{13}} = 0$$

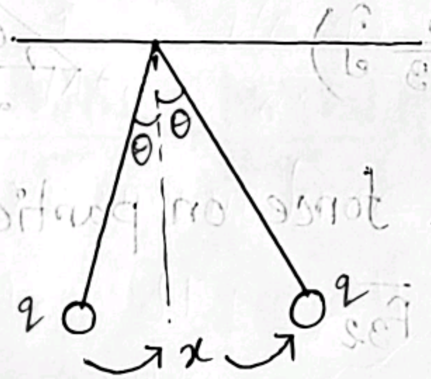
$$\Rightarrow |\vec{F_{32}}| = -\frac{|\vec{F_{31}}| \sqrt{13}}{2\sqrt{2}}$$

$$\Rightarrow Q = ?$$

Case 1: x-axis Q: (−)

Case 2: y-axis Q: (+)

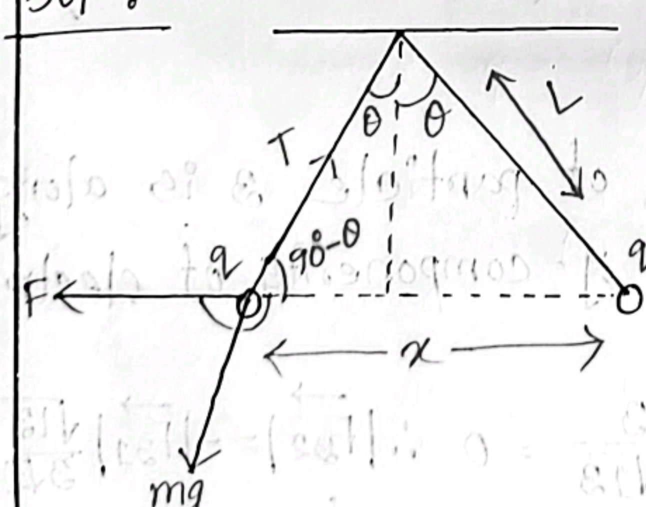
**Problem-42:**




θ is very small and rope is non-conducting. Show that:

$$x = \left(\frac{q^2 L}{2\pi \epsilon_0 mg}\right)^{1/3}$$

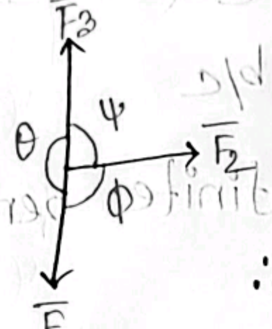
**Sol$^n$:**




Total net force = 0

$$|\vec{F}| = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2}$$

if, $F_{net} = 0$




$$\Rightarrow \frac{F_1}{\sin\theta} = \frac{F_2}{\sin\psi} = \frac{F_3}{\sin\phi}$$

$$\therefore \frac{mg}{\sin(90^\circ+\theta)} = \frac{T}{\sin 90^\circ} = \frac{F}{\sin(180^\circ-\theta)}$$

now,

$$\frac{mg}{\sin(90^\circ+\theta)} = \frac{F}{\sin(180^\circ-\theta)}$$

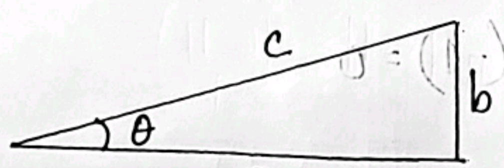
$$\Rightarrow F\, mg\tan\theta \Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q^2}{x^2} \simeq mg\cdot\frac{x/2}{L}$$

$$\Rightarrow x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg}\right)^{1/3}$$

$\theta$ is very small

$\theta \simeq \sin\theta \simeq \tan\theta$

$\downarrow$ (rad) $\quad \sin\theta = \frac{x/2}{L}$




$a^2+b^2=c^2$

$\Rightarrow a = (c^2-b^2)^{1/2}$

$\sin\theta = b/c$

$\tan\theta = b/a$

$$\Rightarrow \left(\frac{b}{c^2-b^2}\right)^{1/2} = b\cdot(c^2-b^2)^{-1/2} = \frac{b}{c}\left(1-b^2/c^2\right)^{-1/2}$$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots$$

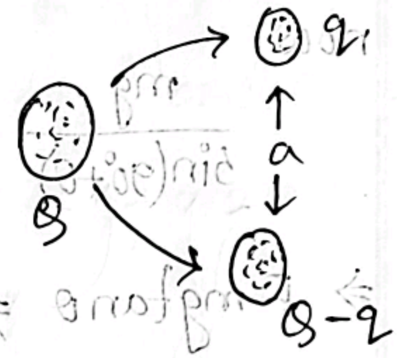
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$$

now, $b/c\,(1-b^2/c^2)^{-1/2} \Rightarrow \frac{b}{c}\left(1-(-1/2)\frac{b^2}{c^2}+\cdots\right)$

$$\simeq b/c$$

if (n) negative $\longrightarrow$ infinite series.

Problem-54: There were seperated at a fixed distance, a. What would be the ratio of Q and q if force between them is greatest?




Sol$^n$: $F(q,a)=\frac{1}{4\pi\epsilon_0}\frac{q(Q-q)}{a^2}$

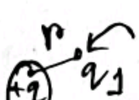
To find the maximum force, we have to find the maxima of F, $\frac{dF}{dq}=0$

$$\Rightarrow Q-q+q(-1)=0$$

$$\Rightarrow q=Q/2$$

Chapter-22

## Electric field:




$$\boxed{E=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

When we place a charged object an electric field of is produced in the surroundings.

$$E=\frac{F}{q_0}\quad\frac{qq_0}{r^2q_0}$$

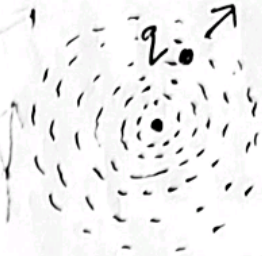
$$\boxed{E=\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

## Electric Field:

// <u>Field:</u> Function of space $(x, y, z)$, possibly time, $t$. It can be either scaler or vector.

test charge

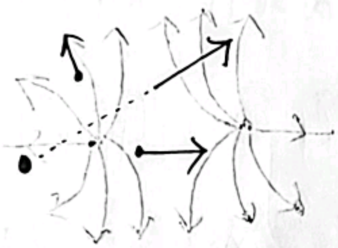
$q_0$

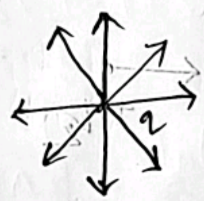


$$\vec{E} = \frac{\vec{F}_{q_0}}{q_0}$$

// For a single positive charge: $\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$

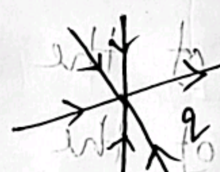
//





For positive charge it is outward.

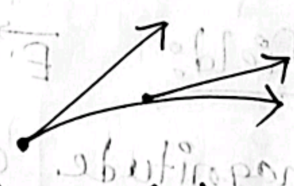
// For negative charge: $\boxed{\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}\hat{r}}$



For negative charge it is inward.

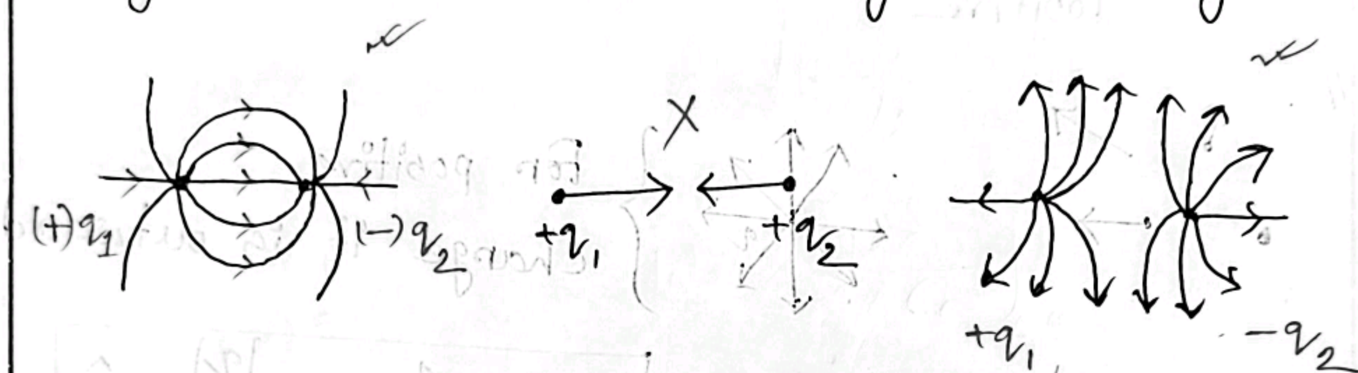
## Properties of electric field:

* <u>Lines:</u> • Electric field is directed along the tangent of field lines. For curve line tangent works.

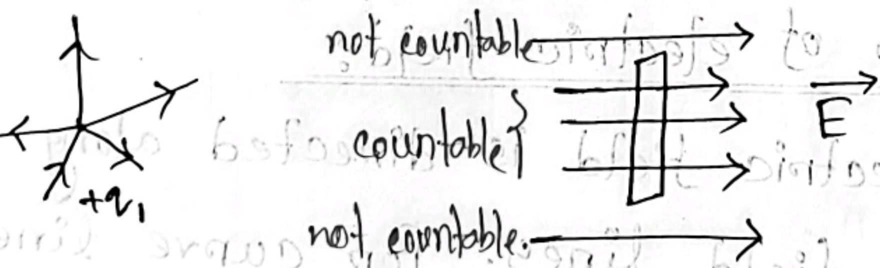


• No two field lines intersect with each other. b field lines repel each other.

• Electric field lines originate from positive charges & terminate in negative charges.



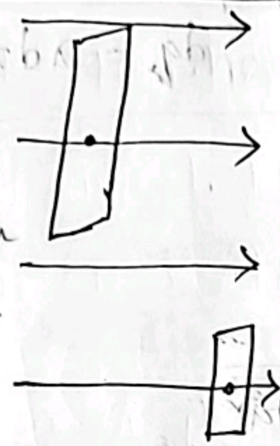

// In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field.



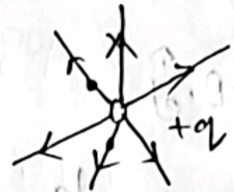

// Uniform Electric Field: $\vec{E}$ = constant.

(direction & magnitude constant).

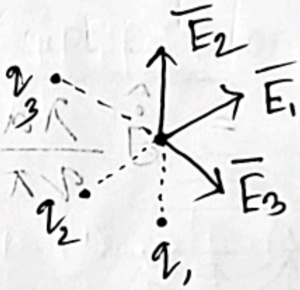
## Non-Uniform Electric Field:



Parallel to each others.



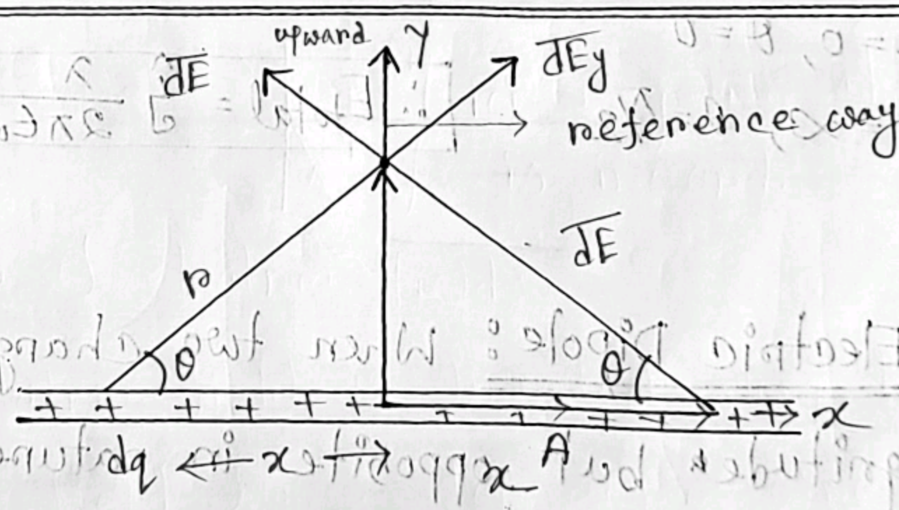

- single charge
- magnitude will not be same.
- if distance equal then same.




// Electric field obeys superposition principle.

// Electric field for a group of charge, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$

## ☐ Electric field for an ~~initiate~~ infinite line of charge:




$|\vec{dE}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$

$|\vec{dE'}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$

$\vec{dE} = dE\cos\theta\,\hat{i} + dE\sin\theta\,\hat{j}$

$\vec{dE} = dE\cos\theta\,(-\hat{i}) + dE\sin\theta\,\hat{j}$

$\vec{dE_y} = 2dE\sin\theta\,\hat{j}$

$\boxed{x\text{-axis symetric}}$

$$\vec{E}_{total} = \int d\vec{E_y}$$

Charge per unit length $= \lambda$

$dq = \lambda dx$

$$= \int 2dE \sin\theta \cdot \hat{j}$$

$$= \hat{j} \cdot 2 \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \sin\theta$$

$$= \hat{j} / 4\pi\epsilon_0 \int_{x=0}^{\infty} \frac{\lambda dx}{x^2+a^2} \cdot \frac{a}{\sqrt{x^2+a^2}}$$

$$= \hat{j} \frac{\lambda a}{2\pi\epsilon_0} \int_0^{\infty} \frac{dx}{(x^2+a^2)^{3/2}}$$

Let,

$x = a\tan\theta$

$\therefore dx = a\sec^2\theta d\theta$

when,

$x = 0, \theta = 0$

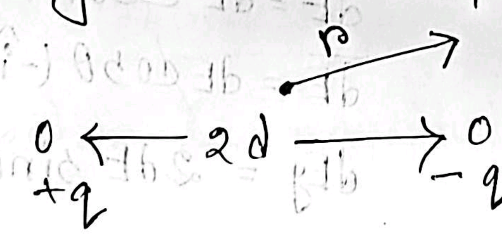
$x = \infty, \theta = \pi/2$

$$E_{total} = \frac{\hat{j}\lambda a}{2\pi\epsilon_0} \int_{\theta=0}^{\pi/2} \frac{a\sec^2\theta}{a^3\sec^3\theta} d\theta$$

$$= \hat{j} \frac{\lambda a}{2\pi\epsilon_0 a^2} \left[+\sin\theta\right]_{\theta=0}^{\pi/2}$$

$$\boxed{\therefore E_{total} = \hat{j} \frac{\lambda}{2\pi\epsilon_0 a}}$$

▣ <u>Electric Dipole:</u> When two charges of same magnitude but opposite in nature stay close enough, they form a dipole.



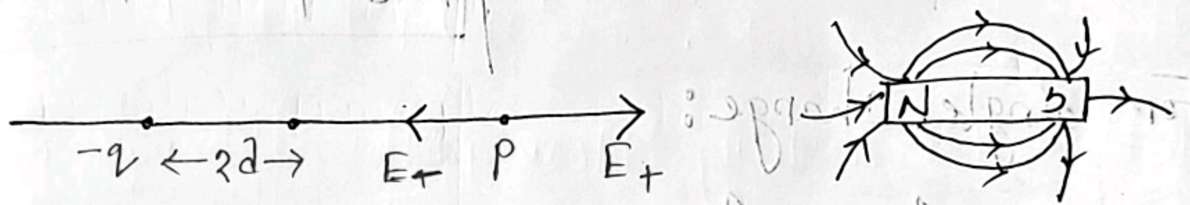

$\boxed{r >> 2d}$

Two Conditions:

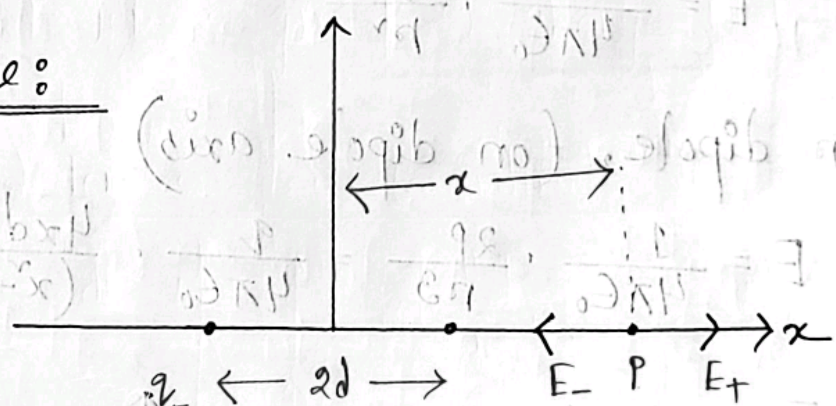
i) Charges should be opposite but have some magnitude.

ii) The distance between them must be very small than the distance of concerned point.

## Electric field for an electric dipole: on dipole axis.




Reference Frame:




$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(x-d)^2}$$

$$E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{(x+d)^2}$$

$$|E_{total}| = \vec{E_+} + \vec{E_-}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(x-d)^2} - \frac{1}{(x+d)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4xd}{(x-d)^2-(x+d)^2}\right]^2$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4xd}{(x^2-d^2)^2}$$

To be a dipole, $2d << x$.

So, $|\vec{E}_{total}| \simeq \frac{q}{4\pi\epsilon_0} \cdot \frac{4xd}{x^4}$

$\therefore$ $E_{total} = \frac{q}{4\pi\epsilon_0} \cdot \frac{4d}{x^3}$ | $\left[x^2\left(x^2 - \cancel{\frac{d^2}{x^2}}^{\,0}\right)\right]^2$

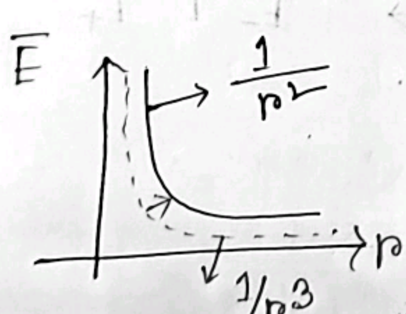
$$\simeq \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{x^3}$$ | $\boxed{q \times 2d = p = \text{dipole moment}}$

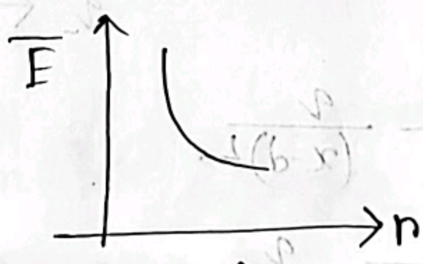
For single charge:

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

For dipole, (on dipole axis)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} = \frac{q}{4\pi\epsilon_0} \cdot \frac{4xd}{(x^2-d^2)^2}$$



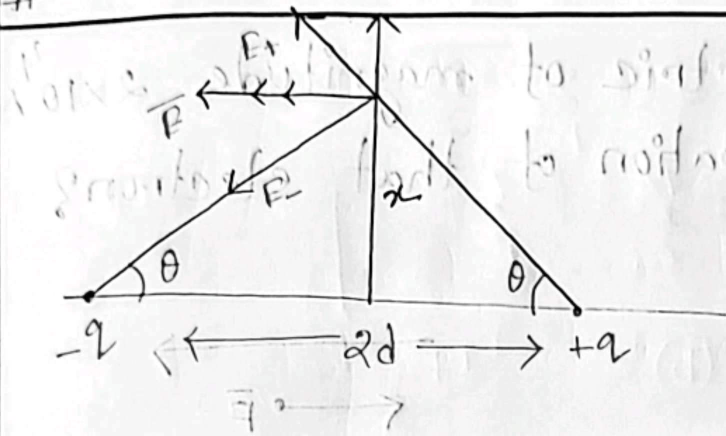




Dipole

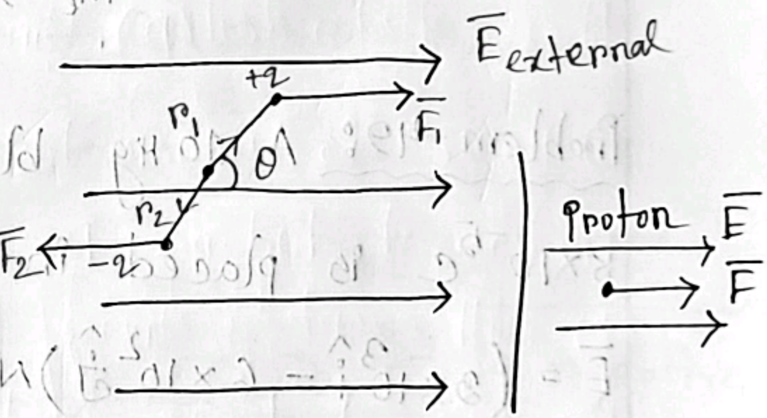
For single charge      $E \propto \frac{1}{r^3}$

$$E \sim \frac{1}{r^2}$$

#




# Electric dipole on an extremal uniform electric field:




$\overline{F} = q\overline{E}$

$\overline{\tau} = \overline{p} \times \overline{F}$

$= (\overline{r_1} \times \overline{F_1}) + (\overline{r_2} \times \overline{F_2})$

$= (\overline{r_1} \times \overline{F_1}) + (-\overline{r_1}) \times (-\overline{F_1})$

$= 2\overline{r_1} \times \overline{F_1}$

$= 2\overline{r_1} \times q\overline{E}$

$= 2\overline{r}q \times \overline{E}$

$\overline{r_1} = -\overline{r_2}$

$\overline{F_1} = -\overline{F_2}$

$\vec{p} = 2dq$ (ıı)

direction will always from negative to positive.

$$\boxed{\begin{aligned} &\therefore \overline{\tau} = \overline{P} \times \overline{E} \\ &\therefore |\overline{\tau}| = PE\sin\theta \end{aligned}}$$

## Potential Energy:

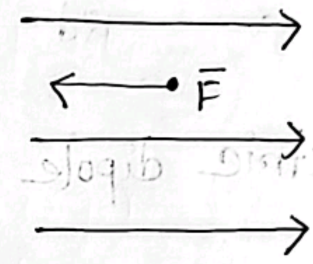
$u = E_p = \int \tau \, d\theta$

$$\boxed{u = \overline{P}.\overline{E}}$$

**Problem-48:** An electron is released from rest in an uniform electric of magnitude $2\times10^{4}\,NC^{-1}$. What is the acceleration of that electron?

**Ans:** $\bar{F} = q\vec{E}$

$m|\vec{a}| = |\bar{F}| = e|\bar{E}|$




$|\vec{a}| = \dfrac{e|\bar{E}|}{m}$ $\quad m_e = 9\times10^{-31}\,kg$

**Problem-49:** A 10 kg block with a charge of $8\times10^{-5}\,C$ is placed in an electric field $\bar{E} = (3\times10^{3}\hat{i} - 6\times10^{2}\hat{j})\,NC^{-1}$. What are the a) magnitude & b) direction (relative to $+x$ axis) of the electrostatic force? If the block is released from rest at the origin at time $t=0$, what are it's a) $x$ and b) $y$ coordinates at $t = 3s$?

**Ans:** $\bar{F} = q\times\bar{E}$

$$= 8\times10^{-5}(3\times10^{3}\hat{i} - 6\times10^{2}\hat{j})$$

$$= \underbrace{24\times10^{-2}\hat{i}}_{F_x\hat{i}} - \underbrace{48\times10^{-3}\hat{j}}_{F_y\hat{j}}$$

$\therefore |F| = \sqrt{F_x^2 + F_y^2}$ ; $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}\left(\frac{-48 \times 10^{-3}}{24 \times 10^{-2}}\right)$

origin

$t = 0$

$= -\tan^{-1}(1/5)$

$F = m\bar{a}$

$\therefore a = \bar{F}/m$

$x = \overset{0}{v_0 t} + \frac{1}{2} a_x t^2$ | $y = \frac{1}{2} a_y t^2 + \overset{0}{v_0 t}$

$= \frac{1}{2} a_x t^2$ | $= \frac{1}{2} a_y t^2$

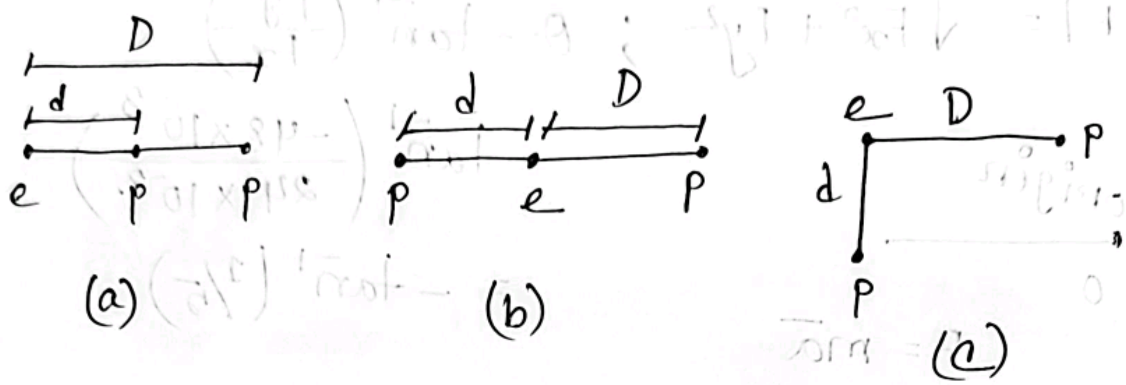
**Chapter-21**

Checkpoint-2: a) Force due to the $\bar{e}$: As $e^-$ is negative force and central is positive force. so, they attract each other. For central protion it is directed to the left.


b) Force due to the protion: as proton is positive charged so it will repel the central proton. so, it is directed to the left.

c) Net force: As both are left directed so it will also left directed.

Checkpoint-3:




Part-A

a) The force of distance (d) is stronger from proton than distance (D). ~~Since, the p is closer~~ lower net force

b) The $e^-$ is closer to one proton from further proton at distance D. The closer proton of distance (d) will exert a strong force leading to a larger net force on the $e^-$ compared to (a).

c) In this case direction & distance are different so, net force will be the sum of these forces.

(b) > (c) > (a)

Part-B - the force from the closer proton is stronger, it will dominate the direction of net force. Here for distance d is less than 45° and vertical component is stronger than horizontal. net force closer to vertical.

Checkpoint-4: Charge flow between them untill they reach the same electric potential. since, the sphere are identical in size, they will share the total charge equally.

$$\text{Total charge} = (-50+20) = -30e$$

$$\text{sphere, A} = -30/2 = -15e.$$

Problem-10:

$$F_{14} = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(\sqrt{2}d)^2} = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2d^2}$$

$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{d^2}, \quad F_{13} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{d^2}$$

$$F_{23,net} = \sqrt{F_{12}^2 + F_{13}^2}$$

$$= \sqrt{2}\,\frac{1}{4\pi\epsilon_0}\frac{Qq}{d^2}$$

now,

$$\sqrt{2}\,\frac{1}{4\pi\epsilon_0}\frac{Qq}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2d^2}$$

$$\Rightarrow \sqrt{2}\,Qq = Q^2/2$$

$$\Rightarrow Q = 2\sqrt{2}\,q$$

$$\therefore Q/q = 2\sqrt{2}$$

Problem-22: $\theta = 30^\circ$, $d = 2.00\,cm$

$q_1 = 0C$ $\quad q_2 = 8.00\times10^{-19}C$

$\cancel{F=}$ $\quad q_3 = q_4 = -1.60\times10^{-19}C$

$\cancel{q_2} = \frac{q_1 \cancel{q_2}}{\cancel{d^2}} \frac{1}{4\cancel{\pi}\epsilon_0}$ $\quad r_{13} = r_{14} = \frac{d}{\cos 30^\circ} = \frac{2d}{\sqrt{3}}$

$\Rightarrow \cancel{8}\times10^{\cancel{-19}} = \cancel{D}$ $\quad \underline{\underline{a}}$

$$F_{net1} = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_3|}{(2d/\sqrt{3})^2}\cos 30^\circ + \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_4|}{(2d/\sqrt{3})^2}\cos 30^\circ - \frac{1}{4\pi\epsilon_0}\frac{|q_1|\,|q_2|}{(d+D)^2} = 0$$

$$\Rightarrow 2\left[\frac{1}{4\pi\epsilon_0}\frac{3|q_1||q_3|}{4d^2}\left(\frac{\sqrt{3}}{2}\right)\right] - $$

$\therefore F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$, $|q_3| = |q_4| = 1.6\times10^{-19}C$

~~Problem-3~~: $\frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{(d+D)^2} = 0$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}|q_1|\left[\frac{3\sqrt{3}\,|q_3|}{4d^2} - \frac{|q_2|}{(d+D)^2}\right] = 0$$

$$\Rightarrow \frac{3\sqrt{3}\,(0.2)}{4d^2} = \frac{1}{(d+D)^2}$$

$$\Rightarrow \sqrt{0.259}\,D = d - \sqrt{0.259}\,d$$

$$\Rightarrow D = \frac{2\times10^{-2}\left(1-\sqrt{0.259}\right)}{\sqrt{0.259}}$$ $\quad \therefore D = 1.92\,cm$

**b.**

As angle decreases, its ~~contains~~ cos increases, resulting in a larger contribution from the charges on the y axis. The force exerted by $q_2$ must be made stronger, so that it must be closer to $q_1$. Thus, D must be decreased.

Problem 39:

$q_1 = 4e$, $q_2 = 6e$, $d_1 = 2.00\ mm$, $d_2 = 6.00\ mm$

$= 0.002\ m$ $\quad = 0.006\ m$

$$r = \sqrt{d_1^2 + d_2^2} = \sqrt{(0.002)^2 + (0.006)^2} = 6.32 \times 10^{-3}\ m$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 \cdot q_2|}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{24 \times (1.6 \times 10^{-19})^2}{(6.32 \times 10^{-3})^2}$$

$$= 1.38 \times 10^{-22}$$

$$\therefore F_x = F \cdot \frac{d_2}{r} = 6.\cancel{9}$$

$$= 1.38 \times 10^{-22} \cdot \frac{0.006}{6.32 \times 10^{-3}}$$

$$= 1.31 \times 10^{-22}\ N$$

**Problem-7:** $a = 5\,cm$, $q_1 = 10\,nc$, $q_2 = -20\,nc$, $q_3 = 20\,nc$, $q_4 = -10\,nc$.

$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{|q_1|}{\left(\frac{a}{\sqrt{2}}\right)^2} - \frac{|q_2|}{\left(\frac{a}{\sqrt{2}}\right)^2} + \frac{|q_3|}{\left(\frac{a}{\sqrt{2}}\right)^2} - \frac{|q_4|}{\left(\frac{a}{\sqrt{2}}\right)^2}\right]\cos 45^\circ$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{10\times10^{-9}}{\left(\frac{0.05}{\sqrt{2}}\right)^2} - \frac{20\times10^{-9}}{1.25\times10^{-3}} + \frac{20\times10^{-9}}{1.25\times10^{-3}} - \frac{10\times10^{-9}}{1.25\times10^{-3}}\right]\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{1}{4\pi\epsilon_0}\times\frac{1}{1.25\times10^{-3}}\left(-1.27\times10^{-8}\right)$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}}$$

$$\Rightarrow E_x = 0$$

$$E_y = \frac{8.99\times10^9}{\left(\frac{0.05}{\sqrt{2}}\right)^2}\left[-10\times10^{-19} + 20\times10^{-19} + 20\times10^{-19} - 10\times10^{-9}\right]\cdot\frac{1}{\sqrt{2}}$$

$$\therefore E_y = 1.02\times10^5\,NC^{-1}$$

$$\overline{E} = E_y\hat{j} = (1.02\times10^5)\,NC^{-1}\,\hat{j}$$

$q_1 = q_2 = 5$ , $q_3 = 3$ , $q_4 = 12$ , $d = 5.0\ \mu m$

Problem-8: ~~on x-axis: -q = -3.20×10⁻¹⁹ C, x = -3.00 m and q = 3.20×10⁻¹⁹ C at x = 3.00 m~~

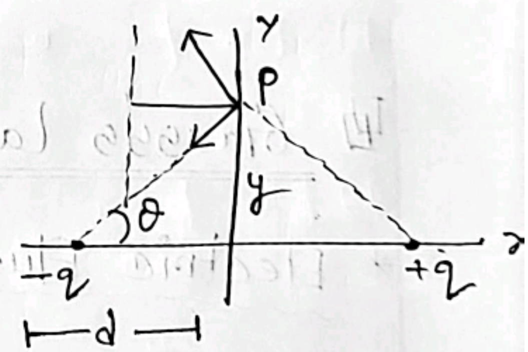
$$\bar{E}_{net} = -\bar{E}_1 + \bar{E}_2 - \bar{E}_3 + \bar{E}_4 \qquad E = k\frac{|Q|}{r^2}$$

$$= -k\frac{|q_1|}{d^2}(\hat{i}) + k\frac{|q_2|}{d^2}(\hat{i}) - k\frac{|q_3|}{d^2}(\hat{j}) + k\frac{|q_4|}{(2d)^2}(\hat{j})$$

$$\bar{E}_{net} = k\left[-\frac{|q_3|}{d^2} + \frac{|q_4|}{(2d)^2}\right](\hat{j}) = k\left[\frac{\overset{3}{\cancel{3.20\times10^{-19}}}}{d^2} - \frac{\overset{12}{\cancel{3.20\times10^{-19}}}}{4d^2}\right]\hat{j}$$

$$= 0$$

Problem-9: on x axis: $-q = -3.20\times10^{-19}C$, at $x = -3.00m$ & $q = 3.20\times10^{-19}C$ at $x = +3.00m$.




a) $$F_{net,x} = 2\left[k\frac{|q|}{(\sqrt{d^2+y^2})^2}\right]\cos\theta$$

$$= 2\left[k\frac{|q|}{(d^2+y^2)}\right]\left(\frac{d}{\sqrt{d^2+y^2}}\right)$$

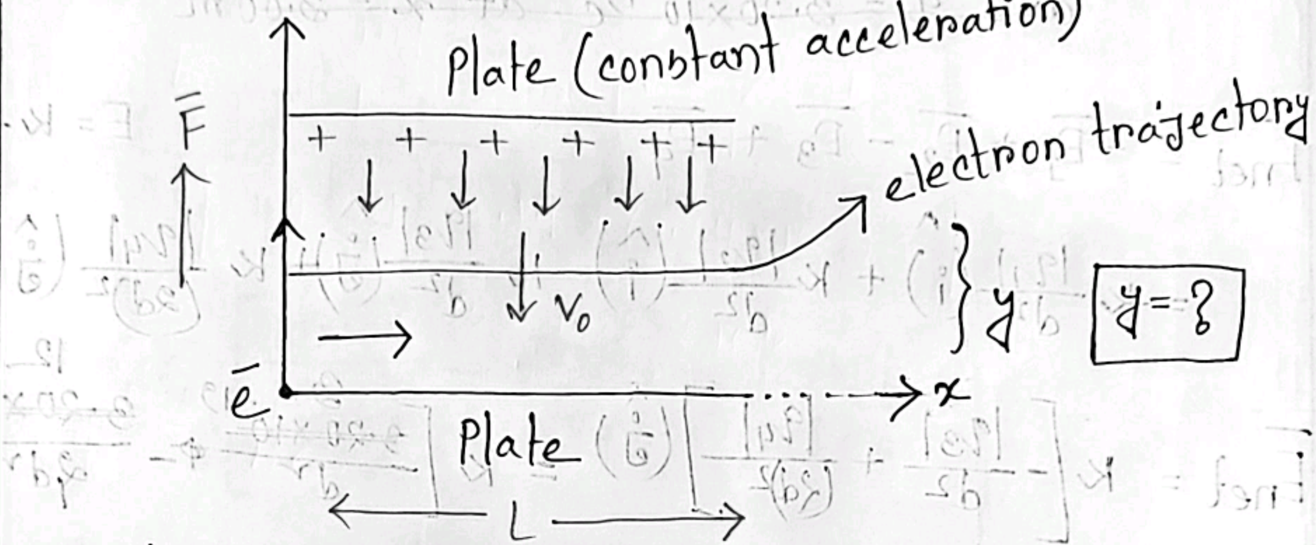
$$E = k\frac{|Q|}{r^2}$$

$$= 2\left[k\frac{|q|\,d}{(d^2+y^2)^{3/2}}\right]$$

$$= 2\,\frac{(8.99\times10^9)(3.2\times10^{-19})(3.0)}{[(4.0)^2+(3.0)^2]^{3/2}} = 1.38\times10^{-10}\ NC^{-1}$$

b) The net electric field points in the $-x$ direction, 180° counterclockwise from the +

## Sample Problem - 22.04:




$t = \dfrac{L}{V_0}$

$y = \cancel{V_{0y}t}^{\,0} + \frac{1}{2} a_y t^2$

$F = ma = eE$

$\therefore \boxed{a = \dfrac{eE}{m}}$

$= \frac{1}{2} \cdot \dfrac{Ee}{m} \cdot \dfrac{L^2}{V_0^2}$

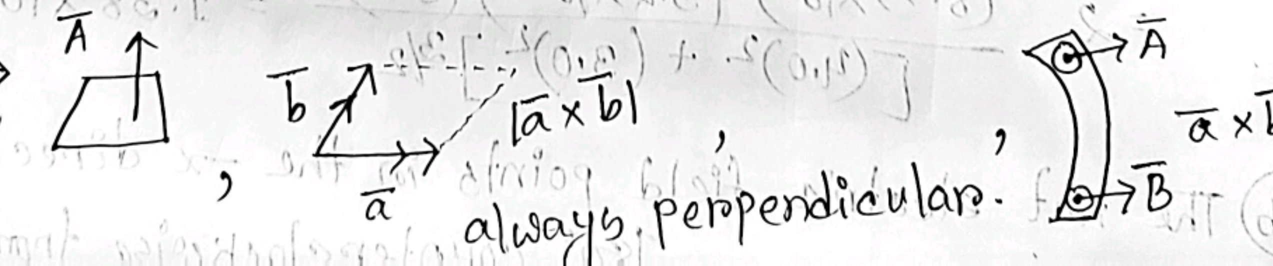
## ⊞ Gauss's Law:

* Electric Flux: → Area is a vector.

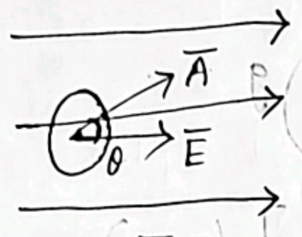
→ magnitude = area.

→ Electric Flux, $\phi$ = (Component of $\bar{E}$ along $\bar{A}$) × Area

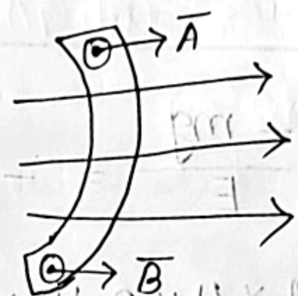
$\Rightarrow E\cos\theta \times A = \bar{E} \cdot \bar{A}$

→ 

→ Uniform:




$\overline{E} \cdot \overline{A}$




$\boxed{\phi, \text{Electric Flux} = \int \overline{E} \cdot d\overline{A}}$

(Electric Flux: Scaler.)

$\phi = \overline{E} \cdot \overline{A}$

→ If the surface is closed, $\boxed{\phi = \oint \overline{E} \cdot d\overline{A}}$

Earth: Closed.

## Chapter-22 [Problems]

Problem-15: $q_1 = q_2 = +e$, $q_3 = +2e$, $a = 6.00\,\mu m$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

a) $$E = k \frac{|q_3|}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{1}{4\pi\epsilon_0} \frac{2e}{\frac{a^2}{2}}$$

$$= \frac{8.99 \times 10^{9} \times (2 \times 1.6 \times 10^{-19})}{\frac{(6.0 \times 10^{-6})^2}{2}} = 160\, NC^{-1}$$

b) This field points at $45°$, counterclock wise from the $x$-axis.

## Problem-39:

$$q = \frac{-mg}{E} = -\left(\frac{\rho \times \frac{4}{3}\pi r^3}{E}\right) g$$

$$\Rightarrow q = \frac{\left[851 \times 4 \times 3.14 \times (1.64 \times 10^{-6})^3\right](9.8)}{3 \times 1.92 \times 10^5}$$

$$\therefore q = -5e$$

## Problem-56:

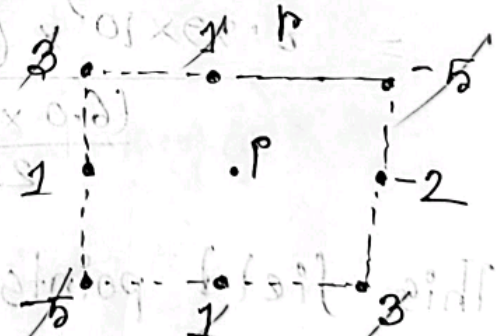
$$\tau = pE\sin\theta = (qd)E\sin\theta$$

a) $\tau = \left[(2 \times 1.6 \times 10^{-19})(0.78 \times 10^{-9})\right](3.4 \times 10^6)\sin 0^\circ = 0$

b) $\tau = [\;\text{''}\;\;\;\text{''}\;]\,(\;\text{''}\;)\sin 90^\circ = 8.5 \times 10^{-22}\ N$

c) $\tau = [\;\text{''}\;\;\;\text{''}\;]\,(\;\text{''}\;)\sin 180^\circ = 0$

## Problem-64:




$$E = \frac{3q}{4\pi\epsilon_0 r^2}\,\hat{i}$$

$$= \frac{3(1.6 \times 10^{-19})}{4\pi(8.89 \times 10^{-12}) \times (1.02)^2}\,\hat{i}$$

$= \quad$ Ans $\hat{i}$

Problem-87: $q_1 = 1.00\,pC = 1\times 10^{-12}\,C$

$q_2 = -2.00\,pC = -2\times 10^{-12}\,C$

$d = 5.00\,cm = 0.05\,m$

$$\vec{E}_1 = \frac{8.99\times 10^{9}\times 1\times 10^{-12}}{\left(\frac{0.05}{2}\right)^2} = 14.38$$

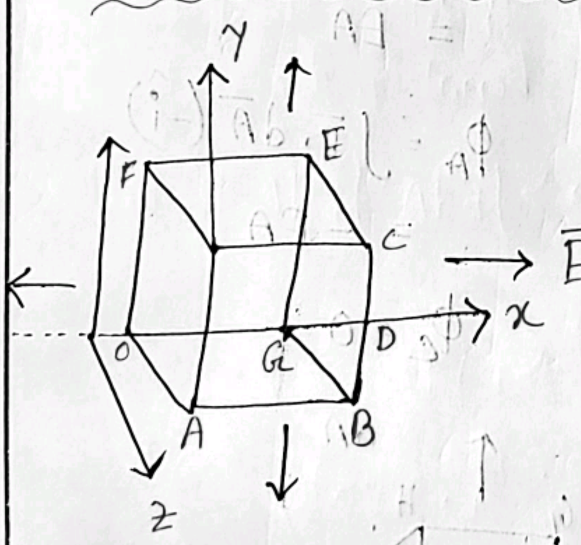
$$E_2 = \frac{8.99\times 10^{9}\times (-2\times 10^{-12})}{\left(\frac{0.05}{2}\right)^2} = -28.77$$

$\therefore E_{net} = 14.38 - 28.77$

$= -14.39$

Chapter - 23

# Uniform electric field:




Electric Flux:

$$\phi = \int \bar{E}\cdot d\bar{A}$$

ABDG Surface:

$$\phi_f = \int \vec{E}\cdot d\bar{A}$$

$\hat{i}\cdot\hat{k} = 0$

$$= \int E\hat{i}\cdot d\bar{A}\hat{k}$$

$$= 0$$

Right surface: BCED

$$\phi = \int E\hat{i}\cdot dA\hat{i}$$

$$= EA$$

FECO surface:

$$\phi_r = \int \bar{E}\cdot d\bar{A}$$

$$= \int E\hat{i}\cdot d\bar{A}(-\hat{k})$$

$$= 0$$

Left surface : (OFGA)

$\phi = \int E\hat{i} \cdot dA(-\hat{i})$

$= -EA$

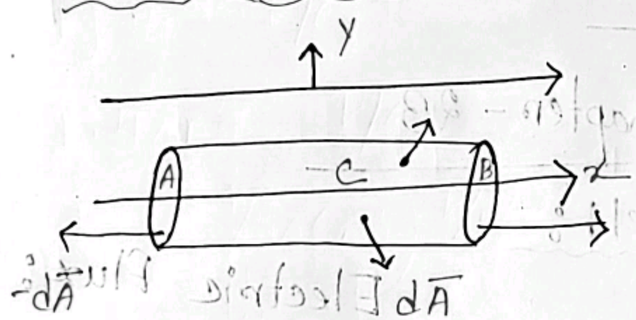
$\phi_t = \phi_b = 0$

$\phi_{total} = \oint \bar{E} \cdot d\bar{A}$

$= \phi_l + \phi_r + \phi_t + \phi_b + \phi_f + \phi_d$

$= 0$

# Uniform electric field:




$\phi_B = \int \bar{E} \cdot d\bar{A}$

$= \int E\hat{i} \cdot dA\,\hat{i}$

$= EA$

$\phi_A = \int \bar{E} \cdot d\bar{A}(-\hat{i})$

$= -EA$

$\phi_C = 0$

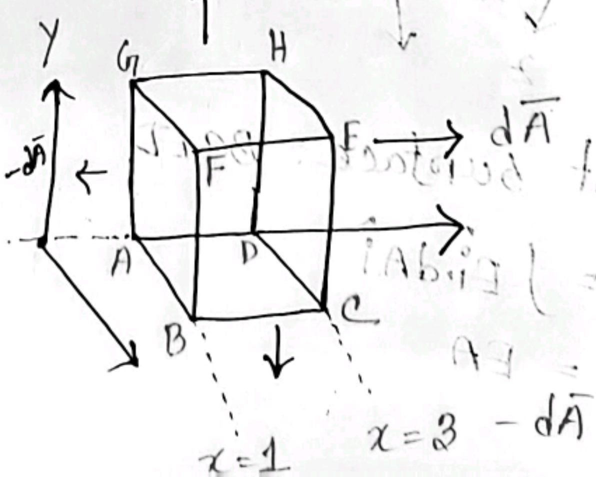
$\therefore \phi_{total} = \oint \bar{E} \cdot d\bar{A}$

$= 0$

Sample problem-23.02:

$\vec{E} = 3x\hat{i} + 4\hat{j}$

↳ for this it is non-uniform



CDEH:

$$\phi_R = \oint \bar{E} \cdot d\bar{A}$$

$$= \int (3x\hat{i} + 4\hat{j}) \cdot dA\,\hat{i}$$

$$= \int 3x\, dA$$

$$= 9 \int dA$$

$$= 9 \times 2^2$$

$$= 36$$

ABFG:

$$\phi_L = \int \bar{E} \cdot d\bar{A}$$

$$= \int (3x\hat{i} + 4\hat{j}) \cdot dA(-\hat{i})$$

$$= -\int 3x\, dA$$

$$= -3 \times 4$$

$$= -12\ Nm^2 C^{-1}$$

ABCD:

$$\phi_b = \int (3x\hat{i} + 4\hat{j}) \cdot dA(-\hat{j})$$

$$= -4 \times 4$$

$$= 16$$

EFGH:

$$\phi_t = \int (3x\hat{i} + 4\hat{j}) \, dA \cdot \hat{j}$$

$$= 16$$

BCEF:

$$\phi_f = \int (3x\hat{i} + 4\hat{j}) \cdot dA \cdot \hat{k} = 0$$

Rear: ADHG

$$\phi_r = 0$$

$\oint \bar{E} \cdot d\bar{A} = 36 + (-12) = 24$; because of '+' it means in < out.

Gauss's Law:

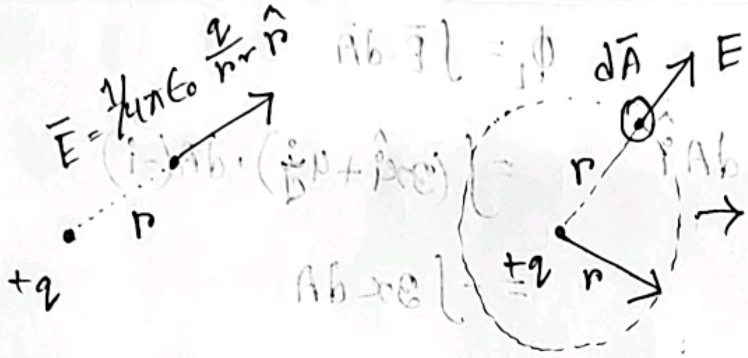
$$\oint \bar{E} \cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0} = \frac{24\,\epsilon_0}{\epsilon_0}$$

Changes outside the surface do not matter.

2

2

# Coulomb's law from Gauss's law : Spherical surface.




→ Gaussian law

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0}$$

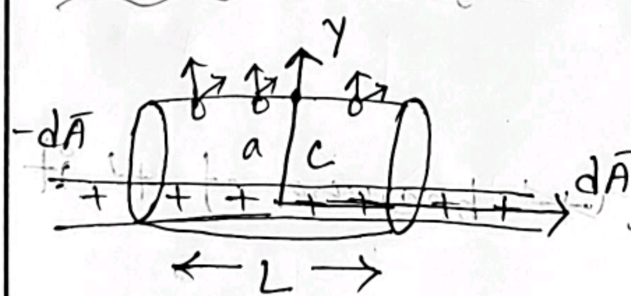
$$\Rightarrow \int E dA = \frac{q}{\epsilon_0}$$

$$\Rightarrow E\int dA = q/\epsilon_0$$

$$\Rightarrow E\cdot 4\pi r^2 = q/\epsilon_0$$

$$\Rightarrow E = 1/4\pi\epsilon_0 \cdot q/r^2$$

# Infinite line of charge




Transitional symmetry

↓

Cylindrical surface

↓

Gaussian surface

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0}$$

Charge density = $\lambda$

$q_{enclosed} = \lambda L$

$$\Rightarrow \underbrace{\int_A \bar{E} d\bar{A}}_{0} + \underbrace{\int_B \bar{E}\cdot d\bar{A}}_{0} + \int_C \bar{E}\cdot d\bar{A} = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E\int dA = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow E\cdot 2\pi a L = \frac{\lambda L}{\epsilon_0}$$

$$\therefore E = \frac{\lambda L}{2\pi\epsilon_0 a}$$

Gauss's law: $q_{enclosed}$ = Charge enclosed by the gaussian surface.

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0}$$

↓

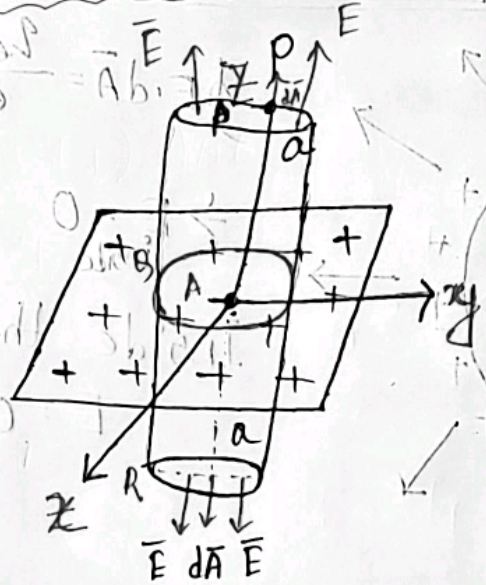
Integration on the Gaussian law

* Planner symmetry.

i) Find the symmetry of the system.

ii) We have to choose gaussian surface in such a way that ~~every~~ every electric field on every point on the surface is constant.

Infinite Sheet




* Surface charg density = $\sigma$

* $q_{enclosed} = \sigma A$

Use Gauss's law:

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$\Rightarrow \int_P \bar{E}\cdot d\bar{A} + \int_Q \bar{E}\cdot d\bar{A} + \int_R \bar{E}\cdot d\bar{A} = \frac{\sigma A}{\epsilon_0}$$

(the $\int_Q$ term marked 0)

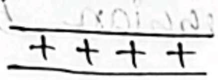
$$\Rightarrow E\int dA + E\int dA = \frac{\sigma A}{\epsilon_0}$$

Constant

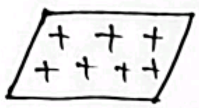
$$\Rightarrow 2EA = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{\therefore\ E = \frac{\sigma}{2\epsilon_0}}$$

** For infinite sheet electric field always constant.

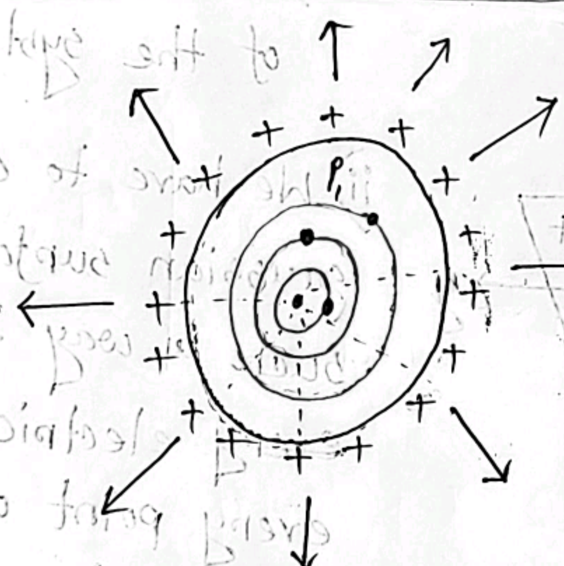


$\Rightarrow E \propto \frac{1}{r} = r^{-1}$



$\Rightarrow E = \text{constant} = r^{0}$

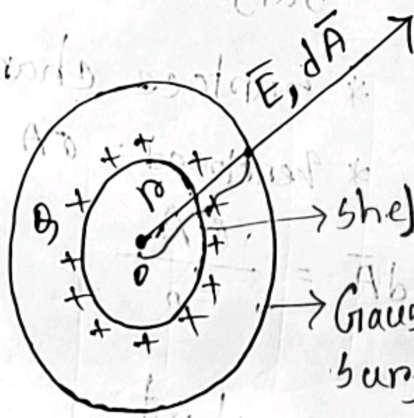
## Spherical Shell / Hollow Sphere: Conducting Surface




$\oint E \cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0} = 0$

$\Rightarrow E_{inside} = 0$

Inside the sphere, $E = 0$




→ shell

→ Gaussian surface

**Outside:**

$\oint E \cdot d\bar{A} = \frac{Q}{\epsilon_0}$

$\Rightarrow E\int dA = E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$

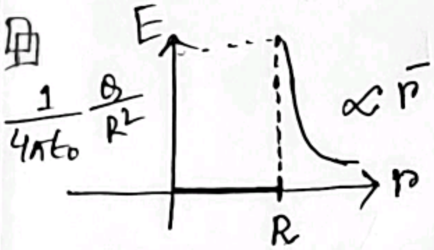
$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

→ It seems that all the charges are at the center of the sphere.

→ Inside the sphere, $E = 0$

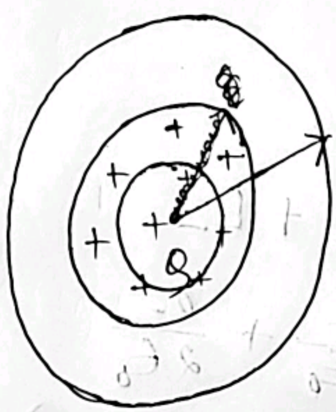
Outside the sphere, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$

At " " , $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2}$ } → 




$r$ = distance from the center of the shell.

## Solid Sphere:




Outside the sphere, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$

At " " , $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2}$

~~Inside~~

Inside " " , $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} \cdot r$

$R$ = Radius of the sphere.


$$\oint E \cdot d\bar{A} = 4\pi r^2 = \frac{q_{enclosed}}{\epsilon_0}$$

Volume charge density $= \rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$$q_{enclosed} = \rho v' = \frac{Q}{4/3\,\pi R^3} \cdot \frac{4}{3}\pi r^3$$

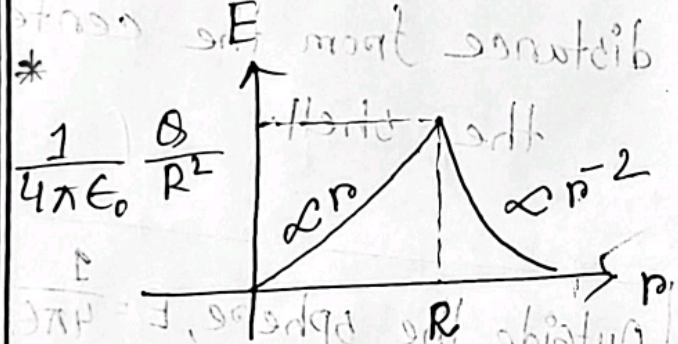
$$\therefore \oint \vec{E}\cdot d\vec{A} = E\cdot 4\pi r^2$$

$$= \frac{q_{enclosed}}{\epsilon_0}$$

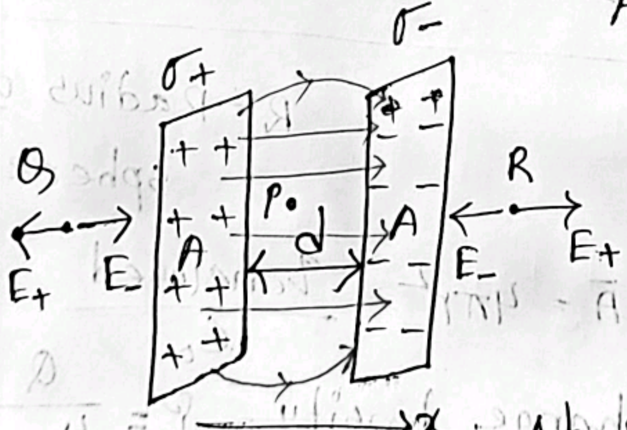
$$= \frac{Q\left(\frac{r}{R}\right)^3}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\,r}$$

*




## Two conducting plate:




$A >> d^2$

At point P:

$$E_P = E_+ + E_-$$

$$= \frac{\sigma_+}{2\epsilon_0} + \frac{\sigma_-}{2\epsilon_0}$$

$$= \frac{1}{2\epsilon_0}(\sigma_+ + \sigma_-)$$

At point Q:

$$E_Q = \frac{\sigma_-}{2\epsilon_0} - \frac{\sigma_+}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_- - \sigma_+)$$

$$E_R = \frac{1}{2\epsilon_0}(\sigma_+ - \sigma_-)$$

If charge densities are same: $\sigma_+ = \sigma_- = \sigma$

$$E_P = \frac{\sigma}{\epsilon_0} \quad , \quad E_R = E_Q = 0$$