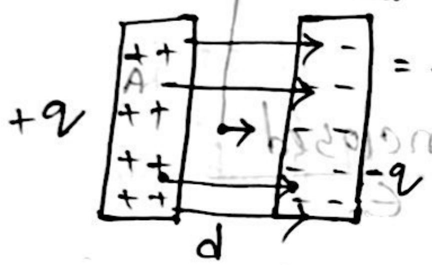
Final

Capacitor: A capacitor is a device which can store energy. How? By storing devices.

Parallel Plate Capacitor:

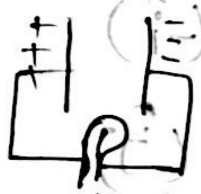
$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$



$$\underbrace{V_f - V_i}_{\text{Potential difference}} = -\int_i^f \vec{E} \cdot d\vec{s}$$

$q \propto V$

$\Rightarrow \boxed{q = CV}$ → Capacitor



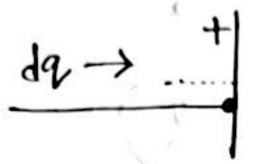
*** Important: Capacitor only depends on the geometry of the capacitor.

$$V_f - V_i = -E\int ds = -Ed$$

$$\therefore V = |V_f - V_i| = Ed = \frac{\sigma d}{\epsilon_0} = \frac{q d}{A\epsilon_0}$$

$$\Rightarrow C = \frac{q}{V} = \frac{A\epsilon_0}{d}$$

Stored Energy: $dW = V\,dq = \frac{q}{C}\,dq$



Work done to store Q charge in the capacitor is,

$$W = \int dW = \int_{q=0}^{Q} \frac{q}{C}\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_{q=0}^{Q}$$

$$\boxed{\therefore W = \tfrac{1}{2}\frac{Q^2}{C}}$$

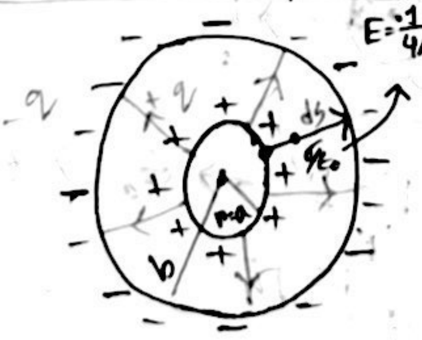
$$\boxed{\begin{aligned} W &= U = \text{stored energy} \\ &= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}VQ \end{aligned}$$

Energy per unit volume:

$$u = \frac{U}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\frac{A\epsilon_0/d}{Ad}V^2$$

$$= \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\epsilon_0 E^2$$

Spherical Capacitor:



$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ // density different & magnitude same.

// $V_f - V_i = \int_i^f \bar{E}\cdot d\bar{s}$

$\Rightarrow V_f - V_i = -\int \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr$

$$= \frac{-q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_a^b$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$$

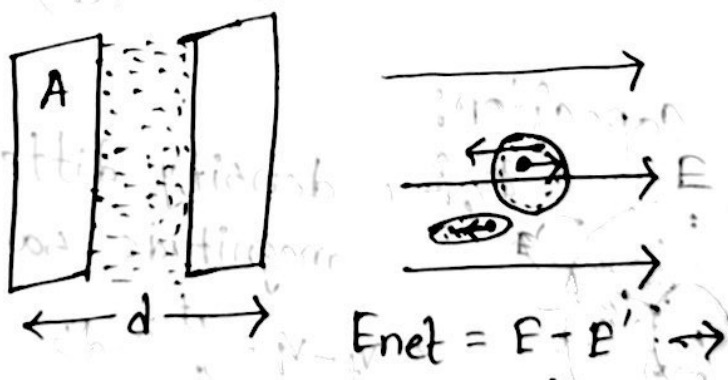
$$V = |V_f - V_i|$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)} = 4\pi\epsilon_0\left(\frac{1}{a}-\frac{1}{b}\right)^{-1}$$

$\Rightarrow$ if $b \to \infty$, $\boxed{C = 4\pi\epsilon_0 a}$ $\to$ radius of that shell.

<u>H.W.:</u> Calculate energy per unit volume for a spherical conductor?



$$E_{net} = E - E' \to \epsilon_0 \to \epsilon$$

$$V_f - V_i = -\int_i^f \bar{E}\cdot d\bar{s}$$

$\epsilon_0 \to \epsilon_0$

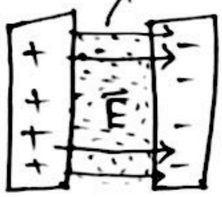
dielectric constant, $k = \frac{\epsilon}{\epsilon_0} = \frac{F_0}{F} \to \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$

$\hookrightarrow \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$

$c = \frac{A\epsilon}{d}$

Capacitor with dielectric :

$\hookrightarrow$ which are not non conductor

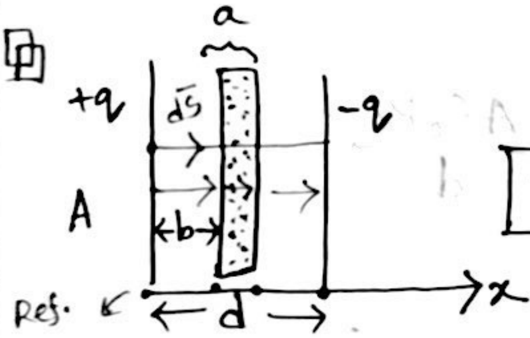


$c = \frac{A\epsilon}{d}$ $\boxed{\epsilon_0 \to \epsilon}$

$\boxed{\epsilon = \epsilon_0 k}$ $\downarrow$ dielectric constant

$k = \frac{\epsilon}{\epsilon_0} = \frac{c}{c_0} \geq 1$

$c_0 \downarrow$ Capacitor with vacuum



$\boxed{q = cv}$

$K = 1 \Rightarrow \epsilon = \epsilon_0 \to$ Vacuum

Potential difference, $v_f - v_i = -\int_i^f \vec{E} \cdot d\vec{s}$

$v_f - v_i = \left[-\int_0^b E_0 \cdot d\vec{s} + \int_b^{a+b} \vec{E} \cdot d\vec{s} + \int_{a+b}^{d} \vec{E}_0 \cdot d\vec{s}\right]$

$= -\left[E_0 \int_0^b d\vec{s} + E\int_b^{a+b} ds + E_0 \int_{a+b}^{d} ds\right]$

$\nearrow$ Vacuum

$E_0 = \frac{q}{A\epsilon_0}$

$E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon}$

$= \frac{q}{A\epsilon_0 K}$

$$\therefore V_f - V_i = -\left[\frac{q}{A\epsilon_0}\cdot b + \frac{q}{A\epsilon_0 K}(a+b-b) + \frac{q}{A\epsilon_0}(d-a-b)\right]$$

$$= -\frac{q}{A\epsilon_0}\left(\frac{a}{k} + d - a\right)$$

$$= -\frac{q}{A\epsilon_0}\left\{a\left(\frac{1}{k}-1\right)+d\right\}$$

$$\therefore V = |V_f - V_i|$$

$$= \frac{q}{A\epsilon_0}\left[d + a\left(\frac{1}{K}-1\right)\right]$$

$$C = \frac{q}{V}$$

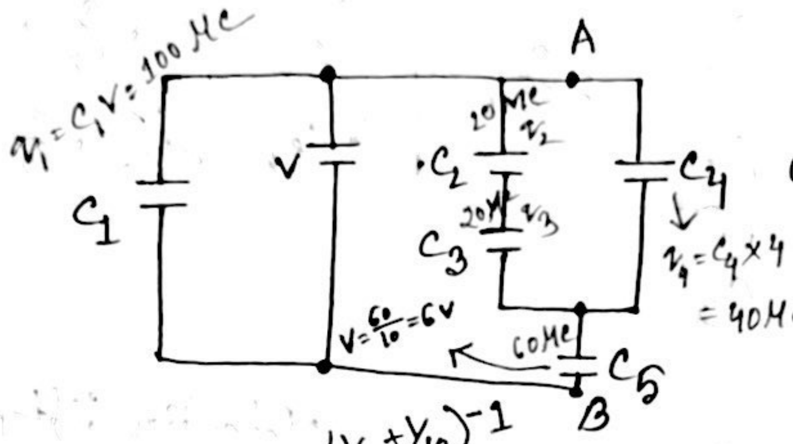
$$= A\epsilon_0\left[d+a\left(\frac{1}{k}-1\right)\right]^{-1}$$

Limiting Case: $(a=d)$, $C = \frac{A\epsilon_0 K}{d}$

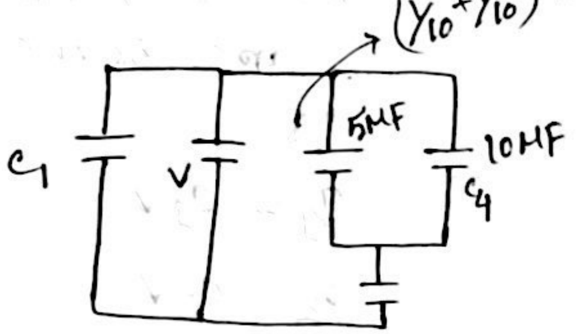
Problem-14:

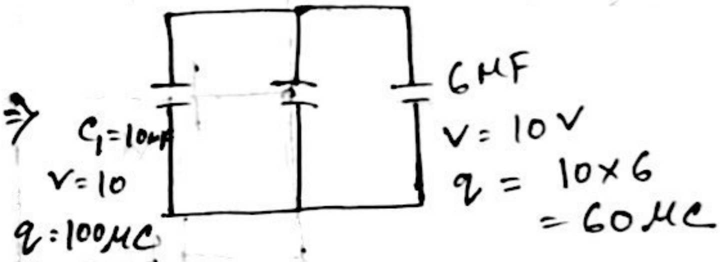
$C_1 = C_2 = C_3 = C_4 = C_5 = 10\,\mu F$

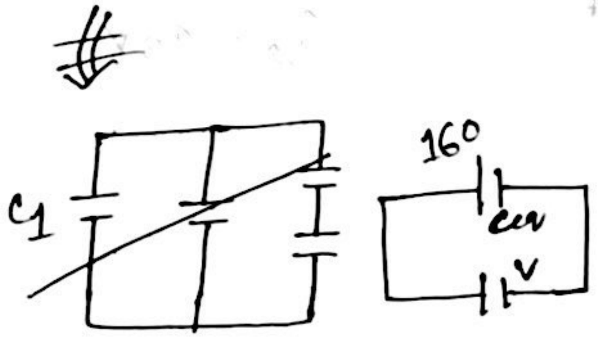
$V = 10\,V$

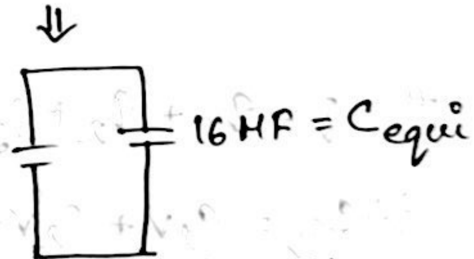


Q: What is change on the capacitors?









$q = CV$

$= 10 \times 16\,\mu F$

$= 160\,\mu C$

Equivalent ~~of~~ capacitor of. C_2 & C_3,

$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = 5\,\mu F$$

$$C_{eq} = \left\{(C_{23} + C_4)^{-1} + C_5^{-1}\right\}^{-1} + C_1$$

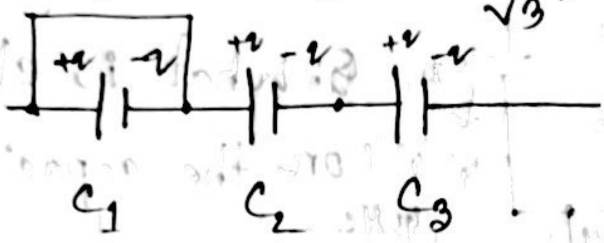
$$= (6 + 10)\,\mu F$$

$$= 16\,\mu F$$

$$q_{eq} = C_{eq} \times V = (10 \times 16)\,\mu C = 160\,\mu C$$

Series: changes are same.

$V_1 = \frac{q}{C_1}$ $V_2 = \frac{q}{C_2}$ $V_3 = \frac{q}{C_3}$

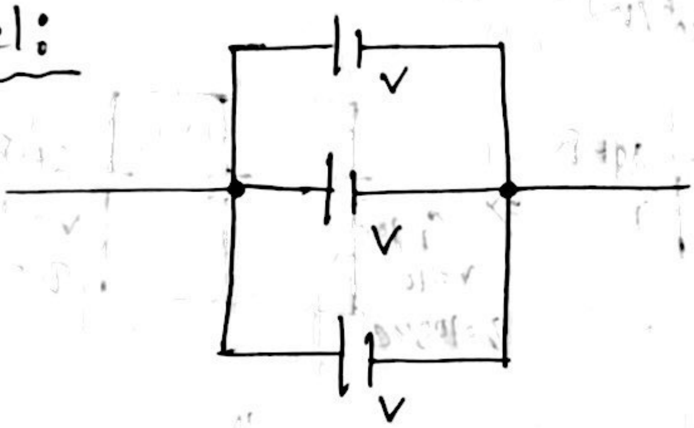


$$V = V_1 + V_2 + V_3$$

$$\Rightarrow \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow C_{eq} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

Parallel:



• Potential differences are same.

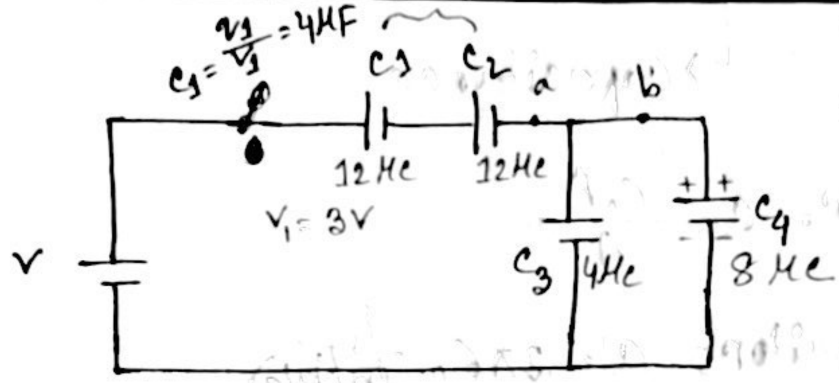
$$q_1 = C_1 V$$
$$q_2 = C_2 V$$
$$q_3 = C_3 V$$

$q = q_1 + q_2 + q_3$

$$\Rightarrow V C_{eq} = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow C_{eq} = C_1 + C_2 + C_3$$

Problem-19: $V_2 = \frac{q_2}{C_2} = \frac{12}{3} = 4V$



$C_2 = 3\ \mu F$

$C_4 = 4\ \mu F$, $V = 9V$

Charge passes through point a & b are 12 μC and 8 μC, respectively. $C_3 = ?$, $C_1 = ?$

$V_4 = \cancel{q_4} \ q_4/C_4 = 2V$, $V_3 = 2V$

$C_3 = \frac{q_3}{V_3} = \frac{4}{2} = 2\ \mu F$

$\cancel{C_{eq} = \left\{ (C_3 + C_4)^{-1} + C_1^{-1} + C_2^{-1} \right\}^{-1}}$

Capacitor: $q = CV \rightarrow$ Potential difference between plates
$\hookrightarrow$ capacitance

Parallel Capacitor: $C = \frac{\epsilon_0 A}{d}$

Cylindrical Capacitor: $C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$

Spherical capacitor: $C = 4\pi\epsilon_0 \frac{ab}{b-a}$

Isolated Sphere: $C = 4\pi\epsilon_0 R$

Series: $V = V_1 + V_2 + V_3$

$$\Rightarrow \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow C_{eq} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

Parallel: $q = q_1 + q_2 + q_3$

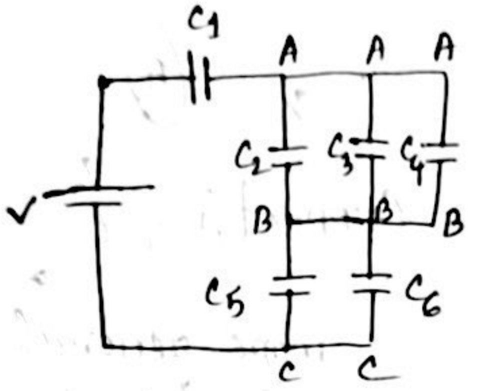
$$\Rightarrow VC_{eq} = C_1V + C_2V + C_3V$$

$$\Rightarrow C_{eq} = C_1 + C_2 + C_3$$

Potential energy: $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$; density $\Rightarrow u = \frac{1}{2}\epsilon_0 E^2$

$$C = \frac{A\epsilon_0 k}{d} ;(a=d)$$

Problem-64:



$V = 10A$

$C_1 = C_5 = C_6 = 6\,\mu F$

$C_2 = C_3 = C_4 = 4\,\mu F$

a) Find net charge at the capacitors.

b) The charge on capacitor 4.

Equivalent capacitor,

$$C_{eq} = \left\{ (C_2 + C_3 + C_4)^{-1} + (C_5 + C_6)^{-1} + \frac{1}{C_1} \right\}^{-1}$$

$$= 3\,\mu F$$

a) So, $q_{eq} = C_{eq} \times V = (3 \times 10)\,\mu C = 30\,\mu C$

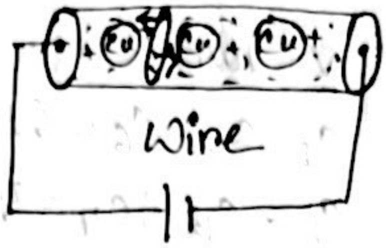
b) $q_{234} = 30\,\mu C$

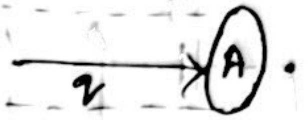
$\Rightarrow V_{234} = \dfrac{q_{234}}{(4+4+4)} = \dfrac{15}{6} v \quad \therefore q_4 = C_4 V_{234} = 4 \times \dfrac{15}{6} = 10\,\mu C$

Chapter.

Current:

Free electron model.





Current, $i = \frac{q}{t}$

more general

$$\boxed{i = \frac{dq}{dt}}$$

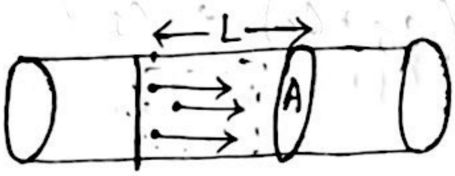
$\mathcal{E}$ = EMF = Electromotive force

$\hookrightarrow$ Unit jule.

Current Density:

$J = \frac{i}{A} \xrightarrow{\text{more general}} i = \int \vec{J} \cdot d\vec{A}$

Drift Speed:



$t = \frac{L}{V_d}$, $i = \frac{q}{t} = \frac{q V_d}{L}$

$q = \boxed{LA}\, e$

number density = $\frac{\text{number}}{\text{volume}}$

$\Rightarrow n \times LA = \text{number}$

$= \frac{nAeLV_d}{L}$

$= nAeV_d$

$$\boxed{\therefore V_d = \frac{i}{nAe} = \frac{J}{ne}}$$

Resistance:

$R = \frac{V}{i}$

Resistivity, $\rho = \frac{E}{J}$

Conductivity, $\sigma = \frac{1}{\rho} = \frac{J}{E}$

$$\therefore \rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{V}{i} \times \frac{A}{L}$$

$$\Rightarrow \boxed{R = \rho \frac{L}{A}}$$

// ρ only depends on the material of the wire.

ρ varies with temperature:

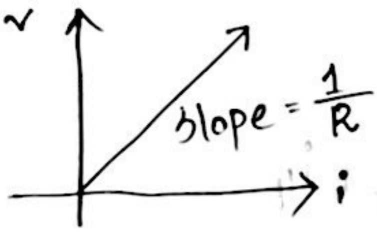
$$\rho_f = \rho_i(1 + \alpha \Delta\theta)$$

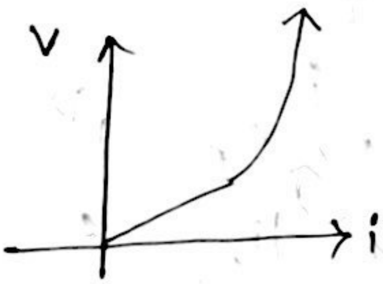
ρ_f = Resistity at final temperature

ρ_i = " " initial "

α = constant $\Delta\theta = \theta_f - \theta_i$

$V = iR$; Ohm's Law.

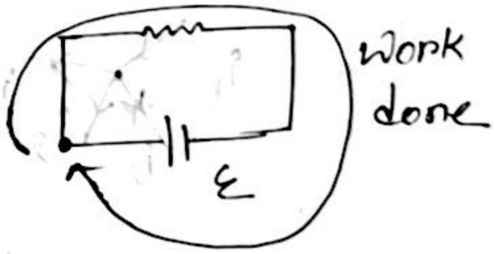




Junction

Ohm's law not work.

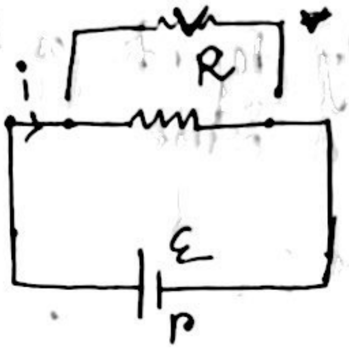
EMF:



Work done by external force to move one 1C charge to the whole circuit.

Circuit:

EMF: Electromotive force



$V = iR$

$V' = ir$

$\mathcal{E} = iR + ir$

- Work done to move 1C charge in a whole circuit.

" If there is no internal resistance, $r = 0$, $\mathcal{E} = iR$

Kirchhoff's law:

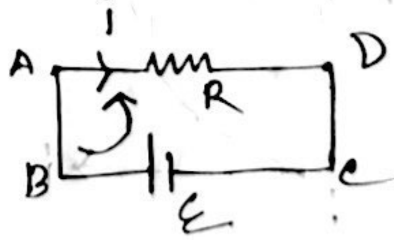
1. Current law (KCL)

- Algebric sum of the currents at any junction of a circuit is zero.

$i_1 + i_3 = i_2 + i_4$

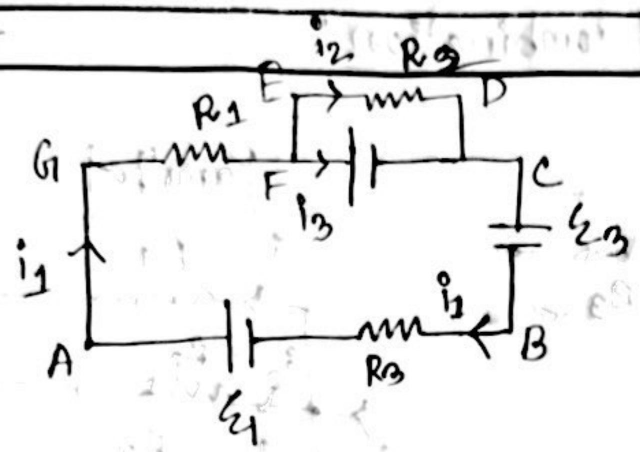
$\Rightarrow i_1 + i_3 - i_2 - i_4 = 0$

Kirchhoff's Voltage law:



ABCDA loop: $-\mathcal{E} + iR = 0$

*



AGFCBA :

$$-i_1R_1 - \mathcal{E}_2 - \mathcal{E}_3 - i_1R_3 + \mathcal{E}_1 = 0$$

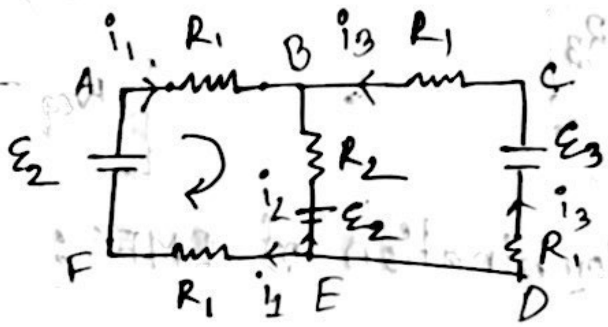
EFCD:

$$-\mathcal{E}_2 + i_2R_2 = 0$$

* at point F: $i_1 = i_2 + i_3$

SP-27.04:

***



$\mathcal{E}_3 = \mathcal{E}_1$

$\mathcal{E}_1 = 3V$

$\mathcal{E}_2 = 6V$

$R_1 = 2\Omega$

$R_2 = 4\Omega$

Loop ABEFA:

$$-i_1R_1 + i_2R_2 - i_1R_1 + \mathcal{E}_1 = 0$$

$$\Rightarrow -2R_1i_1 + R_2i_2 + i_3 \cdot 0 = \mathcal{E}_2 - \mathcal{E}_1 \quad \text{—— (i)}$$

Loop BCDEB : $i_3R_1 - \mathcal{E}_3 + i_3R_1 + \mathcal{E}_2 - i_2R_2 = 0$

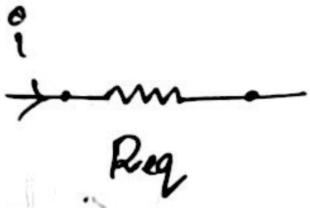
Using KCL at junction B: $i_1 + i_2 + i_3 = 0$

Find the potential difference between A and D.

$$V_D - V_A = -\mathcal{E}_1 + i_1R_1$$

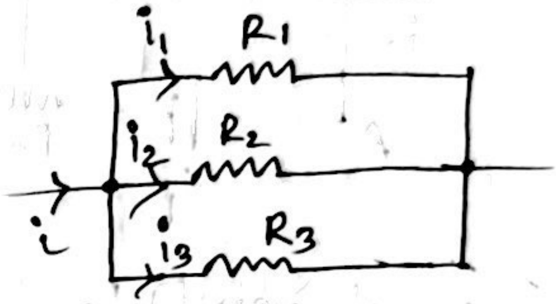
Series Connection / Combination:

⇓



$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

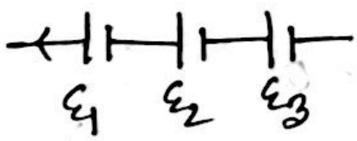
Parallel



⇓

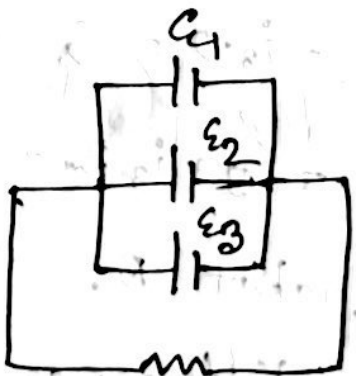
$$\boxed{R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}}$$

Series Combination of EMF'S:



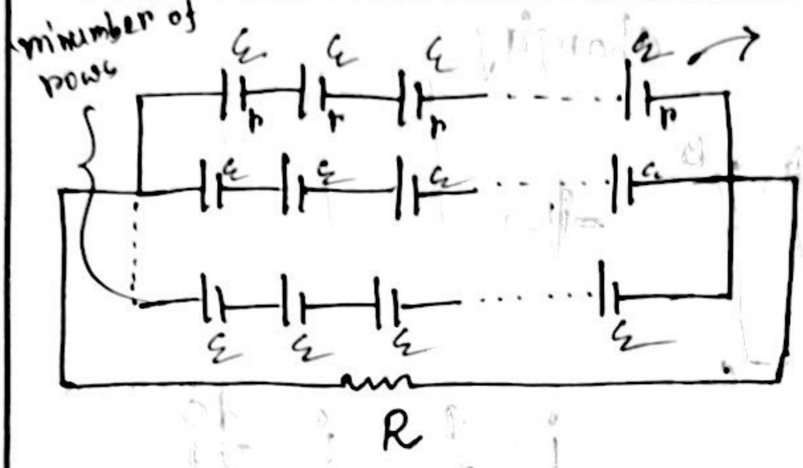
$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3}$$

Parallel:



$$\boxed{\varepsilon_{eq} = \varepsilon}$$

Mix Combination :

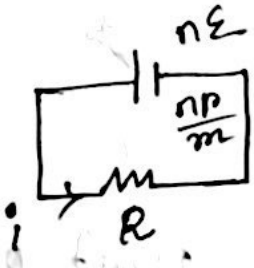


n number of batteries

$$E_{eq} = n\varepsilon$$

$$r_{eq} = \frac{nr}{m}$$

$$r_{eq} = \Big(\underbrace{\frac{1}{nr} + \frac{1}{nr} + \dots + \frac{1}{nr}}_{m \text{ number of terms.}}\Big)^{-1}$$



$$\boxed{i = \frac{n\varepsilon}{\frac{nr}{m} + R}}$$

* Find the condition when i will be maximum?

$$nm = \text{constant} = N$$

$$i(n) = \frac{n\varepsilon}{\frac{nr}{N/n} + R} = \frac{nN\varepsilon}{n^2 R + NR}$$

$$\frac{di}{dn} = 0 = \frac{\cancel{(n^2R+NR)}\,(n^2r+NR) - nN\varepsilon \cdot 2nR}{(n^2r+NR)^2}$$

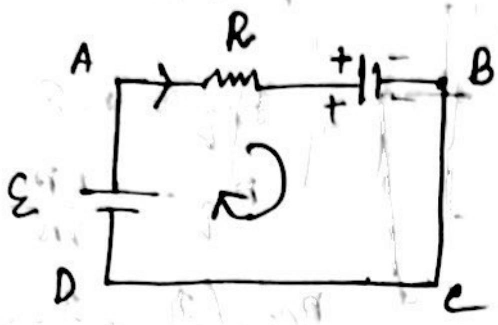
$$\Rightarrow n^2 r + NR - 2n^2 R = 0$$

$$\Rightarrow Nr = n^2 R$$

$$\Rightarrow mr = nR$$

RC Circuit:

Charging



$\cancel{-iR}$

$$i = \frac{q}{t}, \quad i = \frac{dq}{dt}$$

$$-iR - V + \varepsilon = 0$$

$$\Rightarrow -iR - \frac{q}{C} + \varepsilon = 0$$

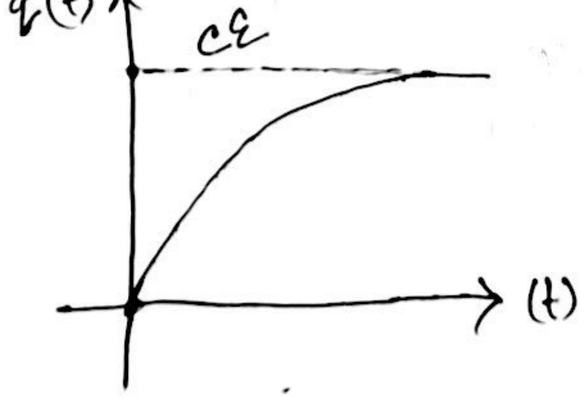
$$\Rightarrow -\frac{dq}{dt}R - \frac{1}{C}q + \varepsilon = 0$$

$$\Rightarrow \frac{dq}{dt} + \left(\frac{1}{RC}\right)q = \frac{\varepsilon}{RC}$$ → First order non-homogenious ordinary differential equation with constant coefficient.

Initial Condition:

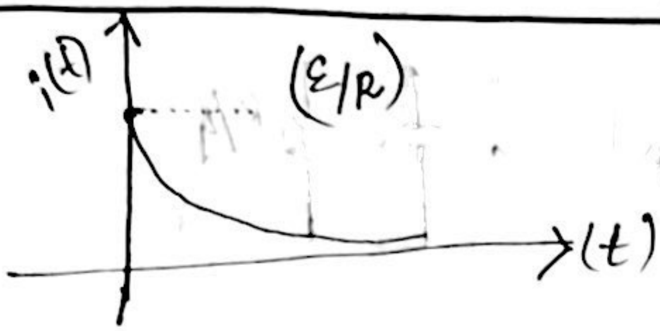
At $t=0$, $q=0$

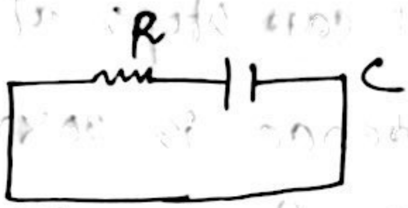
$$\therefore \boxed{q = C\varepsilon\left(1 - e^{-t/RC}\right)}$$ → characteristic time.



$$i = \frac{dq}{dt}$$

$$\boxed{i = \frac{\varepsilon}{R} - e^{-t/RC}}$$





Discharge

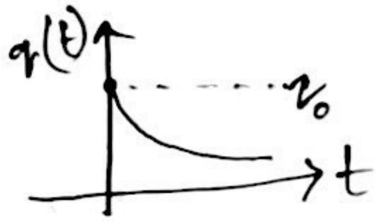
* $-iR - V = 0$

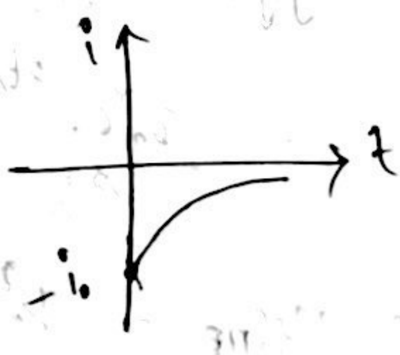
$\Rightarrow -iR - \frac{q}{C} = 0$

Initial condition: $t = 0$, $q = q_0$ = maximum charge

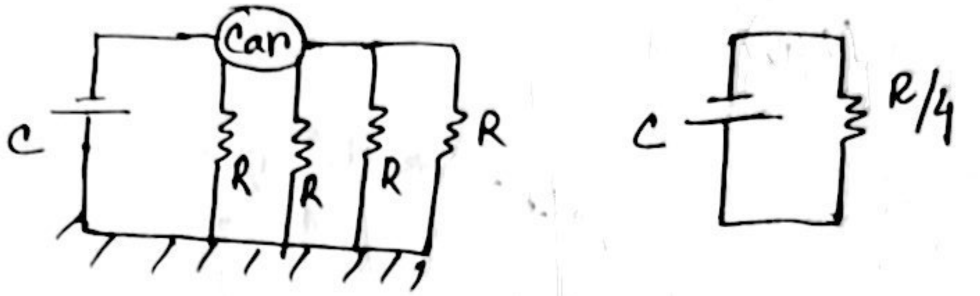
$q(t) = q_0 e^{-t/RC}$

$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right) e^{-t/RC} = -i_0 e^{-t/RC}$





RC Circuit Problem:



when car stops at $t=0$, car ground potential difference is 30 kV. Car ground capacitance 500 pF. $R_{time} = 100\,G\Omega$. How much time the car take to discharge through the times to drop below U_{fire}?

Discharging- $q(t) = q_0 e^{-\frac{t}{RC}}$

V_0 = Potential difference at $t=0$ = 30 kV

Stored energy, $U = \frac{q^2}{2C}$

$$= \frac{q_0^2 e^{-2t/RC}}{2C}$$

Now, $U = U_{fire} = \frac{q_0^2 e^{-2t/RC}}{2C}$

$$\Rightarrow e^{-2t/RC} = \frac{2CU_{fire}}{q_0^2}$$

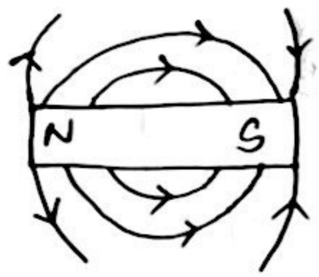
$$\Rightarrow -2t/RC = \ln \frac{2CU_{fire}}{q_0^2} = 9.45$$

$q_0 = C V_0 = 500 \times 10^{-12} \times 30 \times 10^{3}$

V_{fire} = the critical value = 50 m

Chapter-28

Magnetic Field:

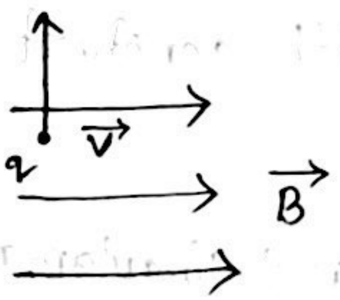


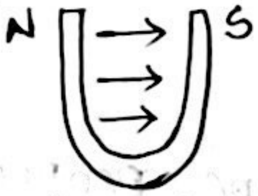
There is no magnetic monopole.

N	S
(+)	(−)

Magnetic Field → $\vec{B}$ → Unit T

Change particle in an external magnetic field:





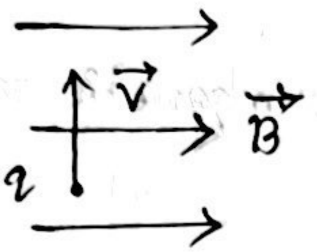
magnetic force on the particle $\vec{F_B} = q(\vec{v} \times \vec{B})$

If both magnetic field and electric field and present, the total force on the charged particle will be : $F_L = q\vec{E} + q(\vec{v} \times \vec{B})$

Lorentz Force: $F_L = q(\vec{E} + \vec{v} \times \vec{B})$

☐ Particle enters an external magnetic field region →

Perpendicular case: —



$$F_B = qvB = F_C = \frac{mv^2}{r}$$

$$\Rightarrow r = \frac{mv}{qB}$$

☐ <u>Charged Particle in an external magnetic field:</u>

⇒ If a charged particle enters an uniform magnetic field ($\vec{B}$) region with constant velocity, $\vec{v}$

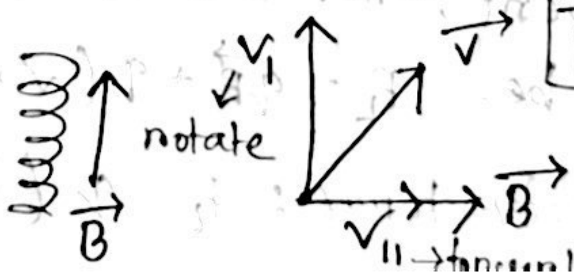
1. $\vec{v}$ and $\vec{B}$ are perpendicular ⇒ Circular motion.

$$\boxed{r = \frac{mv}{qB}}$$

2. If $\vec{v}$ and $\vec{B}$ are <u>not</u> perpendicular ⇒

Helical motion

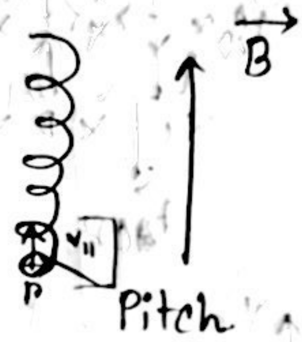
$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$



$v_\perp$ = responsible for circular motion.

$v_\parallel$ = responsible for translation motion.

$$\boxed{r = \frac{m v_\perp}{qB}}$$



Pitch → Distance covered by the charged particle along B in a complete cycle.

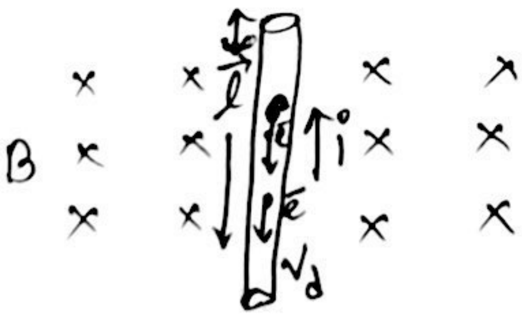
⇒ $v_\parallel T \longrightarrow$ Time Period

⇒ $\boxed{T = \frac{2\pi r}{v_\perp} = \frac{2\pi m}{qB}}$

$$\boxed{\therefore\ P = v_\parallel \cdot \frac{2\pi r}{v_\perp}}$$

$$\boxed{= \frac{2\pi m}{qB} v_\parallel}$$

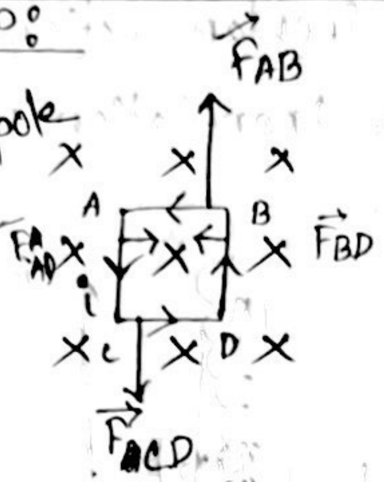
Current carrying wire in a magnetic field :

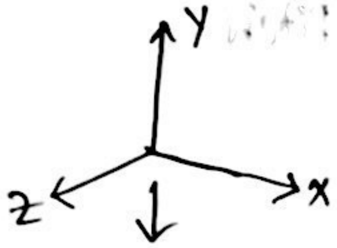


$\vec{F} = i(\vec{l} \times \vec{B})$

$|\vec{l}|$ = length of wire.

Loop:
↓
magnetic dipole





$\vec{B} \to (-\hat{K})$

$l = (-\hat{j})$

$\vec{l} \times \vec{B} = \hat{j} \times \hat{K} = (\hat{i})$

$AD = CD$

$\Rightarrow |\vec{F}_{AD}| = |\vec{F}_{CD}|$

$\vec{\tau} = \vec{m} \times \vec{B}$

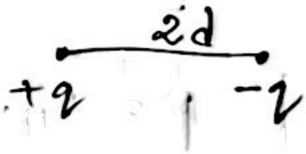
$\vec{m} = \vec{\mu}$ = magnetic dipole moment.

$\vec{m} = N i \vec{A}$

N = Number of turns

$\vec{A}$ = Area vector.

$\vec{\tau} = \vec{P} \times \vec{E}$; $P' = 2dq$



⇒ Potential energy stored in the loop is ——

$U = -\vec{m} \cdot \vec{B}$

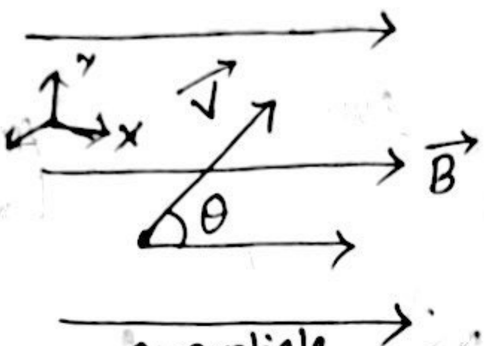
$U = -\vec{P} \cdot \vec{E}$

Problem-4:

$|\vec{v}| = 550\,ms^{-1}$

$|\vec{B}| = 0.0045\,T$

$\theta = 45°$



α-particle

(Helical motion)

as not perpendicular

a. $\vec{F_B} = ?$

b. acceleration ?

c. Does the speed change ?

$\alpha = He^{2+}$

a/

$\vec{F_B} = q(\vec{v} \times \vec{B})$

$|\vec{F_B}| = qvB\sin\theta$

$= \cancel{3.2\times10}$

$= 3.2 \times 10^{-19} \times 550 \times 0.0045 \times \frac{1}{\sqrt{2}}$

$q_\alpha = 2 \times 1.6 \times 10^{-19}$

$\vec{B} = 0.0045\,\hat{i}$

$\vec{v} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$

$= 550\cos45°\,\hat{i} + 550\cos45°\,\hat{j}$

b/

$\cancel{a =}\ |\vec{F_B}| = ma$

$\Rightarrow a = \frac{|\vec{F_B}|}{m}$

c/

$\Delta E_K = W$

$\Rightarrow \frac{1}{2}m(v_f^2 - v_i^2) = W$

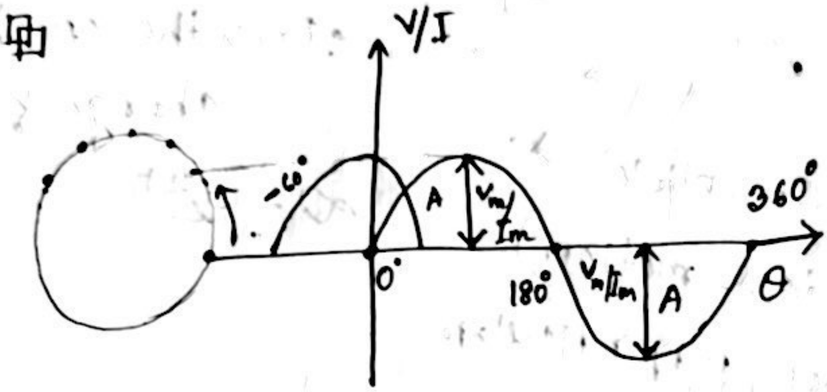
$W = \int \vec{F}\cdot d\vec{x}$

$= 0$

- velocity Change
- speed not change

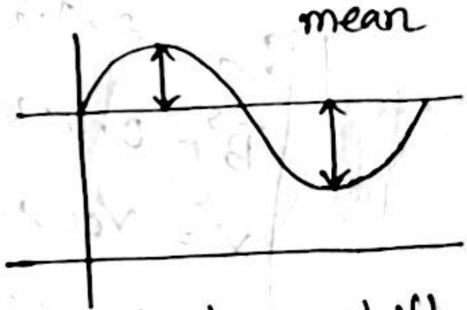
→ Magnetic field cannot change the speed of a particle.

250



$A\sin\theta \to$ angle
↳ Amplitude

$t\omega = \theta$
↳ angular velocity
$rad\,s^{-1}$



$V = V_m \sin\theta$
$I = I_m \sin\theta$

Phase shift

$\Rightarrow V = V_m A \sin(\omega t \pm \phi)$
↑angle

$\Rightarrow V_{(t)} = 10\sin(\underbrace{50t}_{rad} - \underbrace{30°}_{deg})$

$= 10\sin(50t - \frac{\pi}{6})$

$360° = 2\pi$

$\Rightarrow 1 = \frac{2\pi}{360}$

$\Rightarrow 30° = \frac{2\pi \times 30°}{360°} = \frac{\pi}{6}$

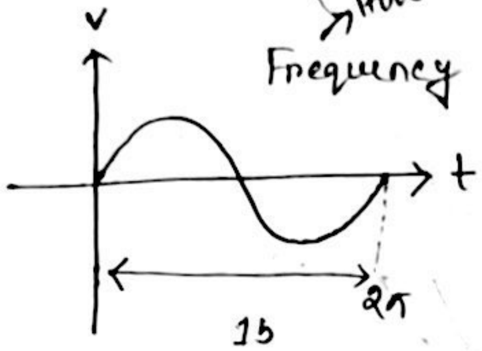
Natural / Linear :

Frequency → How many oscillation in 1 sec

50Hz

$f = 1/T$

$\omega = 2\pi f$



$v = V_m \sin(\omega t - \phi)$

$1\ Hz \rightarrow 2\pi\ rad\ s^{-1}$

$50\ H \rightarrow (2\pi \times 50)\ rad s^{-1}$

$V_m \sin(2\pi f t - \phi)$

$= V_m \sin\left(\frac{2\pi t}{T} - \phi\right)$

$v(t) = 5\sin(100\pi t - 30°)$

$360° \rightarrow 2\pi$

$1 \rightarrow \frac{2\pi}{360} = \frac{\pi}{180°}$, $V_m = 5v$, $\omega = 100\ rad s^{-1}$

$f = 50\ Hz$, $\phi = 30°$, $2\pi f = \omega$

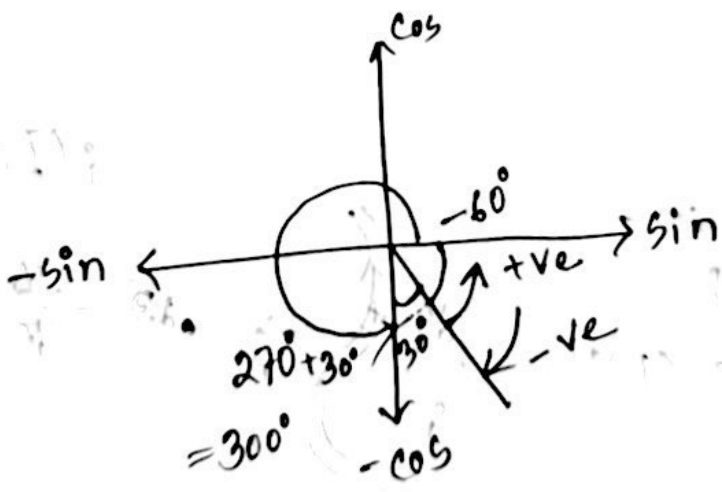
$\Rightarrow f = \frac{\omega}{2\pi}$

$\Rightarrow -180° \leq \phi \leq 180°$

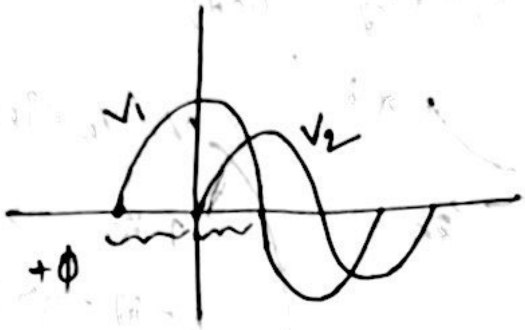
$10\sin(50\pi t - 60°)$

$10\cos(50\pi t - 150°)$

#

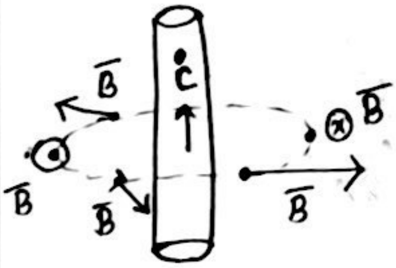


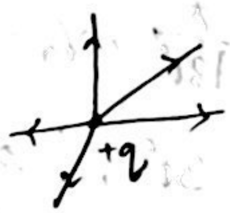
Leading & lagging:



Magnetic field due to current:

chapter 29



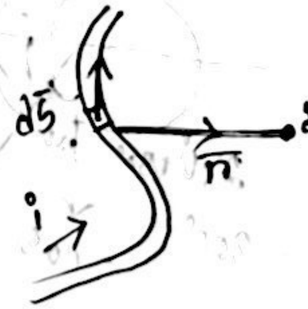


Magnetic fields are along the tangent of this circle.

$i(\bar{l} \times \bar{B})$

Bio savant law:

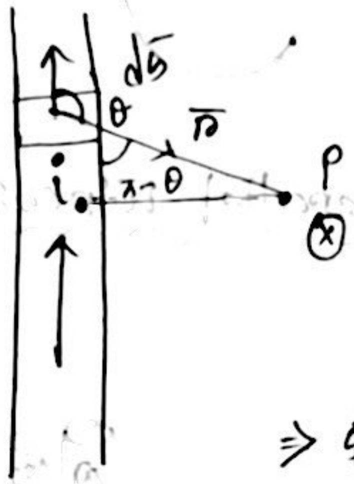
μ_0 = Permiability of free space
$= 4\pi \times 10^{-7} \, TmA^{-1}$



$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{i d\bar{s} \times \hat{r}}{r^2}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

Magnetic field for an infinite / very long straight wire:



$$dB = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\sin\theta}{r^2}$$

$$\therefore B = \int dB = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

$$r^2 = s^2 + a^2$$

$$\Rightarrow \sin(\pi-\theta) = \frac{a}{r}$$

$$\Rightarrow \sin\theta = \frac{a}{r} = \frac{a}{\sqrt{s^2+a^2}}$$

$$\therefore B = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{i\,ds\sin\theta}{r^2}$$

$$= \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty} ds\,\frac{a}{(s^2+a^2)^{3/2}} \rightarrow \frac{2}{a}$$

$$s = a\tan\phi$$

$$ds = a\sec^2\phi\, d\phi$$

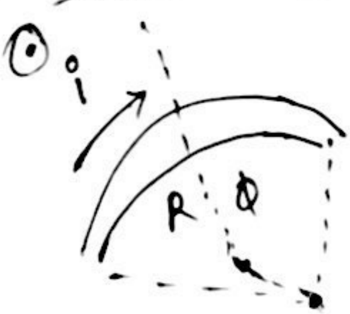
$$s = +\infty,\ \phi = \pi/2$$

$$s = -\infty,\ \phi = -\pi/2$$

$$B = \frac{\mu_0 i}{2\pi a}$$

$$\frac{1}{a}\int_{-\pi/2}^{\pi/2}\cos\phi\, d\phi$$

Magnetic field for a circular Arc:



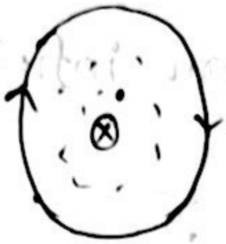
$$B = \frac{\mu_0 i}{4\pi}\frac{\phi}{R}$$

ϕ = Angle produced by the arc at the center.

Circular case:

$\phi = 2\pi$

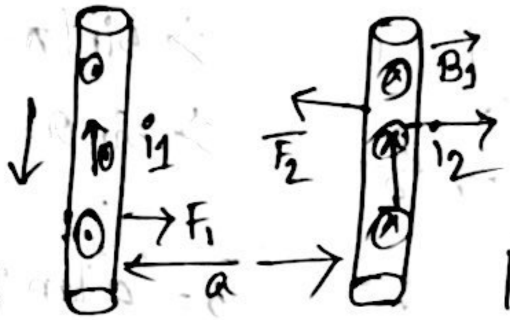
$B = \dfrac{\mu_0 i}{2R}$



Inside the circle, magnetic field is same for every point and its value is $\dfrac{\mu_0 i}{2R}$

Parallel / Antiparallel Wires:

Parallel:



$\vec{F_2} = i_2 \, \vec{l_2} \times \vec{B_2}$

$B_1 = \dfrac{\mu_0}{2\pi a}$

$|\vec{F_2}| = \dfrac{\mu_0}{2\pi} \dfrac{i_1 i_2}{a} l_2$

$|\vec{F_1}| = \dfrac{\mu_0 i_1 i_2}{2\pi a} l_1$

Force per unit length:

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{\mu_0 i_1 i_2}{2\pi a}$$

Parallel currents attract each other.

Ampere's law:

$$\oint \bar{B}\cdot d\bar{s} = \mu_0 i_{elelosed}.$$

$d\bar{s}$ = infinitesimal line element.

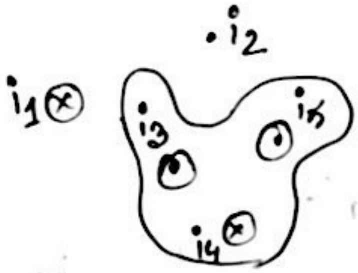
Gauss's law

$$\oint \bar{E}\cdot d\bar{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$\downarrow$

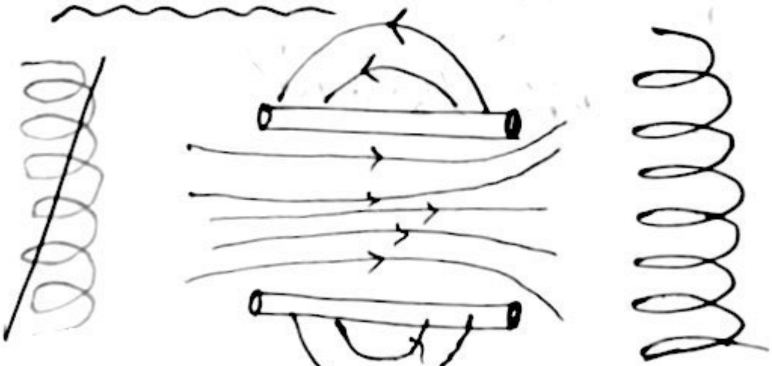
Area surface element.

Anti parallel currents repel each other.

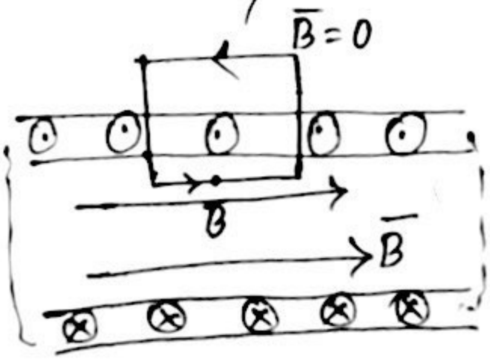


$$\oint \bar{B}\cdot d\bar{s} = \mu_0 (i_3 - i_4 + i_5)$$

Solenoid:



- Inside the solenoid $\bar{B}$ is uniform
- Outside the solenoid $\bar{B} = 0$



$ab = dc = h$

$ad = bc = d$

$$\left(\int_a^b + \int_b^c + \int_c^d + \int_d^a\right)\vec{B}\cdot d\bar{s} = \mu_0 i_{enclosed}$$

$$\int_a^b \bar{B}\cdot d\bar{s} = B\int_a^b ds = Bh$$

$$\int_b^c \bar{B}\cdot d\bar{s} = B\int_b^c ds = 0 \int_d^a \bar{B}\cdot d\bar{s}$$

$$\int_c^d \bar{B}\cdot d\bar{s} = 0$$

$Bh = \mu_0 i_{enclosed}$

per unit

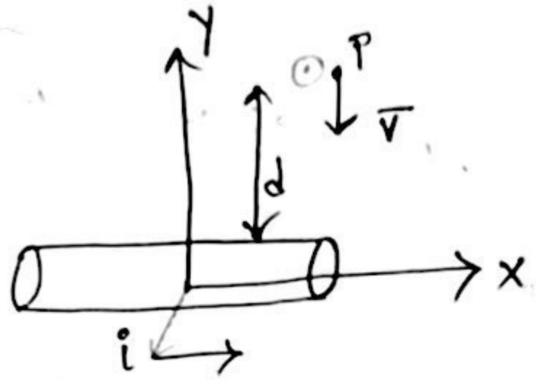
$n =$ number of turns ~~in~~ length ~~h = n~~

Number of turns in h length $= nh$

$$B\cancel{h} = \mu_0 \cdot i \cdot n\cancel{h}$$

$$\boxed{B = \mu_0 n i}$$

Problem-23:



$|\vec{v}| = 200$

$d = 3\ cm$

$i = 100\ mA$

$F_p = ?$

$$B = \frac{\mu_0 i}{2\pi d}$$

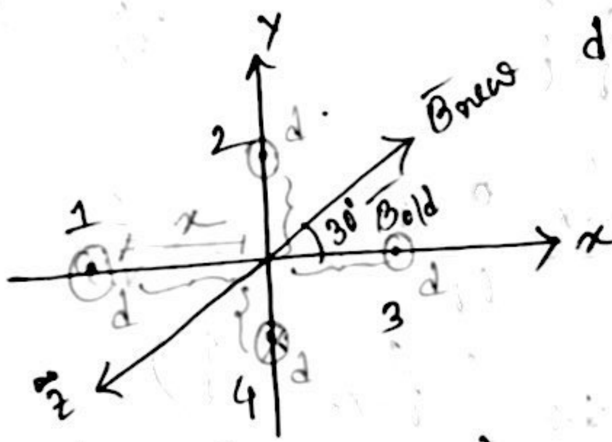
$$\bar{B} = \frac{\mu_0 i}{2\pi d}\hat{k}$$

$$\bar{v} = -200\,\hat{j}$$

$$F_B = q(\bar{v} \times \bar{B})$$

$$= \frac{-200\,q}{2\pi d}\,\hat{i}$$

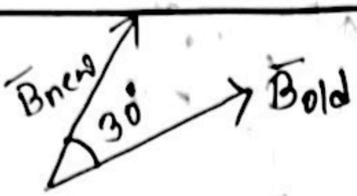
Problem-24:



$d = 10\ cm$

All wires carry same currents.

a) To what value of x must you move wire 1 along the x axis in order to rotate $\bar{B}$ counter-clockwise 30° ?



$$B = \frac{\mu_0 i}{2\pi d}$$

At the origin:

$$\overline{B_1} = \frac{\mu_0 i}{2\pi d}\,\hat{j}$$

$$\overline{B_2} = \frac{\mu_0 i}{2\pi d}\,\hat{i}$$

$$\overline{B_3} = \frac{\mu_0 i}{2\pi d}\,(-\hat{j})$$

$$\overline{B_4} = \frac{\mu_0 i}{2\pi d}\,\hat{i}$$

$$\overline{B_{old}} = 2\times\frac{\mu_0 i}{2\pi d}\,\hat{i}$$

$$\overline{B_1} = \frac{\mu_0 i}{2\pi(d-x)}\,\hat{i}$$

$$\overline{B_{new}} = \overline{B_1'} + \overline{B_2} + \overline{B_3} + \overline{B_4}$$

$$= \frac{\mu_0 i}{2\pi}\left(\frac{2}{d}\hat{i} + \left(\frac{1}{d-x} - \frac{1}{d}\right)\hat{j}\right)$$

$$\tan\theta^{\to 30^\circ} = \frac{\frac{1}{d-x} - \frac{1}{d}}{2/d} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2}{d} = \sqrt{3}\left(\frac{1}{d-x} - \frac{1}{d}\right)$$

$$\Rightarrow \frac{2}{10} = \sqrt{3}\left(\frac{1}{10-x} - \frac{1}{10}\right)$$

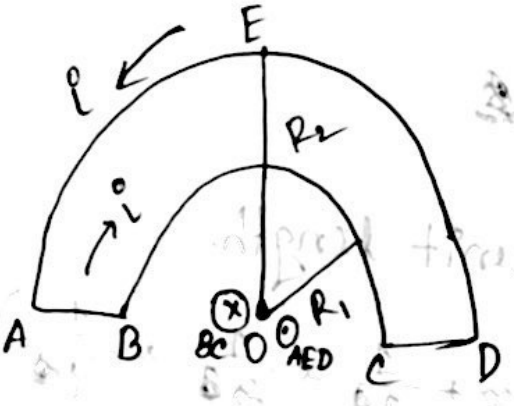
$\therefore\ x =$ Ans.

Due to symmetry wire must be moved toward the x-distance origin by.

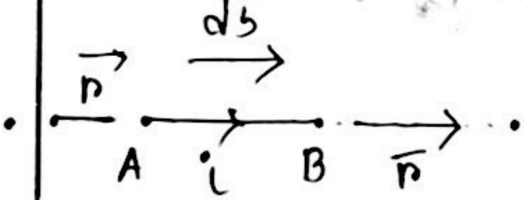
(b) With wire 1 in that position to what value of x must you move wire 3 to rotate it back to the original position.

Magnetic fields due to currents:

Problem-8



• what is the Magnetic field at point O?



$$d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{i\,d\vec{s} \times \hat{r}}{r^2}$$

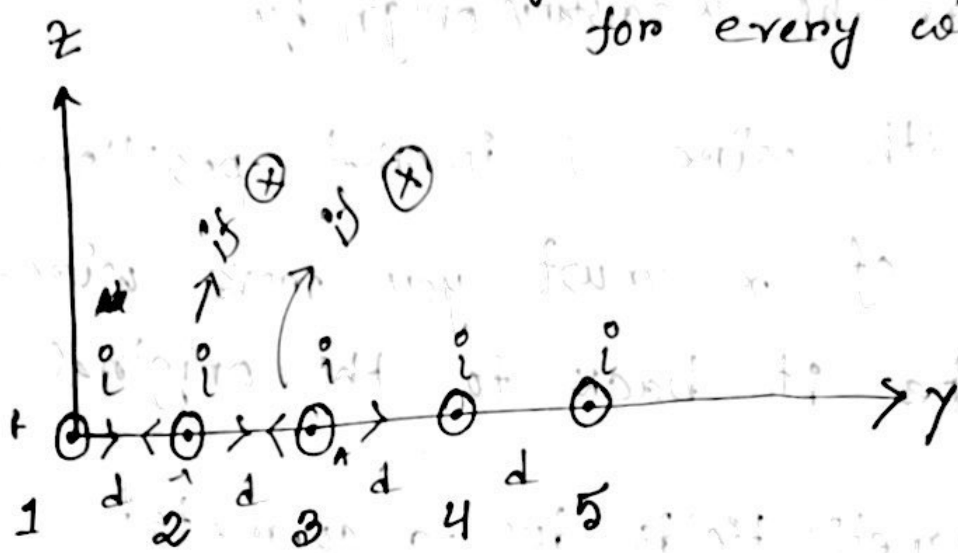
$$B_{BC} = \frac{\mu_0 i}{4R_1} \quad \text{[semi-circular arc]}$$

$$B_{AED} = \frac{\mu_0 i}{4R_2}$$

$\therefore B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and will be directed into the board/page.

Problem: 36

Magnetic force per unit length for every wire?



$$F = \frac{\mu_0 i_1 i_2}{2\pi d}$$

↳ force per unit length.

$$\vec{F_1} = \frac{\mu_0 i^2}{2\pi d}\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$$

$$\vec{F_2} = \frac{\mu_0 i^2}{2\pi d}\left(\frac{1}{2} + \frac{1}{3}\right)$$

$$\vec{F_3} = 0$$

$$\vec{F_4} = -f_2 = \frac{\mu_0 i^2}{2\pi d}\left(\frac{1}{2} + \frac{1}{3}\right)(-\hat{j})$$

$$\vec{F_5} = -F_1$$