Assignment - 02
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Answer to the question no-1

a) Node, $n = 3$

$\therefore$ degree $= n-1 = 2$

$\therefore P_2(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$

when, $x = 2$

$$P_2(x) = a_0 + 2a_1 + 4a_2 = 10$$

when, $x = 4$

$$P_2(x) = a_0 + 4a_1 + 16a_2 = 20$$

when, $x = 6$

$$P_2(x) = a_0 + 6a_1 + 36a_2 = 25$$

Now,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 20 \\ 25 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 8.75 \\ -0.625 \end{bmatrix}$$

Now,

$$P_2(x) = a_0 + a_1 x + a_2 x^2 = V_t(x)$$

$$\therefore a_t(x) = V_t'(x) \text{ where, } x = 7$$

$$= 0 + a_1 + 2 \cdot a_2 x$$

$$= 8.75 + 2 \times (-0.625) \times 7$$

$$= 0 \ ms^{-1}$$

(Ans)

b) here,

	x	$f(x)$
$x_0 =$	2	10
$x_1 =$	4	20
$x_2 =$	6	25

$$\therefore P_2(x) = f(x_0)\, l_0(x) + f(x_1)\, l_1(x) + f(x_2)\, l_2(x)$$

$$l_0(x) = \frac{x-4}{2-4} \times \frac{x-6}{2-6} = \frac{(x-4)(x-6)}{8}$$

$$l_1(x) = \frac{x-2}{4-2} \times \frac{x-6}{4-6} = \frac{(x-2)(x-6)}{-4}$$

$$l_2(x) = \frac{x-2}{6-2} \times \frac{x-4}{6-4} = \frac{(x-2)(x-4)}{8}$$

$$\therefore P_2(x) = 10\left[\frac{(x-4)(x-6)}{8}\right] + 20\left[\frac{(x-2)(x-6)}{-4}\right] + 25\left[\frac{(x-2)(x-4)}{8}\right]$$

$$= \frac{10}{8}(x^2-10x+24) - 5(x^2-8x+12) + \frac{25}{8}(x^2-6x+8)$$

$\therefore P_2(x) = -\frac{5}{8}x^2 + \frac{35}{4}x - 5$

(c)

Currenty, node, $n=3$

if we add 1 more data,

then $n' = 4$

$\therefore$ degree $= n'-1 = \boxed{3}$ (Ans)

So, if a new data point is added, I will use the "Newton's" Divided method for interpolating the new polynomial because of its efficiency.

(Ans)

Answer to the question no-2

$f(x) = 2\cos x + 3\sin x$

$x_0 = -\pi/3 \quad , x_1 = 0 \quad , x_2 = \pi/3$

$n = 2$

Now,

$$|f(x) - P_2(x)| \leq \left| \frac{f''' \eta}{(n+1)!} (x+\tfrac{\pi}{3})(x-0)(x-\tfrac{\pi}{3}) \right|_{max}$$

$$\leq \frac{1}{3!} |f'''(\eta)|_{max} \left| x^3 - \frac{\pi^2}{9} x \right|_{max}$$

here,

$f(\eta) = 2\cos\eta + 3\sin\eta$

$f'(\eta) = -2\sin\eta + 3\cos\eta$

$f''(\eta) = -2\cos\eta - 3\sin\eta$

$f'''(\eta) = 2\sin\eta - 3\cos\eta$

Again,

$w(x) = x^3 - \frac{\pi^2}{9} x$

$w'(x) = 3x^2 - \frac{\pi^2}{9}$

$3x^2 - \frac{\pi^2}{9} = 0$

$\therefore x = \pm \frac{\pi}{3\sqrt{3}}$

$\left| w\left(\frac{\pi}{3\sqrt{3}}\right) \right|_{max} = 0.442011$ $w(x)$ max

$\left| w\left(-\frac{\pi}{3\sqrt{3}}\right) \right| = 0.442011$

Here again,

As the interval is not mentioned, we need to maximize each part of $f'''(\eta)$ to find out $f'''(\eta)_{max}$

$\therefore f'''(\eta) = 2\sin\eta - 3\cos\eta$

$\therefore \max(3\cos\eta) = 3 \cdot \cos(0)$
$= 3$

$\max(2\sin\eta) = 2\sin(\pi/2)$
$= 2$

$\therefore |f'''(\eta)|_{max} = |2-3|_{max}$
$= 1$

finally,

$|f(x) - P_2(x)| \leq \frac{1}{3!} \times 1 \times 0.442011$
≤ 0.0736685 (Ans)