Assignment - 03

22299079_12

1

Given, $f(x) = x \cdot \ln x$

a) $F.D = \frac{f(x+h) - f(x)}{h}$ here, $x=1$, $h=0.1$

$= \frac{f(1+0.1) - f(1)}{0.1}$

$= 1.0484$

$C.D = \frac{f(x+h) - f(x-h)}{2h}$

$= \frac{f(1+0.1) - f(1-0.1)}{2 \times 0.1}$

$= 0.99833$

(Ans)

b) $f(x) = x \cdot \ln x$

$f'(x) = 1 + \ln x$

$f''(x) = \frac{1}{x}$

$f'''(x) = -\frac{1}{x^2}$

at $x = 1$, $h = 0.1$

$f''(x-h) = \frac{1}{1-0.1} = \frac{10}{9} \rightarrow max^m$

$f''(x) = 1$

$f''(x+h) = -\frac{100}{121}$

$$f'''(x-h) = -\frac{1}{(1-0.1)^2} = -1.23456$$

$\therefore$ Upper bound of truncation error,

$$\text{for B.D} = \frac{f''(\eta)}{2} \times h$$

$$= \frac{10/9}{2} \times 0.1$$

$$= 0.55556$$

$$\text{for C.D} = \frac{f'''(\xi)}{3!} \times h^2$$

$$= \frac{1.23456}{6} \times (0.1)^2$$

$$= 0.0020567 \text{ (Ans)}$$

c) $$D_h^{\circ} = \frac{f(x+h) - f(x-h)}{2h}$$

$$= f'(x) + \frac{f^3(x)}{3!} h^2 + \frac{f^5(x)}{5!} h^4 + O(h^6) \quad \text{--- (I)}$$

$$D_{\frac{4h}{3}} = f'(x) + \frac{f^3(x)}{3!}\left(\frac{4h}{3}\right)^2 + \frac{f^5(x)}{5!}\left(\frac{4h}{3}\right)^4 + O\left(\left(\frac{4h}{3}\right)^6\right)$$

$$= f'(x) + \frac{f^3(x)}{3!}\frac{16h^2}{9} + \frac{f^5(x)}{5!}\frac{16 \cdot 16 h^4}{9 \cdot 9} + O(h^6)$$

$$\frac{9}{16} D_{\frac{4h}{3}} = \frac{9}{16} f'(x) + \frac{f^3(x)}{3!} h^2 + \frac{f^5(x)}{5!}\frac{16h^4}{9} + O(h^6) \quad \text{--- (II)}$$

(ii) - (i) $\Rightarrow$

$$\frac{9}{16} D_{\frac{4h}{3}} - D_h = \frac{9}{16} f'(x) - f'(x) + \frac{f^5(x)}{5!}\left(\frac{16}{9} - 1\right)h^4 + O(h^6)$$

$$= \left(\frac{9}{16} - 1\right) f'(x) + \frac{f^5(x)}{5!} h^4 \left(\frac{16}{9} - 1\right) + O(h^6)$$

$$\Rightarrow \frac{\frac{9}{16} D_{\frac{4h}{3}} - D_h}{\frac{9}{16} - 1} = f'(x) + \frac{\left(\frac{16}{9} - 1\right) f^5(x)}{\left(\frac{9}{16} - 1\right) 5!} h^4 + O(h^6)$$

$$\therefore D_h' = f'(x) - \frac{16}{9} \frac{f^5(x)}{5!} h^4 + O(h^6)$$

(Ans)

d) $Dh' = f'(x) - \frac{16}{9} \frac{f^5(x)}{5!} h^4 + O(h^6)$

Upper bound of truncation error,

$$= -\frac{16}{9} \frac{f^5(x)}{5!} h^4$$

$f^3(x) = -\frac{1}{x^2}$

$f^4(x) = \frac{2}{x^3}$

$f^5(x) = -\frac{6}{x^4}$

$\hookrightarrow f^5(0.9) = -9.1445$

$\therefore f^5(1.1) = -4.0980$

∴ upper bound trancation error,

$$= -\frac{16}{9}\frac{f^5(0.9)}{5!}h^4$$

$$= -\frac{16}{9}\frac{-9.1445}{5!}(0.1)^4$$

$$= 0.000013547 \text{ (Ans)}$$

2

$$f(x) = x\cos x - x + \sin x$$

$$f'(x) = -x\sin x + \cos x - 1 + \cos x$$

$$= -x\sin x + 2\cos x - 1$$

$$f'(1.2) = -1.2\sin(1.2) + 2\cos(1.2) - 1$$

$$= -1.39373$$

$$C.D = \frac{f(1.3) - f(1.1)}{2(0.1)} = -1.39445$$

Relative error $= |(\text{Traditional } f'(1.2) - \text{C.D} f'(1.2)|$

$$= |-1.39\cancel{3}73 - (-1.39445)|$$

$$= 0.00007200 \text{ (Ans)}$$

3

a) Given, $f(x) = 4x^3 - 9x^{7x}$

$$f'(x) = \frac{f(2.8) - f(2.6)}{2(0.1)}$$

here, $x = 2.7$, $h = 0.1$

$= -1.10258 \times 10^{10}$

$$f'(x) = \frac{f(2.9) - f(2.5)}{h(0.2)}$$

here, $x = 2.7$, $h = 0.2$

$= -1.3839 \times 10^{10}$

$$\therefore D'_{0,2} = \frac{2^2(-1.10258 \times 10^{10}) - (-1.3839 \times 10^{10})}{2^2 - 1}$$

$= -1.009 \times 10^{10}$ (Ans)

b) $D_h^2 = \dfrac{2^4 D'_{\frac{h}{2}} - D'_h}{2^4 - 1}$

$D'_{\frac{h}{2}} = D'_{0.1} = \dfrac{2^2 D_{h/2} - D_h}{2^2 - 1}$

$D_{h/2} = D_{0.05} = \dfrac{f(2.7+0.05) - f(2.7-0.05)}{2(0.05)}$

$= -1.03833 \times 10^{10}$

$D'_{0.1} = \dfrac{2^2(-1.03833\times 10^{10}) - (-1.10258\times 10^{10})}{2^2 - 1}$

$= -1.0169 \times 10^{10}$

$D^2_{0.2} = \dfrac{2^4 D_{0.1} - D'_{0.2}}{2^4 - 1}$

$= \dfrac{2^4(-1.0169\times 10^{10}) - (-1.009\times 10^{10})}{2^4 - 1}$

$= -1.017 \times 10^{10}$ (Ans)