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Fitting rotational frequency of polytropes as a function of their oblateness (unsubmitted version)

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ABSTRACT

(This preliminary version for RNAAS submission fits all the polytropic data ($1 \leq N \leq 3$) with a single Padé formula, as well as a single polynomial.)
Giving a simple functional relation between the rotational frequency and the deformation for compressible stellar models is useful both in observational and theoretical astrophysics. Here I present simple formulae for the rotational frequency of polytropic stars as a function of the oblateness, which is valid for a different polytropic index in $1 \leq N \leq 3$ and the speed of rotation up to the mass-shedding limit.

Keywords: Stellar rotation

In Yoshida (2022), I presented assessment of the Roche and the Darwin-Radau approximation which are often used in studying rotating stars. Here I report a simple formula of a dimensionless stellar angular frequency ω as a function of its oblateness, which is derived by fitting numerical hydrostatic equilibria for a range of polytropic equation of state (EOS).

The equilibrium models are computed by a numerical code for rotating hydrostatic two-dimensional equilibria. The code is written by the author and tested against the existing results (Hachisu 1986). The code self-consistently takes into account the inhomogeneous gas distribution and its self-gravity. The assumption of slow-rotation is not necessary and the models up to the mass-shedding limit are obtained.

I focus on the property of rotating polytropes whose EOS is

$$p = K\rho^{1+\frac{1}{N}}, \tag{1}$$

where p and ρ are the pressure and the density. K is a constant and N is a constant called polytropic index.

Given N, the rotational angular frequency ω normalized by the factor $\sqrt{GM/R_e^3}$ is fitted, where M is the mass and R_e is the equatorial radius. As for the independent parameter, the normalized oblateness, $f_{\rm obl}/f_{\rm obl,MS} \equiv \tilde{f}_o$, is chosen, where $f_{\rm obl} = 1 - R_p/R_e$ and R_p is the polar radius or the object. Notice that $0 \leq \tilde{f}_o \leq 1$. $f_{\rm obl,MS}$ is the oblateness at the mass-shedding limit for the N value. The oblateness at the mass-shedding limit depends on the EOS. Thus I also make a fitting formula of $f_{\rm obs,MS}(\Gamma)$, where $\Gamma = 1 + 1/N$.

Assumed functional form of the fitting formula of ω^2 is the Padè form,

$$\omega^2(\tilde{f}_o) = \frac{\tilde{f}_o(a_0 + a_1\tilde{f}_o)}{1 + a_2\tilde{f}_o} \tag{2}$$

and that of $f_{\rm obl,MS}$ as,

$$f_{\rm obl,MS}(\Gamma) = \frac{b_0 + b_1\Gamma}{1 + b_2\Gamma} \tag{3}$$

I use the nonlinear least square fitting module of `scipy.optimize.curve_fit` to fit the numerical data. [1] For the fitting of Eq.(2), I compute 694 models of polytropes with $N = (1.0, 1.1, 1.2, 1.4, 1.5, 1.6, 1.8, 2.0, 2.2, 2.5, 3.0)$ from non-rotating to maximally rotating (mass-shedding) cases.

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[1] https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html

The left panel of Fig.1 shows the fitting of the ω^2 data with an analytic form. All the data of different N is weighted equally in the fitting. It is seen that the simple form of Eq.(2) approximates the numerical data within a few percent of errors, though the error increases steeply as the model approaches the mass-shedding limit. The right panel of Fig.1 is $f_{\rm MS}$ and its fitting curve.

The fitting parameters obtained for ω^2 are,

$$a_0 = 0.6042,\ a_1 = -0.07372,\ a_2 = -0.4891, \tag{4}$$

and for $f_{\rm MS}$ are,

$$b_0 = 0.1820,\ b_1 = 0.05912,\ b_2 = -0.1619. \tag{5}$$

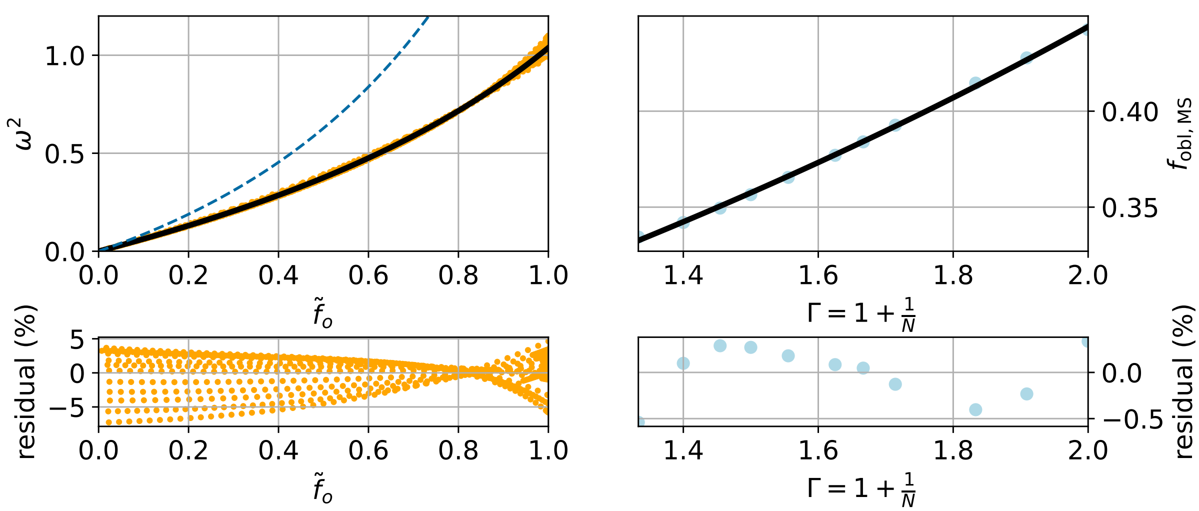

Figure 1. Left: Normalized ω^2 versus normalized oblateness $\tilde{f}_o$. Orange points are numerical data and the solid black line is the fitted formula by Eq.(2). The dashed line is the curve for the incompressible Maclaurin spheroid (Chandrasekhar 1987) in our normalization (for which $f_{\rm obl,MS} = 1$) . The bottom panel shows the relative residuals (in percent) of the fitting. Notice that the large residual for $\tilde{f}_o \to 0$ is an artifact of the behavior of $\omega^2 \to 0$. Right: The oblateness at the mass-shedding limit versus the adiabatic index Γ. The light blue points are numerical data, while the solid black line is the fitted formula by Eq.(3). The bottom panel shows the residuals.

Alternatively, if a simple polynomial form,

$$\omega^2 = \tilde{f}_0 \sum_{j=0} p_j \tilde{f}_o^j, \tag{6}$$

is chosen, a fitting with the minimal number of terms is done in a similar size of the error with,

$$p_0 = 0.6020,\ p_1 = 0.2655,\ p_2 = -0.04464,\ p_3 = 0.2148. \tag{7}$$

REFERENCES

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