Homework #1

Task description:

Your report should include:
- Codes you have run to obtain the results
- Graphs and Tables of results
- Discussion on results

1. Consider the cost function:

$$f(x) = x_1 \exp\{-(x_1^2 + x_2^2)\} + \frac{1}{20}(x_1^2 + x_2^2) \tag{1}$$

First, plot the contour of the cost function in $x_1 - x_2$-plane. Then, implement the steepest descent and Newton's method to find local (or global) minimum. For comparison, consider three initial guess:

$$(i) \quad x_0 = \begin{bmatrix} -0.4 \\ 0.6 \end{bmatrix}, \quad (ii) \quad x_0 = \begin{bmatrix} 1.0 \\ 0.1 \end{bmatrix}, \quad (iii) \quad x_0 = \begin{bmatrix} 1.6 \\ 1.9 \end{bmatrix}$$

Explain what happens in the results. (Finding local minima or not? How many iterations? Does the Hessian positive or not? etc)

2. Consider the cost function (1) subject to the inequality constraint:

$$g(x) = \frac{x_1 x_2}{2} + (x_1 + 2)^2 + \frac{(x_2 - 2)^2}{2} - 2 \leq 0$$

Find the minimum which satisfies the constraint by the numerical methods. Explain the methods you have used to obtain the results.

Tips

Stepsize

There are variations of rules for selecting stepsize $\alpha^{(k)}$:

Constant stepsize $\alpha(k) = a > 0$. (may not converge if it is too large or may converge too slow if it is too small)

Exact line search choose $\alpha^{(k)} \in [0, b]$ so that f is minimized, i.e.

$$f(x^{(k)} + \alpha^{(k)} d^{(k)}) = \min_{\alpha \in [0,b]} f(x^{(k)} + \alpha d^{(k)})$$

which involves a one-dimensional search (line search) over $[0, b]$.

Successive step size reduction (Armijo rule) Start with $\alpha_k = 1 > 0$, $0 < \beta < 1$ and $0 < \sigma < 1$. If

$$f(x^{(k)}) - f(x^{(k)} + \alpha_k d^{(k)}) \geq -\sigma \alpha_k \nabla f(x^{(k)})^T d^{(k)}$$

then stop. If not, set $\alpha_k \leftarrow \beta \alpha_k$ and repeat. In practice the following choices are used: β : 1/2 to 1/10, $\sigma \in [10^{-5}, 10^{-1}]$ (see Fig.1). The steepest descent algorithm will demonstrate global convergence properties under the Armijo rule.

Regularized Newton's method

In theory, Newton's method requires a positive definite Hessian to work. An simple modification of Newton's method is so called regularization, i.e. to add a regularizing term to Hessian to be positive definite. (You can also implement BFGS rule.)

Let $H = \nabla^2 f(x^{(k)})$ and $\lambda_i,\ i = 1, 2$ be eigenvalues of H. A simple way to add regularizing term is

$$H_{new} = H + (|\lambda_{min}| + \delta) I$$

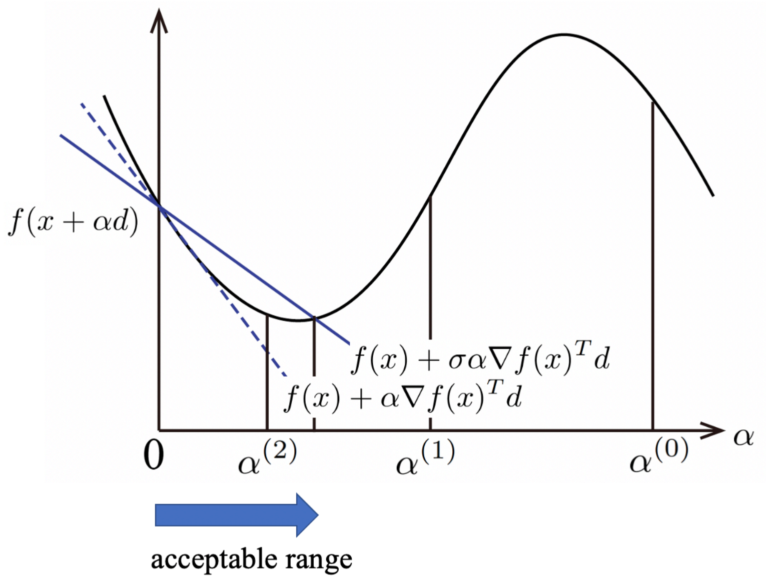

Figure 1: Armijo rule

where $\delta > 0$ is a small constant.

Due: May 16 (Fri.)
Please submit the report in .pdf format on Moodle.