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Homework #11

$$
\begin{aligned}
e ::= & \cdots \\
& \mid \{\text{throw}\} \\
& \mid \{\text{try } e \text{ catch } e\} \\
& \mid \{\text{withtype } \{t\ \{x\ \tau\}\ \{x\ \tau\}\}\ e\} \\
& \mid \{\text{cases } t\ e\ \{x\ \{x\}\ e\}\{x\ \{x\}\ e\}\}
\end{aligned}
$$

$$
\begin{aligned}
\tau ::= & \cdots \\
& \mid t \\
& \mid \text{anyT}
\end{aligned}
$$

a) Write the operational semantics of the form $\boxed{\sigma \vdash e \Rightarrow v}$ for the expressions

$$\sigma \vdash \{\text{throw}\} \Rightarrow exception$$

$$\frac{\sigma \vdash e_1 \Rightarrow v}{\sigma \vdash \{\text{try } e_1 \text{ catch } e_2\} \Rightarrow v}$$

$$\frac{\sigma \vdash e_1 \Rightarrow exception \quad \sigma \vdash e_2 \Rightarrow v}{\sigma \vdash \{\text{try } e_1 \text{ catch } e_2\} \Rightarrow v}$$

$$\frac{\sigma[x_1 \mapsto constructorV(false),\ x_2 \mapsto constructorV(true)] \vdash e \Rightarrow v}{\sigma \vdash \{\text{withtype } \{t\ \{x_1\ \tau_1\}\ \{x_2\ \tau_2\}\}\ e\} \Rightarrow v}$$

$$\frac{\sigma \vdash e_0 \Rightarrow variantV(false, v) \qquad \sigma[y_1 \mapsto v] \vdash e_1 \Rightarrow v'}{\sigma \vdash \{\text{cases } t\ e_0\ \{x_1\ \{y_1\}\ e_1\}\{x_2\ \{y_2\}\ e_2\}\} \Rightarrow v'}$$

b) Write the typing rules for the expressions

$$\Gamma \vdash \{\text{throw}\} : anyT$$

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \{\text{try } e_1 \text{ catch } e_2\} : \tau}$$

$$\frac{\Gamma' = \Gamma[t = x_1@\tau_1 + x_2@\tau_2,\ x_1 : (\tau_1 \to t), x_2 : (\tau_2 \to t)] \quad \Gamma' \vdash \tau_1 \quad \Gamma' \vdash \tau_2 \quad \Gamma' \vdash e : \tau_0}{\Gamma \vdash \{\text{withtype } \{t\ \{x_1\ \tau_1\}\ \{x_2\ \tau_2\}\}\ e\} : \tau_0}$$

$$\frac{\Gamma = [\ldots t = x_1@\tau_1 + \ x_2@\tau_2 \ldots] \quad \Gamma \vdash \ e_0 : t \quad \Gamma[x_3{:}\tau_1] \vdash e_1 : \ \tau_0 \quad \Gamma[x_4{:}\tau_2] \vdash e_2 : \ \tau_0}{\Gamma \vdash \{\text{cases} \ t \ \ e_0 \ \ \{x_1 \{x_3\} \, e_1\} \{x_2 \ \ \{x_4\} \ \ e_2\}\} : \ \tau_0}$$

c) Draw the type derivation of the following expression:

```
{withtype {fruit {apple num} {banana (bool -> num)}}
          {cases fruit {banana {fun {x:num} 3}}
             {apple  {n} n}
             {banana {f} {f true}}}}
```

$$\frac{\Gamma' = \Gamma\begin{bmatrix} fruit = apple@\text{num} + banana@(\text{bool} \to \text{num}), \\ apple{:}\,(\text{num} \to fruit), \\ banana{:}\,((\text{bool} \to \text{num}) \to fruit) \end{bmatrix} \quad \Gamma' \vdash \text{num} \quad \Gamma' \vdash (\text{bool} \to \text{num}) \quad \dfrac{\begin{array}{c}\Gamma'[\ldots fruit = apple@\text{num} + banana@(\text{bool} \to \text{num}) \ldots] \\ \Gamma' \vdash \ \{banana \ \{fun \ \ \{x{:}\,\text{num}\} \ 3\}\}{:}\ fruit \\ \Gamma'[n{:}\,\text{num}] \vdash n : \text{num} \\ \Gamma'[f{:}\,(\text{bool} \to \text{num})] \vdash \{f \ true\} : \text{num}\end{array}}{\begin{array}{l}\Gamma' \vdash \{\text{cases} \ fruit \ \{banana \ \{fun \ \ \{x{:}\,\text{num}\} \ \ 3\}\} \\ \qquad \{apple \quad \{n\} \, n\} \\ \qquad \{banana \ \{f\} \, \{f \ true\}\}\} :\end{array} \ \text{num}}}{\begin{array}{l}\Gamma \vdash \{\text{withtype} \ \{fruit \ \ \{apple \ \text{num}\} \{banana \ \ (\text{bool} \to \text{num})\}\} \\ \qquad \{\text{cases} \ fruit \ \ \{banana \ \ \{fun \ \ \{\text{x}{:}\,\text{num}\} \ \ 3\}\} \\ \qquad\quad \{apple \quad \{n\} \, n\} \\ \qquad\quad \{banana \ \ \{f\} \, \{f \ \ true\}\}\}\}\end{array} \ : \ \text{num}}$$