

Homework #10

a) `{{fun {x:num y:num} {+ x y}} 8}`

$$\frac{\frac{[x:num\ y:num] \vdash x:num \quad [x:num\ y:num] \vdash y:num}{[x:num\ y:num] \vdash \{+x\ y\} : num}}{\emptyset \vdash \{fun\ \{x:num\ y:num\}\ \{+x\ y\}\} : (num\ num \rightarrow num)} \quad \emptyset \vdash 8 : num$$

$$\emptyset \vdash \{\{fun\ \{x:num\ y:num\}\ \{+x\ y\}\} 8\} : no\ type$$

b) `{{if false {fun {x:num} x} {fun {y:num} 2}} 57}`

$$\frac{\emptyset \vdash false : bool \quad \frac{\frac{[x:num] \vdash x : num}{[x:num] \vdash x : num} \quad \frac{[y:num] \vdash y : num}{[y:num] \vdash 2 : num}}{\emptyset \vdash \{fun\ \{x:num\}\ x\} : (num \rightarrow num) \quad \emptyset \vdash \{fun\ \{y:num\}\ 2\} : (num \rightarrow num)}}{\emptyset \vdash \{\{if\ false\ \{fun\ \{x:num\}\ x\}\ \{fun\ \{y:num\}\ 2\}\} 57\} : (num \rightarrow num)}$$

`{{fun {x:num y:num->bool} {y {+ x 17}}}`
`42 {fun {x: num} {> x 72}}}`

c) where the type of `>` is `(num num -> bool)`

$$\frac{\frac{[x:num\ y:num \rightarrow bool] \vdash y : (num \rightarrow bool) \quad \frac{[x:num\ y..] \vdash x:num \quad [x:num\ y..] \vdash 17:num}{[x:num\ y:num \rightarrow bool] \vdash \{+x\ 17\} : num}}{[x:num\ y:num \rightarrow bool] \vdash \{y\ \{+x\ 17\}\} : (num\ (num \rightarrow bool) \rightarrow bool)}}{\emptyset \vdash \{fun\ \{x:num\ y:num \rightarrow bool\}\ \{y\ \{+x\ 17\}\}\} : (num \rightarrow bool)} \quad \emptyset \vdash 42 : num$$

$$\frac{\frac{[x..] \vdash > : (num\ num \rightarrow bool) \quad [x..] \vdash num \quad [x..] \vdash 17 : num}{[x:num] \vdash \{>\ x\ 72\} : bool}}{\emptyset \vdash \{fun\ \{x:num\}\ \{>\ x\ 72\}\} : (num \rightarrow bool)}$$

$$\emptyset \vdash \{\{fun\ \{x:num\ y:num \rightarrow bool\}\ \{y\ \{+x\ 17\}\}\} 42\ \{fun\ \{x:num\}\ \{>\ x\ 72\}\}\}$$

$y..$ is simplified term for $y : (num \rightarrow bool)$, and $x..$ is simplified term for $x : num$ because of the lack of space...