Homework 8

20130538 Taesoo Lim

Operational Semantics of BFAE

Expressions

$e ::= n$	(num n)
$\mid e + e$	(add l r)
$\mid e - e$	(sub l r)
$\mid x$	(id s)
$\mid \lambda x.e$	(fun x b)
$\mid e\ \ e$	(app f a)
$\mid ref\ e$	(newbox e)
$\mid e := e$	(setbox b e)
$\mid !e$	(openbox e)
$\mid e\ ;\ e$	(seqn f s)
$\mid (e)$	

Values and Environment

$$a \in Addr$$

$$\sigma \in Env = \mathrm{Var} \xrightarrow{fin} Val$$

$$< \lambda x.e, \sigma > \in Closure = Exp \times Env$$

$$v \in Val = \mathbb{Z} + Closure + Addr$$

$$M \in Store = Addr \xrightarrow{fin} Val$$

Semantics

```
(define (interp bfae ds st)
  (type-case BFAE bfae
    [num (n)       (v*s (numV n) st)]
```

…

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\sigma, M \vdash n \Rightarrow n, M$$

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```
[add (l r)     (interp-two l r ds st
                           (lambda (vl v2 st1) (v*s (num+ vl v2) st)))]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 + e_2 \Rightarrow n_1 + n_2, M_2}$$

```
[sub (l r)     (interp-two l r ds st
                           (lambda (vl v2 st1) (v*s (num- vl v2) st)))]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e_1 \Rightarrow n_1, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow n_2, M_2}{\sigma, M \vdash e_1 - e_2 \Rightarrow n_1 - n_2, M_2}$$

```
[id (s)        (v*s (lookup s ds) st)]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{x \in Domain(\sigma)}{\sigma, M \vdash x \Rightarrow \sigma(x), M}$$

```
[fun (x b)     (v*s (closureV x b ds) st)]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\sigma, M \vdash \lambda x.e \Rightarrow < \lambda x.e, \sigma >, M$$

```
[app (f a)     (interp-two f a ds st
                           (lambda (f-val a-val st1)
                             (interp (closureV-body f-val)
                                     (aSub (closureV-param f-val)
                                           a-val
                                           (closureV-ds f-val))
                                     st1)))]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e_1 \Rightarrow < \lambda x.e, \sigma >, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \quad \sigma'[x \mapsto v_2], M_2 \vdash e \Rightarrow v, M_3}{\sigma, M \vdash e_1\ e_2 \Rightarrow v, M_3}$$

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```
[newbox (v)    (type-case Value*Store (interp v ds st)
                 [v*s (vl stl)
                      (local [(define a (malloc stl))]
                        (v*s (boxV a)
                             (aSto a vl stl)))])]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e \Rightarrow v, M' \quad a \notin Domain(M)}{\sigma, M \vdash ref\ e \Rightarrow\ a, M'[a \mapsto v]}$$

```
[setbox (b v) (interp-two b v ds st
                          (lambda (b-val v-val stl)
                            (v*s v-val
                                 (aSto (boxV-address b-val)
                                       v-val
                                       stl)))))]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e_1 \Rightarrow\ a, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \quad a \in Domain(M_2)}{\sigma, M \vdash\ e := e \Rightarrow v_2, M_2[a \mapsto v_2]}$$

Even though the implementation in the textbook does not replace the value at the address a into new given value, the intention of "setbox" is supposed to be replacement of the value at that address. Thus, it is reasonable to write down the operational semantics as shown in the above. (which specifies the condition of a as a member of domain of M_2 and then change the value of a)

```
[openbox (b)   (type-case Value*Store (interp b ds st)
                 [v*s (b stl)
                      (v*s (store-lookup (boxV-address b)
                                         stl)
                           stl)])]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e \Rightarrow\ a, M_1 \quad a \in Domain(M_1)}{\sigma, M \vdash\ !e \Rightarrow M_1(a), M_1}$$

```
[seqn (a b)    (interp-two a b ds st
                           (lambda (vl v2 stl) (v*s v2 stl)))]
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\frac{\sigma, M \vdash e_1 \Rightarrow v_1, M \quad \sigma, M_1 \vdash\ e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1; e_2 \Rightarrow v_2, M_2}$$