第五章 历年真题

2003—2004

1. $f(x)$ 在 $[-a,a]$ 上连续，则 $f(x)$ 为奇函数是积分 $\int_{-a}^{a} f(x)dx=0$ 的（B）

A 必要. B. 充分. C. 充要. D. 非充分非必要.

2. $f(x)=\begin{cases} x+1, & x<0 \\ x^2, & x\geq 0 \end{cases}$ 则 $\int_{-1}^{1} f(x)dx=$ ______

解. $\int_{-1}^{1} f(x)dx=\int_{-1}^{0}(x+1)dx+\int_{0}^{1}x^2dx=\left[\frac{x^2}{2}+x\right]_{-1}^{0}+\left[\frac{1}{3}x^3\right]_{0}^{1}=\frac{5}{6}$.

3. 由定积分定义知，和式极限 $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{n}{n^2+k^2}=$ ______

解. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{n}{n^2+k^2}=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{1+(\frac{k}{n})^2}\cdot\frac{1}{n}=\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{k=1}^{n}\frac{1}{1+(\frac{k}{n})^2}=\int_0^1\frac{1}{1+x^2}dx=[\arctan x]_0^1=\frac{\pi}{4}$

补: $\int_a^b f(x)dx=\lim\limits_{\lambda\to 0}\sum\limits_{i=1}^{n}f(\xi_i)\Delta x_i$

取 $a=0$, $b=1$. $\Delta x_i=\frac{1}{n}$. $\xi_i=x_i=\frac{i}{n}$. $\lambda\to 0 \Leftrightarrow \Delta x_i=\frac{1}{n}\to 0 \Leftrightarrow n\to\infty$

$\int_0^1 f(x)dx=\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}f(\frac{i}{n})\cdot\frac{1}{n}=\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}f(\frac{i}{n})$

$$\begin{cases} \lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}f(\frac{i}{n})=\int_0^1 f(x)dx, & \xi_i \text{ 取 } [x_{i-1},x_i] \text{ 右端点. } \xi_i=x_i=\frac{i}{n} \\ \lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}f(\frac{i-1}{n})=\int_0^1 f(x)dx, & \xi_i \text{ 取 } [x_{i-1},x_i] \text{ 左端点. } \xi_i=x_{i-1}=\frac{i-1}{n} \end{cases}$$

例1. $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}\sqrt{1+\frac{i}{n}}=\int_0^1\sqrt{1+x}\,dx=\left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1=\frac{4}{3}\sqrt{2}-\frac{2}{3}$

2. $p>0$. $\lim\limits_{n\to\infty}\frac{1^p+2^p+\cdots+n^p}{n^{p+1}}=\lim\limits_{n\to\infty}\frac{1}{n}\left[(\frac{1}{n})^p+(\frac{2}{n})^p+\cdots+(\frac{n}{n})^p\right]=\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}(\frac{i}{n})^p$

$=\int_0^1 x^p dx=\left[\frac{x^{p+1}}{p+1}\right]_0^1=\frac{1}{p+1}$.

3. $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1^2}}+\frac{1}{\sqrt{n^2+2^2}}+\cdots+\frac{1}{\sqrt{n^2+n^2}}\right)=\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{i=1}^{n}\frac{1}{\sqrt{1+(\frac{i}{n})^2}}=\int_0^1\frac{dx}{\sqrt{1+x^2}}\xlongequal{x=\tan t}\int_0^{\frac{\pi}{4}}\sec t\,dt$

$=[\ln|\sec t+\tan t|]_0^{\frac{\pi}{4}}=\ln(\sqrt{2}+1)$.

$I=\lim\limits_{n\to\infty}f(n)$. $f(n)=\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}$ $\quad \frac{n}{\sqrt{n^2+n}}\leq f(n)\leq\frac{n}{\sqrt{n^2+1}}$ 由夹逼准则 $\lim\limits_{n\to\infty}f(n)=1$

4. $\int_0^{\frac{\pi}{4}} \cos^5 2x\,dx$

$= \int_0^{\frac{\pi}{4}} \cos^4 2x \cdot \cos 2x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos^4 2x\,d\sin 2x = \frac{1}{2}\int_0^{\frac{\pi}{4}} (1-\sin^2 2x)^2\,d\sin 2x$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}} [1-2\sin^2 2x+\sin^4 2x]\,d\sin 2x = \frac{1}{2}[\sin 2x - \frac{2}{3}\sin^3 2x + \frac{1}{5}\sin^5 2x]_0^{\frac{\pi}{4}} = \frac{4}{15}$

5. $\int_{-\infty}^{0} \frac{dx}{x^2+2x+2}$

$= \int_{-\infty}^{0} \frac{d(x+1)}{(x+1)^2+1} = [\arctan(x+1)]_{-\infty}^{0} = \arctan 1 - \lim_{x\to-\infty} \arctan(x+1) = \frac{\pi}{4} - (-\frac{\pi}{2}) = \frac{3}{4}\pi$

6. 证明. $\int_0^{\frac{\pi}{2}} f(\sin x)\,dx = \int_0^{\frac{\pi}{2}} f(\cos x)\,dx$

思路. 证 $\int_0^{\frac{\pi}{2}} f(\sin x)\,dx = \int_0^{\frac{\pi}{2}} f(\cos t)\,dt$.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \|$

$$\int_0^{\frac{\pi}{2}} f[\sin(\tfrac{\pi}{2}-t)]\,dt$$

证明. $\int_0^{\frac{\pi}{2}} f(\sin x)\,dx \xlongequal{\text{令} x=\frac{\pi}{2}-t} \int_{\frac{\pi}{2}}^{0} f[\sin(\tfrac{\pi}{2}-t)]\cdot(-dt)$

$= -\int_{\frac{\pi}{2}}^{0} f[\sin(\tfrac{\pi}{2}-t)]\,dt = \int_0^{\frac{\pi}{2}} f(\cos t)\,dt = \int_0^{\frac{\pi}{2}} f(\cos x)\,dx$ #.

2004–2005.

1. 设 $f(x)$ 在 $[a,b]$ 连续. 且 $\int_a^b f(x)\,dx = 0$. 则 (C)

A. 在 $[a,b]$ 的某个小区间上 $f(x)=0$. B. $[a,b]$ 上一切 x 均使 $f(x)=0$.

C. 在 $[a,b]$ 内至少有一点 x. 使 $f(x)=0$. D. $[a,b]$ 内不一定有 x. 使 $f(x)=0$.

积分中值定理. $\exists\,\xi\in[a,b]$. $\int_a^b f(x)\,dx = f(\xi)(b-a)$ $\quad\therefore f(\xi)=0$.

2. $\int_1^2 \frac{\sqrt{4-x^2}}{x^2}\,dx$

$\xlongequal{\text{令} x=2\sin t} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2\cos t}{4\sin^2 t}\cdot 2\cos t\,dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t\,dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\csc^2 t - 1)\,dt$

$= [-\cot t - t]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \sqrt{3} - \frac{\pi}{3}$.

3. 证明 $\int_a^b f(x)dx=(b-a)\int_0^1 f[a+(b-a)x]dx$

思路. $\int_a^b f(x)dx=(b-a)\int_0^1 f[a+(b-a)t]dt$

证: $\int_a^b f(x)dx \xlongequal{x=a+(b-a)t} \int_0^1 f[a+(b-a)t]\cdot(b-a)dt$

$$=(b-a)\int_0^1 f[a+(b-a)t]dt=(b-a)\int_0^1 f[a+(b-a)x]dx$$

2005-2006.

1. 设 $f(x)$ 连续. $F(x)=\int_x^{e^{-x}} f(t)dt$. 则 $F'(x)=$ (A)

A. $-e^{-x}f(e^{-x})-f(x)$.　　B. $-e^{-x}f(e^{-x})+f(x)$

C. $e^{-x}f(e^{-x})-f(x)$.　　D. $e^{-x}f(e^{-x})+f(x)$.

$$F'(x)=f(e^{-x})\cdot(e^{-x})'-f(x)=-e^{-x}f(e^{-x})-f(x)$$

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)dt\right]=f[b(x)]\cdot b'(x)-f[a(x)]\cdot a'(x)$$

2. $\int_0^{+\infty}\frac{1}{x^2+4x+8}dx=$ $\underline{\quad\frac{\pi}{8}\quad}$.

解: $=\int_0^{+\infty}\frac{dx}{(x+2)^2+2^2}=\left[\frac{1}{2}\arctan\frac{x+2}{2}\right]_0^{+\infty}=\frac{1}{2}\cdot\frac{\pi}{2}-\frac{1}{2}\frac{\pi}{4}=\frac{\pi}{8}$

3. $f(x)=\begin{cases} x\sin x. & x>0 \\ -1. & x\le 0.\end{cases}$ 求 $\int_0^{2\pi} f(x-\pi)dx$

解: $\int_0^{2\pi} f(x-\pi)dx \xlongequal{令x-\pi=t} \int_{-\pi}^{\pi} f(t)dt=\int_{-\pi}^0 -dx+\int_0^\pi x\sin x\,dx$

$$=[-x]_{-\pi}^0+\int_0^\pi -x\,d\cos x=-\pi-[x\cos x]_0^\pi+\int_0^\pi \cos x\,dx$$

$$=-\pi+\pi+[\sin x]_0^\pi=0.$$

4. 估计 $\int_0^2 e^{x^2-x}dx$ 的值.

解. 记 $u=x^2-x$. $u'=2x-1$　$x=\frac{1}{2}$ 为驻点. $u''=2>0$. $x=\frac{1}{2}$ 为极小值点.

$u(\frac{1}{2})=-\frac{1}{4}$. $u(0)=0$. $u(2)=2$. $\therefore -\frac{1}{4}\le u(x)\le 2$. $\therefore e^{-\frac{1}{4}}\le e^{x^2-x}\le e^2$.

$$2e^{-\frac{1}{4}}\leq e^{-\frac{1}{4}}(2-0)\leq\int_0^2 e^{x^2-x}dx \leq e^2(2-0)=2e^2$$

$$\therefore\ 2e^{-\frac{1}{4}}\leq\int_0^2 e^{x^2-x}dx\leq 2e^2 \quad \#.$$

2006-2007

1. $\frac{d}{dx}\left(\int_1^2 2x\cos x\,dx\right)=$ $\underline{0}$.

2. $\lim\limits_{x\to 0}\frac{\int_0^x \cos^2 t\,dt}{\sin x}=$ $\underline{1}$.

解 $\overset{洛}{=}\lim\limits_{x\to 0}\frac{\cos^2 x}{\cos x}=\lim\limits_{x\to 0}\cos x=1$

3. 反常积分 $\int_1^{+\infty}\frac{dx}{x^p}$ 当 $\underline{p>1}$ 时收敛.

4. $\int_0^1 e^{\sqrt{x}}dx \xlongequal{令\sqrt{x}=t} \int_0^1 e^t\cdot 2t\,dt=2\int_0^1 t\,de^t=\left[2te^t\right]_0^1-2\int_0^1 e^t\,dt$

$=2e-\left[2e^t\right]_0^1=2e-(2e-2)=2$

5. 设 $f(x)$ 在 $[a,b]$ 上连续. 证明：若 $[a,b]$ 上, $f(x)\geq 0$ 且 $\int_a^b f(x)dx=0$. 则在 $[a,b]$ 上, $f(x)\equiv 0$.

证明：(反证). 设 $\exists\xi\in[a,b]$. $f(\xi)>0$.

若 $\xi\in(a,b)$. $\because f(x)$ 在 $[a,b]$ 连续. $\therefore\exists\delta>0$ 当 $x\in(\xi-\delta,\xi+\delta)$ 有 $f(x)>0$. $\int_a^b f(x)dx=\underbrace{\int_a^{\xi-\delta}f(x)dx}_{\geq 0}+\underbrace{\int_{\xi-\delta}^{\xi+\delta}f(x)dx}_{>0}+\underbrace{\int_{\xi+\delta}^b f(x)dx}_{\geq 0}>0$.

从而 $\int_{\xi-\delta}^{\xi+\delta}f(x)dx>0$.

与 $\int_a^b f(x)dx=0$. 矛盾.

若 $\xi=a$. $\exists\delta>0$. 当 $x\in[a,a+\delta]$. $f(x)>0$. $\int_a^{a+\delta}f(x)dx>0$.

从而 $\int_a^b f(x)dx>0$. 矛盾.

若 $\xi=b$. $\exists\delta>0$. 当 $x\in[b-\delta,b]$. $f(x)>0$. $\int_{b-\delta}^b f(x)dx>0$.

从而 $\int_a^b f(x)dx>0$. 矛盾. #

2007-2008.

1. 由几何意义. $\int_{-1}^{1}\sqrt{1-x^2}dx = (\ B\ )$

A. π B. $\frac{\pi}{2}$. C. 1. D. 0.

2. $\int_0^a (2x-1)dx = \frac{1}{4}$. $a=$ ____.

解: $\frac{1}{4} = \int_0^a (2x-1)dx = [x^2-x]_0^a = a^2-a$ ~~$=a(a-1)$~~

$a^2-a-\frac{1}{4}=0$. $(a-\frac{1}{2})^2=\frac{1}{2}$. $a=\frac{1}{2}\pm\frac{\sqrt{2}}{2}$

3. $\int_0^{+\infty} e^{-ax}dx = \underline{\frac{1}{a}}$ $a>0$.

解: $= -\frac{1}{a}\int_0^{+\infty} e^{-ax}d(-ax) = -\frac{1}{a}[e^{-ax}]_0^{+\infty} = -\frac{1}{a}\lim_{x\to+\infty} e^{-ax} + \frac{1}{a}e^0 = \frac{1}{a}$

(当 $a<0$. ∞. 发散).

4. $\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}dx$

$\xlongequal{x=\sin t} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos t}{\sin^2 t}\cdot\cos t\,dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2 t\,dt \xlongequal{同2004-2005} \sqrt{3}-\frac{\pi}{3}$

2008-2009.

1. 下列广义积分中发散的是 (A)

A. $\int_1^{+\infty}\frac{dx}{x}$ B. $\int_1^{+\infty}\frac{dx}{x\sqrt{x}}$. C. $\int_1^{+\infty}\frac{dx}{x^2}$. D. $\int_1^{+\infty}\frac{dx}{x^2\sqrt{x}}$

$a>0$. $\int_a^{+\infty}\frac{dx}{x^p} = \begin{cases} 收 & p>1 \\ 发 & 0<p\le 1 \end{cases}$

2. $\lim_{x\to 0}\frac{\int_0^x \arctan t\,dt}{x^2} = \underline{\frac{1}{2}}$.

解: $\xlongequal{\frac{0}{0}} \lim_{x\to 0}\frac{\arctan x}{2x} \xlongequal{\frac{0}{0}} \lim_{x\to 0}\frac{\frac{1}{1+x^2}}{2} = \frac{1}{2}$

3. $\int_0^{2\pi}|\sin x|dx$.

$=\int_0^{\pi}\sin x dx-\int_{\pi}^{2\pi}\sin x dx=[-\cos x]_0^{\pi}+[\cos x]_{\pi}^{2\pi}=2+2=4$.

4. 设 $f(x)$ 是在 $(-\infty,+\infty)$ 上连续的奇函数. 且 $[0,+\infty)$ 上单调增加. 令 $F(x)=\int_0^x(2t-x)f(x-t)dt$. 证明: ①. $F(x)$ 是奇函数 ②. $F(x)$ 在 $[0,+\infty)$ 上单调减少的

证明. $f(-x)=-f(x)$. $x\in(-\infty,+\infty)$.

$f'(x)>0$. $x\in[0,+\infty)$.

$F(x)=\int_0^x(2t-x)f(x-t)dt \overset{u=x-t}{=\!=\!=} \int_x^0[2(x-u)-x]f(u)\cdot(-du)$

$=\int_0^x(x-2u)f(u)du=\int_0^x(x-2t)f(t)dt$

①. $F(-x)=\int_0^{-x}(-x-2t)f(t)dt \overset{令u=-t}{=\!=\!=} \int_0^x(-x+2u)f(-u)(-du)$

$=\int_0^x(2u-x)f(u)du=\int_0^x(2t-x)f(t)dt=-F(x)$.

$\therefore F(x)$ 为奇函数

②. $F(x)=x\int_0^x f(t)dt-\int_0^x 2tf(t)dt$

$F'(x)=\int_0^x f(t)dt+xf(x)-2xf(x)=\int_0^x f(t)dt-xf(x)$

$F'(0)=0$

$F''(x)=f(x)-f(x)-xf'(x)=-xf'(x)<0$. $x\in[0,+\infty)$.

$F'(x)$ 单调递减 在 $[0,+\infty)$ 上 $\therefore F'(x)\leqslant F'(0)=0$

$\therefore$ 在 $[0,+\infty)$ 上. $F'(x)\leqslant 0$. 从而 $F(x)$ 在 $[0,+\infty)$ 上单调递减. #

2008—2009.

1. 下之积分收敛的是（ D ）

A. $\int_0^1 \frac{dx}{x}$ B $\int_0^1 \frac{dx}{x\sqrt{x}}$ C. $\int_0^1 \frac{dx}{x^2}$ D $\int_0^1 \frac{dx}{\sqrt{x}}$

$$\int_0^1 \frac{dx}{x^p} = \begin{cases} \text{收} & 0<p<1 \\ \text{发} & p\geq 1 \end{cases}$$

2. $f(x)$ 在 $[a,b]$ 上连续. $f(x)>0$. 证明: $F(x)=\int_a^x f(t)\,dt+\int_b^x \frac{1}{f(t)}dt$ 的导函数 $F'(x)\geq 2$. 以及 $F(x)=0$ 在 (a,b) 内有且仅有一个根.

证明: $F'(x)=f(x)+\frac{1}{f(x)}$ 记 $u=f(x)$. $G(u)=u+\frac{1}{u}$. $u>0$.

$G'(u)=1-\frac{1}{u^2}=0$. $u=1$ 、 $u=-1$ (舍).

$G''(u)=\frac{2}{u^3}>0$. $\therefore u=1$ 为极小值点. $G(u)\geq G(1)=2$.

从而 $F'(x)\geq 2$.

$F(a)=\int_a^a f(t)\,dt+\int_b^a \frac{1}{f(t)}dt=0-\int_a^b \frac{1}{f(t)}dt<0$ （$\because f(t)>0$）

$F(b)=\int_a^b f(t)dt+\int_b^b \frac{1}{f(t)}dt=\int_a^b f(t)\,dt+0>0$ （$\because f(t)>0$）

$F'(x)\geq 2 \Rightarrow F'(x)>0. \Rightarrow F(x)$ 在 $[a,b]$ 上单调增

$\Longrightarrow F(x)=0$ 在 (a,b) 内有且仅有一个根. #

2009—2010.

1. $f(x)\in C[-a,a]$. 正确的是（ C ）

A. $\int_{-a}^a f(x)\,dx=0$ B. $\int_{-a}^a f(x)dx=\int_0^a [f(x)-f(-x)]dx$

C. $\int_{-a}^a f(x)dx=\int_0^a [f(x)+f(-x)]dx$. D $\int_{-a}^a f(x)dx=2\int_0^a f(x)dx$

2. 收敛的是(C)

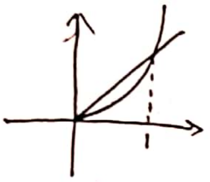
A. $\int_{-\infty}^{+\infty}\sin x\,dx$. B. $\int_{1}^{+\infty}\frac{dx}{\sqrt{x}}$ C. $\int_{-\infty}^{0}e^{x}dx$. D. $\int_{0}^{+\infty}\frac{x^2}{1+x^3}dx$

$[-\cos x]_{-\infty}^{+\infty}$ 发散. $P=\frac{1}{2}$ 发散 $[e^x]_{-\infty}^{0}=1-0=1$ $\left[\frac{1}{3}\ln|1+x^3|\right]_{0}^{+\infty}=\infty$ 发散

3. 正确的是(B)

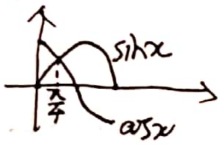
A. $\int_0^1 x\,dx<\int_0^1 x^2dx$. B. $\int_1^2 x\,dx<\int_1^2 x^2dx$ C. $\int_0^1 e^x dx<\int_0^1 e^{x^2}dx$

D. $\int_0^{\frac{\pi}{4}}\sin x\,dx>\int_0^{\frac{\pi}{4}}\cos x\,dx$



当 $x\in[0,1]$. $x>x^2$ $\begin{cases} e^x>e^{x^2}. \int_0^1 e^x dx>\int_0^1 e^{x^2}dx \text{ . C} \times. \\ \int_0^1 x\,dx>\int_0^1 x^2dx \quad \text{A} \times \end{cases}$

$x\in[1,2]$. $x^2>x$. $\int_1^2 x^2dx>\int_1^2 x\,dx$ B ✓.



$x\in[0,\frac{\pi}{4}]$. $\cos x>\sin x$. $\int_0^{\frac{\pi}{4}}\cos x\,dx>\int_0^{\frac{\pi}{4}}\sin x\,dx$. D ×.

4. $\int_0^{x^2}\sin\sqrt{t}\,dt$ 与 x^a 当 $x\to0^+$ 时是同阶无穷小. 则 $a=$ 3 .

解: $\lim\limits_{x\to0^+}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^a}=\lim\limits_{x\to0^+}\frac{\sin\sqrt{x^2}\cdot2x}{ax^{a-1}}=\lim\limits_{x\to0^+}\frac{2x\sin x}{ax^{a-1}}=\lim\limits_{x\to0^+}\frac{2}{a}\frac{\sin x}{x}\cdot\frac{1}{x^{a-3}}$

$=\frac{2}{a}\lim\limits_{x\to0^+}\frac{1}{x^{a-3}}=$ 常数. $\Longrightarrow a=3$ 或 $\lim\limits_{x\to0^+}\frac{2x^2}{ax^{a-1}}=$ 常数 $\Rightarrow a=3$

5. $f(x)=\int_0^x t(1-t)e^{-2t}dt$ 的极值和单调区间.

解. $f'(x)=x(1-x)e^{-2x}=0$. $x=0$. $x=1$. 定义域 $(-\infty,+\infty)$.

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,+\infty)$
y'	$-$	0	$+$	0	$-$
y	↘	0	↗	$\int_0^1 t(1-t)e^{-2t}dt$	↙

极小值 $f(0)=0$.
极大值 $f(1)=\frac{1}{2}e^{-2}$
单增区间 $[0,1]$
单减区间 $(-\infty,0]$. $[1,+\infty)$.

$f(1)=\int_0^1(t-t^2)e^{-2t}dt=-\frac{1}{2}\int_0^1(t-t^2)\,de^{-2t}=\frac{1}{2}\left[(t^2-t)\cdot e^{-2t}\right]_0^1-\frac{1}{2}\int_0^1 e^{-2t}(2t-1)dt$

$=\frac{1}{4}\int_0^1(2t-1)\,de^{-2t}=\frac{1}{4}\left[(2t-1)e^{-2t}\right]_0^1-\frac{1}{4}\int_0^1 e^{-2t}\cdot2dt=\frac{1}{4}(e^{-2}+1)+\frac{1}{4}\int_0^1 e^{-2t}d(-2t)$

$=\frac{1}{4}(e^{-2}+1)+\frac{1}{4}\left[e^{-2t}\right]_0^1=\frac{1}{2}e^{-2}$

6. $\int_0^4 \frac{\sqrt{x}}{1+\sqrt{x}}dx \xlongequal{令\sqrt{x}=t} \int_0^2 \frac{t}{1+t}\cdot 2t\,dt = 2\int_0^2 \frac{t^2-1+1}{t+1}dt = 2\int_0^2 (t-1+\frac{1}{t+1})dt$

$= 2\left[\frac{t^2}{2} - t + \ln|t+1|\right]_0^2 = 2\ln 3$

7. 设$f(x)$在区间$[-a,a]$ $(a>0)$上有二阶连续导数，$f(0)=0$.

(1) 写出带拉格朗日余项的一阶麦克劳林公式

(2). 证明在$[-a,a]$上存在一点η, 使得 $a^3 f''(\eta) = 3\int_{-a}^{a} f(x)\,dx$.

解: (1). $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \frac{f^{(n+1)}(\xi)}{(n+1)!}x^{n+1}$.

ξ介于0和x之间.

带拉余项一阶麦克劳林公式: $f(x) = f(0) + f'(0)x + \frac{f''(\xi)}{2!}x^2$. ξ介于0和x之间

$\because f(0)=0$. $\therefore f(x) = f'(0)x + \frac{f''(\xi)}{2!}x^2$ ξ介于0和x之间.

证明: (2). $\because f(x) = f'(0)x + \frac{f''(\xi)}{2}x^2$. ξ介于0和x之间

$\therefore \int_{-a}^{a} f(x)\,dx = \int_{-a}^{a}\left[f'(0)x + \frac{f''(\xi)}{2}x^2\right]dx$

$= f'(0)\int_{-a}^{a} x\,dx + \frac{f''(\xi)}{2}\int_{-a}^{a} x^2\,dx$

$= 0 + \frac{f''(\xi)}{2}\left[\frac{x^3}{3}\right]_{-a}^{a}$

$= \frac{a^3}{3}f''(\xi)$. ξ介于0和x之间. x介于a. a之间.

$a^3 f''(\xi) = 3\int_{-a}^{a} f(x)\,dx$. ξ介于$-a$和a之间. #.

(ξ写为η).

2010-2011.

1. 收敛的是（C）

A. $\int_e^{+\infty}\frac{\ln x}{x}dx$. B. $\int_e^{+\infty}\frac{1}{x\ln x}dx$. C. $\int_e^{+\infty}\frac{1}{x(\ln x)^2}dx$. D. $\int_e^{+\infty}\frac{1}{x\sqrt{\ln x}}dx$

解. $A=\int_e^{+\infty}\ln x\,d\ln x=\frac{1}{2}(\ln x)^2\Big|_e^{+\infty}=+\infty-\frac{1}{2}$ 发散.

$B=\int_e^{+\infty}\frac{1}{\ln x}d\ln x=\left[\ln\ln x\right]_e^{+\infty}=\infty$ 发散.

$C=\int_e^{+\infty}\frac{1}{(\ln x)^2}d\ln x=\left[-\frac{1}{\ln x}\right]_e^{+\infty}=0+1=1$.

$D=\int_e^{+\infty}\frac{1}{\sqrt{\ln x}}d\ln x=\left[2\sqrt{\ln x}\right]_e^{+\infty}=\infty-2$ 发散.

2. $f(x)\in C[-a,a]$. $\int_{-a}^{a}[f(x)-f(-x)]\cos x\,dx=\underline{\ 0\ }$.

$g(x)=[f(x)-f(-x)]\cos x$. $g(-x)=[f(-x)-f(x)]\cos(-x)=-[f(x)-f(-x)]\cos x=-g(x)$

3. $\lim\limits_{x\to0}\frac{\int_0^{x}\sin t^3dt}{\int_0^{x^2}(e^t-1)dt}\overset{\frac{0}{0}}{=\!=}\lim\limits_{x\to0}\frac{\sin x^3}{2x(e^{x^2}-1)}=\lim\limits_{x\to0}\frac{x^3}{2x\cdot x}=\lim\limits_{x\to0}\frac{x}{2}=0$.

4. $\int_0^{\frac{\pi}{2}}e^{2x}\cos x\,dx=\int_0^{\frac{\pi}{2}}e^{2x}d\sin x=\left[e^{2x}\sin x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}2e^{2x}\cdot\sin x\,dx$

$=e^{\pi}+2\int_0^{\frac{\pi}{2}}e^{2x}d\cos x=e^{\pi}+\left[2e^{2x}\cos x\right]_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}2e^{2x}\cos x\,dx$

$=e^{\pi}-2-4\int_0^{\frac{\pi}{2}}e^{2x}\cos x\,dx$

$\int_0^{\frac{\pi}{2}}e^{2x}\cos x\,dx=\frac{1}{5}(e^{\pi}-2)$.

5. $f(x)\in C[a,b]$. $f(x)$ 在 (a,b) 可导. $f'(x)\leq0$. $F(x)=\frac{1}{x-a}\int_a^{x}f(t)dt$.

证. $F'(x)\leq0$. $x\in(a,b)$.

证明. $F'(x)=\frac{f(x)(x-a)-\int_a^{x}f(t)dt}{(x-a)^2}$

记 $g(x)=f(x)(x-a)-\int_a^{x}f(t)dt$ $x\in(a,b)$.

$g'(x)=f'(x)(x-a)+f(x)-f(x)=f'(x)(x-a)\le 0.\quad x\in(a,b)$

$g(x)$ 在 (a,b) 单调减．$g(x)\le g(a)=0$．

$\therefore F'(x)\le 0.\quad x\in(a,b)$．#.

2013－2014

1. $f''(x)\in C[a,b]$．$f'(a)=b$．$f'(b)=a$．则 $\int_a^b f'(x)f''(x)dx=$ (D)

A. $a-b$．　B. $\frac{1}{2}(a-b)$．　C. a^2-b^2．　D $=\frac{1}{2}(a^2-b^2)$．

解：$\int_a^b f'(x)f''(x)dx=\int_a^b f'(x)df'(x)=\frac{1}{2}[f'(x)]^2\Big|_a^b=\frac{1}{2}[f'(b)]^2-\frac{1}{2}[f'(a)]^2$

$=\frac{1}{2}(a^2-b^2)$

2. $f(x)$ 连续．$\lim\limits_{x\to a}\frac{x}{x-a}\int_a^x f(t)dt$

$=\lim\limits_{x\to a}\frac{\left[x\int_a^x f(t)dt\right]'}{(x-a)'}=\lim\limits_{x\to a}\left[\int_a^x f(t)dt+xf(x)\right]=af(a)$．

3. $\int_1^{\sqrt{3}}\frac{x+\arctan x}{1+x^2}dx=\int_1^{\sqrt{3}}\frac{x}{1+x^2}dx+\int_1^{\sqrt{3}}\frac{\arctan x}{1+x^2}dx$

$=\frac{1}{2}\int_1^{\sqrt{3}}\frac{1}{1+x^2}d(1+x^2)+\int_1^{\sqrt{3}}\arctan x\,d\arctan x$

$=\frac{1}{2}[\ln(1+x^2)]_1^{\sqrt{3}}+\frac{1}{2}[(\arctan x)^2]_1^{\sqrt{3}}=\frac{1}{2}\ln 2+\frac{5\pi^2}{288}$

4. $\int_1^2\frac{\sqrt{4-x^2}}{x^2}dx\ \overset{x=2\sin t}{=\!=\!=}\ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{2\cos t}{4\sin^2 t}\cdot 2\cos t\,dt=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cot^2 t\,dt$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\csc^2 t\,dt-\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}dt=-[\cot t]_{\frac{\pi}{6}}^{\frac{\pi}{2}}-[t]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=-(0-\sqrt{3})-\frac{\pi}{3}=\sqrt{3}-\frac{\pi}{3}$

5. $f(x)=\begin{cases}\frac{1}{1+e^x}. & x<0\\ \frac{1}{1+x}. & x\ge 0.\end{cases}$　求 $\int_0^2 f(x-1)dx$

解. $\int_0^2 f(x-1)dx\ \overset{x-1=t}{=\!=\!=}\ \int_{-1}^1 f(t)dt=\int_{-1}^0\frac{1}{1+e^x}dx+\int_0^1\frac{1}{1+x}dx$

$$=\int_{-1}^{0}\frac{e^x}{e^x(1+e^x)}dx+\int_0^1\frac{1}{1+x}dx=\int_{-1}^{0}(\frac{1}{e^x}-\frac{1}{1+e^x})de^x+[\ln|1+x|]_0^1$$

$$=[\ln|e^x|-\ln|e^x+1|]_{-1}^{0}+\ln 2=-\ln\frac{1}{e}+\ln\frac{1+e}{e}=\ln(1+e)$$

6. 讨论 $\int_1^2\frac{dx}{x\ln x}$ 敛散性.

解：$x=1$ 为瑕点.

$$\int_1^2\frac{dx}{x\ln x}=\int_1^2\frac{1}{\ln x}d\ln x=[\ln|\ln x|]_1^2=\ln\ln 2-\lim_{x\to1^+}\ln\ln x$$

$$=\ln\ln 2+\infty=\infty \therefore 发散$$

2013-2014 开学查考.

1. $\int_0^1\frac{1}{x^p}dx$ 收敛. 则必有 $P\in(0,1)$

2. $\lim\limits_{x\to0}\frac{\int_0^x\sin t^2dt}{x-\sin x}=\lim\limits_{x\to0}\frac{\sin x^2}{1-\cos x}=\lim\limits_{x\to0}\frac{x^2}{\frac{1}{2}x^2}=2$

3. $\int_0^2|1-x|dx=\int_0^1(1-x)dx+\int_1^2(x-1)dx$

$$=[x-\frac{x^2}{2}]_0^1+[\frac{x^2}{2}-x]_1^2=\frac{1}{2}+\frac{1}{2}=1.$$

2014-2015.

1. $\lim\limits_{x\to0}\frac{\int_0^{x^2}t\cos t^2dt}{x^4}=\lim\limits_{x\to0}\frac{x^2\cos x^4\cdot 2x}{4x^3}=\lim\limits_{x\to0}\frac{1}{2}\cos x^4=\frac{1}{2}$

2. $\int_{-\pi}^{\pi}(x^4\sin x+|x|)dx=\int_{-\pi}^{\pi}x^4\sin x\,dx+\int_{-\pi}^{\pi}|x|dx$

$$=0+2\int_0^{\pi}x\,dx=[x^2]_0^{\pi}=\pi^2.$$

2015-2016.

1. $\lim\limits_{x\to0}\dfrac{\int_{\cos x}^{1}e^{-t^2}dt}{x^2}=\lim\limits_{x\to0}\dfrac{-e^{-\cos^2x}\cdot(-\sin x)}{2x}=\lim\limits_{x\to0}\frac{1}{2}e^{-\cos^2x}\cdot\dfrac{\sin x}{x}=\dfrac{1}{2e}$

2. $\int_1^2\dfrac{2+3\sin\frac{1}{x}}{x^2}dx=2\int_1^2x^{-2}dx+3\int_1^2\dfrac{\sin\frac{1}{x}}{x^2}dx$

$=-\left[\dfrac{2}{x}\right]_1^2-3\int_1^2\sin\frac{1}{x}\,d\frac{1}{x}=1+3\left[\cos\frac{1}{x}\right]_1^2=1+3(\cos\frac{1}{2}-\cos1)$

3. $\int_{-\infty}^{+\infty}\dfrac{1}{x^2+2x+2}dx=\int_{-\infty}^{+\infty}\dfrac{1}{(x+1)^2+1}dx=\left[\arctan(x+1)\right]_{-\infty}^{+\infty}=\frac{\pi}{2}-(-\frac{\pi}{2})=\pi$

2016-2017.

1. $y=\int_0^{2x}\sin t^2dt$. 则 $y'=$ $\underline{2\sin(2x)^2}$

2. $\int_{-\pi}^{\pi}x^6\sin^3x\,dx=$ $\underline{\quad 0\quad}$

3. $\lim\limits_{x\to1}\dfrac{\int_1^xe^{t^2}dt}{\ln x}=\lim\limits_{x\to1}\dfrac{e^{x^2}}{\frac{1}{x}}=e$

4. $\int_0^1\dfrac{\sqrt{x}}{1+\sqrt{x}}dx\xlongequal{\sqrt{x}=t}\int_0^1\dfrac{t}{1+t}\cdot2t\,dt=2\int_0^1\dfrac{t^2-1+1}{1+t}dt$

$=2\int_0^1(t-1+\frac{1}{1+t})dt=2\left[\frac{t^2}{2}-t+\ln|1+t|\right]_0^1=2\ln2-1$

2016-2017 开学重考.

1. $y=\int_0^x\sec t\,dt$. $\dfrac{dy}{dx}=$ $\underline{\sec x}$

2. $\int_{-\pi}^{\pi}\dfrac{x^3\cos x}{1+x^2+x^6}dx=$ $\underline{\quad 0\quad}$

3. $\int_{-\infty}^{+\infty}\dfrac{1}{1+x^2}dx=\left[\arctan x\right]_{-\infty}^{+\infty}=\frac{\pi}{2}-(-\frac{\pi}{2})=\pi$

4. $\lim\limits_{x\to0}\dfrac{\int_0^x\sin t^3dt}{x^4}=\lim\limits_{x\to0}\dfrac{\sin x^3}{4x^3}=\lim\limits_{x\to0}\dfrac{x^3}{4x^3}=\dfrac{1}{4}$

5. $\int_0^1\dfrac{\sqrt[4]{x}}{1+\sqrt{x}}dx\xlongequal{\sqrt[4]{x}=t}\int_0^1\dfrac{t}{1+t^2}\cdot4t^3dt=4\int_0^1\dfrac{t^4-1+1}{t^2+1}dt=4\int_0^1(t^2-1+\frac{1}{t^2+1})dt$

$=4\left[\frac{t^3}{3}-t+\arctan t\right]_0^1$

$=\pi-\frac{8}{3}$

2017-2018

1. $y=\int_0^{x^2}\sqrt{1+x^3}\,dx$. $dy=$ $\underline{2x\sqrt{1+x^6}\,dx}$

2. $\int_{-\pi}^{\pi}x^2\ln(x+\sqrt{1+x^2})dx=$ $\underline{\ 0\ }$

$f(x)=\ln(x+\sqrt{1+x^2})$. $f(-x)=\ln(-x+\sqrt{1+x^2})=\ln\frac{1+x^2-x^2}{x+\sqrt{1+x^2}}=\ln\frac{1}{x+\sqrt{1+x^2}}$

$=-\ln(x+\sqrt{1+x^2})=-f(x)$.

3. $P<1$. $\int_1^2\frac{1}{(x-1)^P}dx$是 $\underline{收敛}$

4. $\lim\limits_{x\to0}\frac{\int_0^x e^{t^2}\tan t\,dt}{\sqrt{1+x^2}-1}=\lim\limits_{x\to0}\frac{e^{x^2}\tan x}{\frac{x}{\sqrt{1+x^2}}}=\lim\limits_{x\to0}e^{x^2}\cdot\sqrt{1+x^2}\cdot\frac{\tan x}{x}=1$.

5. $\int_0^1\frac{1}{\sqrt{(1+x^2)^3}}dx\xlongequal{x=\tan t}\int_0^{\frac{\pi}{4}}\frac{1}{\sec^3 t}\cdot\sec^2 t\,dt=\int_0^{\frac{\pi}{4}}\cos t\,dt=[\sin t]_0^{\frac{\pi}{4}}=\frac{\sqrt{2}}{2}$.

2017-2018 (16周).

1. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x\sqrt{\cos x-\cos^2 x}\,dx=$ $\underline{\ 0\ }$

2. $\int_{-\infty}^{+\infty}\frac{a}{1+x^2}dx=\pi$ 则 $a=$ $\underline{\ 1\ }$

$\pi=a\int_{-\infty}^{+\infty}\frac{1}{1+x^2}dx=a[\arctan x]_{-\infty}^{+\infty}=a\pi$. $a=1$

3. $\lim\limits_{x\to0}\frac{\int_0^x\sin t\,dt}{\int_{\cos x}^1 te^{-2t}dt}=\lim\limits_{x\to0}\frac{\sin x}{-\cos x e^{-2\cos x}\cdot(-\sin x)}=\lim\limits_{x\to0}\frac{e^{2\cos x}}{\cos x}=e^2$

4. $\int_0^4\frac{\sqrt{x}}{1+\sqrt{x}}dx\xlongequal{\sqrt{x}=t}\int_0^2\frac{t}{1+t}\cdot2t\,dt=2\int_0^2\frac{t^2-1+1}{1+t}dt$

$=2\int_0^2(t-1+\frac{1}{1+t})dt=2[\frac{t^2}{2}-t+\ln|1+t|]_0^2$

$=2\ln3$.

2017-2018 开学查考.

1. $y=\int_0^x \sqrt{1+t^4}\,dt$. 则 $dy=$ $\underline{\sqrt{1+x^4}\,dx}$

2. $\int_{-1}^{1} x^{10}\ln(x+\sqrt{1+x^2})\,dx=$ $\underline{\ 0\ }$

3. $p>1$. $\int_1^2 \frac{1}{(x-1)^p}dx$ 是 $\underline{发散}$

4. $\lim\limits_{x\to 0}\frac{\int_0^x(\sin t^2+\sin t)\,dt}{e^x+e^{-x}-2}=\lim\limits_{x\to 0}\frac{\sin x^2+\sin x}{e^x-e^{-x}}=\lim\limits_{x\to 0}\frac{2x\cos x^2+\cos x}{e^x+e^{-x}}=\frac{1}{2}$

5. $\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}\frac{1}{x^2\sqrt{1-x^2}}dx \xlongequal{x=\sin t} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{1}{\sin^2 t\cdot\cos t}\cdot\cos t\,dt=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\csc^2 t\,dt=\cdots$

2018-2019.

1. $f(x)=\int_0^x e^{-t^2}\ln t\,dt$. 则 $f(x)$ 的驻点 $x=$ $\underline{\ 1\ }$

$f'(x)=e^{-x^2}\ln x=0$. $\quad x=1$.

2. $\int_{-10}^{10}x^3\sin x^2\,dx=$ $\underline{\ 0\ }$

3. $p<1$. $\int_2^{+\infty}\frac{1}{x(\ln x)^p}dx$ 是 $\underline{发散}$.

$=\int_2^{+\infty}\frac{1}{(\ln x)^p}d\ln x \xlongequal{u=\ln x}\int_{\ln 2}^{+\infty}\frac{1}{u^p}du$

4. $\lim\limits_{x\to 0}\frac{\sin x-x}{x-\int_0^x e^{t^2}dt}=\lim\limits_{x\to 0}\frac{\cos x-1}{1-e^{x^2}}=\lim\limits_{x\to 0}\frac{-\sin x}{-2xe^{x^2}}=\lim\limits_{x\to 0}\frac{1}{2e^{x^2}}\cdot\frac{\sin x}{x}=\frac{1}{2}$

5. $\int_1^5 e^{-\sqrt{2x-1}}dx \underset{x=\frac{t^2+1}{2}}{\xlongequal{\sqrt{2x-1}=t}} \int_1^3 e^{-t}\cdot t\,dt=-\int_1^3 t\,de^{-t}$

$=-\left[te^{-t}\right]_1^3+\int_1^3 e^{-t}dt=e^{-1}-3e^{-3}-\left[e^{-t}\right]_1^3=2e^{-1}-4e^{-3}$.

2018-2019

1. $f(x)$ 连续. $\frac{d}{dx}\int_1^{2x} f(t)\,dt = \underline{2f(2x)}$

2. $f(x)=\begin{cases}1+x^2, & x<0\\ e^{-x}, & x\geq 0\end{cases}$，求 $\int_1^3 f(x-2)\,dx$

解: $\int_1^3 f(x-2)\,dx \xlongequal{x-2=t} \int_{-1}^1 f(t)\,dt = \int_{-1}^0 (1+x^2)\,dx + \int_0^1 e^{-x}\,dx$

$= \left[x+\frac{x^3}{3}\right]_{-1}^0 - \left[e^{-x}\right]_0^1 = \frac{4}{3} - \frac{1}{e} + 1 = \frac{7}{3} - \frac{1}{e}$.

2018-2019. 开学重考.

1. $f(x)=\int_0^x e^{-t^2}\ln t\,dt$. 则 $f(x)$ 的单增区间 $\underline{[1,+\infty)}$.

解: $f'(x) = e^{-x^2}\ln x > 0$. $x>1$.

2. $\int_{-1}^1 \tan x^3 \cdot \sin x^2\,dx = \underline{\ 0\ }$

3. $p>1$. $\int_2^{+\infty} \frac{1}{x(\ln x)^p}\,dx$ 是 $\underline{收敛}$. 同5-15页 2018-2019. 3.

4. $\lim\limits_{x\to 0}\frac{x-\int_0^x e^{t^2}dt}{\tan x - x} = \lim\limits_{x\to 0}\frac{1-e^{x^2}}{\sec^2 x-1} = \lim\limits_{x\to 0}\frac{-x^2}{\tan^2 x} = \lim\limits_{x\to 0}\frac{-x^2}{x^2} = -1$

$x\to 0$. $e^x-1 \sim x$. $1-e^{x^2} \sim -x^2$.

5. $\int_1^3 \cos\sqrt{2x-2}\,dx \xlongequal[x=\frac{t^2+2}{2}]{\sqrt{2x-2}=t} \int_0^2 \cos t\cdot t\,dt = \int_0^2 t\,d\sin t = [t\sin t]_0^2 - \int_0^2 \sin t\,dt$

$= 2\sin 2 + [\cos t]_0^2 = 2\sin 2 + \cos 2 - 1$

6. 证: 当 $x>0$. $\int_0^x e^{-t^2}dt < x$.

证明: 设 $f(x)=\int_0^x e^{-t^2}dt - x$. $f(0)=0$.

当 $x>0$. $f'(x)=e^{-x^2}-1<0$. ∴当 $x>0$ 时. $f(x)$ 是单调减的.

∴ $f(x)<f(0)=0$. ∴ $\int_0^x e^{-t^2}dt - x < 0$

即 $\int_0^x e^{-t^2}dt < x$. #.

2019-2020.(16周).

1. $\int_{-1}^{1}\frac{1+\sin x}{1+x^2}dx=\int_{-1}^{1}\frac{1}{1+x^2}dx+\int_{-1}^{1}\frac{\sin x}{1+x^2}dx=[\arctan x]_{-1}^{1}+0=\frac{\pi}{2}$

2. $\int_{e}^{+\infty}\frac{1}{x(\ln x)^p}dx$ 收敛. 则P取值范围是 $\underline{(1,+\infty)}$

解: $=\int_{e}^{+\infty}\frac{1}{(\ln x)^p}d\ln x\xlongequal{u=\ln x}\int_{1}^{+\infty}\frac{1}{u^p}du$ 收敛. $P>1$.

3. $\lim\limits_{x\to0}\frac{\int_0^x(1-\cos t)dt}{\sin x^3}=\lim\limits_{x\to0}\frac{\int_0^x(1-\cos t)dt}{x^3}=\lim\limits_{x\to0}\frac{1-\cos x}{3x^2}=\lim\limits_{x\to0}\frac{\frac{1}{2}x^2}{3x^2}=\frac{1}{6}$

4. $\int_0^{\frac{\pi}{2}}f(x-\frac{\pi}{4})dx$. $f(x)=\begin{cases}\frac{2}{1+\cos2x}, & x<0\\ \tan x, & x\geq0\end{cases}$

解: $\int_0^{\frac{\pi}{2}}f(x-\frac{\pi}{4})dx\xlongequal{x-\frac{\pi}{4}=t}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}f(t)dt=\int_{-\frac{\pi}{4}}^{0}\frac{2}{1+\cos2x}dx+\int_0^{\frac{\pi}{4}}\tan x\,dx$

$=\int_{-\frac{\pi}{4}}^{0}\frac{2}{2\cos^2x}dx-[\ln|\cos x|]_0^{\frac{\pi}{4}}=\int_{-\frac{\pi}{4}}^{0}\sec^2x\,dx+\frac{1}{2}\ln2$

$=[\tan x]_{-\frac{\pi}{4}}^{0}+\frac{1}{2}\ln2=1+\frac{1}{2}\ln2$

2019-2020.

1. $f(x)=\int_0^{\sqrt{x}}\cos t^2dt$. 则 $f'(x)=\underline{\cos x\cdot\frac{1}{2\sqrt{x}}}$

2. $I_1=\int_{-\pi}^{\pi}(\sin x)^2dx$. $I_2=\int_{-\pi}^{\pi}(\sin x)^4dx$. $I_1\ \underline{>}\ I_2$

$-\pi<x<\pi$. $-1<\sin x<1$. $0<(\sin x)^2<1$. $(\sin x)^2>(\sin x)^4$.

3. $\lim\limits_{x\to0}\frac{x-\int_0^xe^{t^2}dt}{2x^3}=\lim\limits_{x\to0}\frac{1-e^{x^2}}{6x^2}=\lim\limits_{x\to0}\frac{-x^2}{6x^2}=-\frac{1}{6}$