第四章 历年考题.

2003-2004.

1. 已知 $\frac{\cos x}{x}$ 是 $f(x)$ 的一个原函数. 则 $\int f(x)\frac{\cos x}{x}dx = \underline{\frac{\cos^2 x}{2x^2}+C}$.

解: $\left(\frac{\cos x}{x}\right)' = f(x)$. 记 $u=\frac{\cos x}{x}$. $\int f(x)\frac{\cos x}{x}dx = \int u'\cdot u\,dx = \int u\,du = \frac{u^2}{2}+C$

$$= \frac{\cos^2 x}{2x^2}+C$$

2. $\int\left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}\,dx \xlongequal{令x=t^4} \int\left(1-\frac{1}{t^8}\right)\sqrt{t^4\sqrt{t^4}}\cdot 4t^3 dt = \int \frac{t^8-1}{t^8}\, 4t^6 dt$

$$=4\int\frac{t^8-1}{t^2}dt = 4\int(t^6-t^{-2})dt = 4\left(\frac{t^7}{7}+\frac{1}{t}\right)+C = 4\left(\frac{1}{7}x^{\frac{7}{4}}+x^{-\frac{1}{4}}\right)+C$$

2004-2005.

1. 若 $\int f(x)dx = F(x)+C$. $u=\psi(x)$. 则 $\int f(u)du = \underline{F(\psi(x))+C}$.

2. $\int \arcsin x\,dx = x\arcsin x - \int x\cdot\frac{1}{\sqrt{1-x^2}}dx = x\arcsin x + \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}d(1-x^2)$

$$= x\arcsin x+\sqrt{1-x^2}+C$$

2005-2006.

1. 正确的是（ B ）

A. $\int df(x) = f(x)dx$.　　B. $\int f'(x)dx = f(x)+C$

C. $\frac{d}{dx}\int f(x)dx = f(x)+C$.　　D. $d[\int f(x)dx] = f(x)$.

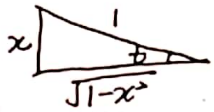
2. 若 e^{-x} 是 $f(x)$ 一个原函数. 则 $\int xf(\ln x)dx = \underline{-x+C}$.

解. $f(x) = (e^{-x})' = -e^{-x}$. $f(\ln x) = -e^{-\ln x} = -e^{\ln x^{-1}} = -x^{-1} = -\frac{1}{x}$.

$$\int xf(\ln x)dx = -\int x\cdot\frac{1}{x}dx = -\int dx = -x+C$$

3. $\int \frac{x\arcsin x}{\sqrt{1-x^2}}dx$　令 $x=\sin t$　$-\frac{\pi}{2}<t<\frac{\pi}{2}$. $\sqrt{1-x^2}=\cos t$.　$dx=\cos t\,dt$

$t=\arcsin x$.



$= \int \frac{\sin t \cdot t}{\cos t}\cdot \cos t\,dt$

$= \int t\sin t\,dt = -\int t\,d\cos t = -t\cos t + \int \cos t\,dt = -t\cos t + \sin t + C$.

$= -\arcsin x \cdot \sqrt{1-x^2} + x + C$.

补. $\int \frac{\arcsin x}{\sqrt{1-x^2}}dx = \int \arcsin x\,d\arcsin x = \frac{1}{2}(\arcsin x)^2 + C$.

2006-2007.

1. $\int \frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx = \int \frac{1}{\sqrt[3]{\sin x-\cos x}}d(\sin x-\cos x) = \frac{3}{2}(\sin x-\cos x)^{\frac{2}{3}}+C$.

2007-2008.

1. 若 $f(x)$ 的导函数为 $\sin x$，则 $f(x)$ 的一个原函数为（ B ）

A. $1+\sin x$.　B. $1-\sin x$.　C. $1+\cos x$.　D. $1-\cos x$.

解. $f'(x)=\sin x$.　$f(x)=\int \sin x\,dx = -\cos x + C_1$

$\int f(x)dx = \int(-\cos x + C_1)dx = -\sin x + C_1x + C_2$

2. $F(x)=\int f(2x+1)dx$ 的导函数 $F'(x)=$ $\underline{f(2x+1)}$

3. $\int \tan^3 x\cdot \sec x\,dx = \int \tan^2 x\cdot \tan x\cdot \sec x\,dx = \int \tan^2 x\,d\sec x$

$= \int(\sec^2 x - 1)\,d\sec x = \int \sec^2 x\,d\sec x - \int d\sec x$

$= \frac{1}{3}\sec^3 x - \sec x + C$.

2008-2009

1. 若$\frac{2}{3}\ln\cos 2x$是$f(x)=k\tan 2x$的一个原函数，则$k=$（ D ）

A. $\frac{2}{3}$.　B. $-\frac{2}{3}$.　C. $\frac{4}{3}$.　D. $-\frac{4}{3}$.

解：$k\tan 2x=(\frac{2}{3}\ln\cos 2x)'=\frac{2}{3}\cdot\frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2=-\frac{4}{3}\tan 2x$.

2. 正确的是（ D ）

A. $\int f'(x)dx=f(x)$　B. $\frac{d}{dx}\int f(x)dx=f(x)+C$

C. $\frac{d}{dx}\int_a^b f(x)dx=f(x)$.　D. $\frac{d}{dx}\int_a^b f(x)dx=0$.

解：$\int f'(x)dx=f(x)+C$. $\frac{d}{dx}\int f(x)dx=f(x)$. $\int_a^b f(x)dx$为常数，求导为0.

3. $\int(\frac{1}{\sqrt{x}}+\frac{2x^2}{1+x^2}-e^{2x})dx$

$=\int\frac{1}{\sqrt{x}}dx+2\int\frac{x^2}{1+x^2}dx-\int e^{2x}dx=2\sqrt{x}+2\int\frac{1+x^2-1}{1+x^2}dx-\frac{1}{2}\int e^{2x}d(2x)$

$=2\sqrt{x}+2\int dx-2\int\frac{1}{1+x^2}dx-\frac{1}{2}e^{2x}=2\sqrt{x}+2x-2\arctan x-\frac{1}{2}e^{2x}+C$.

2008-2009 开学查考.

1. $\int(\sqrt{x}+\frac{2x^2}{1+x^2}-\cos 3x)dx$

$=\int\sqrt{x}dx+2\int\frac{x^2}{1+x^2}dx-\int\cos 3x dx=\frac{2}{3}x^{\frac{3}{2}}+2\int dx-2\int\frac{1}{1+x^2}dx-\frac{1}{3}\int\cos 3x d(3x)$

$=\frac{2}{3}x^{\frac{3}{2}}+2x-2\arctan x-\frac{1}{3}\sin 3x+C$

2. $\int e^x\sin 3x dx=\int\sin 3x de^x=e^x\sin 3x-\int e^x d\sin 3x$

$=e^x\sin 3x-3\int e^x\cos 3x dx=e^x\sin 3x-3\int\cos 3x de^x$

$=e^x\sin 3x-3e^x\cos 3x+3\int e^x d\cos 3x=e^x\sin 3x-3e^x\cos 3x-9\int e^x\sin 3x dx$

$10\int e^x\sin 3x dx=e^x\sin 3x-3e^x\cos 3x+C_1$.

$\int e^x\sin 3x dx=\frac{1}{10}(e^x\sin 3x-3e^x\cos 3x)+C$　$C=\frac{C_1}{10}$

2009-2010.

1. 正确的是（ B. ）

A. $d(\int f(x)dx)=f(x)$　　B. $\int df(x)=f(x)+C$.

C. $\frac{d}{dx}(\int f(x)dx)=f(x)+C$　　D. $\int f'(x)dx=f(x)$.

2. $\int e^{2x}\sin 3x\,dx=\frac{1}{2}\int \sin 3x\,de^{2x}=\frac{1}{2}e^{2x}\sin 3x-\frac{1}{2}\int e^{2x}d\sin 3x$

$=\frac{1}{2}e^{2x}\sin 3x-\frac{1}{2}\times 3\int e^{2x}\cos 3x\,dx=\frac{1}{2}e^{2x}\sin 3x-\frac{3}{2}\cdot\frac{1}{2}\int \cos 3x\,de^{2x}$

$=\frac{1}{2}e^{2x}\sin 3x-\frac{3}{4}e^{2x}\cos 3x+\frac{3}{4}\int e^{2x}d\cos 3x$

$=\frac{1}{2}e^{2x}(\sin 3x-\frac{3}{2}\cos 3x)-\frac{9}{4}\int e^{2x}\sin 3x\,dx$

$(1+\frac{9}{4})\int e^{2x}\sin 3x\,dx=\frac{1}{2}e^{2x}(\sin 3x-\frac{3}{2}\cos 3x)+C_1$

$\int e^{2x}\sin 3x\,dx=\frac{2}{13}e^{2x}(\sin 3x-\frac{3}{2}\cos 3x)+C$.　$C=\frac{4}{13}C_1$.

2010-2011.

1. $\int f(x)e^{-\frac{1}{x}}dx=-e^{-\frac{1}{x}}+C$. 则 $f(x)$ 等于（ C ）

A. $\frac{1}{x}$.　B. $\frac{1}{x^2}$.　C. $-\frac{1}{x^2}$.　D. $-\frac{1}{x}$.

解. $f(x)e^{-\frac{1}{x}}=(-e^{-\frac{1}{x}})'=-e^{-\frac{1}{x}}\cdot[-(-\frac{1}{x^2})]=-\frac{1}{x^2}e^{-\frac{1}{x}}$

2. $\int \frac{\sqrt[4]{x}}{1+\sqrt{x}}dx \xlongequal[x=t^4]{\sqrt[4]{x}=t} \int \frac{t}{1+t^2}\cdot 4t^3dt=4\int \frac{t^4}{t^2+1}dt$

$=4\int \frac{t^2(t^2+1)-(t^2+1)+1}{t^2+1}dt=4\int (t^2-1+\frac{1}{t^2+1})dt$

$=4(\frac{t^3}{3}-t+\arctan t)+C=4(\frac{1}{3}x^{\frac{3}{4}}-x^{\frac{1}{4}}+\arctan\sqrt[4]{x})+C$.

2013-2014.

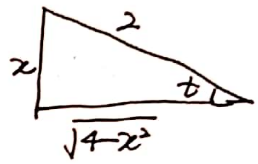
1. $\int(\sqrt{x}+\frac{1}{\sqrt{x}})dx=$ ______

解：$=\int\sqrt{x}dx+\int\frac{1}{\sqrt{x}}dx=\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}+C$.

2013-2014 开学重考.

1. $\int\sqrt{4-x^2}\,dx$　令 $x=2\sin t$. $\sqrt{4-x^2}=2\cos t$. $dx=2\cos t\,dt$

$=\int 2\cos t\cdot 2\cos t\,dt=4\int\cos^2 t\,dt=2\int(1+\cos 2t)dt$



$=2\int dt+\int\cos 2t\,d(2t)=2t+\sin 2t+C$

$=2\arcsin\frac{x}{2}+2\sin t\cos t+C=2\arcsin\frac{x}{2}+x\cdot\frac{\sqrt{4-x^2}}{2}+C$.

2. $\int e^x\cos x\,dx=\int\cos x\,de^x=e^x\cos x+\int e^x\sin x\,dx=e^x\cos x+\int\sin x\,de^x$

$=e^x\cos x+e^x\sin x-\int e^x\cos x\,dx$

$\int e^x\cos x\,dx=\frac{1}{2}e^x(\cos x+\sin x)+C$.

2014-2015.

1. $\int(x+\sqrt{x})dx=\underline{\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}+C}$

2. $\int\frac{3x^2-4}{x^2+1}dx=3\int\frac{x^2+1-\frac{7}{3}}{x^2+1}dx=3\int dx-7\int\frac{1}{x^2+1}dx$

$=3x-7\arctan x+C$.

3. $\int x^2e^{-x}dx=-\int x^2de^{-x}=-x^2e^{-x}+\int e^{-x}\cdot 2x\,dx$

$=-x^2e^{-x}-2\int x\,de^{-x}=-x^2e^{-x}-2xe^{-x}+2\int e^{-x}dx$

$=-x^2e^{-x}-2xe^{-x}-2e^{-x}+C$

2015-2016.

1. $f(x)=e^{-2x}$. $\int \frac{f'(\ln x)}{x}dx = $ (A)

A. $\frac{1}{x^2}+C$. B. $-\frac{1}{x^2}+C$. C. $\ln x+C$. D. $-\ln x+C$

解: $\int \frac{f'(\ln x)}{x}dx = \int f'(\ln x)d\ln x = f(\ln x)+C = e^{-2\ln x}+C = \frac{1}{x^2}+C$.

2. $\int x^2\cos x\,dx = \int x^2 d\sin x = x^2\sin x - \int \sin x \cdot 2x\,dx$

$= x^2\sin x + 2\int x\,d\cos x = x^2\sin x + 2x\cos x - 2\int \cos x\,dx$

$= x^2\sin x + 2x\cos x - 2\sin x + C$.

2016-2017.

1. $\int e^{-x}\sin e^{-x}dx = $ $\underline{\cos e^{-x}+C}$.

解: $= -\int \sin e^{-x}de^{-x} = \cos e^{-x}+C$.

2. $\int (e^{2x}+3\sec^2 x - \frac{1}{\sqrt{4-x^2}})dx$

$= \int e^{2x}dx + 3\int \sec^2 x\,dx - \int \frac{1}{\sqrt{4-x^2}}dx$

$= \frac{1}{2}e^{2x} + 3\tan x - \arcsin\frac{x}{2} + C$

3. $\int x\arctan x\,dx = \frac{1}{2}\int \arctan x\,dx^2$

$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int x^2 d\arctan x$

$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$

$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int (1-\frac{1}{1+x^2})dx$

$= \frac{1}{2}x^2\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C$.

2016—2017 开学考

1. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \underline{-2\cos\sqrt{x}+C}$.

解. $2\int \sin\sqrt{x}\,d\sqrt{x} = -2\cos\sqrt{x}+C$

2. $\int(e^{3x}+3\cos x-\frac{1}{1+x})dx$

$=\int e^{3x}dx+3\int\cos x dx-\int\frac{1}{1+x}dx$

$=\frac{1}{3}e^{3x}+3\sin x-\ln|1+x|+C$

3. $\int x^2e^x dx=\int x^2 de^x = x^2e^x+\int e^x\cdot 2x dx = x^2e^x+2\int x de^x$

$=x^2e^x+2xe^x-2\int e^x dx = x^2e^x+2xe^x-2e^x+C$.

2017—2018.

1. $\sin x^2$是$f(x)$的一个原函数. 则$\int\frac{1}{\sqrt{x}}f(\sqrt{x})dx = \underline{2\sin x+C}$.

解. $\int f(x)dx=\sin x^2+C$. $\int\frac{1}{\sqrt{x}}f(\sqrt{x})dx=2\int f(\sqrt{x})d\sqrt{x} = 2\sin(\sqrt{x})^2+C=2\sin x+C$

2. $\int[e^{2x}+\sec x(\sec x-\tan x)]dx$

$=\int(e^{2x}+\sec^2x-\sec x\tan x)dx=\frac{1}{2}e^{2x}+\tan x-\sec x+C$.

3. $\int x\sin(\ln x)dx = \frac{1}{2}\int\sin(\ln x)dx^2=\frac{1}{2}x^2\sin(\ln x)-\frac{1}{2}\int x^2\cdot\cos(\ln x)\cdot\frac{1}{x}dx$

$=\frac{1}{2}x^2\sin(\ln x)-\frac{1}{4}\int\cos(\ln x)dx^2=\frac{1}{2}x^2\sin(\ln x)-\frac{1}{4}x^2\cos(\ln x)+$

$\frac{1}{4}\int x^2\cdot[-\sin(\ln x)]\frac{1}{x}dx = \frac{1}{2}x^2\sin(\ln x)-\frac{1}{4}x^2\cos(\ln x)-\frac{1}{4}\int x\sin(\ln x)dx$

$\frac{5}{4}\int x\sin(\ln x)dx=\frac{1}{2}x^2[\sin(\ln x)-\frac{1}{2}\cos(\ln x)]+C_1$

$\int x\sin(\ln x)dx=\frac{2}{5}x^2[\sin(\ln x)-\frac{1}{2}\cos(\ln x)]+C$.

2017-2018.（16周）.

1. $\int xe^{-x^2}dx=-\frac{1}{2}\int e^{-x^2}d(-x^2)=\underline{-\frac{1}{2}e^{-x^2}+C}$

2. $\int(\frac{1}{1-2x}+\sec^2x-\frac{1}{\sqrt{4-x^2}})dx=\int\frac{1}{1-2x}dx+\int\sec^2xdx-\int\frac{1}{\sqrt{4-x^2}}dx$

$=-\frac{1}{2}\ln|1-2x|+\tan x-\arcsin\frac{x}{2}+C$

2017-2018 开学查考

1. $\int\frac{1}{x}\sin(\ln x)dx=\int\sin(\ln x)d\ln x=-\cos(\ln x)+C$.

2. $\int[\cos2x+\sec x(\sec x-\tan x)]dx=\int\cos2xdx+\int\sec^2xdx-\int\sec x\tan xdx$

$=\frac{1}{2}\sin2x+\tan x-\sec x+C$.

3. $\int x\arctan xdx=\frac{1}{2}\int\arctan xdx^2=\frac{1}{2}x^2\arctan x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$

$=\frac{1}{2}x^2\arctan x-\frac{1}{2}\int(1-\frac{1}{1+x^2})dx=\frac{1}{2}x^2\arctan x-\frac{1}{2}x+\frac{1}{2}\arctan x+C$.

2018-2019.

1. $\cos\sqrt{x}$ 是 $f(x)$ 一个原函数. 则 $\int xf(x^2)dx=\underline{\frac{1}{2}\cos x+C}$.

解. $\int f(x)dx=\cos\sqrt{x}+C$. $\int xf(x^2)dx=\frac{1}{2}\int f(x^2)dx^2=\frac{1}{2}\cos x+C$

2. $\int[xe^{x^2}+\sec x(\sec x-\sin x)]dx=\int(xe^{x^2}+\sec^2x-\frac{\sin x}{\cos x})dx$

$=\frac{1}{2}\int e^{x^2}dx^2+\int\sec^2xdx+\int\frac{1}{\cos x}d\cos x$

$\int\tan xdx=-\ln|\cos x|+C$

也可直接代公式

$=\frac{1}{2}e^{x^2}+\tan x+\ln|\cos x|+C$

2018-2019

1. 正确的是（ C ）

A. $\int f(x)dx = f(x)$　　B. $\int df(x) = f(x)$

C. $(\int f(x)dx)' = f(x)$　　D. $d\int f(x)dx = f(x)$

2. $\int \frac{1+\ln x}{x}dx = \int (1+\ln x)\,d\ln x = \ln x + \frac{1}{2}\ln^2 x + C$

或 $= \int (1+\ln x)\,d(\ln x+1) = \frac{(1+\ln x)^2}{2} + C$

3. $f(x)$ 的一个原函数为 $\frac{\cos x}{x}$，求 $\int x f'(x)dx$

解：∵ $f(x)$ 的一个原函数为 $\frac{\cos x}{x}$

∴ $f(x) = \left(\frac{\cos x}{x}\right)' = \frac{-x\sin x - \cos x}{x^2}$．$\int f(x)dx = \frac{\cos x}{x} + C$

$\int x f'(x)dx = \int x\,df(x) = xf(x) - \int f(x)dx = x\frac{-x\sin x - \cos x}{x^2} - \frac{\cos x}{x} + C'$

$= -\sin x - \frac{2\cos x}{x} + C'$.

2018-2019 开学查考.

1. e^x 是 $f(x)$ 的一个原函数，则 $\int \frac{1}{x} f(\ln x)dx =$ $\underline{\quad x+C \quad}$.

解：$\int f(x)dx = e^x + C$．$\int \frac{1}{x} f(\ln x)dx = \int f(\ln x)\,d\ln x = e^{\ln x} + C = x + C$

2. $\int [x\sin x^2 + \sec x(\cos x - \tan x)]dx$

$= \int x\sin x^2 dx + \int dx - \int \sec x\tan x\,dx$

$= \frac{1}{2}\int \sin x^2 dx^2 + x - \sec x$

$= -\frac{1}{2}\cos x^2 + x - \sec x + C$.

2019-2020 (16周).

1. $F'(x)=\frac{1}{\sqrt{1-x^2}}$. $F(1)=\frac{3}{2}\pi$. 则 $F(x)=$ $\underline{\arcsin x+\pi}$.

解: $F(x)=\int F'(x)dx=\int\frac{1}{\sqrt{1-x^2}}dx=\arcsin x+C$.

$\frac{3}{2}\pi=F(1)=\arcsin 1+C=\frac{\pi}{2}+C$. $\therefore C=\pi$. $\therefore F(x)=\arcsin x+\pi$.

2. $\int(\frac{1}{\sqrt{9+x}}+\frac{1}{\sqrt{9+x^2}})dx=\int(9+x)^{-\frac{1}{2}}d(x+9)+\int\frac{1}{\sqrt{3^2+x^2}}dx$

$=2\sqrt{9+x}+\ln(x+\sqrt{9+x^2})+C$ $\quad\int\frac{dx}{\sqrt{x^2+a^2}}=\ln(x+\sqrt{x^2+a^2})+C$.

若不记得公式, 设 $x=3\tan t\ (-\frac{\pi}{2}<t<\frac{\pi}{2})$ 做.

3. $\int\sin\sqrt{x}\,dx \xlongequal[x=t^2]{令\sqrt{x}=t} \int\sin t\cdot 2t\,dt=-2\int t\,d\cos t=-2t\cos t+2\int\cos t\,dt$

$=-2t\cos t+2\sin t+C=-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$.

2019-2020.

1. $\int(\frac{e^{\arctan x}}{1+x^2}-\frac{1}{x^2}\cos\frac{1}{x})dx=\int\frac{e^{\arctan x}}{1+x^2}dx-\int\frac{1}{x^2}\cos\frac{1}{x}dx$

$=\int e^{\arctan x}d\arctan x+\int\cos\frac{1}{x}d\frac{1}{x}$

$=e^{\arctan x}+\sin\frac{1}{x}+C$