

Assignment of Data Mining (5) May 17, 2022

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For parameter $p \in (0,1)$, an object function to be maximized is defined as

$$f(p) = -p \log p - (1-p) \log(1-p).$$

1. Find the update rule by Newton-Raphson algorithm to optimize the parameter p .

$$f'(p) = -\log p - 1 - \{-\log(1-p) - 1\}$$

$$= \log\left(\frac{1-p}{p}\right)$$

$$f''(p) = \frac{1}{\frac{1}{p} - 1} \left(-\frac{1}{p^2}\right)$$

$$= -\frac{1}{p - p^2}$$

よって

$$d = -\frac{f'(p)}{f''(p)} = (p - p^2) \log\left(\frac{1-p}{p}\right)$$

以上より update rule は

$$p_{k+1} = p_k + d = p_k + (p_k - p_k^2) \log\left(\frac{1-p_k}{p_k}\right)$$

2. Compute 5 updated values p^1, p^2, \dots, p^5 by Newton-Raphson method, from initial value $p^0 = 0.1$.

$$p^1 = p^0 + (p^0 - p^{0^2}) \log\left(\frac{1-p^0}{p^0}\right) = 0.29775$$

$$p^2 = p^1 + (p^1 - p^{1^2}) \log\left(\frac{1-p^1}{p^1}\right) = 0.477161$$

$$p^3 = p^2 + (p^2 - p^{2^2}) \log\left(\frac{1-p^2}{p^2}\right) = 0.499968$$

$$p^4 = p^3 + (p^3 - p^{3^2}) \log\left(\frac{1-p^3}{p^3}\right) = 0.5$$

$$p^5 = p^4 + (p^4 - p^{4^2}) \log\left(\frac{1-p^4}{p^4}\right) = 0.5$$