Assignment of Data Mining (2) April 19, 2022

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1. Find the maximum likelihood estimator of parameter $\boldsymbol{\mu}$ of a polynomial distribution.

(See p.17-18 in the lecture slides)

$$Multi(m|\mu) = \frac{N!}{\Pi_{k=1}^{K} m_k!} \Pi_{k=1}^{K} \mu_k^{m_k}$$

対数をとると，

$$L(\mu) = logMulti(m|\mu) = \Sigma_{k=1}^{K}(logN! - \Sigma_{k=1}^{K} logm_k! + \Sigma_{k=1}^{K} m_k log\mu_k)$$

$L(\mu)$ は凹関数であるため，$L(\mu)$の極値をとるμを求めればよい．

$\Sigma_{k=1}^{K}\mu_k - 1 = 0$より，

$$L(\mu) = logMulti(m|\mu) = \Sigma_{k=1}^{K}(logN! - \Sigma_{k=1}^{K} logm_k! + \Sigma_{k=1}^{K} m_k log\mu_k) + \lambda(\Sigma_{k=1}^{K}\mu_k - 1)$$

両辺μで微分すると，

$$\frac{\delta}{\delta\mu} L(\mu) = \Sigma_{k=1}^{K} m_k \frac{1}{\mu_k} + \lambda = 0$$

$$\mu_k = -\frac{1}{\lambda} \Sigma_{k=1}^{K} m_k$$

$\mu_1 + \mu_2 +$ ・・・ $+ \mu_k = 1$，$m_1 + m_2 +$ ・・・ $+ m_k = N$より，

$$\Sigma_{k=1}^{K}\mu_k = -\frac{1}{\lambda} \Sigma_{k=1}^{K} m_k = 1$$

$$\lambda = -N$$

よって最尤推定は

$$\mu_{ML} = \frac{\Sigma_{k=1}^{K} m_k}{N}$$