Experiment No. 8
Single Source Shortest Path using Dynamic Programming (Bellman-Ford Algorithm)
Date of Performance:
Date of Submission:

Experiment No: 8

Title: Single Source Shortest Path: Bellman Ford

Aim: To study and implement Single Source Shortest Path using Dynamic Programming: Bellman Ford

Objective: To introduce Bellman Ford method

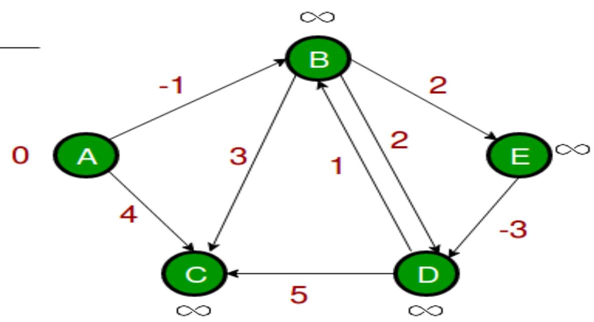
Theory:

Given a graph and a source vertex source in graph, find shortest paths from src to all vertices in the given graph. The graph may contain negative weight edges.We have discussed Dijkstra's algorithm for this problem. Dijkstra's algorithm is a Greedy algorithm and time complexity is O(VLogV) (with the use of Fibonacci heap). Dijkstra doesn't work for Graphs with negative weight edges, Bellman-Ford works for such graphs. Bellman-Ford is also simpler than Dijkstra and suites well for distributed systems. But time complexity of Bellman-Ford is O(VE), which is more than Dijkstra.

Example:

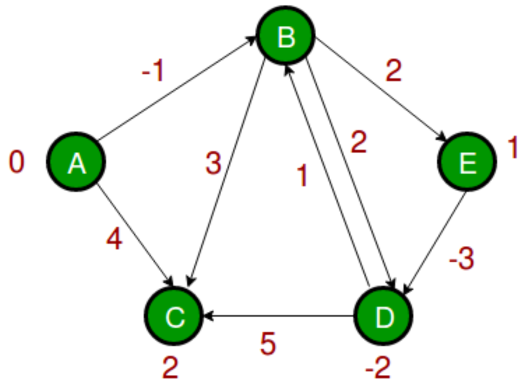
Let the given source vertex be 0. Initialize all distances as infinite, except the distance to the source itself. Total number of vertices in the graph is 5, so all edges must be processed 4 times.

A	B	C	D	E
0	∞	∞	∞	∞

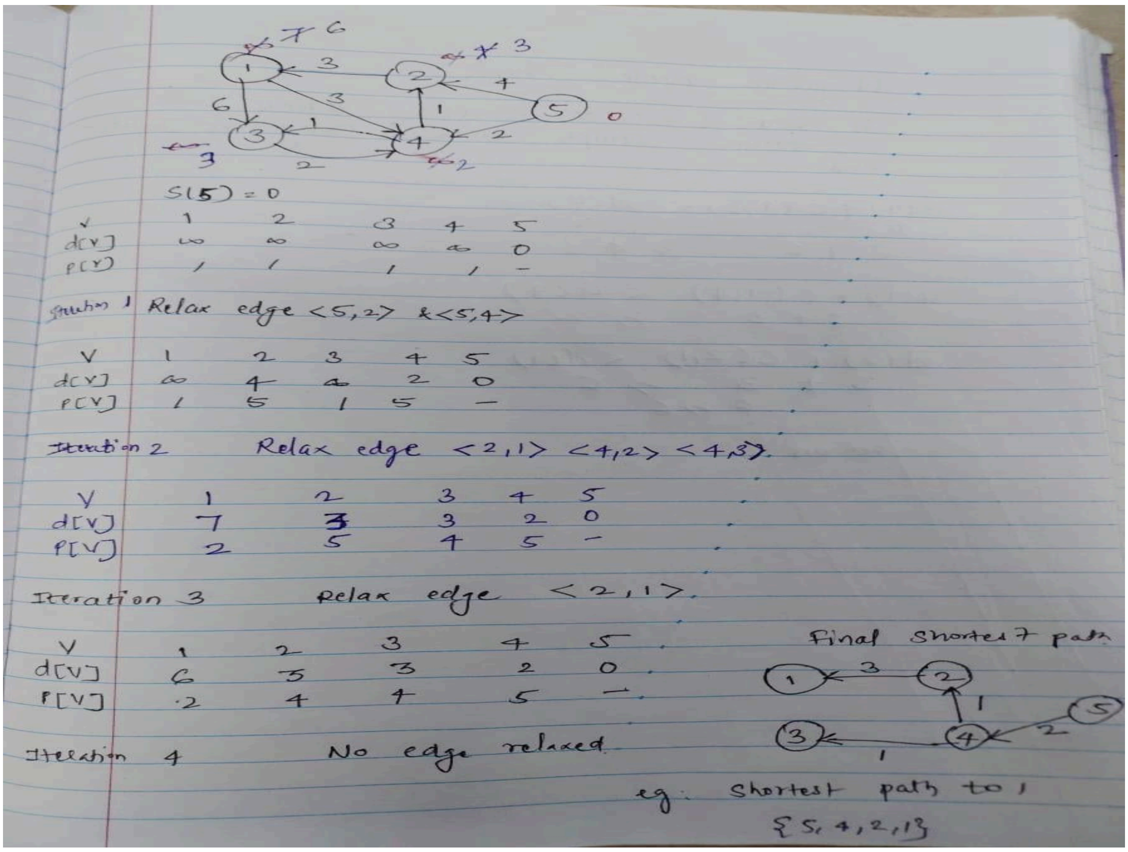


Let all edges are processed in the following order: (B, E), (D, B), (B, D), (A, B), (A, C), (D, C), (B, C), (E, D). We get the following distances when all edges are processed the first time. The first row shows initial distances. The second row shows distances when edges (B, E), (D, B), (B, D) and (A, B) are processed. The third row shows distances when (A, C) is processed. The fourth row shows when (D, C), (B, C) and (E, D) are processed.

A	B	C	D	E
0	∞	∞	∞	∞
0	-1	∞	∞	∞
0	-1	4	∞	∞
0	-1	2	∞	∞
0	-1	2	∞	1
0	-1	2	1	1
0	-1	2	-2	1



The second iteration guarantees to give all shortest paths which are at most 2 edges long. The algorithm processes all edges 2 more times. The distances are minimized after the second iteration, so third an fourth iterations don't update the distances.



Algorithm:

```
function Bellman_Ford(list vertices, list edges, vertex source, distance[], parent[])

// Step 1 – initialize the graph. In the beginning, all vertices weight of
// INFINITY and a null parent, except for the source, where the weight is 0

for each vertex v in vertices
   distance[v] = INFINITY
   parent[v] = NULL

distance[source] = 0
// Step 2 – relax edges repeatedly
   for i = 1 to V-1    // V – number of vertices
      for each edge (u, v) with weight w
         if (distance[u] + w) is less than distance[v]
            distance[v] = distance[u] + w
            parent[v] = u

// Step 3 – check for negative-weight cycles
for each edge (u, v) with weight w
   if (distance[u] + w) is less than distance[v]
      return "Graph contains a negative-weight cycle"

return distance[], parent[]
```

Output:

Shortest path from source (5)

Vertex 5 -> cost=0 parent=0

Vertex 1-> cost=6 parent=2

Vertex 2-> cost=3 parent=4

Vertex 3-> cost =3 parent =4

Vertex 4-> cost =2 paren=5

Implementation:

```
#include <bits/stdc++.h>
using namespace std;
struct Edge {
   int src, dest, weight;
};
struct Graph {
    int V, E;
   struct Edge* edge;
};
struct Graph* createGraph(int V, int E)
{
   struct Graph* graph = new Graph;
   graph->V = V;
   graph->E = E;
   graph->edge = new Edge[E];
   return graph;
}
void printArr(int dist[], int n)
{
   printf("Vertex   Distance from Source\n");
   for (int i = 0; i < n; ++i)
      printf("%d \t\t %d\n", i, dist[i]);
}
void BellmanFord(struct Graph* graph, int src)
{
   int V = graph->V;
   int E = graph->E;
   int dist[V];
```

```
    for (int i = 0; i < V; i++)
        dist[i] = INT_MAX;
    dist[src] = 0;
    for (int i = 1; i <= V - 1; i++) {
        for (int j = 0; j < E; j++) {
            int u = graph->edge[j].src;
            int v = graph->edge[j].dest;
            int weight = graph->edge[j].weight;
            if (dist[u] != INT_MAX
                && dist[u] + weight < dist[v])
                dist[v] = dist[u] + weight;
        }
    }
    for (int i = 0; i < E; i++) {
        int u = graph->edge[i].src;
        int v = graph->edge[i].dest;
        int weight = graph->edge[i].weight;
        if (dist[u] != INT_MAX
            && dist[u] + weight < dist[v]) {
            printf("Graph contains negative weight cycle");
            return; // If negative cycle is detected, simply
                    // return
        }
    }

    printArr(dist, V);

    return;
}
```

```
int main()
{
     int V = 5; // Number of vertices in graph
    int E = 8; // Number of edges in graph
    struct Graph* graph = createGraph(V, E);
    graph->edge[0].src = 0;
    graph->edge[0].dest = 1;
    graph->edge[0].weight = -1;
    graph->edge[1].src = 0;
    graph->edge[1].dest = 2;
    graph->edge[1].weight = 4;
    graph->edge[2].src = 1;
    graph->edge[2].dest = 2;
    graph->edge[2].weight = 3;
    graph->edge[3].src = 1;
    graph->edge[3].dest = 3;
    graph->edge[3].weight = 2;
    graph->edge[4].src = 1;
    graph->edge[4].dest = 4;
    graph->edge[4].weight = 2;
    graph->edge[5].src = 3;
    graph->edge[5].dest = 2;
    graph->edge[5].weight = 5;
    graph->edge[6].src = 3;
    graph->edge[6].dest = 1;
    graph->edge[6].weight = 1;
    graph->edge[7].src = 4;
    graph->edge[7].dest = 3;
    graph->edge[7].weight = -3;
    BellmanFord(graph, 0);
```

```
    return 0;
}
```

Output:

```
Output
/tmp/HNSwVsixpU.o
Vertex   Distance from Source
0        0
1        -1
2        2
3        -2
4        1


=== Code Execution Successful ===
```

Conclusion: The implementation of Bellman-Ford involves iterative relaxation steps to find the shortest paths from a single source vertex while handling negative edge weights and detecting negative cycles. It's a fundamental algorithm in graph theory due to its versatility and ability to handle a wide range of graph types.