LAB EXERCISE 1 - SOLUTIONS

QUESTION 1

a) The probability that Alice rolls above 15 on a 20-sided die is:

P(D > 15) = 5/20 = 1/4 = 0.25

since there are 5 successful outcomes (16, 17, 18, 19, 20) out of 20 possible outcomes.

b) If Alice can reroll once (advantage), the probability that she rolls above 15 in either the first or second roll is calculated using the complement rule:

P(D1 > 15 or D2 > 15) = 1 - P(D1 <= 15 and D2 <= 15) = 1 - P(D1 <= 15) P(D2 <= 15)

P(D <= 15) = 15/20 = 3/4.

P(D1 > 15 or D2 > 15) = 1 – 3/4 * 3/4 = 7/16 = 0.4375

QUESTION 2

The probability that all tasks A, B, and C are completed on time is the product of their individual probabilities since they are independent:

P(all on time) = P(A) * P(B) * P(C)

= 0.8 * 0.7 * 0.9 = 0.504

QUESTION 3

The probability of selecting a product of type A and it being defective is:

P(A and D) = P(A) * P(D | A)

= 0.6 * 0.03 = 0.018

QUESTION 4

Using the law of total probability (i.e., marginalising out the weathermen):

P(rain) = P(rain | A correct) * P(A correct) + P(rain | B correct) * P(B correct)

where:

- P(rain | A correct) = 0.7
- P(rain | B correct) = 0.4
- P(A correct) = 0.8
- P(B correct) = 0.6

P(rain) = (0.7 * 0.8) + (0.4 * 0.6)

= 0.56 + 0.24 = 0.8

QUESTION 5

a) Total flagged transactions (true and false positives):

- Total transactions = 10,000
- Fraudulent transactions = 0.01 * 10,000 = 100
- Legitimate transactions = 0.99 * 10,000 = 9,900
- True positives = 0.95 * 100 = 95
- False positives = 0.05 * 9,900 = 495

Total flagged transactions = 95 + 495 = 590

b) Out of those flagged, the number actually fraudulent:

- True positives = 95

Proportion truly fraudulent = 95 / 590 = 0.161 = 16.1%

c) The rarity of fraud makes the reliability of the AI less effective. Since the vast majority of transactions are legitimate, even a small false positive rate results in many false flags. This inflates the total number of flagged transactions, while the number of true positives remains small, reducing the proportion of flagged transactions that are genuinely fraudulent. Ignoring this when reasoning about the effectiveness of the AI is known as the **base rate fallacy**.