

June 2, 2016

## 1 Greedy Algorithm 2

### 1.1 Stable Marriage

Given  $n$  men and  $n$  women, output a *perfect* (complete bijection) matching  $M$  between men/women s.t. there are no 2  $(m, w), (m', w')$ , s.t.  $m$  prefers  $w'$  to  $w$  and  $w'$  prefers  $m$  to  $m'$ .

```
function stableMatching {
    Initialize all  $m \in M$  and  $w \in W$  to free
    while  $\exists$  free man  $m$  who still has a woman  $w$  to propose to {
         $w$ 
        = first woman on  $m$ 's list to whom  $m$  has not yet proposed
        if  $w$ 
        is free
             $(m, w)$  become engaged
        else some pair  $(m', w)$ 
        already exists
            if  $w$  prefers  $m$  to  $m'$ 
                 $m'$  becomes
                free
             $(m, w)$  become engaged
        else
             $(m', w)$ 
        remain engaged
    } }
```

Invariants:

This terminates because the while loop runs at most  $O(n^2)$  times,  $n$  times for each man as he goes down his list. Women will always be matched with a partner (since they'll be reached in someone's list eventually), and since the cardinalities of the sets are equal, men will always find a partner too.

This is stable as men always propose in order of their preferences. If there were another woman he hasn't proposed to, it must've been a woman that he proposed to already, then she either dumped him or she rejected him in the first place.