8

May 26, 2016

1 Greedy Algorithm 2

1.1 Stable Marriage

Given n men and n women, output a *perfect* (complete bijection) matching M between men/women s.t. there are no 2 (m, w), (m', w'), s.t. m prefers w' to w and w' prefers m to m'.

```
   function stableMatching {     Initialize all m ∈ M and w ∈ W to free
while ∃ free man m who still has a woman w to propose to {        w
= first woman on m’s list to whom m has not yet proposed       if w
is free          (m, w) become engaged        else some pair (m', w)
already exists          if w prefers m to m'           m' becomes
free            (m, w) become engaged           else           (m', w)
remain engaged     } }
```

Invariants:

This terminates because the while loops runs at most $O(n^2)$ times, n times for each man as he goes down his list. Women will always be matched with a partner (since they'll be reached in someone's list eventually), and since the cardinalities of the sets are equal, men will always find a partner too.

This is stable as men always propose in order of their preferences. If there were another women he hasn't proposed to, it must've been a women that he proposed to already, then she either dumped him or she rejected him in the first place.