Procedures for Bayesian Trader Prediction Market Simulation

- Hyperparameters:
  - A random event X with all kinds of outcome x and some probability distribution p;
  - Agent's belief for this event, $p(x;\theta)$. This belief has a general form:

    $$p(x;\theta) = \exp\left(\langle \theta, \phi(x) \rangle - T(\theta)\right),$$

    $T(\theta)$ as the log-partition function.
    ***Note.* Agent's belief on the probability distribution of this event is not reflected directly from $p(x;\theta)$. Instead, it will be reflected from its prior, i.e, the distribution of the parameter θ.**
  - Agent's prior for this event. Prior should be of the form

    $$p(\theta; b_0) = \exp\left(\langle n\nu, \theta \rangle + nT(\theta) - \psi(\nu, n)\right),$$

    where $b_0 = \begin{bmatrix} n\nu \\ n \end{bmatrix}$.

- Procedures:
  1. According to the current outstanding shares and the cost function, assume the current market price for the security (contract) is
     $$\nabla C(\theta) = \nu.$$
  2. One agent comes into the market with some prior $p(\theta; b_0)$ where $b_0 = \begin{bmatrix} n\nu \\ n \end{bmatrix}$.
  3. He is provided with a *private* set of data points of size m and mean $\hat{\mu}$.
  4. His posterior belief $p(\theta; b_1)$ is updated to be of same form but $b_1 = \begin{bmatrix} n\nu + m\hat{\mu} \\ n + m \end{bmatrix}$.
  5. He would like to buy/sell some number δ of security (contract) such that
     $$\nabla C(\theta + \delta) = \frac{n\nu + \hat{\mu}}{n+1}.$$
  6. Repeat the above steps until all agents have traded in the market.

- Questions and concerns:
  1. Why is the fact that every agent would have a prior with a parameter such that **its first entry is always ν times larger than the second entry?**
  2. When calculating $p(x;\theta) = p(\theta; b_0)$, since the second term of θ is $-\frac{1}{2\sigma^2}$, should we use $\frac{\mu}{\sigma}$ in calculation? (which is the current method).