Summary about Prediction Market

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1 Exponential Family

2 Scoring Rules

Scoring rules is the simplest form of prediction mechanism. For every agent with some information, a scoring rule evaluates how close the information is from the actual outcome, and pays the agent in return for his/her information.

2.1 Motivations

Taking an event $\mathbf{X}$ with outcome space $\mathcal{X}$, we want to know something about each agent's belief $p(\mathbf{x})$ on the actual outcome $\mathbf{x} \in \mathcal{X}$, compare it with the actual outcome, and see how accurate the agent's prediction is. However, it is impossible to ask agent for his/her entire $p(\mathbf{x})$ probability distribution. What should we do?

We've already known that sufficient statistic is a very suitable parameter to characterize a probability distribution. Therefore, we can use it to represent agent's belief. We just ask each agent for the sufficient statistic $\mathbf{u}(\mathbf{x})$ as a representation of his/her belief $p(\mathbf{x})$. Then we are able to measure how accurate the agent can predict with a scoring rule based on the actual outcome.

Assume $\hat{\boldsymbol{\mu}} = \mathbb{E}_p[\mathbf{u}(\mathbf{x})]$ is the *expected* sufficient statistic from agent's belief $p(\mathbf{x})$. Then, with the actual outcome denoted $\mathbf{x}$, the scoring rule takes the form:

$$S(\hat{\boldsymbol{\mu}}, \mathbf{x}).$$

We can see that the scoring rule is a measure of how close the agent's belief is from the actual outcome.

2.2 Incentive Compatibility

Incentive compatibility is a property of prediction mechanism with which the best strategy for an agent to earn the most profit is to *honestly* report all the information as soon as he/she has it. As a prediction mechanism, a proper scoring rule should leverage **incentive compatibility** in order to get real information from agents.

Assume that we already set the sufficient statistic to be $\mathbf{u}(\mathbf{x})$. Then for each $p \in \mathcal{P}$ that takes this $\mathbf{u}(\mathbf{x})$ as its sufficient statistic, we can calculate its *expected* sufficient statistic $\boldsymbol{\mu} = \mathbb{E}_p[\mathbf{u}(\mathbf{x})]$. A scoring rule is thus called **proper** if it satisfies the following, for all such p, for all $\hat{\boldsymbol{\mu}} \neq \boldsymbol{\mu}$:

$$\mathbb{E}_p[S(\boldsymbol{\mu}, \mathbf{x})] \geqslant \mathbb{E}_p[S(\hat{\boldsymbol{\mu}}, \mathbf{x})].$$

Any scoring rule with this property actually encourages agents to report a probability distribution as close to the actual probability distribution of $\mathbf{X}$ as possible, which is in accordance with the essence of incentive compatibility.

2.3 Logarithmic Scoring Rule

Suppose that we have set a form of the sufficient statistic. A classic logarithmic scoring rule takes the form:

$$S(\boldsymbol{\mu}, \mathbf{x}) = \ln p(\mathbf{x}; \boldsymbol{\mu}),$$

where $\boldsymbol{\mu} = \mathbb{E}_p[\mathbf{u}(\mathbf{x})]$ is the expected sufficient statistic over $p(\mathbf{x}; \boldsymbol{\mu})$.

3 Cost Function based Prediction Market with Bayesian Traders