

$$\begin{aligned}
P(x; \mu, \sigma^2) &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
&= \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
&= \frac{1}{\sqrt{2\pi}} e^{\ln \frac{1}{\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\
&= \frac{1}{\sqrt{2\pi}} e^{-\ln(\sigma) - \frac{(x-\mu)^2}{2\sigma^2}} \\
&= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2} + \frac{\mu x}{\sigma^2} - \frac{\mu^2}{2\sigma^2} + \ln\left(\frac{1}{\sigma}\right)} \\
&= \frac{1}{\sqrt{2\pi}} e^{\begin{bmatrix} \frac{\mu}{\sigma^2} & -\frac{1}{2\sigma^2} \end{bmatrix} \begin{bmatrix} x & x^2 \end{bmatrix}^T - \left(\frac{\mu^2}{2\sigma^2} + \ln \sigma\right)}
\end{aligned}$$

For

$$P(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}} \exp \left( \begin{bmatrix} \frac{\mu}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{bmatrix} \cdot \begin{bmatrix} x \\ x^2 \end{bmatrix} - \left( \frac{\mu^2}{2\sigma^2} + \ln \sigma \right) \right) \quad (1)$$

In Eq (1), counting measure is  $1/\sqrt{2\pi}$ , natural parameter  $\eta$  is  $\begin{bmatrix} \frac{\mu}{\sigma^2} \\ -\frac{1}{2\sigma^2} \end{bmatrix}$ , sufficient statistic  $t(x)$  is  $\begin{bmatrix} x \\ x^2 \end{bmatrix}$ ,  $a(\eta)$  is  $\left(\frac{\mu^2}{2\sigma^2} + \ln \sigma\right)$ .

$$\begin{aligned}
\eta_1 &= \frac{\mu}{\sigma^2} \\
\eta_2 &= -\frac{1}{2\sigma^2}
\end{aligned}$$

Thus,

$$\mu = \frac{\eta_1}{-2\eta_2}$$

And,

$$\begin{aligned}
a(\eta) &= \left[ \frac{\left(\frac{\eta_1}{2\eta_2}\right)^2}{2\left(\frac{1}{-2\eta_2}\right)^2} + \ln \sqrt{\frac{1}{-2\eta_2}} \right] = -\frac{1}{2}\eta_1^2 \frac{1}{2\eta_2} - \frac{1}{2} \ln(-2\eta_2) \\
&= \frac{\eta_1^2}{4\eta_2} - \frac{1}{2} \ln(-2\eta_2)
\end{aligned}$$