Procedures for Bayesian Trader Prediction Market Simulation

- Hyperparameters:
  - A random event X with all kinds of outcome x and some probability distribution p;
  - Market maker's belief for this event, $p(x;\theta)$. This belief has a general form:

    $$p(x;\theta) = \exp\left(\langle\theta, \phi(x)\rangle - T(\theta)\right),$$

    $T(\theta)$ as the log-partition function.
  - Market maker's prior for this event. Prior should be of the form

    $$p(\theta; b_0) = \exp\left(\langle n\nu, \theta\rangle + nT(\theta) - \psi(\nu, n)\right),$$

    where $b_0 = \begin{bmatrix} n\nu \\ n \end{bmatrix}$.
  - Posterior update:

    $$\mathbf{E}_{\theta\sim b_0}\mathbf{E}_{x\sim\theta}[\phi(x)] = \nu,$$

    therefore ν is the posterior mean, because posterior is essentially $p(\theta|x)$. *With regards to cost-function prediction market, we can also see that ν is the price.*

- Procedures:
  1. One agent comes into the market and purchase some amount of contracts (securities). Assume the agent possess dataset of size m and mean $\hat{\mu}$.
  2. Update the posterior mean (the new price):

     $$\nu' = \frac{n\nu + m\hat{\mu}}{n+m}.$$

  3. Repeat the above steps until all agents have traded in the market.