

## Procedures for Bayesian Trader Prediction Market Simulation

- Hyperparameters:

- A random event  $X$  with all kinds of outcome  $x$  and some probability distribution  $p$ ;
- Agent's belief for this event,  $p(x; \theta)$ . This belief has a general form:

$$p(x; \theta) = \exp(\langle \theta, \phi(x) \rangle - T(\theta)),$$

$T(\theta)$  as the log-partition function.

**Note.** Agent's belief on the probability distribution of this event is not reflected directly from  $p(x; \theta)$ . Instead, it will be reflected from its prior, i.e, the distribution of the parameter  $\theta$ .

- Agent's prior for this event. Prior should be of the form

$$p(\theta; b_0) = \exp(\langle n\nu, \theta \rangle + nT(\theta) - \psi(\nu, n)),$$

$$\text{where } b_0 = \begin{bmatrix} n\nu \\ n \end{bmatrix}.$$

- Procedures:

1. According to the current outstanding shares and the cost function, assume the current market price for the security (contract) is

$$\nabla C(\theta) = \nu.$$

2. One agent comes into the market with some prior  $p(\theta; b_0)$  where  $b_0 = \begin{bmatrix} n\nu \\ n \end{bmatrix}$ .
3. He is provided with a *private* set of data points of size  $m$ .
4. He calculates the average payoff by

$$\hat{\mu} = \frac{1}{m} \sum_{i=1}^m \phi(x_i).$$

5. His posterior belief  $p(\theta; b_1)$  is updated to be of same form but  $b_1 = \begin{bmatrix} n\nu + m\hat{\mu} \\ n + m \end{bmatrix}$ .
6. He would like to buy/sell some number  $\delta$  of security (contract) such that

$$\nabla C(\theta + \delta) = \frac{n\nu + \hat{\mu}}{n + 1}.$$

7. Repeat the above steps until all agents have traded in the market.

- Questions and concerns:

1. Why is the fact that every agent would have a prior with a parameter such that **its first entry is always  $\nu$  times larger than the second entry**?
2. When calculating  $p(x; \theta) = p(\theta; b_0)$ , since the second term of  $\theta$  is  $-\frac{1}{2\sigma^2}$ , should we use  $\frac{\mu}{\sigma}$  in calculation (which is the current method)?
3. How to determine  $\nu_2$  when we calculate the  $[n\nu, n]$  pair?