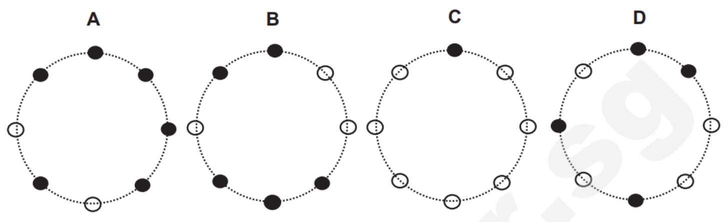
Electrostatics Problem Set

1 Part A

Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Low field

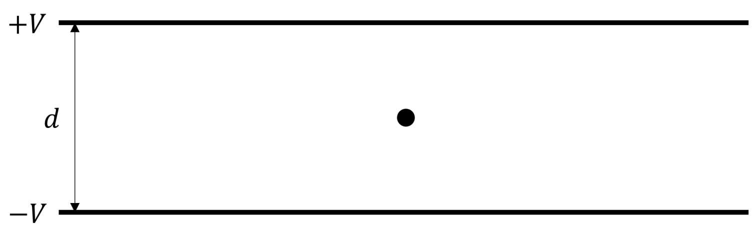
A combination of eight electrons (unshaded circle) and protons (shaded circle) are fixed



in place and evenly spread around the circumference of the circle as shown. Which of the combinations has the lowest resultant electric field magnitude at the center of the circle?

Problem A.2. Current in wire

An oil drop is vertically suspended in between two parallel plates as shown in the figure.



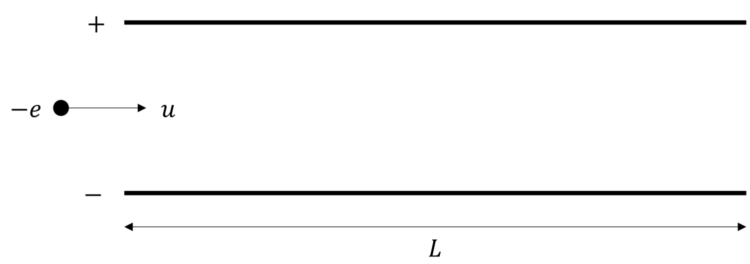
Given that the oil is in vertical equilibrium, what is the sign of its charge? Find the charge to mass ratio of the drop.

Problem A.3. Accelerating a charge

An electron of charge $-e$ and mass m is acclerated from rest through a potential difference of V. Find the final velocity u of the electron.

Problem A.4. Parallel plate deflection

An electron is passed a pair of oppositely charged parallel plates as shown in the figure. The electron has charge $-e$, mass m and initial horizontal velocity u. In which direction will the electron deflect after passing through the plates? What is the magnitude of the deflection?



Problem A.5. Current in wire

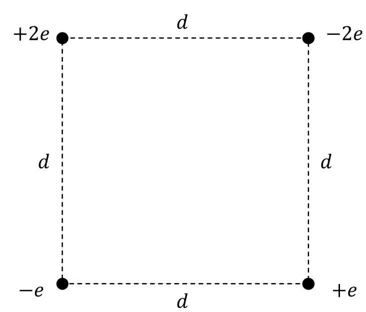
The number density of electrons in a wire is n. The cross-sectional area of the wire is A. The drift velocity of the electrons is v. Find the current in the wire.

2 Part B

Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Charge on Square

4 point charges are arranged at the corners of a square of side length d. The charges are



as indicated on the diagram. What is the electric potential V and the magnitude of the electrostatic force F felt by a point charge of $-e$ placed at the centre of the square?

Problem B.2. Approximate pd

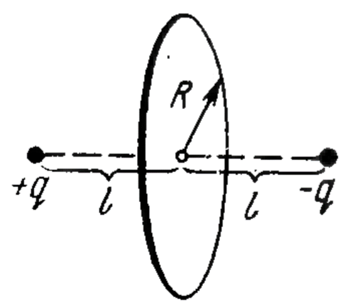
Consider a large sphere of charge Q and radius r. What is the magnitude of the potential difference between the surface of the sphere and a point a small distance d outside (i.e. away from) the surface of the sphere? Assume $d \ll r$.

Problem B.3. Passing through

Simplifying Thomsons plum pudding model, consider a sphere Au with radius, $r = 1.44 \times$

10^{-10}m, charge $79e$ uniformly distributed within the sphere and a point charge He with negligible radius and charge $2e$. Considering only electrostatics,
(a) how much kinetic energy does He need to have initially, when it was far away, in order to just reach the surface of Au?
(b) how much kinetic energy does He need to have initially, when it was far away, in order to just pass through Au in a head-on collision?

Problem B.4. Electric Flux
Two point charges q and $-q$ are separated by the distance $2l$ (see figure). Find the flux of



the electric field strength vector across a circle of radius R at a distance l from both charges.

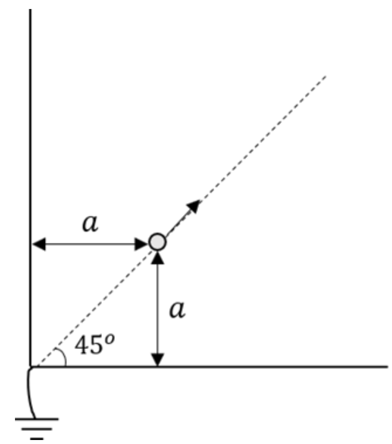
Problem B.5. Ring Oscillations
A particle of mass m and charge q is placed in the center of a fixed ring carrying charge Q homogeneously distributed around the ring.
(a) Find the electric field magnitude along the axis of the ring.
(b) Find the location along the axis at which the electric field is maximum.
(c) The particle is then displaced a small distance Δx perpendicular to the plane containing the ring. Under what conditions will the particle exhibit simple harmonic motion? Find the period of these oscillations.

3 Part C

Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!

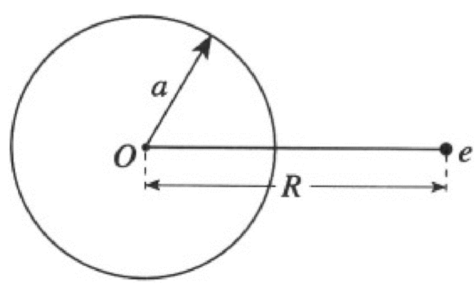
Problem C.1. Image Charge 1
(a) A particle carrying charge q is situated a distance a away from an infinite grounded

plane. What is the force experienced by the particle?



(b) A particle carrying charge q is situated a distance a away from the sides of two perpendicular walls (figure above). The particle is then moved along the diagonal as shown below. What is the work required to move the particle to infinity?

Problem C.2. Image Charge 2



A point charge is placed at a distance R from the center of a metallic sphere of radius with (see figure). The sphere is insulated and is electrically neutral.
(a) Find the electric potential on the surface of the sphere.
(b) Find the force acting on the charge.

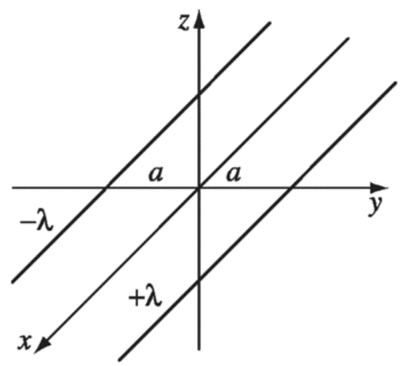
Problem C.3. Hollowed Out

A sphere of radius R_1 has charge density ρ uniform within its volume, except for a small spherical hollow region of radius R_2 located a distance a from the center. Find the electric field distribution within the hollow region.

Problem C.4. Hemisphere Force

Find the force the bottom half of a conducting sphere carrying charge Q (uniformly distributed) exerts on the upper half.

Problem C.5. Equipotential Lines



Two infinitely long wires running parallel to the x axis carry uniform charge densities $+\lambda$ and $-\lambda$ (see figure).

(a) Find the potential at any point (x, y, z) setting $V = 0$ at the origin.

(b) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.

4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. Potential Ratio

Find the ratio of the electric potential V_1 at the center of a uniformly charged cube to the electric potential V_2 at the corner of the same cube.

Problem D.2. Shell's Theorem

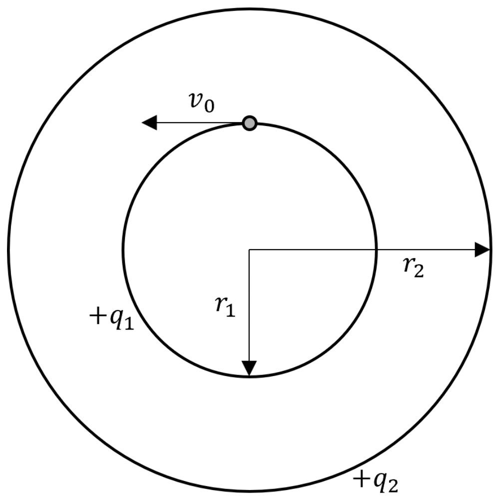
Show, without Gauss' Law, that the electric field inside a uniformly charged shell is zero and that the field outside the shell is equivalent to consider the whole shell as just a single point charge at its center.

5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.

Out next week!

Problem E.1. Electrical orbit

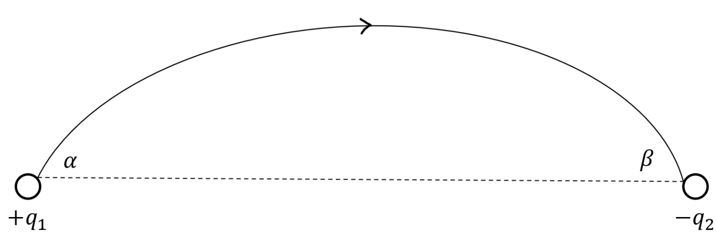


A electron is launched tangential to a metal sphere with radius r_1 carrying charge $+q_1$ such that an outer metal net of radius r_2 barely catches the electron. Find the launch speed v_0.

Problem E.2. Moving out

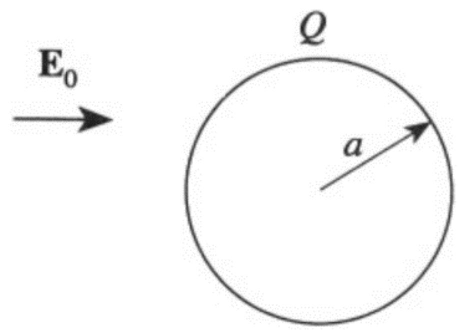
A point charge q is at the center of an uncharged spherical conducting shell, of inner radius a and outer radius b. How much work would it take to move the charge out to infinity (through a tiny hole drilled in the shell)?

Problem E.3. Field lines



An electric field line emerges from charge $+q_1$ at an angle α to the line connecting $+q_1$ to $-q_2$. What is the angle β at which the field line will enter $-q_2$?

Problem E.2. Polarization



A conducting sphere of radius on whose surface resides a total charge Q is placed in a uniform electric field (see figure). Find the potential at all points in space exterior to the sphere. What is the surface charge density?

Problem E.5. Electric dipole

An electric dipole consists of two charges separated by some distance. Consider a dipole arrangement with $+q$ situated at $(0, 0, d/2)$ and $-q$ situated at $0, 0, -d/2$. Find the electric field produced a large distance away. Work in spherical coordinates.