Dynamics Problem Set

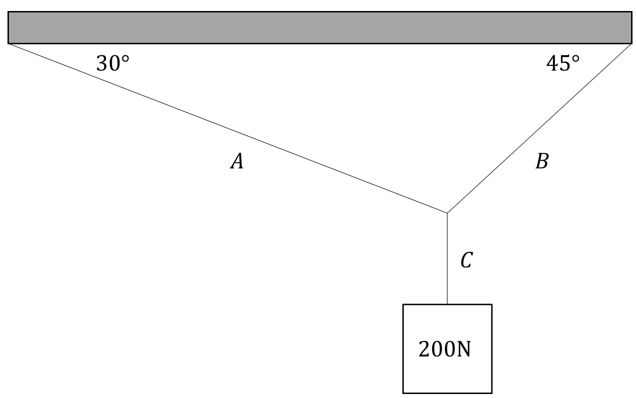
1 Part A

Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Pulled Aside

A 5.0kg mass hangs from a fixed point O by a light inextensible string. It is pulled aside by a horizontal force P and rests in equilibrium with the string inclined at an angle 30° to the vertical. Find the magnitude of force P.

Problem A.2. Suspended Weight



Find the tension in each cord (A, B, C) in the figure below if the weight of the suspended mass is 200N.

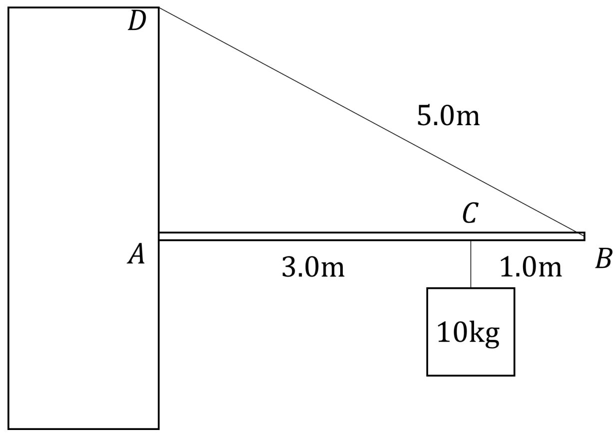
Problem A.3. Leaning Ladder

A 15.0m uniform ladder of mass 5.0kg rests against a frictionless wall and makes an angle 60° with the horizontal.

(a) Determine the horizontal and vertical forces that the ground exerts on the base of the ladder when a 800N firefighter is 4.00m from the bottom.

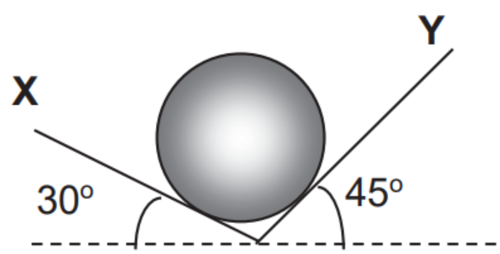
(b) If the ladder is on the verge of slipping when the firefighter is 9.00m up, what is the coefficient of static friction between the ladder and the ground?

Problem A.4. Strut



Determine the tension in the cable BD and the horizontal and vertical components exerted on the strut AB at pin A. Find the magnitude and direction of this force. Assume the weight of the strut to be negligible.

Problem A.5. Two walls



Determine the ratio of the normal contact force exerted by wall X on the ball to that exerted by wall Y on the ball.

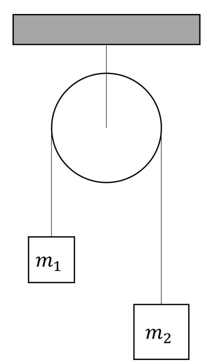
2 Part B

Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Falling Chain

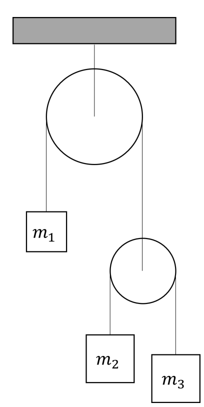
A chain of mass M and length L is suspended vertically with its lower end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale when a length x of the chain has fallen? Neglect the size of the individual links.

Problem B.2. Atwood Machine



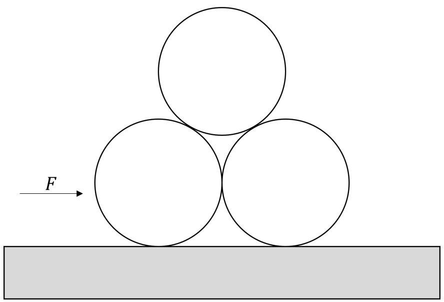
Two masses m_1 and m_2 are hung over a fixed, frictionless pulley. Find the acceleration of the masses in terms of m_1, m_2 and g.

Problem B.3. Double Atwood Machine



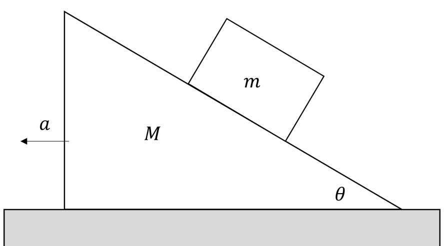
Find the acceleration of the each of the masses in terms of m_1, m_2, m_3 and g. Assume all pulleys are frictionless.

Problem B.4. Accelerating Cylinder



Three identical cylinders are arranged in a triangle as shown above, with the bottom two lying on the ground. The ground and the cylinders are frictionless. A constant horizontal force is applied to the left cylinder. Let a be the acceleration given to the system. For what range of a will all three cylinders remain in contact with each other?

Problem B.5. Moving Plane

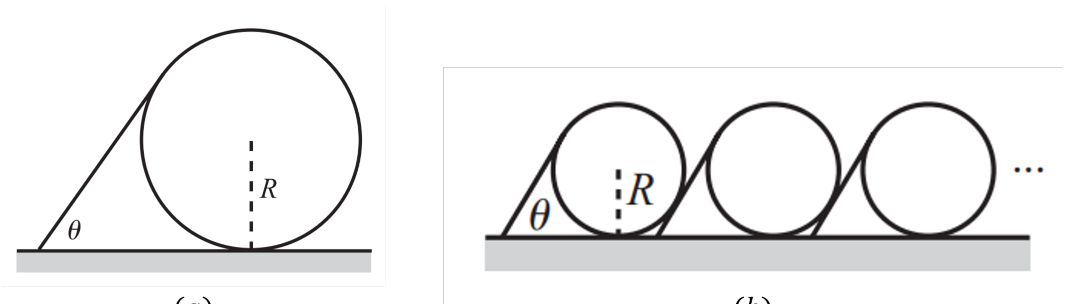


A block of mass m is held motionless on a frictionless plane of mass M and angle θ with the horizontal. The plane rests on a frictionless horizontal surface. What is the horizontal acceleration a of the plane after the block is released?

3 Part C

Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!

Problem C.1. Sticks and Circles

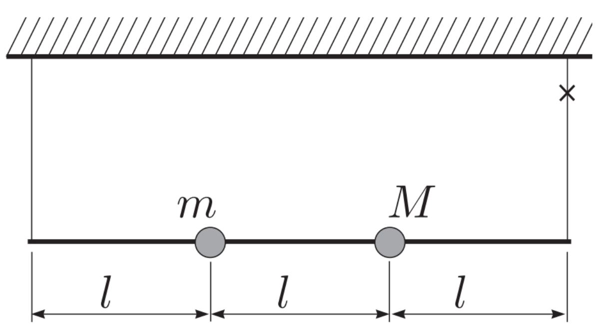


(a) (b)

(a) A stick of mass density per unit length ρ rests on a circle of radius R. The stick makes an angle θ with the horizontal and is tangent to the circle at its upper end. Friction exists at all points of contact, and assume that it is large enough to keep the system at rest. Find the friction force between the ground and the circle.

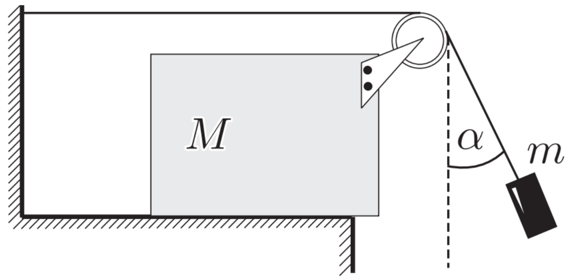
(b) A large number of sticks (with mass density per unit length ρ) and circles (with radius R) lean on each other. Each stick makes an angle θ with the horizontal and is tangent to the next circle at its upper end. The sticks are hinged to the ground, and every other surface is frictionless (unlike in the previous problem). In the limit of a very large number of sticks and circles, what is the normal force between a stick and the circle it rests on, very far to the right? Assume that the last circle leans against a wall, to keep it from moving.

Problem C.2. Initial Tension



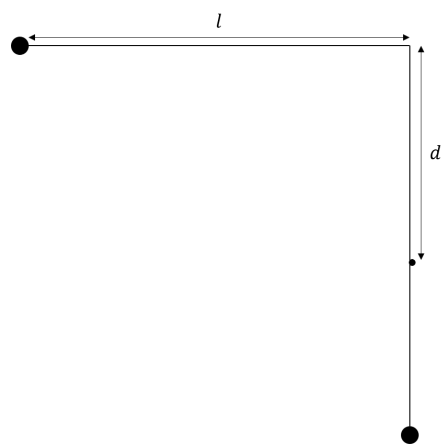
A light rod with length $3l$ is attached to the ceiling by two strings with equal lengths. Two balls with masses m and M are fixed to the rod, the distance between them and their distances from the ends of the rod are all equal to l. Find the tension in the second string right after the first has been cut.

Problem C.3. Unchanging Angle

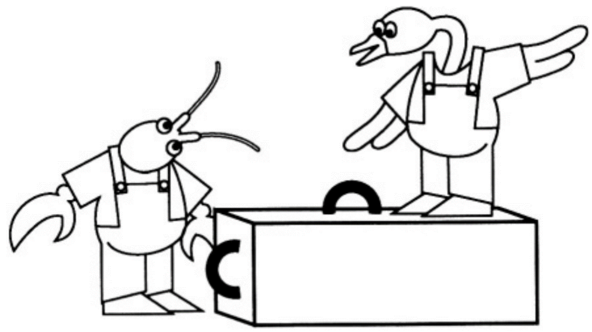


A block with mass M is on a slippery horizontal surface. A thread extends over one of its corners. The thread is attached to the wall at its one end and to a little block of mass m, which is inclined by an angle α with respect to the vertical, at the other. Initially the thread is stretched and the blocks are held in place. Then the blocks are released. For which ratio of masses will the angle α remain unchanged throughout the subsequent motion?

Problem C.4. Striking the peg



As shown in the figure above, a pendulum of string length l is released from horizontal level. When the string is vertical, it strikes a peg a distance d from its pivot. What is the ratio d/l that would allow the pendulum bob to swing back and strike the peg?



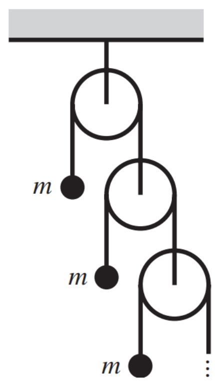
Problem C.5. Swan and Crawfish

Two movers, Swan and Crawfish, from Swan, Crawfish, and Pike, Inc., must move a long, low, narrow dresser, along a rough surface, with a coefficient of friction $\mu = 0.5$. The mass of the dresser is $M = 150$kg. Swan can apply a maximum force of 700N, and Crawfish 350N. Obviously, together they can move the dresser; however, each of them insists on his own way of moving the darn thing, and they cannot agree. Show that by using his own method, each of them can move the dresser alone. What are these methods?

4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. Infinite Atwood Machine



Consider the infinite Atwoods machine shown in the figure above. A string passes over each pulley, with one end attached to a mass and the other end attached to another pulley. All the masses are equal to m, and all the pulleys and strings are massless. The masses are held fixed and then simultaneously released. What is the acceleration of the top mass?

Problem D.2. Watch the video

Here is the link: https://www.youtube.com/watch?v=1Xp_imnO6WE

A uniform rod of mass m and length l is supported horizontally at its ends by my two forefingers. Whilst I am slowly bringing my fingers together to meet under the centre of the rod, it slides on either one or other other of them. How much work do I have to do in the process if the coefficient of static friction is μ_{stat} and the coefficient of kinetic friction is μ_{kin} ($\mu_{kin} < \mu_{stat}$)?

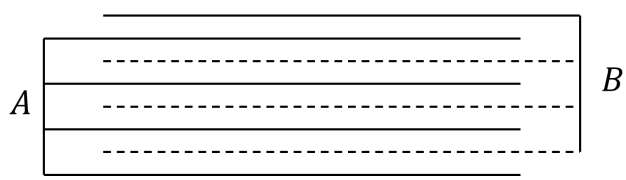
5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.

Problem E.1. Capstan Equation

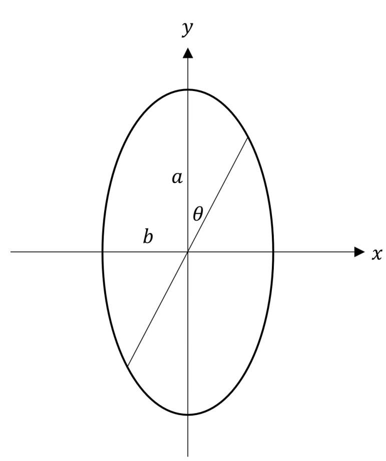
A rope wraps an angle θ around a pole. You grab one end and pull with a tension T_0. The other end is attached to a large object, say, a boat. If the coefficient of static friction between the rope and the pole is μ, what is the largest force the rope can exert on the boat, if the rope is not to slip around the pole?

Problem E.2. Paper Vice



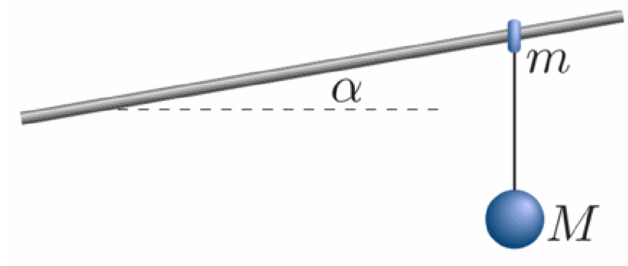
As shown in the figure above, two books are placed such that their pages are interlocking in an alternating manner. The coefficient of static friction μ_s between the pages is 0.3. If each page has a mass of 5g, and each book has 200 pages, what is the force F needed to pull B away from A, which is fixed.
Ans: 1170N

Problem E.3. Fastest descent



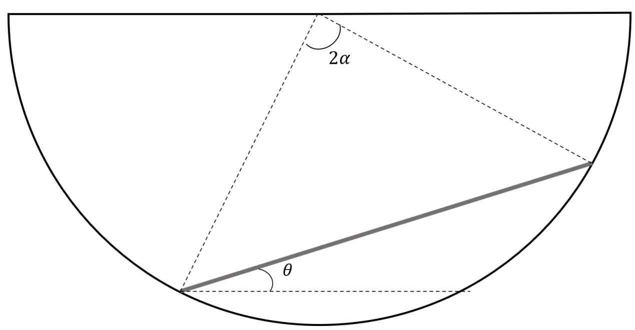
As shown in the figure above, mass m is made to slide down an ellipse with semi-major axis a and semi-minor axis b through a diagonal of angle θ to the vertical. What is the angle θ for which the mass makes the fastest descent?

Problem E.4. Initial Accleration



A slippery rod is positioned at an angle α with respect to the horizon. A little ring of mass m can slide along the rod, to which a long thread is attached. A small sphere of size M is attached to the thread. Initially the ring is held motionless, and the thread hangs vertically. Then the ring is released. What is the acceleration of the sphere immediately after that?

Problem E.5. Rod in a bowl



As shown in the figure above, a rod is placed in a hemispherical bowl, where the rod lies in the plane containing the center of the bowl. If the static coefficient of friction between the rod and the bowl is β, show that the maximum value that θ can take is

$$\theta = \arctan\left[\frac{\tan(\alpha+\beta) - \tan(\alpha-\beta)}{2}\right] \tag{1}$$