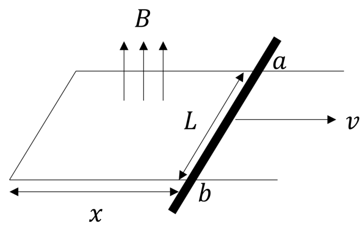
Electromagnetic Induction Problem Set

1 Part A

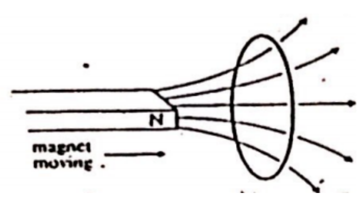
Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Sliding Rod



Determine the direction of induced current by Lenz's law.

Problem A.2. Moving magnet



(a) Determine the direction of induced current by Lenz's law.

(b) Show how this flow is consistent with the principle of conservation of energy.

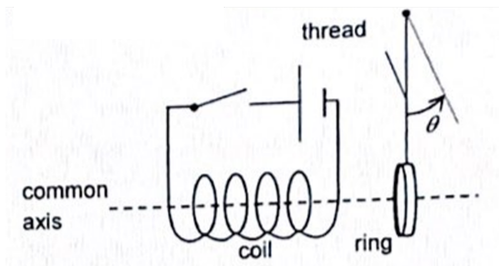
Problem A.3. Rotating coil

A rectangular coil of 800 turns has an area of 0.050m^2. It is placed at right angles to a magnetic field of flux density 4.0×10^{-5}T. It is then rotated through $180°$ in 0.20s. Find the average emf induced in the coil.

Problem A.4. Moving rod

Calculate the induced emf between the ends of a uniform horizontal rod CD, which is 1.5m long and moving at 30ms^{-1} horizontally to the right, assuming the Earth's magnetic field strength is 6.0×10^{-5}T and acts downwards at $65°$ to the horizontal. Which end of CD is at a higher potential?

Problem A.5. Electromagnet

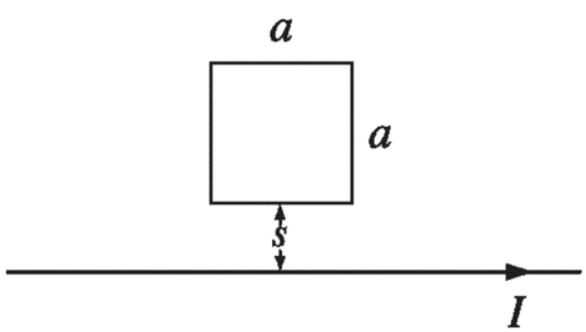


A soft-iron ring hangs vertically from a thread and has its axis aligned with a coil as shown in the figure. The current in the coil is switched on at the time t_0. Sketch the $\theta - t$ graph for the ensuing motion.

2 Part B

Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Square loop



A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I.

(a) Find the magnetic flux through the loop.

(b) If someone now pulls the loop directly away from the wire, at speed v, what emf is generated? In what direction (clockwise or counterclockwise) does the current flow?

(c) What if the loop is pulled to the right at speed v?

Problem B.2. Faraday's law

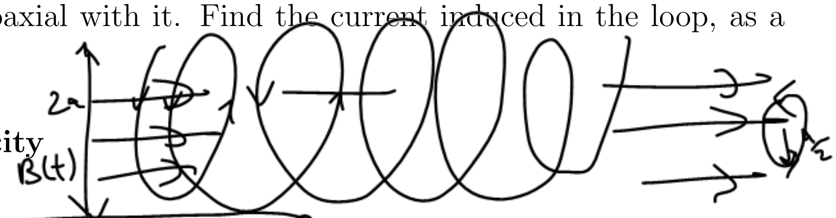
A square loop of wire, with sides of length a, lies in the first quadrant of the $x-y$ plane, with one corner at the origin. In this region, there is a nonuniform time-dependent magnetic field $B(y,t) = ky^3t^2\boxed{\hat{z}}$ (where k is a constant). Find the emf induced in the loop.

unit vector in z-direction

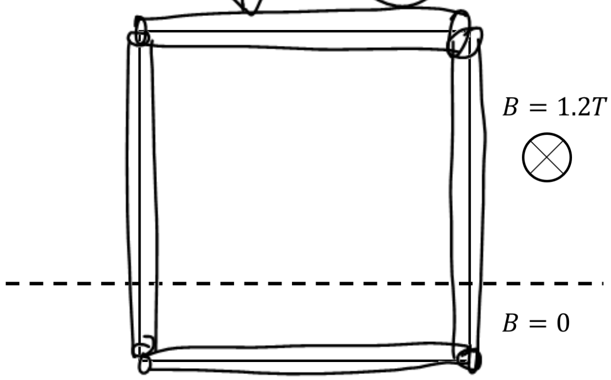
Problem B.3. Faraday's law again

$\boxed{B = \mu n I}$

A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $B(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.



Problem B.4. Terminal Velocity



A vertical square loop of copper wire with sides of length 10cm is falling as shown from a region where the magnetic field is horizontal and of magnitude 1.2T into a region where the field is zero. The wire has a diameter of 1mm.

(a) Calculate the magnitude of the current round the loop in terms of the velocity v of the fall, and indicate its direction.

(b) What is the magnetic force acting on the loop, again expressed in terms of v?

(c) If the velocity of fall reaches a steady value while the upper arm of the circuit remains in the field, calculate this velocity.

The resistivity of copper is $1.7 \times 10^{-8}\Omega$m; the density is 8960kgm^{-3}

Problem B.5. Magnetic field by capacitor

A circular parallel-plate capacitor of radius a and plate separation d is connected in series with a resistor R and a switch, initially open, to a constant voltage source V_0. The switch is closed at time $t = 0$. Assuming that the charging time of the capacitor, $\tau = RC$, is very long compared with a/c and that $d << a$ (C is the capacitance and c is the speed of light),

(a) Find an expression for the displacement current density as a function of time.

(b) Obtain an expression for the magnetic flux density B as a function of time and position between the capacitor plates.

3 Part C

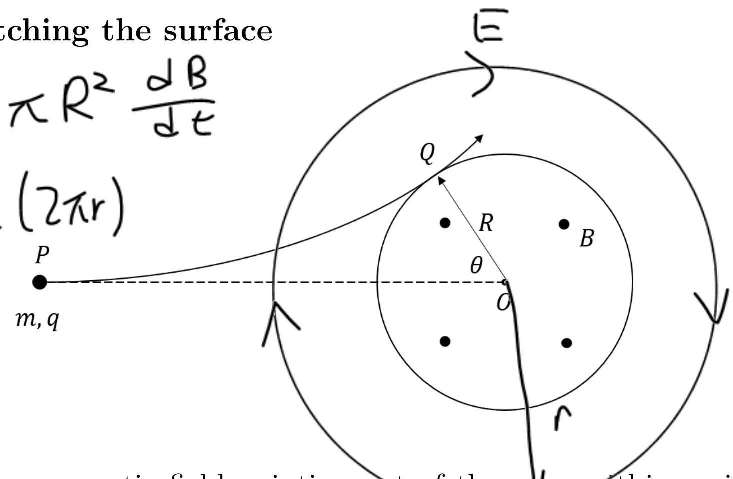
Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!

Problem C.1. Scratching the surface

$\Sigma = -\frac{d\Phi}{dt} = -\pi R^2 \frac{dB}{dt}$

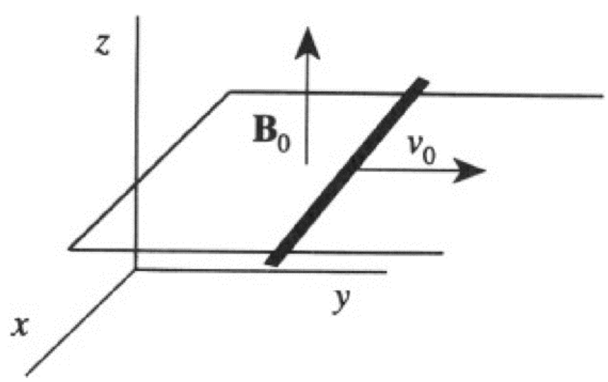
↓

voltage change $= V = E(2\pi r)$



In the figure above, the magnetic field pointing out of the page within a circular region of radius R is changing at a rate of $dB/dt = k > 0$. A charge of mass m and charge q is shot towards the circular region with its initial velocity directed towards the centre of circle O at time $t = 0$. At time $t = \tau$ the particles path is tangent to the circle at point Q where $\angle POQ = \theta$. Find the initial speed of the particle v_0.

Problem C.2. Sliding rod



A copper rod slides on frictionless rails in the presence of a constant magnetic field $B_0\hat{z}$ At $t = 0$ the rod is moving in the y direction with velocity v_0 (see figure).
(a) What is the subsequent velocity of the rod if its conductivity is σ and the mass density of copper is ρ_m.
(b) For copper, $\sigma = 5 \times 10^{17}\text{s}^{-1}$ and $\rho_m = 8900\text{kgm}^{-3}$. If $B_0 = 1$ Gauss, estimate the time it takes for the rod to stop.
(c) Show that the rate of decrease of the kinetic energy of the rod per unit volume is equal to the ohmic heating rate per unit volume.

Problem C.3. Terminal velocity of a loop

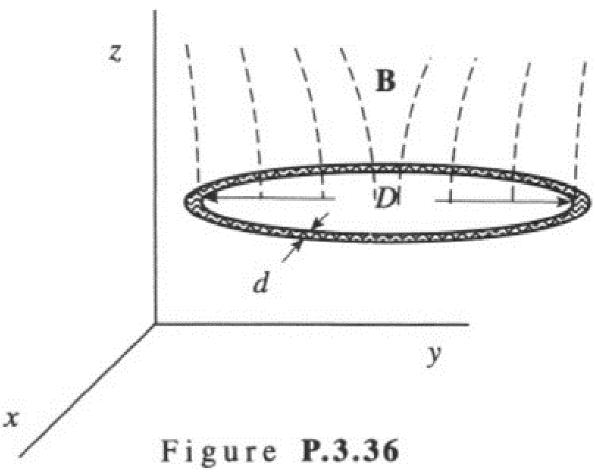


Figure P.3.36

A conducting circular loop made of wire of diameter d, resistivity ρ and mass density ρ_m is falling from a great height h in a magnetic field $B_z = B_0(1 + \kappa z)$ with a component where κ is some constant. The loop of diameter D is always parallel to the $x - y$ plane. Disregarding air resistance, find the terminal velocity of the loop.

Problem C.4. General loop (Griffiths 5.42)

A plane wire loop of irregular shape is situated so that part of it is in a uniform magnetic field B (in Fig. 5.57 the field occupies the shaded region, and points perpendicular to the plane of the loop). The loop carries a current I. Show that the net magnetic force on the loop is $F = IBw$, where w is the chord subtended. Generalize this result to the case where the magnetic field region itself has an irregular shape. What is the direction of the force?

Problem C.5. Various olympiads

Honestly EMI is probably one of the olympiad favorites which can make for challenging

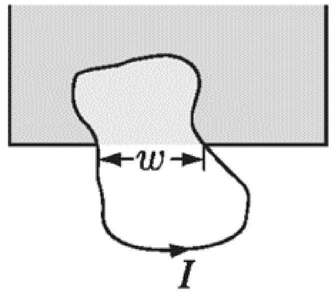


FIGURE 5.57

problems at times. Some good references include IdPhO 2020 Q1, EuPhO 2022 Q3, and IZhO questions (e.g. 2020 and 2015).

4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. Motional emf
Show that if the magnetic field in a given frame is constant in time, then for a loop of any shape moving in any manner, the emf $\mathcal{E}$ around the loops is related to the magnetic flux Φ through the loop by

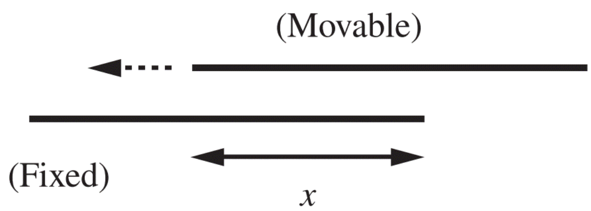
$$\mathcal{E} = -\frac{d\Phi}{dt} \tag{1}$$

Problem D.2. Infinite surfaces
An infinite number of different surfaces can be fit to a given boundary line, and yet, in defining the magnetic flux through a loop, $\Phi = \int \mathbf{B} \cdot \mathbf{da}$, I never specified the particular surface to be used. Justify this apparent oversight.

5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.

Problem E.1. Force on a capacitor plate (Purcell 3.26)



A parallel-plate capacitor consists of a fixed plate and a movable plate that is allowed to slide in the direction parallel to the plates. Let x be the distance of overlap, as shown in the figure below. The separation between the plates is fixed.

(a) Assume that the plates are electrically isolated, so that their charges $\pm Q$ are constant. In terms of Q and the (variable) capacitance C, derive an expression for the leftward force on the movable plate. Hint: Consider how the energy of the system changes with x.

(b) Now assume that the plates are connected to a battery, so that the potential difference is held constant. In terms of ϕ and the capacitance C, derive an expression for the force.

(c) If the movable plate is held in place by an opposing force, then either of the above two setups could be the relevant one, because nothing is moving. So the forces in (a) and (b) should be equal. Verify that this is the case.

Problem E.2. Solenoid induction

An infinite solenoid has radius R and n turns per unit length. The current grows linearly with time, according to $I(t) = Ct$. Find the electric field as a function of r, both inside and outside the solenoid.

Problem E.3. Mutual inductance solenoid

The figure above shows a solenoid of radius a_1 and length b_1 located inside a longer solenoid of radius a_2 and length b_2. The total number of turns is N_1 on the inner coil, N_2 on the outer. Work out an approximate formula for the mutual inductance M.

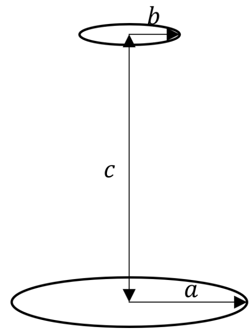
Problem E.4. Rotating coil 2

Consider a closed circuit of wire formed into a coil of N turns with radius a, resistance R, and self-inductance L. The coil rotates in a uniform magnetic field B about a diameter perpendicular to the field.

(a) Find the current in the coil as a function of B for rotation at a constant angular velocity ω. Here $d(t) = \omega t$ is the angle between the plane of the coil and B.

(b) Find the externally applied torque required to maintain this uniform rotation. (In both parts you should assume that all transient effecta have died away.)

Problem E.5. Mutual inductance circular loop (Lim Yung Kuo)



Two circular loops of radius b and a respectively are placed coaxial with a distance z between them. Find their mutual inductance.