Kinematics Problem Set

1 Part A

Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Throwing stones
At the top of a cliff of height 100m, a boy throws a stone vertically upwards at a velocity of 15ms^{-1}. How much time later should he drop a second stone from rest such that both stones reach the bottom of the cliff at the same time?

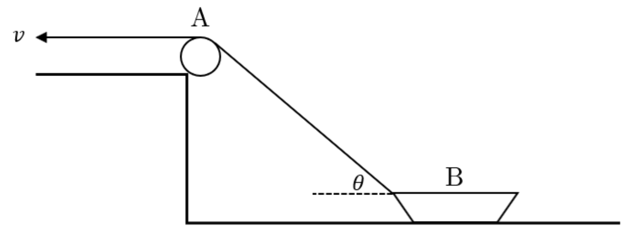
Problem A.2. Relative distances
At the top of a cliff of height 100m, a boy drops a stone. Find t, the time taken for the stone to reach the ground. Also, find h_1, the distance covered by the stone in the first $t/2$ seconds of its motion, and h_2, the distance covered by the stone in the second $t/2$ seconds of its motion.

2 Part B

Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Over a wall
The launch point of a projectile is situated a distance d away from a wall of height h. What is the angle at which the projectile should be launched so as to minimize the velocity requried to clear the wall? What is the corresponding minimum velocity?

Problem B.2. Reeling In
Referring to the diagram on the following page, a man standing at A reels in a boat B via the connecting rope as shown in the figure below. At this instant, the rope is making an angle θ with the horizontal, the rope is pulled back at A with a constant velocity v, and the vertical distance between A and B is h. Find the speed and acceleration of the boat at this instant.



Problem B.3. Triangular snails

Three small snails start off at the vertex of an equilateral triangle of side 60cm. The first set out towards the second, the second towards the third, and the third towards the first, with uniform speed of 5cms^{-1}. During their motion they always head directly towards their respective targets.

(a) How much time would have elapsed before they meet?

(b) What distance would each snail have travelled?

(c) Express the equation for their paths in polar coordinates, with the origin being the centroid of the triangle.

Problem B.4. On a Carousel

Ann is sitting on the edge of a carousel that has a radius of 6m and is rotating steadily. Bob is standing still on the ground at a point that is 12m from the centre of the carousel. At a particular instant, Bob observes Ann moving directly towards him with a speed of 1ms^{-1}. With what speed does Ann observe Bob to be moving at that same moment?

3 Part C

Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!

Problem C.1. Always moving away

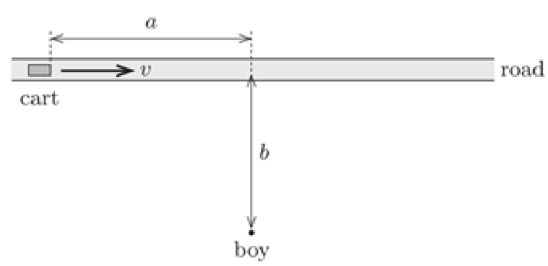
What is the maximum angle to the horizontal at which a stone can be thrown such that it is always moving away from the launch point?

Problem C.2. Over a tree trunk

A tree trunk of diameter 20cm lies in a horizontal field. A lazy grasshopper wants to jump over the tree trunk. What is the minimum take-off speed required?

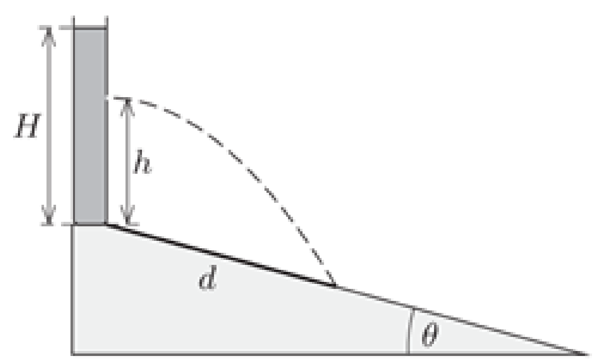
Problem C.3. Boy to Cart

A cart is moving on a straight road with constant velocity v. A boy, standing in an adjoining meadow, spots the cart and hopes to get a ride on it.



(a) In which direction should he run to catch the cart? Solve the problem generally: denote the speed of the cart by v, the maximal speed of the boy by u, and take the initial positions of the cart and boy to be as shown in the figure.
(b) Unlike in (a), the boy now always runs directly towards the cart with a uniform speed u. How long does the boy take to reach the cart?

Problem C.4. Water Jet



At the top of a long incline that makes an angle θ with the horizontal, there is a cylindrical vessel containing water to a depth H. A hole is to be drilled in the wall of the cylinder, so as to produce a water jet that lands a distance d down the incline. How far, h, from the bottom of the vessel should the hole be drilled in order to make d as large as possible? What is this maximum value of d?

4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. Maximum Projectile Length

For a given initial velocity v, at what angle θ to the horizontal should a projectile be launched at to maximize the length of its trajectory.

Problem D.2. Non-linear drag force

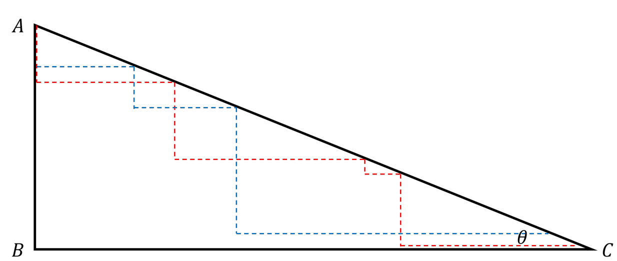
A ball is dropped from $y = H$ at $t = 0$. Suppose that in addition to its own weight, the ball experiences a non-linear drag force of the form $F = -kv^2$. Find the height of the ball $y(t)$ as a function of time.

5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.

Problem E.1. Infinite Paths

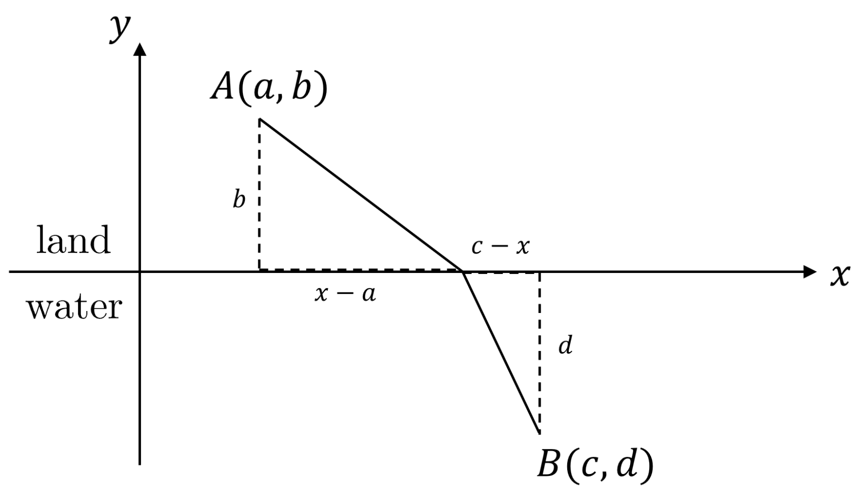
In the figure below, a particle can move from point A to C through a combination of horizontal and vertical motion as depicted by the dotted lines.



(a) Find the angle $\angle ACB = \theta$ such that t_{ABC}, the time taken for the path $A \to B \to C$, equals t_{AC}, the time taken for the direct path $A \to C$.

(b) For this same θ, we now allow the movement of the particle along arbitrary horizontal and vertical dotted lines as previously mentioned. Find the ratio t_{max}/t_{min}, where t_{max} is the maximum possible time for the particle to reach C from A, while t_{min} is the minimum possible time.

Problem E.2. Snell's Law



In the diagram below, Kelvin is currently at point A and wishes to get to point B in the shortest time possible. He runs (on land) two times as fast as he swims (in water). At what point x should he enter the water?

Problem E.3. Accurate Projectile

Daniel is an artillery operator and wishes to hit a target some distance away. In his first attempt, the launch angle was set to α and the projectile landed a distance a in front of the target. In his second attempt, the launch angle was set to β and the projectile landed a distance b behind the target. Show that the launch angle θ required to hit the target is

$$\theta = \frac{1}{2}\sin^{-1}(\frac{a\sin 2\beta + b\sin 2\alpha}{a+b}). \tag{1}$$

Problem E.4. Wheels?

A small object is initially at rest on the edge of a horizontal table. It is pushed (by an sharp impulse) in such a way that it falls off the other side of the 1m wide table after 2s. What is the maximum possible coefficient of kinetic friction between the object and the table?

Problem E.5. Train window

Gareth was bored on the train ride and is looking out the window. When the train is at rest, the raindrops make and angle θ_0 to the left of the vertical. When the train is moving at speed v_1 to the left, the raindrops now make an angle θ_1 to right of the vertical. The train then accelerates further and at speed v_2 to the left, the raindrops make an angle θ_2 to the vertical. Find the ratio v_1/v_2.