Conservation Laws Problem Set

1 Part A

Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Elastic collision

A 5kg mass moving to the right with initial velocity 1ms^{-1} collides head on with a 2kg mass initially at rest. What are their velocities after the collision. Assume the collision is elastic.

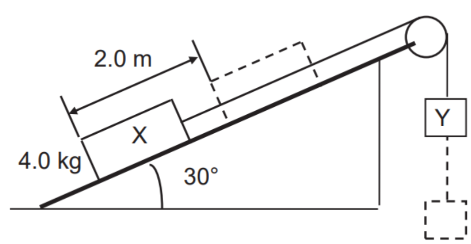
Problem A.2. Inelastic collision

A 5kg mass moving to the right with initial velocity 1ms^{-1} collides head on with a 2kg mass initially at rest. What are their velocities after the collision. Assume the collision is completely inelastic.

Problem A.3. Relative speeds

For a completely elastic collision between masses m_1 and m_2, derive the canonical result that relative speed of approach equals relative speed of separation, from conservation of energy and momentum.

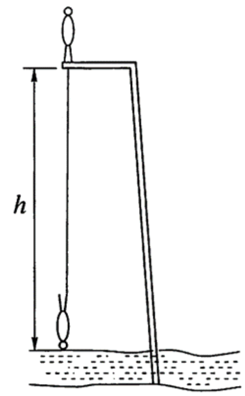
Problem A.4. Pulley energy



The diagram shows two bodies X and Y connected by a light cord passing over a light, free-running pulley. X starts from rest and moves on a slope inclined at 30° to the horizontal. X has a mass of 4.0kg, and Y has a mass of 5.0kg. Friction between X and the slope is 5.0N. What is the total kinetic energy of the system when X has travelled 2.0m along the plane?

Problem A.5. Bungee jumping

I have some reservations about the placement of this problem because although you can help yourself numerically with all the values provided, the calculations aren't that easy compared



to previous Part A questions. But nonetheless I believe everyone should give this a try!
A man of height $h_0 = 2$m is bungee jumping from a platform situated at height $h = 25$m above a lake. One end of an elastic rope is attached to his foot and the other end is fixed to the platfrom. He starts falling from rest is a vertical position. The length and elastic properties of the rope are chosen so that his speed will have been reduced to zero at the instant when his head reaches the surface of the water. Ultimately the jumper is hanging from the rope, with his head 8m above the water.
(a) Find the unstretched length of the rope.
(b) Find the maximum speed and acceleration achieved during the jump.

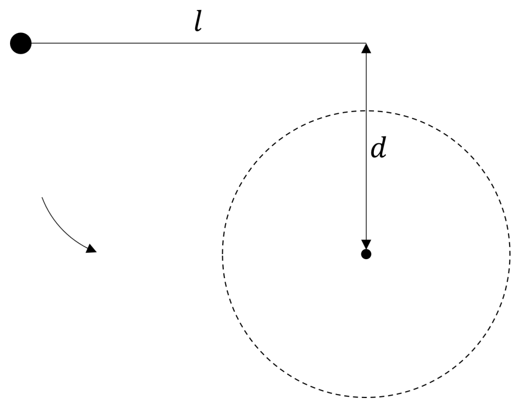
2 Part B

Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Falling Chain
A chain of mass M and length L is suspended vertically with its lower end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale when a length x of the chain has fallen? Neglect the size of the individual links.

Problem B.2. Losing Contact 1
A solid sphere of radius r is initially placed at rest on top of a fixed hemisphere, also of radius r. The solid sphere is then given a slight push and begins rolling down the hemisphere. At what angle θ from the vertical will the sphere lose contact with the hemisphere? Assume the sphere always rolls without slipping.

Problem B.3. Always taut

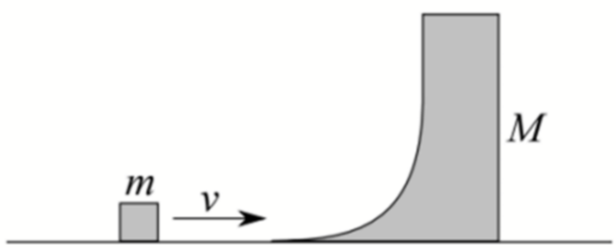


A pendulum of mass m and length l is released from rest from a horizontal position. A nail a distance d below the pivot causes the mass to move in a circle as drawn in the dotted line. Find the minimum distance d for which the mass will swing completely round the circle.

Problem B.4. Orbital ascension

A mass m is attached to one end of a massless string hung from the ceiling and held in the $x-z$ plane, at angle θ to the vertical. The mass is struck in the y direction with some initial velocity u such that the mass barely scrapes the ceiling. Find u.

Problem B.5. Shooting up a curve

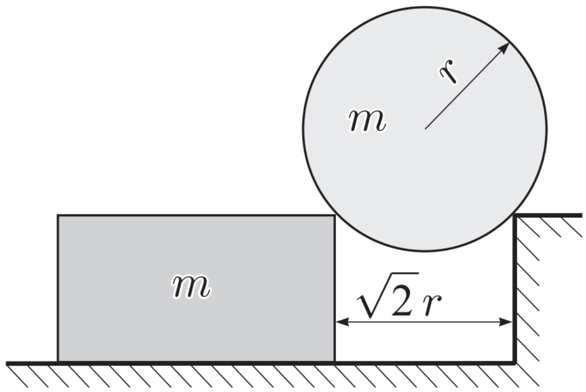


A block of mass m is launched horizontally onto a curved wedge of mass M at a velocity v. What is the maximum height reached by the clokc after it shoots of ther vertical segment of the wedge? Assume all surfaces are frictionless; both the block and the curved wedge are free to move. The curved wedge does not tilt or topple.

3 Part C

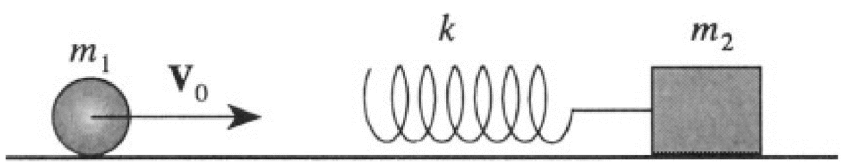
Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like *these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!*

Problem C.1. Losing Contact



Two slippery horizontal surfaces form a step. A block with the same height as the step is pushed near the step, and a cylinder with radius r is placed on the gap. Both the cylinder and the block have mass m. Find the normal force N between the cylinder and the step at the moment when distance between the block and the step is $\sqrt{2}r$. Initially, the block and the step were very close together and all bodies were at rest. Friction is zero everywhere. Will the cylinder first separate from the block or the step?

Problem C.2. Spring Mass Collision



A mass m_1 with initial velocity v_0 strikes a mass-spring system m_2 initially at rest but able to recoil. The spring is massless with spring constant k. There is no friction.

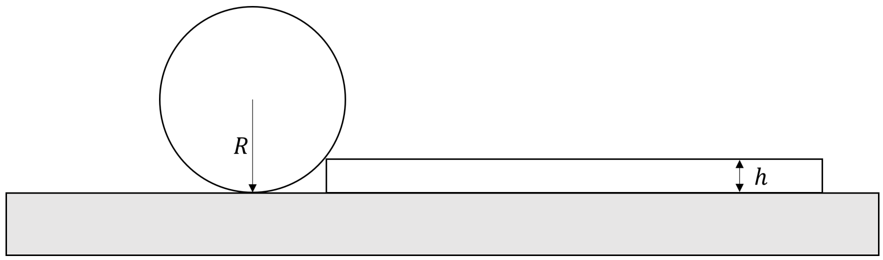
(a) What is the maximum compression of the spring?

(b) If long after the collision the masses travel in the same direction, what are the final velocities v_1, v_2 of m_1, m_2 respectively?

Problem C.3. Losing Contact 3

A ladder of length l and uniform mass density ρ stands on a frictionless floor and leans against a frictionless wall. It is initially held motionless, with its bottom end an infinitesimal distance from the wall. It is then released, whereupon the bottom end slides away from the wall, and the top end slides down the wall. When it loses contact with the wall, what is the horizontal component of the velocity of the center of mass?

Problem C.4. Rolling Over



The figure above shows a solid cylinder of mass M and radius R. You wish to roll the cylinder over a step of height h.
(a) The cylinder is pushed over by continuosly applying some force to some point on the cylinder. What is the minimum force required to roll the cylinder over?
(b) The cylinder is launced with some initial horizontal velocity v. What is the minimum velocity required to launch the cylinder over the step?
Assume pure rolling and no contact loss in both instances described above.

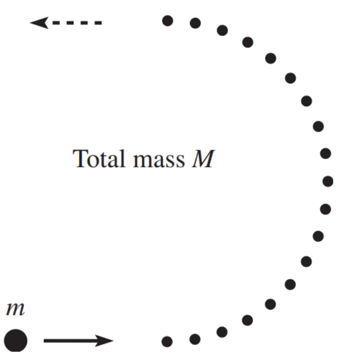
Problem C.5. Maximal deflection
A mass M collides with a stationary mass m. If $M < m$, then it is possible for M to bounce directly backward. However, if $M > m$, then there is a maximal angle of deflection of M. Show that this maximal angle equals $\sin^{-1}(m/M)$.

4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. π balls 1

N identical balls lie equally spaced in a semicircle on a frictionless horizontal table, as shown. The total mass of these balls is M. Another ball of mass m approaches the semicircle from



the left, with the proper initial conditions so that it bounces (elastically) off all N balls and finally leaves the semicircle, heading directly to the left.
(a) In the limit $N \to \infty$, find the minimum value of M/m that allows the incoming ball to come out heading directly to the left.
(b) In the minimum M/m case found in part (a), show that the ratio of m's final speed to initial speed equals $e^{-\pi}$

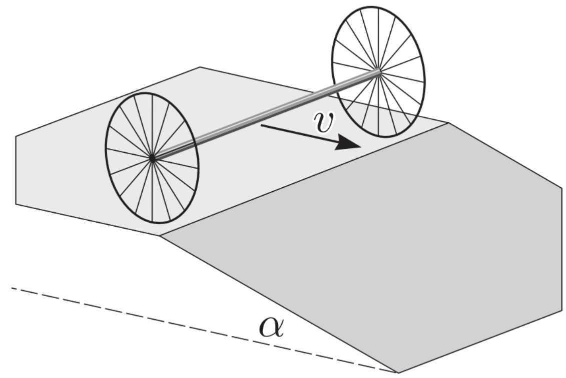
Problem D.2. π balls 2
A block with large mass M slides with speed v_0 on a frictionless table toward a wall. It collides elastically with a ball with small mass m, which is initially at rest at a distance L from the wall. The ball slides toward the wall, bounces elastically, and then proceeds to bounce back and forth between the block and the wall.
(a) How close does the block come to the wall?
(b) How many tiems does the ball bounce off the block, by the time the block makes its closests approach to the wall?
Assume $M \gg m$, and give your answers to leading order in m/M

5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.

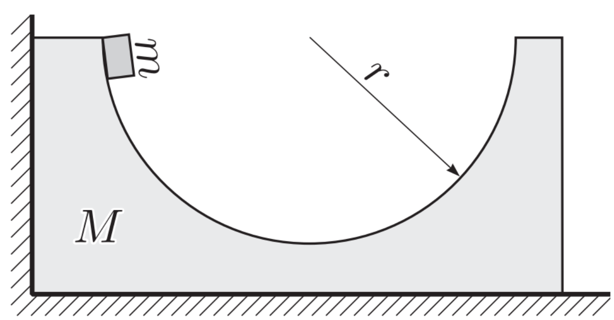
Problem E.1. Introduction to Oscillations
A mass m is placed on a horizontal frictionless ground and attached to a wall with a spring with spring constant k. Find the equations of motion and the frequency of oscillations through applying conservation of energy.

Problem E.2. Wheel down Slope



Light wheels with radius R are attached to a heavy axle. The system rolls along a horizontal surface which suddenly turns into a slope with angle α. For which angles α will the wheels move without lifting off, i.e. touch the surface at all times? Mass of the wheels can be neglected. The axle is parallel to the boundary between horizontal and sloped surfaces and has velocity v.

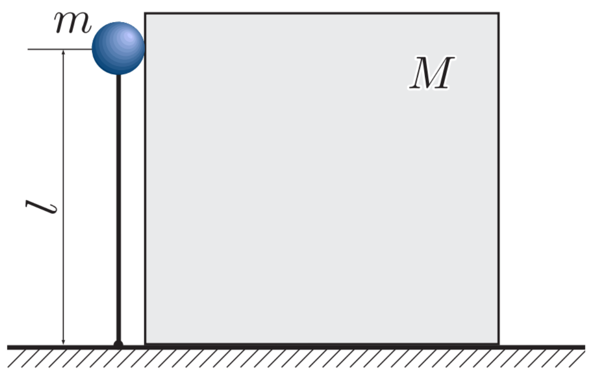
Problem E.3. Hemisphere sliding



A block with mass M lies on a horizontal slippery surface and also touches a vertical wall. In the upper surface of the block, there is a cavity with the shape of a half-cylinder with radius r. A small pellet with mass m is released at the upper edge of the cavity, on the side closer to the wall. What is the maximum velocity of the block during its subsequent motion? Friction can be neglected.

Problem E.4. Separation

A light rod with length l is connected to the horizontal surface with a hinge; a small sphere of mass m is connected to the end of the rod. Initially the rod is vertical and the sphere rests against the block of mass M. The system is left to freely move and after a certain time



the block loses contact with the surface of the block at the moment when the rod forms an angle $\alpha = \pi/6$ with the horizontal. Find the ratio of masses M/m and the velocity u of the block at the moment of separation.

Problem E.5. Bouncing Ball

A ball falls down from height h, initially the ball's horizontal velocity was v_0 and it wasn't rotating.

(a) Find the velocity and the angular velocity of the ball after the following collision against the floor: the ball's deformation against the floor was absolutely elastic, yet there was friction at the contact surface such that the part of the ball that was in contact with the floor stopped.

(b) Answer the same question with the assumption that the velocities of the surfaces in contact never homogenized and that throughout the collision there was friction with coefficient
.