Magnetic Fields Problem Set

1 Part A

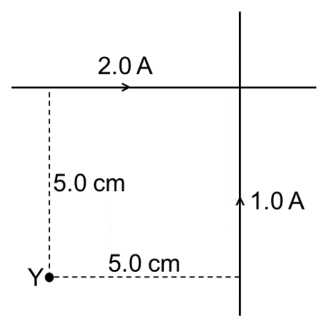
Part A problems are hands-on application problems for students to practice directly applying the concepts and formulas to mathematically understand simple physical scenarios.

Problem A.1. Solenoid field

Find the magnetic field at the center of a solenoid with 10 turns per cm, when a current of $I = 2A$ is flowing through the solenoid.

Problem A.2. Crossed wires

Find the magnetic field at point Y labelled in the diagram below.



Problem A.3. Velocity selector

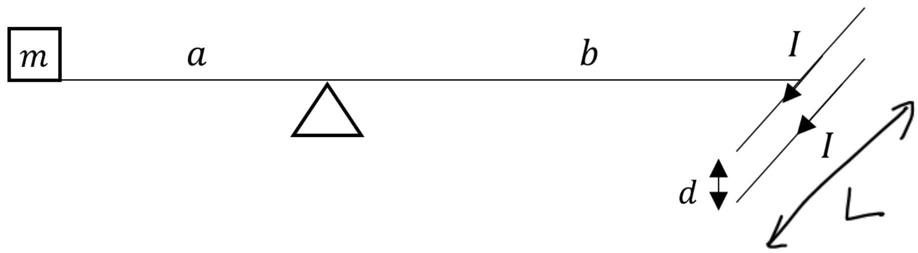
What is the velocity u at which the electron will pass through the magnetic field region undeflected?

Problem A.4. Current force

Two wires carrying currents I_1 and I_2 in opposite directions respectively are placed a distance l from each other. Find the force of interaction between the wires. Is the force attractive or repulsive?

per unit length

Problem A.5. Weighing balance

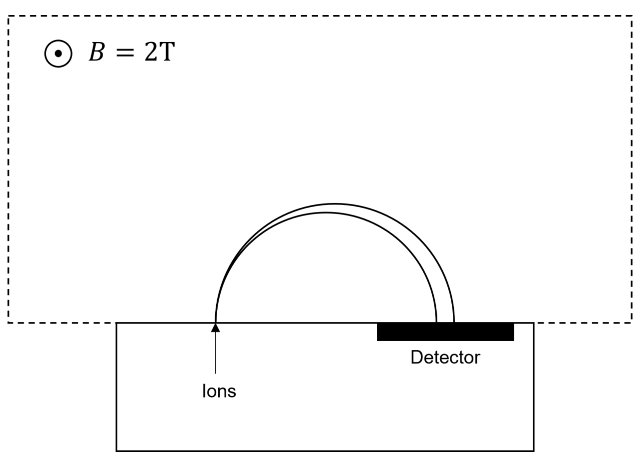


A weighing balance consists of the set-up shown in the figure above. Find the current I needed to keep the set-up level in its current configuration. The gravitational field strength is g.

2 Part B

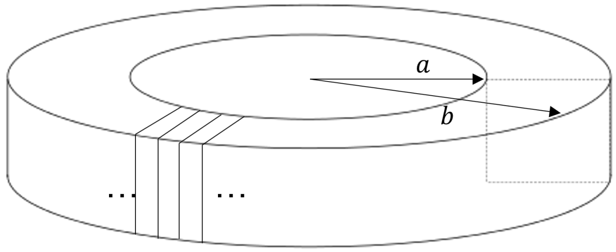
Intermediate problems like those in Part B tend to be more subtle, but these are the ones, when practiced in bulk, develop physical intuition and a tighter grasp on the abstract formulas and equations.

Problem B.1. Ion detector



Two ions, neon-20 and neon-22, both with charge $+e$, is passed through a region of magnetic field $B = 2\text{T}$ directed out of the page, with the same initial velocity of $2 \times 10^5 \text{ms}^{-1}$. What is the distance between their impact points on the detector? Use $e = 1.6 \times 10^{-19}\text{C}$ and $u = 1.66 \times 10^{27}\text{kg}$.

Problem B.2. Toroid field



(a) Find the magnetic field at the center of a rectangular toroid with inner radius a, outer radius b, height h carrying current I through N turns of wire.
(b) Find the the magnetic flux through its rectangular cross section.

Problem B.3. Circular wire
Find the magnetic field B as a function of z on the axis of a circular loop of radius R, with its center at the origin, carrying current I in the anti-clockwise direction.

Problem B.4. Line field
Find the magnetic field a distance z away from the center of a wire of length l carrying current I.

Problem B.5. Square field
Find the magnetic field at the center of a square loop of side length a carrying current I.

3 Part C

Here in Part C you should find the most conceptually complex and mathematically intensive problems, but you should still savour the process of carefully thinking through questions like these, for this is the essence which makes problem-solving in physics enjoyable. And don't forget to give yourself a pat on the back after solving these!

Problem C.1. Spinning shell
Find the magnetic field at the center of a uniformly charged spherical shell, of radius R and total charge Q, spinning at constant angular velocity ω.

Problem C.2. Line charges

Suppose you have two infinite straight line charges λ, a distance d apart, moving along at

charge per unit length

$B = \frac{\mu_0 I}{2\pi r}$

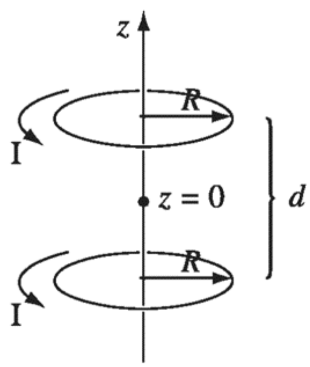
① Express I in terms of λ, v

② Use Gauss' Law and Ampere's Law

$E = \frac{\lambda}{2\pi r \varepsilon_0}$

a constant speed v. How great would v have to be in order for the magnetic attraction to balance the electrical repulsion?

Problem C.3. Helmholtz coil (Griffiths 5.47)



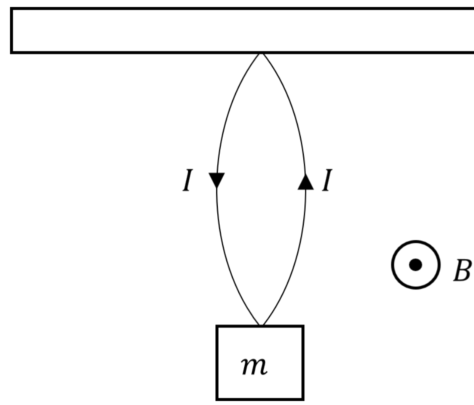
(a) Find the magnetic field B as a function of z.
(b) Show that $\partial B/\partial z = 0$ at $z = 0$.
(c) Find d such that $\partial^2 B/\partial z^2 = 0$ at $z = 0$.

Problem C.4. Exotic motion

A particle of positive charge q and mass m is released from rest at the origin in a region at which magnetic field B points in the x-direction and electric field E points in the z-direction. Find the resulting motion path of the particle. (Find its position as a function of time)

Problem C.5. Lifting up (Estonian-Finnish 2012)

A mass m is originally suspended from a two-part wire, each part of length l, such that the mass was originally a length l from the ceiling. A magnetic field B is uniformly directed out of the page. A switch is then turned on such that a current I flows in the wires (figure on next page). What current I is needed to raise the mass by a height of $\Delta h = l(1 - 3/\pi)$



4 Part D

**Bonus Part D problems span a variety: from quick brain teasers to challenging ones employing even more advanced techniques than those found in Part C. They are usually out of the scope of even the most accelerated high school curriculums, and will likely not be tested for olympiads either. But nonetheless, here's one such problem to keep you entertained if you are done with all the above.*

Problem D.1. Magnetic dipole
A magnetic dipole can be modelled as a circular loop of wire of radius r carrying current I. Consider a magnetic dipole at the origin with its axis oriented in the z-direction (that means the loop lies in the $x-y$ plane). Show that the magnetic field far away from the axis can be written in spherical coordinates as

$$B = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\boldsymbol{r}} + \sin\theta\hat{\boldsymbol{\theta}}) \tag{1}$$

Problem D.2. Spinning shell 2
Calculate the magnetic force of attraction between the northern and southern hemispheres of a spinning spherical shell of radius R with surface charge density σ uniformly distributed over its surface. The angular velocity is ω.

5 Part E

Supplementary problems entirely for your own practice and will not be covered in class.
Out next week!

Problem E.1. Cube Magnetic Field (Cahn)
The current I flowing along the edges of one face of a cube (see Figure P.3.31a) produces a

magnetic field in the center of the cube of magnitude B_0. Consider another cube where the current I flows along a path shown in Figure P.3.31 b. What magnetic field will now exist at the center of the cube?

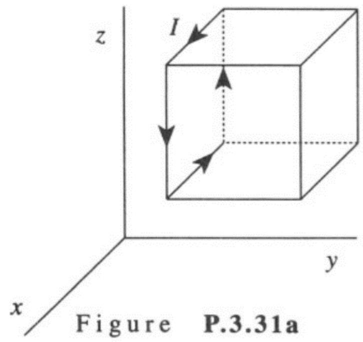


Figure **P.3.31a**

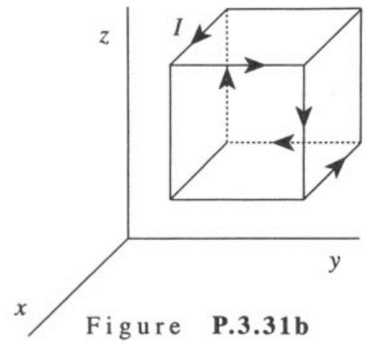


Figure **P.3.31b**

Problem E.2. Levitating Ring

Find the force experienced by a small ring of radius a carrying current I in the z-direction placed in a magnetic field $B = B_0(1 + \alpha z)$.

Problem E.3. Constant B (Purcell)

How should the current density inside a thick cylindrical wire depend on r so that the magnetic field has constant magnitude inside the wire?

Problem E.4. Radial Exit (Griffiths)

A circularly symmetrical magnetic field (**B** depends only on the distance from the axis),

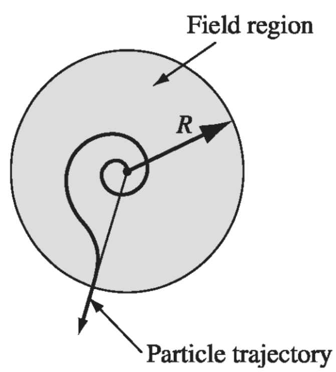


FIGURE 5.58

pointing perpendicular to the page, occupies the shaded region in Fig. 5.58. If the total flux ($\int \mathbf{B} \cdot \mathbf{da}$) is zero, show that a charged particle that starts out at the center will emerge from the field region on a radial path (provided it escapes at all). On the reverse trajectory, a particle fired at the center from outside will hit its target (if it has sufficient energy), though it may follow a weird route getting there. [Hint: Calculate the total angular momentum acquired by the particle, using the Lorentz force law.]

Problem E.4. Radial Exit (Cahn)

A nonrelativistic charged particle is orbiting in a uniform magnetic field of strength B_0 at the center of a large solenoid. The radius of the orbit is R_0. The field is changed slowly to B_1. What is the new radius R_1 of the orbit? If the field is suddenly changed back to what is the final radius R_2?