# Low dimensional examples

Shing Tak Lam

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In this note, we study low dimensional examples of the construction in [1]. We will use the notation as in §2 of the paper.

## 1 Case $k = 1$

In this case, the diagram is

$$0 \underset{\beta_0}{\overset{\alpha_0}{\rightleftarrows}} V_1$$

So there is only one point in $M$, which is $(0, 0)$. In this case, the image of $\Phi^c$ is the zero orbit.

## 2 Case $k = 2$

In this case, we have the diagram

$$0 \underset{\beta_0}{\overset{\alpha_0}{\rightleftarrows}} V_1 \underset{\beta_1}{\overset{\alpha_1}{\rightleftarrows}} V_2$$

In this case, for a point $p = (\alpha_1, \beta_1) \in \mu_c^{-1}(0)$[1], we have $X = \alpha_1\beta_1$, and $\beta_1\alpha_1 = 0$, so $X^2 = 0$. Therefore, all of the Jordan blocks for $X$ have size at most 2.

Since $\operatorname{rank}(X) = \operatorname{rank}(\alpha_1\beta_1) \le \min\{\operatorname{rank}(\alpha_1, \beta_1)\} \le \dim(V_1)$ , and $\operatorname{rank}(X)$ is the number of nonzero Jordan blocks, this gives us a relation between the number of Jordan blocks and the dimension of $V_1$.

## References

[1] Piotr Z. Kobak and Andrew Swann. "Classical nilpotent orbits as hyperkähler quotients". In: *Int. J. Math.* 07.02 (Apr. 1996). Publisher: World Scientific Publishing Co., pp. 193–210. ISSN: 0129-167X. DOI: 10.1142/S0129167X96000116. URL: https://www.worldscientific.com/doi/10.1142/S0129167X96000116 (visited on 07/27/2023).

[1] We omit $\alpha_0, \beta_0$ as they are zero.