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Robust Trajectory Tracking of Delta Parallel Robot Using Sliding Mode Control

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***Abstract*—For purpose of realizing robust trajectory tracking of parallel Delta robot with uncertain model parameters, a sliding mode control approach based on Delta robot dynamics is proposed. Once the reference trajectory is given, we can obtain the sliding mode controller, so that Delta parallel robot can move along the reference trajectory. Meanwhile, the disturbance and uncertainty of robot system are considered in the proposed method, which is beneficial to improve the reliability of controller and practicability of robot. The system stability is proved by Lyapunov method. Simulation experiment results demonstrate that the robust trajectory tracking of parallel Delta robot can be realized by the proposed method, and the designed controller is simple, robust and reliable.**

***Keywords*—parallel robot; sliding mode control; stability; trajectory tracking.**

I. INTRODUCTION

Parallel robots [1], [2] have the advantages of high precision, large stiffness, good dynamic performance and compact structure [3], [4], so they are widely used in various production lines. Delta parallel robot is a widely used manipulator within the niche market of parallel robotics. Delta parallel robots are widely used in the rapid sorting, grabbing and assembly of industrial automation production lines due to their unique parallel structure [5], [6], which have the advantages of fast speed, accurate positioning and high efficiency. The Delta parallel robot usually has 3 degrees of freedom [7], and its end-effector (EE) is able to perform translational motion on three mutually orthogonal axes on the base coordinate system. The driving motor of each joint of Delta robot is usually mounted on the fixed platform, which can greatly reduce the robot inertia during the movement, and realize the acceleration and deceleration motion quickly.

Parallel robot is a multivariable, multi-parameter coupling, multi-degree-of-freedom nonlinear system, and the control strategy of the system is usually complicated. High-precision trajectory tracking is conducive to high-quality manufacturing, so it is important to study parallel robot trajectory tracking control. Over the past few decades, many researchers have studied the parallel robots trajectory tracking control [8]. Generally speaking, the commonly used control strategies for robots are strategies based on kinematic model or dynamic model. Position-based control is one of the commonly used trajectory tracking methods based on kinematic model. In the position-based control method, the parallel robot is divided into several independent active joint systems, and the controller performs position control on each motor respectively. PD control is commonly used for position control. Some scholars consider each degree of freedom of a parallel manipulator as a separate system, and PID control is used for each system respectively, but the control effect is not ideal. The above control method does not consider the dynamics of Delta robot, while the inertia and gravity affects the positioning accuracy. When the position control strategy is adopted to all the independent kinematic chains of the parallel robot, the motion error between the desired trajectory and actual trajectory will force parallel robots to deform. In particular, when the speed or load of the robot is large, the deformation will be more prominent and may even destroy the robot. Therefore, for purpose of getting excellent dynamic quality for high-speed robots, the control strategy based on dynamic model is usually adopted for parallel robot, which generally needs to obtain the inverse dynamic model in advance. The commonly used control methods based on dynamic model for parallel robot include augmented PD control and computed torque control [9], [10]. However, the above control methods do not consider various uncertainties of system. In the working process of the robot system, there are external disturbances and various uncertain factors, such as model error, parameter error, measurement error, joint friction and so on. In order to make the robot system stable under uncertainty conditions, many control strategies have been presented. An optimization design and tuning approach of PID-type interval type-two fuzzy logic

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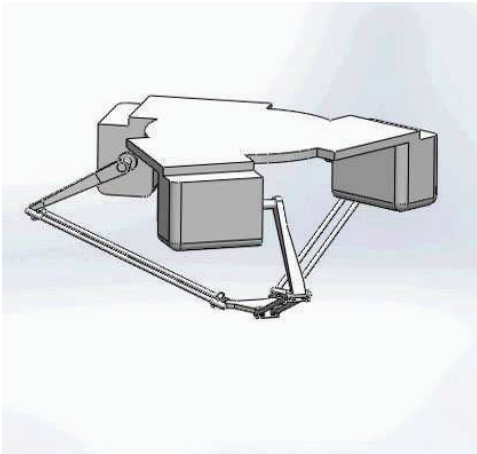

Fig. 1. Delta robot diagram.

controller for trajectory tracking control of Delta parallel robot [11] and a systematic design method based on interval type-two fuzzy logic control method [12] were presented. However, how to choose linguistic rules and ensure the stability of the robot system is still a difficult problem. Sliding mode control is an important class of discontinuous feedback methods for realizing robust stable control of nonlinear systems. Sliding mode control can not only stabilize the system in a limited time, but also have the inherent anti-interference ability. Sliding mode controller can also effectively track the trajectory when the system is subject to parameter perturbation, and its physical implementation is simple. In this article, a sliding mode controller is proposed to realize trajectory tracking of Delta robots with uncertain parameters.

In this article, Delta robot dynamics model is established and analyzed. Then, the sliding mode control strategy is proposed. Moreover, the system stability is proved by Lyapunov theorem. Finally, simulation experiment results demonstrate the effectiveness of the proposed method.

II. Delta Robot Dynamics Model

Firstly, Delta robot dynamics is introduced, on which basis a sliding mode control strategy is proposed. Fig. 1 illustrates the diagram of a Delta robot.

The dynamics of Delta robot is as follows [13]–[15].

$$M(q(t))\frac{d^2q(t)}{dt^2} + G(q(t)) + R(q(t),p(t))\lambda(t) = \tau(t) \quad (1)$$

Where $M(q(t))$ denotes the inertia matrix, $q(t)$, which is $[\ q_1(t) \quad q_2(t) \quad q_3(t)\ ]^{\mathrm{T}}$, represents the actuated joint position vector, $G(q(t))$ denotes the gravitational vector term, $p(t)$, which is $[\ p_x(t) \quad p_y(t) \quad p_z(t)\ ]^{\mathrm{T}}$, denotes the position of EE, $R(q(t),p(t))$ is the kinematic restriction term, $\lambda(t)$, which is $[\ \lambda_1(t) \quad \lambda_2(t) \quad \lambda_3(t)\ ]^{\mathrm{T}}$, denotes the Lagrange multipliers term, $\tau(t)$, which is $[\ \tau_1(t) \quad \tau_2(t) \quad \tau_3(t)\ ]^{\mathrm{T}}$, denotes the input torque.

III. Trajectory Tracking Control

During the movement of Delta robot, there is a certain error between the reference trajectory and actual trajectory. For example, the motion of robot's EE may have deviation between the actual position and the desired position at the initial point due to calibration and measurement errors, etc. Meanwhile the trajectory tracking control method may not consider the dynamics of Delta robot, while the inertia and gravity effects will inevitably affect positioning accuracy. In addition, there are calculation errors, encoder errors, motor nonlinear errors, etc., which will also cause errors between the actual trajectory and the desired trajectory. Therefore, the control method based on dynamics under uncertain conditions is studied. In this article, a sliding mode controller is proposed to meet the requirements for trajectory tracking of the robot system.

The reference trajectory is given at first, on which basis, the dynamics of Delta parallel robot is used to calculated the control force. Then the motor of every active joint is controlled according to the corresponding force so that the Delta robot could move along the reference trajectory.

A. Controller Design

The Delta robot dynamics equation (1) can be rewritten as follows.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (2)$$

In general, the actual model cannot be exactly the same as the nominal model. Therefore, considering the outside interference, dynamics parameter uncertainty and other system uncertainty factors, the model perturbation terms are added to equation (2), and the following model can be obtained.

$$\hat{M}(q)\ddot{q} + \hat{C}(q,\dot{q})\dot{q} + \hat{G}(q) + \tau_d = \tau \quad (3)$$

Where $\hat{M}(q) = M(q) - M_e(q)$ represents the actual inertia matrix, $\hat{C}(q,\dot{q})\dot{q} = C(q,\dot{q})\dot{q} - C_e(q,\dot{q})\dot{q}$ represents the actual Coriolis terms, and $\hat{G}(q) = G(q) - G_e(q)$ represents the actual gravitational vector terms of Delta parallel robot, $M_e(q)$, $C_e(q,\dot{q})$, $G_e(q)$ represent the errors between the actual model and the nominal model caused by uncertainty of model parameters. τ_d represents the external disturbance term. $q_{ref}(t) = [q_{ref1}, q_{ref2}, q_{ref3}]^{\mathrm{T}}$ denotes the reference joint trajectory of the EE of Delta robot. Thus the tracking joint position error e and its derivative $\dot{e}$ can be obtained by

$$\begin{aligned} e &= q_{ref} - q \\ \dot{e} &= \dot{q}_{ref} - \dot{q} \end{aligned} \quad (4)$$

Where $\dot{q}$ represents the actual joint speed of the EE of Delta robot, and $\dot{q}_{ref}$ represents the reference joint speed of the EE of Delta robot.

A robust controller was designed to make the EE of Delta robot move along the desired trajectory as far as possible even if there exists model uncertainty and external interference. The following switching function is selected in proposed controller.

$$s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \dot{e} + De \quad (5)$$

Where D is a diagonal matrix, and the elements on the diagonal of the matrix D are d_1, d_2 and d_3. d_1, d_2 and d_3 are constants greater than zero. So

$$\begin{aligned} \dot{s} &= \ddot{e} + D\dot{e} = \ddot{q}_{ref} - \ddot{q} + D\dot{e} \\ &= \ddot{q}_{ref} - \hat{M}(q)^{-1}\left[\tau - \hat{C}(q,\dot{q})\dot{q} - \hat{G}(q) - \tau_d\right] + D\dot{e} \end{aligned} \tag{6}$$

where $\ddot{q}$ represents the joint acceleration of the Delta robot's EE, and $\ddot{q}_{ref}$ represents the desired joint acceleration of the Delta robot's EE.

For purpose of stabilizing the parallel robot system, the controller for Delta robot system can be defined as follows.

$$\begin{aligned} \tau &= M(q)\ddot{q}_{ref} + C(q,\dot{q})\dot{q}_{ref} + G(q) \\ &+ K_p s + K_i \int s\mathrm{d}t + \begin{bmatrix} n_1 \cdot \mathrm{sgn}(s_1) \\ n_2 \cdot \mathrm{sgn}(s_2) \\ n_3 \cdot \mathrm{sgn}(s_3) \end{bmatrix} \\ &+ M(q)D\dot{e} + C(q,\dot{q})De \end{aligned} \tag{7}$$

Where $K_p > 0$, $K_i > 0$, n_1, n_2 and n_3 are constants.

In order to ensure the convergence of the robot system, candidate Lyapunov function is defined as follows.

$$V = \frac{1}{2}s^{\mathrm{T}}\hat{M}(q)\,s + \frac{1}{2}\left(\int s\mathrm{d}t\right)^{\mathrm{T}} K_i \left(\int s\mathrm{d}t\right) \tag{8}$$

By deriving the Lyapunov function, we can get the following formula.

$$\begin{aligned} \dot{V} &= \frac{1}{2}(\dot{s}^{\mathrm{T}}\hat{M}(q)\,s + s^{\mathrm{T}}\,\dot{\hat{M}}(q)\,s + s^{\mathrm{T}}\hat{M}(q)\,\dot{s}) \\ &+ s^{\mathrm{T}}K_i\left(\int s\mathrm{d}t\right) \\ &= s^{\mathrm{T}}\hat{M}(q)\,\dot{s} + \frac{1}{2}s^{\mathrm{T}}\,\dot{\hat{M}}(q)\,s + s^{\mathrm{T}}K_i\left(\int s\mathrm{d}t\right) \\ &= s^{\mathrm{T}}\hat{M}(q)\,\dot{s} + \frac{1}{2}s^{\mathrm{T}}\left(\dot{\hat{M}}(q) - 2\hat{C}(q,\dot{q})\right)s \\ &+ s^{\mathrm{T}}\hat{C}(q,\dot{q})s + s^{\mathrm{T}}K_i\left(\int s\mathrm{d}t\right) \end{aligned} \tag{9}$$

Since $\dot{\hat{M}}(q) - 2\hat{C}(q,\dot{q})$ is a skew symmetric matrix, therefore the quadratic form of this matrix is zero. So the following equation can be obtained.

$$\dot{V} = s^{\mathrm{T}}\left(\hat{M}(q)\,\dot{s} + \hat{C}(q,\dot{q})s + K_i\int s\mathrm{d}t\right) \tag{10}$$

By substituting formula (6) into the formula (10), the following equation can be obtained.

$$\begin{aligned} \dot{V} &= s^{\mathrm{T}}\begin{pmatrix} \hat{M}(q)\begin{bmatrix} -\hat{M}(q)^{-1}\begin{pmatrix} \tau - \hat{C}(q,\dot{q})\dot{q} \\ -\hat{G}(q) - \tau_d \end{pmatrix} \\ +\ddot{q}_{ref} + D\dot{e} \end{bmatrix} \\ +\hat{C}(q,\dot{q})s + K_i\int s\mathrm{d}t \end{pmatrix} \\ &= s^{\mathrm{T}}\begin{pmatrix} -\tau + (M(q) - M_e(q))\left[\ddot{q}_{ref} + D\dot{e}\right] \\ +(C(q,\dot{q})\dot{q} - C_e(q,\dot{q})\dot{q}) \\ +G(q) - G_e(q) + \tau_d + \\ (C(q,\dot{q}) - C_e(q,\dot{q}))\,(\dot{q}_{ref} - \dot{q} \\ +D(q_{ref} - q)) + K_i\int s\mathrm{d}t \end{pmatrix} \\ &= s^{\mathrm{T}}\begin{pmatrix} -\tau + M(q)\left[\ddot{q}_{ref} + D\dot{e}\right] + G(q) \\ +C(q,\dot{q})(D(q_{ref} - q) + \dot{q}_{ref}) \\ -M_e(q)\left[\ddot{q}_{ref} + D\dot{e}\right] - G_e(q) + \tau_d \\ -C_e(q,\dot{q})(D(q_{ref} - q) + \dot{q}_{ref}) + K_i\int s\mathrm{d}t \end{pmatrix} \\ &= s^{\mathrm{T}}\begin{pmatrix} -\tau + M(q)\ddot{q}_{ref} + C(q,\dot{q})\dot{q}_{ref} + G(q) \\ +M(q)D\dot{e} + C(q,\dot{q})De \\ -M_e(q)\ddot{q}_{ref} - C_e(q,\dot{q})\dot{q}_{ref} - G_e(q) + \tau_d \\ -M_e(q)D\dot{e} - C_e(q,\dot{q})De + K_i\int s\mathrm{d}t \end{pmatrix} \end{aligned} \tag{11}$$

Where $M_e(q)\ddot{q}_{ref} + C_e(q,\dot{q})\dot{q}_{ref} + G_e(q) - \tau_d + M_e(q)D\dot{e} + C_e(q,\dot{q})De$ is caused by external disturbances and model errors, denoted as E_d, which is set as follows.

$$\begin{aligned} E_d &= M_e(q)\ddot{q}_{ref} + C_e(q,\dot{q})\dot{p}_{ref} + G_e(q) - \tau_d \\ &+ M_e(q)D\dot{e} + C_e(q,\dot{q})De \end{aligned} \tag{12}$$

Since the disturbances are always bounded, the item E_d has an upper bound, which is as follows.

$$|E_{di}| \leq \varepsilon_i, \varepsilon_i > 0 \tag{13}$$

Where i is from 1 to 3, and E_{di} is the element on the diagonal of the matrix E_d. Therefore, formula (11) can be simplified.

$$\dot{V} = s^{\mathrm{T}}\begin{bmatrix} -\tau + M(q)\ddot{q}_{ref} + C(q,\dot{q})\dot{q}_{ref} + G(q) \\ + M(q)D\dot{e} + C(q,\dot{q})De \\ - E_d + K_i\int s\mathrm{d}t \end{bmatrix} \tag{14}$$

Substituting formula (7) into the above formula, we can get the following formula.

$$\dot{V} = s^{\mathrm{T}}\left[-K_p s + \begin{bmatrix} -n_1 \cdot \mathrm{sgn}(s_1) \\ -n_2 \cdot \mathrm{sgn}(s_2) \\ -n_3 \cdot \mathrm{sgn}(s_3) \end{bmatrix} - E_d \right] \tag{15}$$

So if $n_i \geq \varepsilon_i$, the following inequality holds.

$$\dot{V} \leq -s^{\mathrm{T}}K_p s \leq 0 \tag{16}$$

Therefore, the tracking joint position error e and its derivative $\dot{e}$ converge asymptotically to 0.

IV. Simulation Experiment Results

The diagram of the robust nonlinear sliding mode controller of Delta robot is illustrated in Fig. 2. Simulink in MATLAB is used for simulation to demonstrate the effectiveness of the presented sliding mode controller. Firstly, the workspace of Delta robot is analyzed, which is demonstrated in Fig. 3.

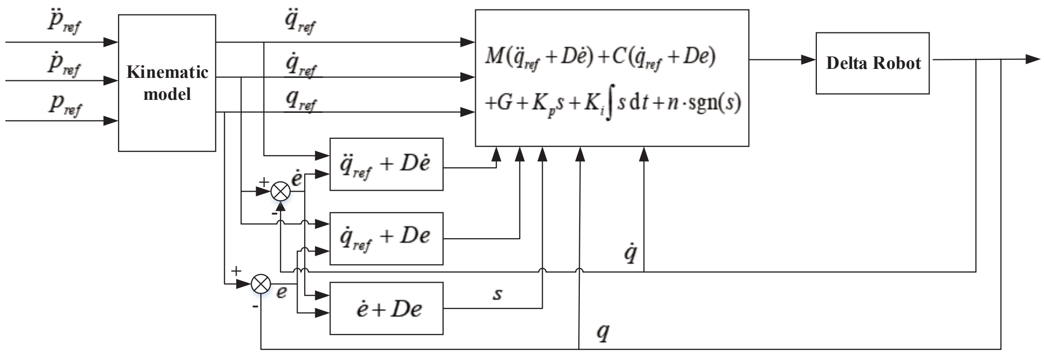

Fig. 2. Diagram of the sliding mode controller of parallel Delta robot.

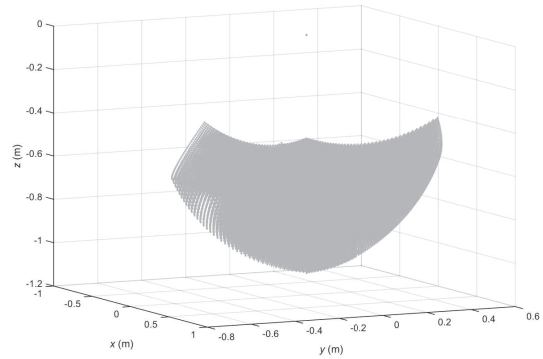

Fig. 3. Workspace of Delta robot.

Fig. 4. Delta robot.

From Fig. 3 we can see that the higher the position of the Delta robot's EE is in the workspace, the larger the range of its movement is in this horizontal plane. In contrast, the closer the position of the Delta robot's EE is to the bottom of the workspace, the smaller the range of its movement is in this horizontal plane. In order to eliminate destroying robot's mechanical structure and make the robot have good kinematic performance, the EE of robot should work within the workspace and as far as possible from the boundary of the workspace.

Delta robot is illustrated in Fig. 4. The geometric parameters and dynamic parameters of the lab's Delta robot are listed in Table I.

TABLE I: THE GEOMETRIC PARAMETERS AND DYNAMIC PARAMETERS OF DELTA ROBOT

Parameter	Value
Length of driving arm	0.325m
Length of follower arm	0.800m
Radius of fixed platform	0.150m
Radius of moveable platform	0.051m
Mass of moveable platform	0.320kg
Mass of driving arm	1.070kg
Mass of follower arm	0.600kg

In simulation experiment, we use the following desired trajectory $(q_1 = 0.55\sin\alpha, q_2 = 0.7\sin\alpha, q_3 = 0.4\sin\alpha)\mathrm{rad}$, where $\alpha = \pi t$. α is determined by the sampling period and accuracy requirements of system. The controller parameters are selected as: $d_1 = d_2 = d_3 = 4$, $n_1 = n_2 = n_3 = 5$. The simulation experiment results of position and speed of each joint based on the proposed sliding mode controller are illustrated in Fig. 5, Fig. 6, Fig. 7 separately. We can see under the proposed sliding mode controller, the errors between the desired trajectory and actual trajectory are very small, and the root mean square error is 0.0039, which indicates that the controller can track the reference trajectory well. That is to say that the proposed method can be used to achieve trajectory tracking of EE of parallel Delta robot. Meanwhile, its control structure is simple, robust and reliable. Moreover, the disturbance and uncertainty of robot system are considered in the proposed method, which is beneficial to eliminate external disturbances and uncertainties, and further improve the reliability of controller and practicability of Delta robot.

V. CONCLUSION

For purpose of achieving the robust trajectory tracking of Delta robot when the Delta robot system has system distur-

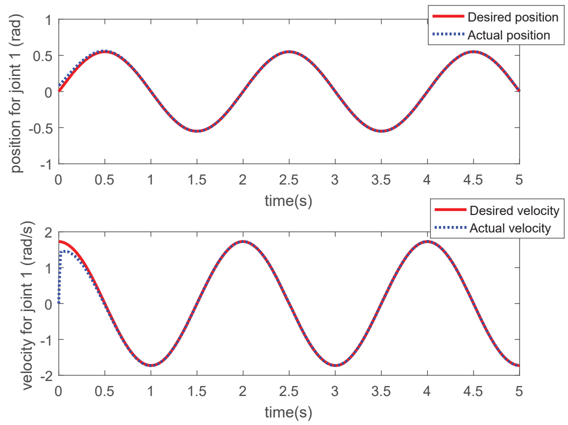

Fig. 5. Results of position and velocity of joint 1.

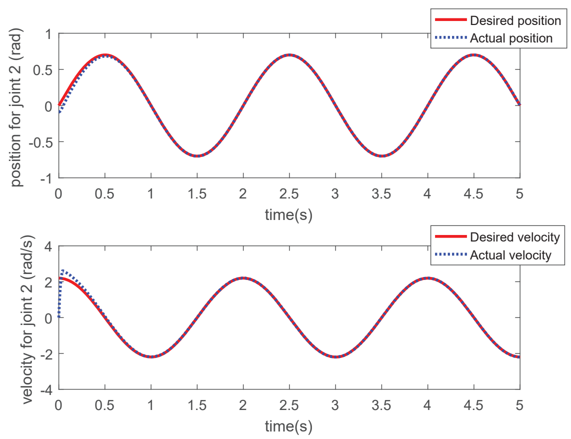

Fig. 6. Results of position and velocity of joint 2.

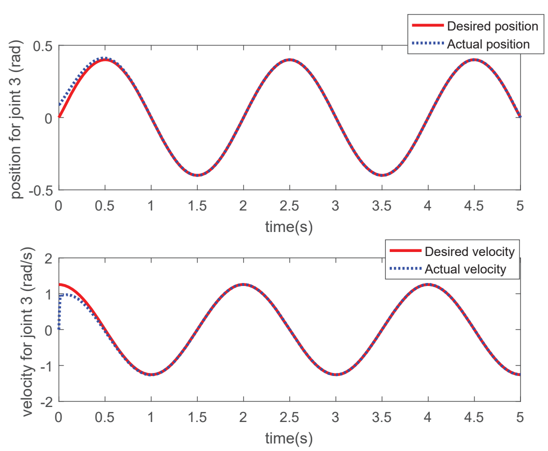

Fig. 7. Results of position and velocity of joint 3.

bance or the model parameters are uncertain, a sliding mode controller of Delta robot is proposed. First, the dynamic model of the Delta robot is given. Then, in order for the robot EE to track the desired trajectory, the controller is designed based on sliding mode control method. The system stability is proved by Lyapunov method. The results of simulation experiment demonstrate that the proposed method can be used to realize the trajectory tracking of the EE of the parallel Delta robot, and the designed controller is simple, robust and reliable. Future works will be focused on how to improve the sliding mode controller to reduce chatter and improve dynamic performance in practical applications.

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