

Fuzzy Sliding Mode Tracking Control for DC Motor Servo System without Uncertainty Information

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Abstract—To deal with the uncertainty in DC motor servo system, a sliding mode controller (SMC) is designed. In order to weaken the chattering of the controlled system, a fuzzy logic reasoning strategy is established. The experimental results show that the proposed fuzzy sliding mode control has better performance in the sense of the suppressed chattering as compared to the traditional sliding mode control and exponential reaching law sliding mode control, and high tracking accuracy as compared to boundary layer SMC.

Keywords—DC servo system; Fuzzy sliding mode control; Exponential reaching law; Boundary layer SMC

I. INTRODUCTION

DC motor servo systems are widely used as the executors in automatic control systems due to their good mechanical properties and regulation characteristics. However, the nonlinearity and uncertainty in the DC servo system is the main obstacle to the high performance control, therefore, the high precision position tracking control of DC motor servo system is still a challenging task^[1].

Because of friction, the inertia variation caused by load change, parameter drift caused by temperature, unmodelled high frequency dynamics, measurement noise and other factors, the DC servo systems modelled by the linear model have the uncertainties. The uncertainties neglected in the controller design might cause the performance of the control system to deteriorate and even lead to instability. As the sliding mode variable structure control can deal with the modelled and unmodelled uncertainties and keep the system stability when the modelling is not accurate, it is very suitable for the control of motor servo system^[2,3].

A PID controller for position control of DC motor servo system was presented in Reference [6]. PID parameters obtained by tuning are generally fit for a specific operating point. For the control system whose model is susceptible to the environment, the PID control is poor in robustness and may not even reach the control goal. A sliding mode variable structure control for position control of DC motor servo system was designed in Reference [7]. The simulation results showed that

the performance of sliding mode variable structure control was superior to that of conventional PID control. An exponential reaching law sliding mode control strategy was used to control the system with a large range of motor parameters perturbation in Reference [8]. The simulation results showed that the method was robust, but the discontinuous switching characteristics of the sliding mode control caused the chattering of the controller output. The unmodelled dynamics of the DC motor servo system was analyzed, and the sliding mode controller of the interpolation smoothing algorithm was designed in Reference [9]. The chattering caused by the unmodelled dynamic of the DC servo system was weakened, but the algorithm was complicated. A new type of disturbance observer was designed in Reference [10]. By the feed-forward compensation of the external interference, the gain of the switching term in the sliding mode controller was reduced, and then the chattering was weakened. Sliding mode variable structure control is an effective method to solve the problem of bounded undetectable perturbation, variable parameters and model uncertainties. In practice, uncertainties or disturbances may be completely unknown, and the SMC can be used by selecting a large enough gain for the robust item in the controller, which maybe cause a enlarged chattering. The chattering of the controller output might be filtered and weakened by the low frequency pass properties of the plant to be controlled. However, the chattering might invoke the unmodelled dynamics of the system and lead to the degradation of the control system performance or even instability of the controlled system. Therefore, weakening or eliminating the chattering of SMC controller is still an important problem. Fuzzy control^[11-14] uses the human experience and fuzzy reasoning to design controller without the knowledge of the plant model. Fuzzy control is also proposed to combine with SMC to reduced the chattering of the controller output for permanent magnet brushless DC motor^[15,16]. And all methods above considered the theoretical analysis and simulation. In this paper, a fuzzy sliding mode controller for DC motor servo system is designed by combining the conventional sliding mode control and fuzzy control. The fuzzy reasoning is used to adjust the gain for the robust item in the SMC. The experimental results show that the fuzzy sliding mode control

method improves the performance of the control system as compared with the conventional SMC, SMC with boundary layer and exponential approximation law SMC.

The following of this paper is arranged as follows: Section II introduces the DC motor servo system and its simplified linear mathematical model briefly. Section III gives the design details of fuzzy SMC. Section IV gives experiment comparison results of the proposed method as compared to other methods. The conclusion is given in Section V.

II. MODEL OF DC SERVO SYSTEM

The schematic diagram of permanent magnet DC motor with a negligible armature inductance is shown in Fig. 1.

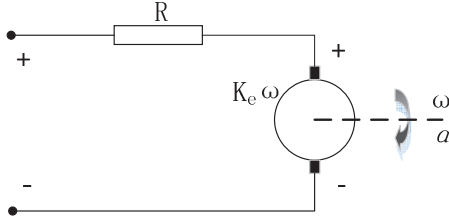


Fig.1 Diagram of the DC motor

According to the diagram of the DC motor in Fig.1, the motor armature voltage equation is given by

$$v(t) = Ri(t) + K_e \omega(t) \quad (1)$$

and rotor mechanical motion equation is given by

$$J\dot{\omega}(t) = K_m i(t) - \beta \omega(t) \quad (2)$$

where $v(t)$ is the armature input voltage, $i(t)$ is the armature current, $\omega(t)$ is the angular velocity of the rotor, R is the resistance of the armature winding, J is the inertia moment of the moving parts, β is the damping coefficient due to the viscous friction, $K_e \omega(t)$ is the back EMF, and $\tau = K_m i(t)$ is the electromechanical torque.

Equation (3) can be obtained by combining Eq.(1) and Eq.(2) ,

$$T_s \dot{\omega}(t) = -\omega(t) + K_{sm} v(t) \quad (3)$$

where $T_s = \frac{RJ}{\beta R + K_e K_m}$, $K_{sm} = \frac{K_m}{\beta R + K_e K_m}$, and the transfer function from the input voltage to the motor speed can be given by

$$G(s) = \frac{\omega(s)}{v(s)} = \frac{K_{sm}}{T_s s + 1} \quad (4)$$

The transfer function from the input voltage to the motor position is given by

$$G(s) = \frac{\alpha(s)}{v(s)} = \frac{K_{sm}}{s(T_s s + 1)} \quad (5)$$

By defining $u(t) = \frac{v(t)}{v_{\max}}$ and $K_s = K_{sm} v_{\max}$, the transfer

functions from the voltage to the angular speed and from the voltage to the angular can be given by

$$G(s) = \frac{\omega(s)}{u(s)} = \frac{K_s}{T_s s + 1} \quad (6)$$

and

$$G(s) = \frac{\alpha(s)}{u(s)} = \frac{K_s}{s(T_s s + 1)} \quad (7)$$

Let $\mathbf{x} = [x_1 \ x_2]^T$ be the state vector, where x_1 is the angle α in [rad] representing the position of the motor shaft, and $x_2 = \omega$ is the angular velocity in [rad/s]. Time t is measured in [s]. The state equations are as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 + bu \end{cases} \quad (8)$$

where $a = -\frac{1}{T_s} < 0$, $b = \frac{K_s}{T_s} > 0$. Equation (8) is the DC servo motor state equations.

III. FUZZY SLIDING MODE CONTROLLER DESIGN

We consider the n order SISO nonlinear system described by:

$$\mathbf{x}^{(n)} = f(\mathbf{x}, t) + g(\mathbf{x}, t)u(t) + d(t) \quad (9)$$

where $\mathbf{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T \in R^n$, $y = x \in R$, $u \in R$, $|d(t)| \leq D$ is an undetermined uncertainty with boundary D .

The sliding mode controller is composed of an equivalent control and a switching control (i.e., the robust item). The equivalent control keeps the system state on the sliding surface, and the switching control forces the system state to slide on the sliding surface.

A Equivalent Sliding Mode Controller Design

The controlled plant is given by

$$\mathbf{x}^{(n)} = f(\mathbf{x}, t) + g(\mathbf{x}, t)u(t) \quad (10)$$

Let $e = x_d - x$ be the tracking error, where x_d is the tracking reference signal. Define a time-varying surface $s(t)$ in the state-space R^n as

$$s = c_1 e + c_2 \dot{e} + \dots + e^{(n-1)} \quad (11)$$

where $C = [c_1, c_2, \dots, c_{n-1}, 1]$ is the coefficients of Hurwitz polynomial. We then obtain

$$\begin{aligned}\dot{s} &= c_1 \dot{e} + c_2 \ddot{e} + \dots + c_{n-1} e^{(n-1)} + x_d^{(n)} - x^{(n)} \\ &= \sum_{i=1}^{n-1} c_i e^{(i)} + x_d^{(n)} - f(\mathbf{x}, t) - g(\mathbf{x}, t) u(t)\end{aligned}\quad (12)$$

To let $\dot{s} = 0$, the equivalent controller is

$$u_{eq} = \frac{1}{g(\mathbf{x}, t)} \left(\sum_{i=1}^{n-1} c_i e^{(i)} + x_d^{(n)} - f(\mathbf{x}, t) \right) \quad (13)$$

B Sliding Mode Controller Design

In order to meet the sliding mode arrival conditions $s\dot{s} \leq -\eta|s|$, $\eta > 0$, switching control must be used. The switching controller is

$$u_{sw} = \frac{1}{g(\mathbf{x}, t)} k \operatorname{sgn}(s) \quad (14)$$

where $\operatorname{sgn}(\cdot)$ is the sign function, and k is the rate at which the state of the system approaches the switching surface $s = 0$. The whole sliding mode controller is

$$u = u_{eq} + u_{sw} \quad (15)$$

We know that, to determine k in (14), we have to know the uncertainty term d , however, we don't know the uncertainty. Consequently, the controller cannot operate. Fortunately, we do know that the controller is stable if parameter k is large enough. So we can choose a large enough k to make the system track the reference output. From Eq. (15) and Eq. (14), we know that the chattering of the controller is caused by the sign function in Eq. (14). The larger k , the larger chattering of the controller. If we can adjust the amplitude of the sign function according to the system state or the slide surface s , the chattering might be reduced provided that the tracking performance is guaranteed. Following this idea, the fuzzy logic is employed to adjust the amplitude of the sign function, by this way, a fuzzy slide mode controller is proposed in the following subsection.

C Fuzzy SMC Design

The fuzzy SMC is proposed as follows

$$u = u_{eq} + \mu(s) u_{sw} \quad (16)$$

If $\mu(s) = 1$, $u = u_{eq} + u_{sw}$, then the control law (16) is the traditional sliding mode control. Because we do not have the information about uncertainty, we do have some knowledge about how to determine the chattering amplitude, k , for example, if the tracking error is large, then k should be large; if the tracking error is small, then k should be small too, and so on. We can construct a fuzzy controller based on these knowledge (experience or rules) to avoid the need for the uncertainty boundary. In addition, to adjust the amplitude of the switching control force might relief the chattering. In the proposed fuzzy reasoning logic, s is used as the input and $\mu(s)$ is the output. The language variable values are respectively: $\{N, Z, P\}$. The membership function of input variable and output variable are shown in Fig.2.

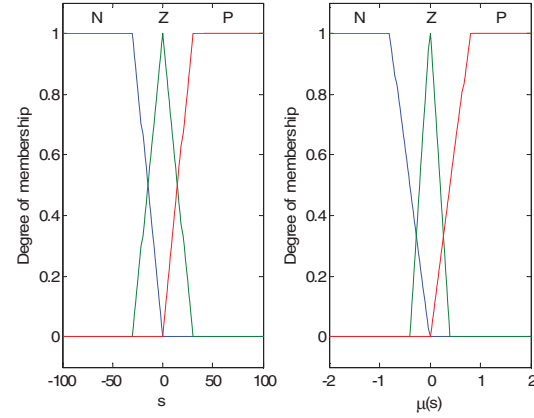


Fig.2 The membership function of input variable and output variable

Three fuzzy rules are given as follows and used in the fuzzy reasoning process to calculate the amplitude of the switching term.

If s is N then $\mu(s)$ is P

If s is Z then $\mu(s)$ is Z

If s is P then $\mu(s)$ is P

The fuzzy reasoning employs Mamdani inference method, the output signal can be obtained by defuzzified using the center-of-gravity method. The block diagram of control system is shown in Fig.3. From the experimental results shown in the next section, we know the simple fuzzy reasoning method can reduce the chattering and improve the performance of the controlled system.

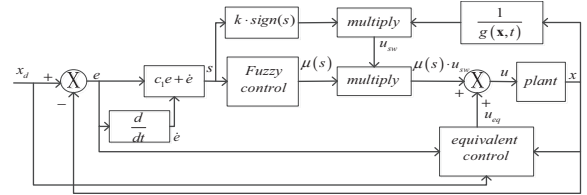


Fig.3 The control block diagram of servo system

IV. EXPERIMENTAL RESULTS

In this paper, DC motor servo system produced by INTECO company is used to test the proposed method and to compare the different methods. The DC motor servo system consists of several modules mounted at the metal rail, including DC motor with tachogenerator, gearbox with the output disk, inertial load, encoder module, etc. as shown in Fig.4.

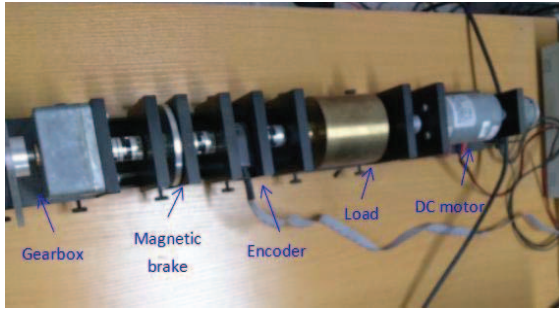


Fig.4 Photo of the DC servo system

The parameters of DC motor servo system are as follows: $K_s = 196[\text{rad/s}]$, $T_s = 1.04[\text{s}]$, which determine $a = -0.96[\text{s}^{-2}]$ and $b = 188.5[\text{rad/s}^2]$ in Eq. (8). The proposed method is compared with the conventional SMC (given in Eq.(15)), boundary layer SMC where the $\text{sgn}(\cdot)$ is replaced by its continuous approximation $\text{sat}(s/\phi)$ ($\phi > 0$ is the width of the boundary layer) in the conventional SMC and the exponential reaching law SMC, which is designed as follows:

The DC servo system can be given as

$$\ddot{x} = f + bu \quad (17)$$

where $f = ax$, the sliding surface is

$$s = \lambda e + \dot{e} \quad (18)$$

where $\lambda > 0$. Using the exponential reaching law, we obtain

$$\dot{s} = -\mu s - \varepsilon \text{sgn}(s) \quad \mu > 0, \varepsilon > 0 \quad (19)$$

and the controller is

$$u = \frac{1}{b}(\ddot{x}_d + \lambda \dot{e} - f + \mu s + \varepsilon \text{sgn}(s)) \quad (20)$$

The exponential reaching law SMC parameters are chosen as follows

$$\mu = 60, \varepsilon = 1, \lambda = 25$$

The boundary layer SMC parameters are chosen as follows

$$c_1 = 20, k = 80, \phi = 1$$

The traditional SMC parameters are chosen as follows

$$c_1 = 30, k = 350$$

The parameters in the conventional SMC and the exponential reaching law SMC are chosen by a fine adjusting procedure to get the performance as good as the designer can.

The fuzzy SMC parameters are chosen as follows

$$c_1 = 30, k = 350$$

The experimental results for the step reference is given in Fig.5, where the reference signal is given by red solid line, the output of exponential reaching law SMC is given by cyan-blue

dotted line, the output of boundary layer SMC is given by blue dot and dash line, the output of the traditional SMC is given by magenta double hyphen, and the output of the proposed method is given by black solid line. The tracking error and control force are shown in Fig.6, Fig.7, respectively.

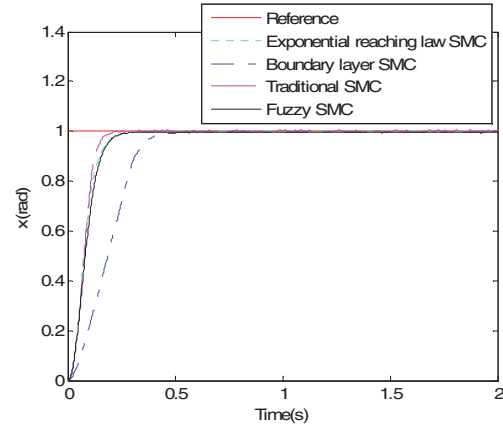


Fig.5 The outputs of the servo system using four methods

From the comparison results shown in Figs.5, 6 and 7, we get some conclusions as follow:

(1) Four kinds of SMCs all track the desired output within the acceptable error range(Fig.5);

(2) Compared with the traditional SMC, the boundary layer SMC effectively attenuates chattering, but the corresponding tracking error is also increased as shown by Figs. 6 and 7.

(3) Fuzzy SMC weighs the chattering and the tracking error, which has the best compromise performance.

To compare the results quantitatively, we define root mean-square error index as follows:

$$RMSE = \sqrt{\frac{1}{N_2 - N_1} \sum_{k=N_1}^{N_2} e_k^2} \quad (21)$$

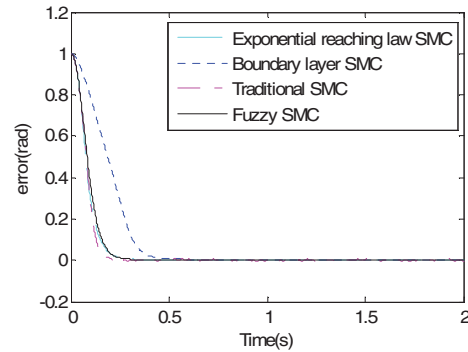


Fig.6 The tracking errors of the servo system using four methods

N_1 is the start time, and N_2 is the end time. The comparisons of the four methods are given in Table I. From the waveform of real-time control and quantitative comparison, we know the proposed fuzzy SMC has the best compromise performance.

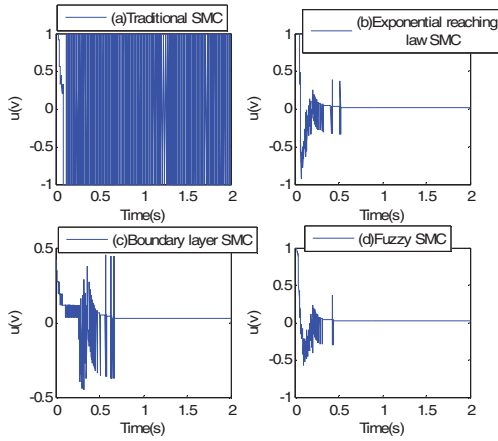


Fig.7 The controller outputs using four methods

TABLE I. PERFORMANCE CRITERIA

Indices	methods			
	traditional SMC	exponential reaching law SMC	boundary layer SMC	fuzzy ISMC
Adjustment time	0.1680s	0.2260s	0.4080s	0.2240s
Variance (controller outputs)	0.9788	0.0432	0.0084	0.0273
RMSE	0.0161	0.1730	0.2600	0.1783

V. CONCLUSION AND DISCUSSION

This paper establishes a set of servo system based on computer control, including Inteco digital servo and open software development environment. The experimental results show that the servo system based on fuzzy sliding mode control has a faster adjustment time compared with boundary layer sliding mode control and exponential reaching law SMC. Compared with the traditional SMC, the rest three methods effectively attenuates chattering, but the root mean-square error of the boundary layer SMC is large. So, we can see the best compromise performance of fuzzy SMC.

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