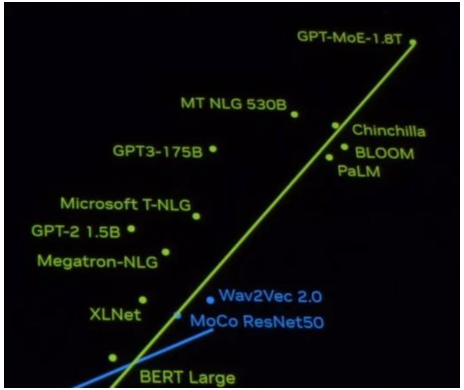
Lecture 4

Mixtures of Experts

CS336

Tatsu H

Mixture of experts



GPT4 (?)

Mistral AI
@MistralAI

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Grok

deepseek

DeepSeekMoE: Towards Ultimate Expert Specialization in Mixture-of-Experts Language Models

deepseek

DeepSeek-V3 Technical Report

OLMoE: Open Mixture-of-Experts Language Models

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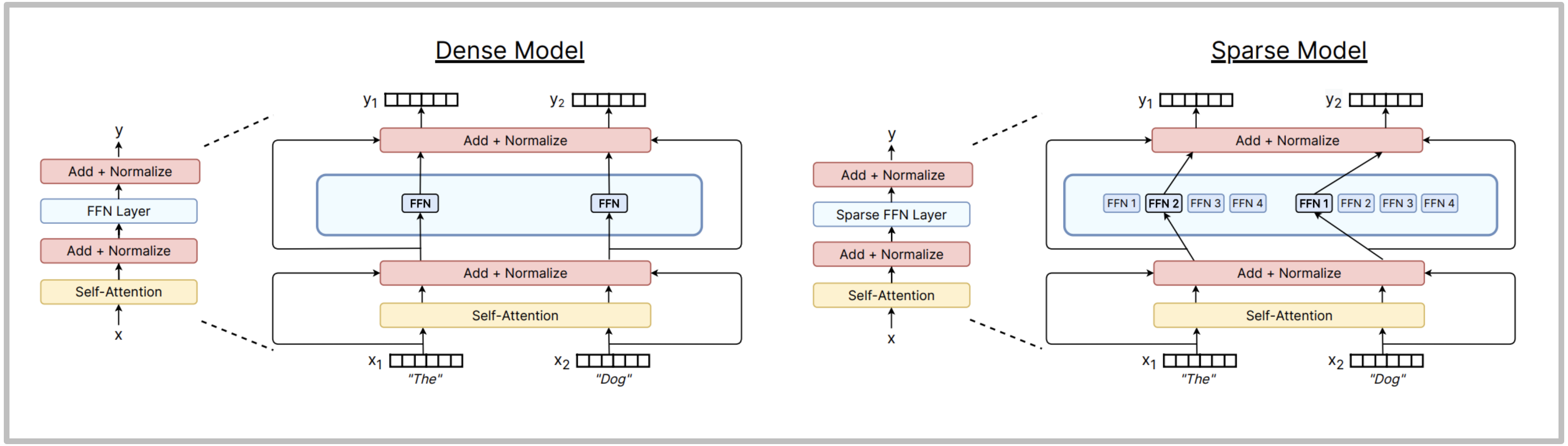
当下东西方的实践来看，MoE 都要比 dense model 性能好

What’s a MoE?

直觉的，我们以为 MoE 是多个专家结合在一起，比如有的专门做 coding。但事实上不是这样，更像是一个架构结构，变化主要发生在 MLP 层



[Fedus et al 2022]

Replace big feedforward with (many) big feedforward networks and a selector layer

You can increase the # experts without affecting FLOPs

Why are MoEs getting popular?

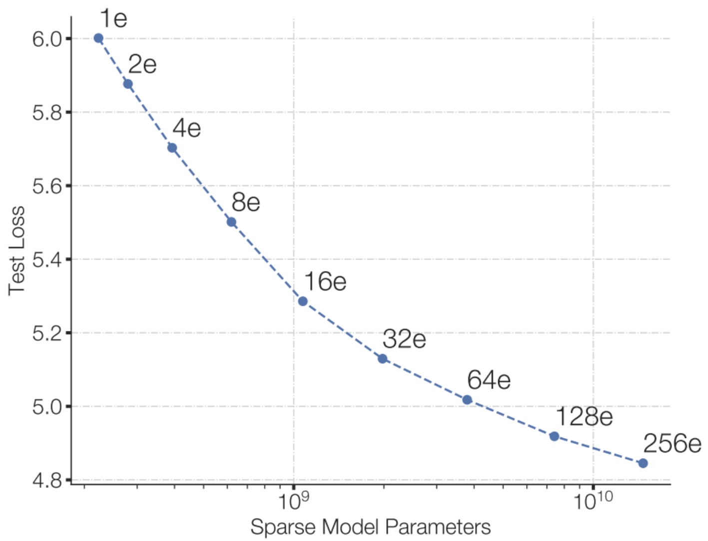
这张图证明了混合专家（Mixture of Experts, MoE）架构的有效性：

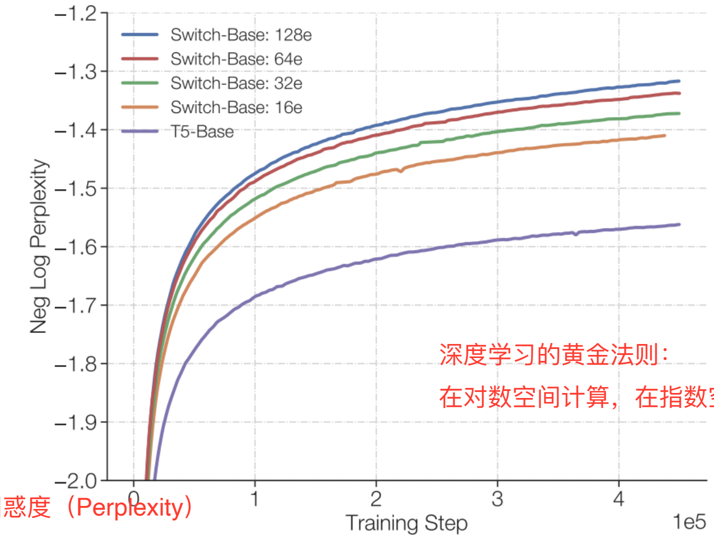
✅ 增加专家数量能持续提升性能

✅ Switch Transformer优于密集模型（T5）

✅ 存在明显的规模效应（scaling law）

Same FLOP, more param does better





深度学习的黄金法则：

在对数空间计算，在指数空间理解 🎯

困惑度（Perplexity）

↓ 指数形式，不直观

对数困惑度（Log Perplexity）

↓ 去掉指数，线性化

负对数困惑度（Neg Log Perplexity）

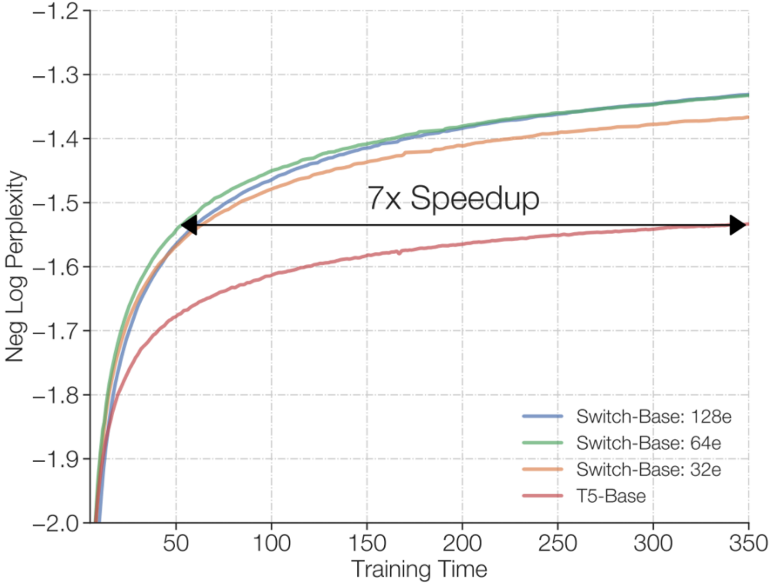
↓ 翻转方向，符合"越高越好"的直觉

左图表明，同样的 FLOP 下，专家数越多，Loss 越低

[Fedus et al 2022]

Why are MoEs getting popular?

Faster to train MoEs



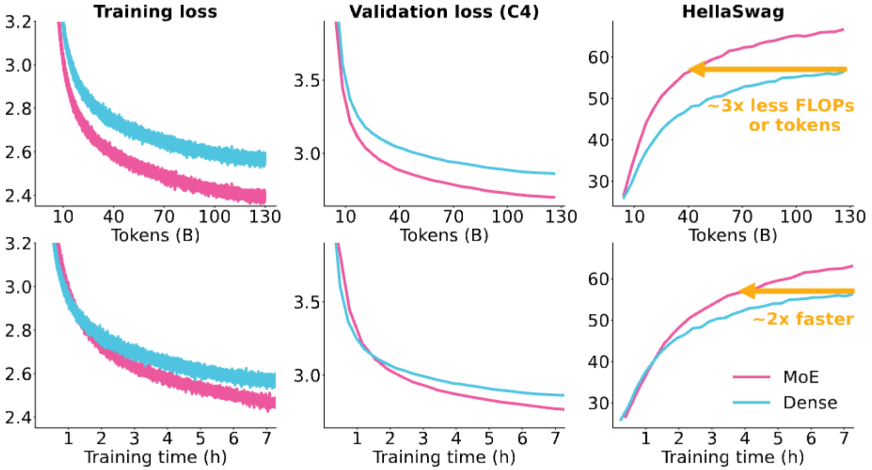
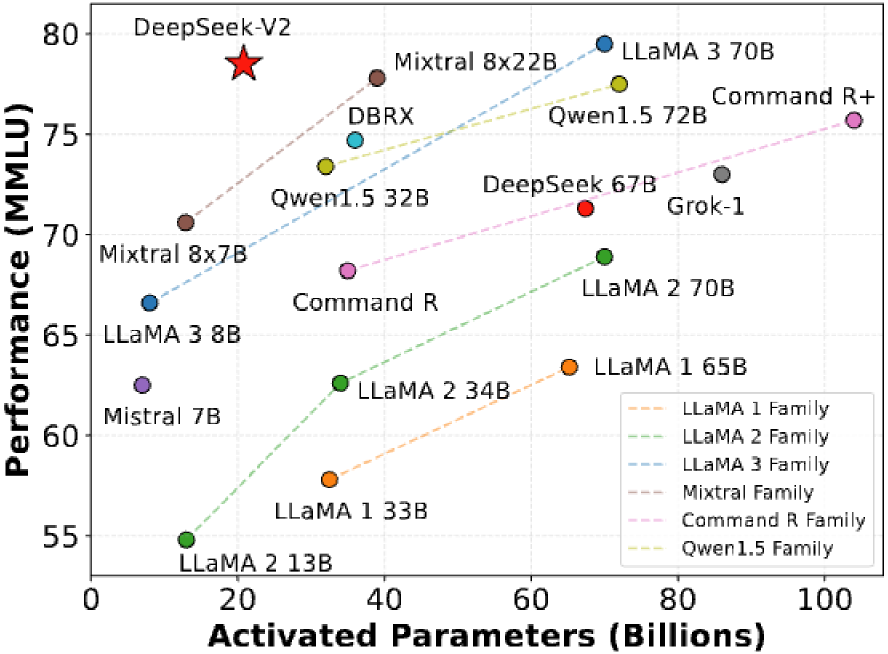


Figure 4: **MoE vs. Dense.** We train a 1.3B parameter dense model and a 1.3B active, 6.9B total parameter MoE model, each on 128 H100 GPUs. Apart from MoE-related changes, we train both

[OlMoE]

Why are MoEs getting popular?



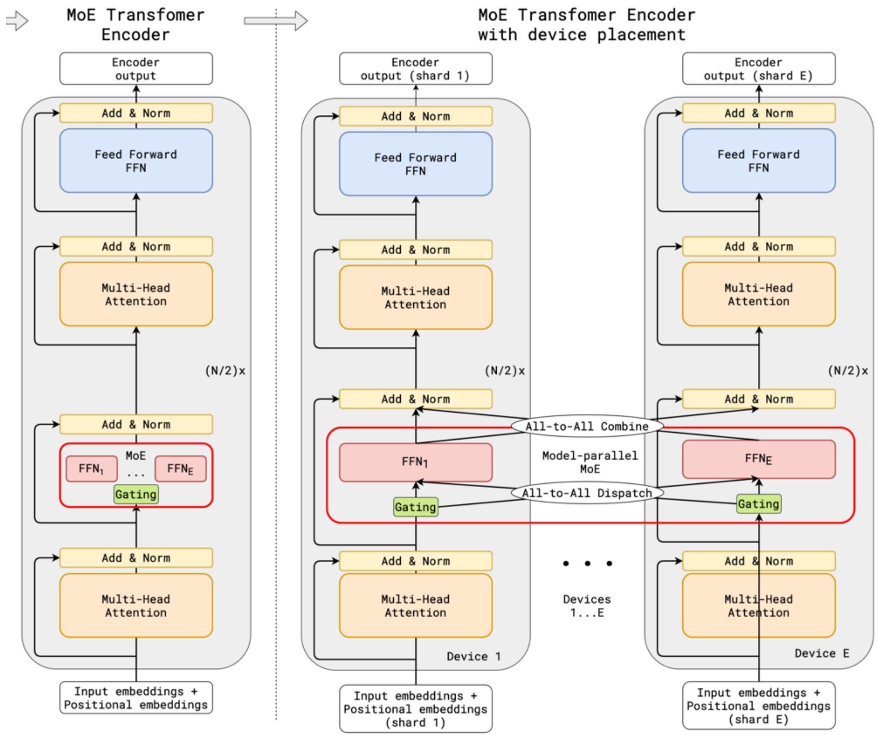
tricked part

Highly competitive vs dense equivalents

Why are MoEs getting popular?

模型分片，然后后把 token 路由到不同的分片上去

Parallelizable to many devices



Some MoE results – from the west

Category Benchmark	Llama 4 Maverick	Gemini 2.0 Flash	DeepSeek v3.1	GPT-4o
Inference Cost Cost per 1M input & output tokens (3:1 blended)	$0.19-$0.49[5]	**$0.17**	$0.48	$4.38
Image Reasoning MMMU	**73.4**	71.7	No multimodal support	69.1
MathVista	**73.7**	73.1		63.8
Image Understanding ChartQA	**90.0**	88.3		85.7
DocVQA (test)	**94.4**	—		92.8
Coding LiveCodeBench (10/01/2024-02/01/2025)	43.4	34.5	**45.8/49.2**[3]	32.3[3]
Reasoning & Knowledge MMLU Pro	80.5	77.6	**81.2**	—
GPQA Diamond	**69.8**	60.1	68.4	53.6

MoEs are most of the highest-performance open models, and are quite quick

Earlier MoE results from Chinese groups – Qwen

Chinese LLM companies are also doing quite a bit of MoE work on the smaller end

Model	MMLU	GSM8K	HumanEval	Multilingual	MT-Bench
Mistral-7B	64.1	47.5	27.4	40.0	7.60
Gemma-7B	64.6	50.9	32.3	-	-
Qwen1.5-7B	61.0	62.5	36.0	45.2	7.60
DeepSeekMoE 16B	45.0	18.8	26.8	-	6.93
Qwen1.5-MoE-A2.7B	62.5	61.5	34.2	40.8	7.17

Model	#Parameters	#(Activated) Parameters
Mistral-7B	7.2	7.2
Qwen1.5-7B	7.7	7.7
Gemma-7B	8.5	7.8
DeepSeekMoE 16B	16.4	2.8
Qwen1.5-MoE-A2.7B	14.3	2.7

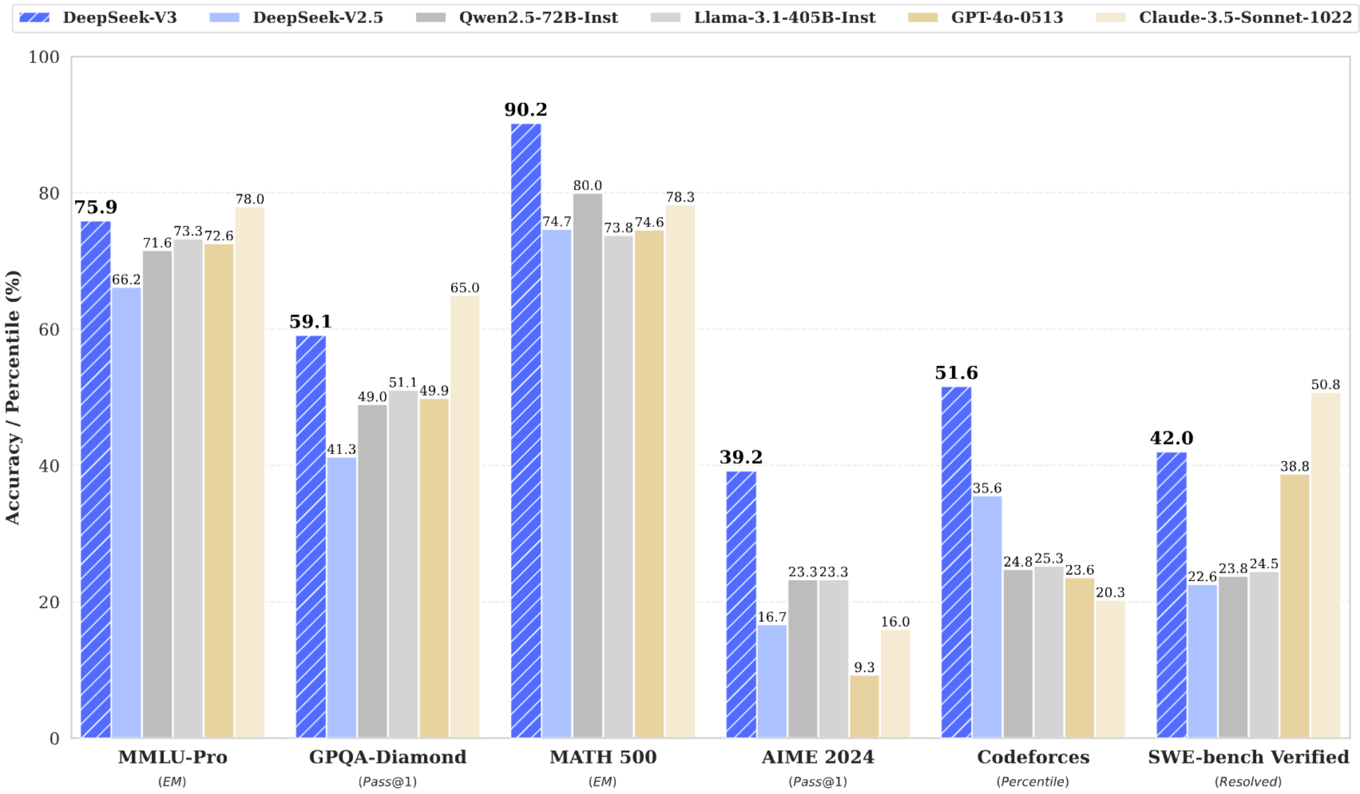
Earlier MoE results from Chinese groups - DeepSeek

There's also some good recent ablation work on MoEs showing they're generally good

DeepSeek 做了详细的比较
1. train dense 模型，发生什么，比如哪些地方会失效
2. naive 的 MoE
3. 更聪明的路由

Metric	# Shot	Dense	Hash Layer	Switch
# Total Params	N/A	0.2B	2.0B	2.0B
# Activated Params	N/A	0.2B	0.2B	0.2B
FLOPs per 2K Tokens	N/A	2.9T	2.9T	2.9T
# Training Tokens	N/A	100B	100B	100B
Pile (Loss)	N/A	2.060	1.932	1.881
HellaSwag (Acc.)	0-shot	38.8	46.2	49.1
PIQA (Acc.)	0-shot	66.8	68.4	70.5
ARC-easy (Acc.)	0-shot	41.0	45.3	45.9
ARC-challenge (Acc.)	0-shot	26.0	28.2	30.2
RACE-middle (Acc.)	5-shot	38.8	38.8	43.6
RACE-high (Acc.)	5-shot	29.0	30.0	30.9
HumanEval (Pass@1)	0-shot	0.0	1.2	2.4
MBPP (Pass@1)	3-shot	0.2	0.6	0.4
TriviaQA (EM)	5-shot	4.9	6.5	8.9
NaturalQuestions (EM)	5-shot	1.4	1.4	2.5

Recent MoE results – DeepSeek v3



Why haven’t MoEs been more popular?

Infrastructure is complex / advantages on multi node

At a high level, sparsity is good when you have many accelerators (e.g. GPU/TPU) to host all the additional parameters that comes when using sparsity. Typically models are trained using data-parallelism where different machines will get different slices of the training/inference data. The machines used for operating on the different slices of data can now be used to host many more model parameters. Therefore, sparse models are good when training with data parallelism and/or have high throughput while serving: training/serving on many machines which can host all of the parameters.

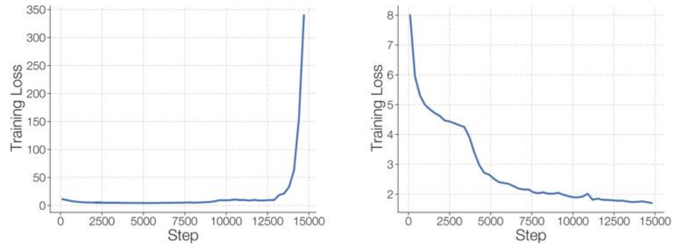
因为太复杂了，且混乱，所以没有成为标准

[Fedus et al 2022]

Training objectives are somewhat heuristic (and sometimes unstable)

令牌的路由不是可微分的，
因为它是离散的

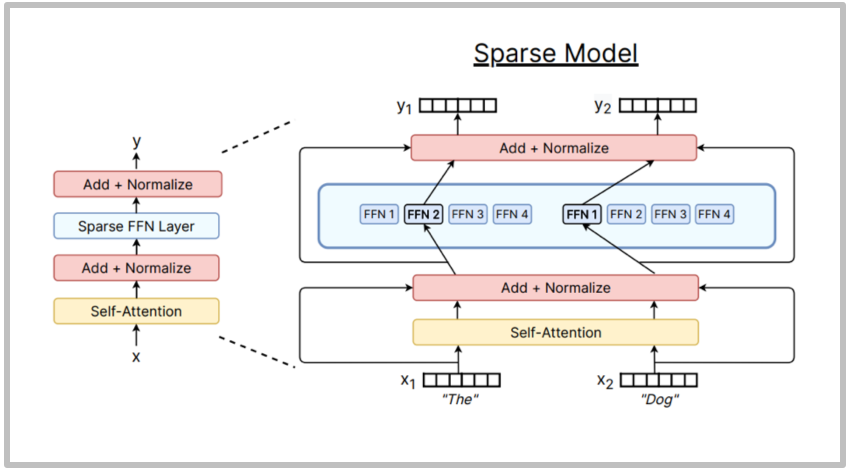
Sparse models often suffer from training instabilities (Figure 1) worse than those observed in standard densely-activated Transformers.



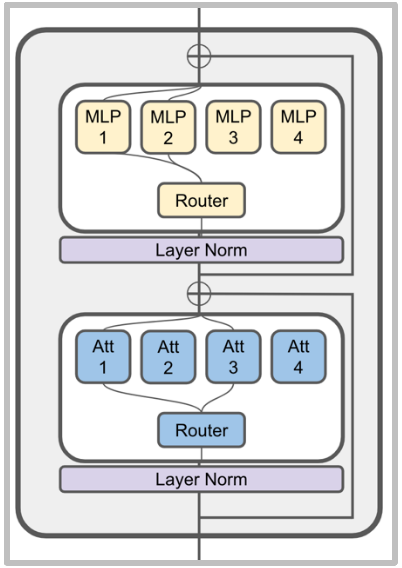
[Zoph et al 2022]

What MoEs generally look like

Typical: replace MLP with MoE layer



Less common: MoE for attention heads



[ModuleFormer, JetMoE]

MoE – what varies?

- Routing function
- Expert sizes
- Training objectives

Routing function - overview

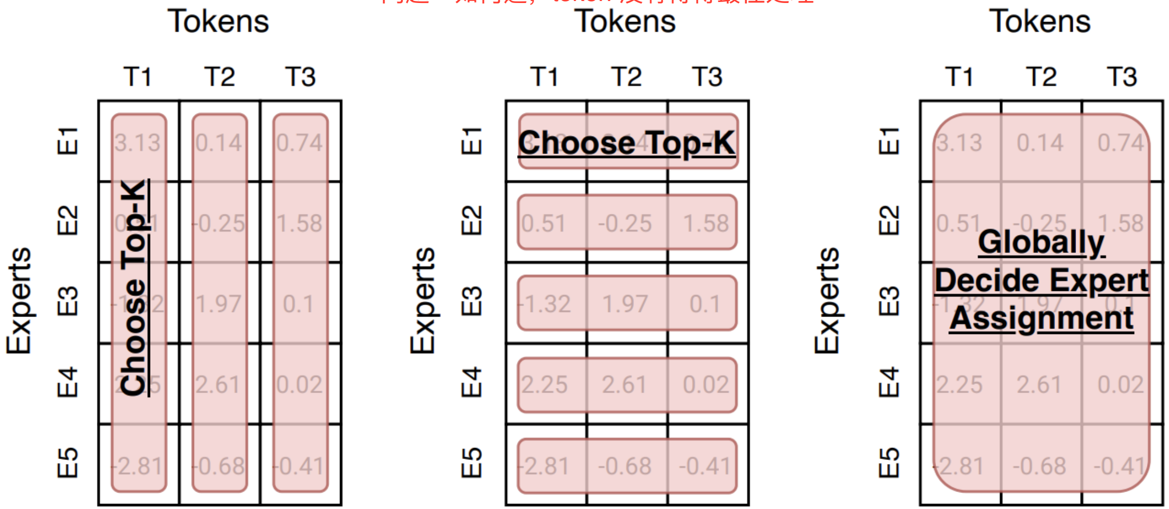
本质问题：token 和专家如何匹配

Many of the routing algorithms boil down to ‘choose top k’

全局优化

token 选最合适的 top-k 专家，
问题：有些专家会忙死，有些会闲死

专家选 top-k token
问题：如何选，token 没有得得最佳处理



Token chooses expert

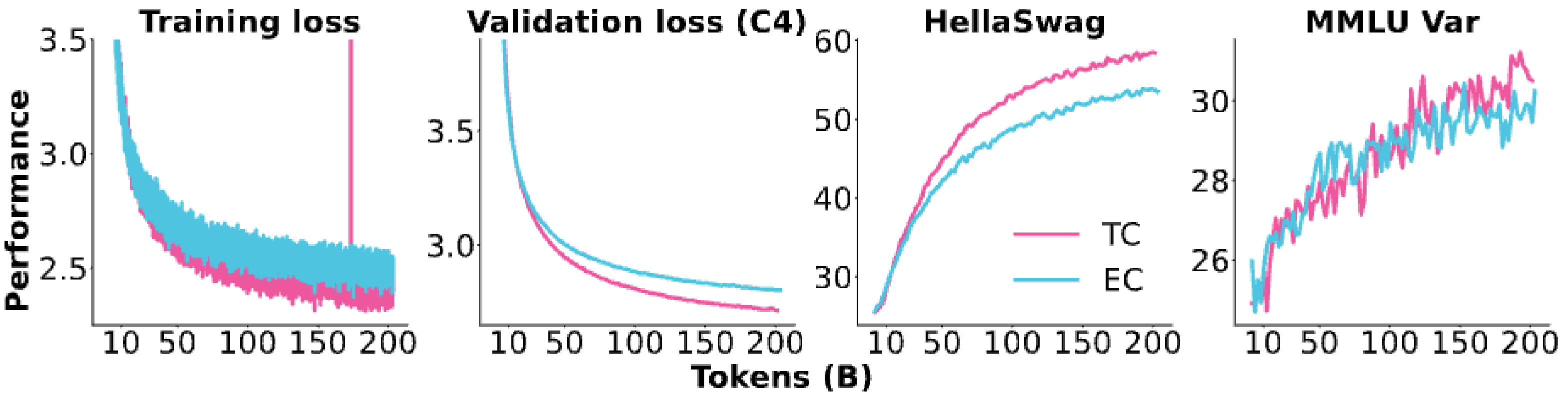
Expert chooses token

Global routing via optimization

[Fedus et al 2022]

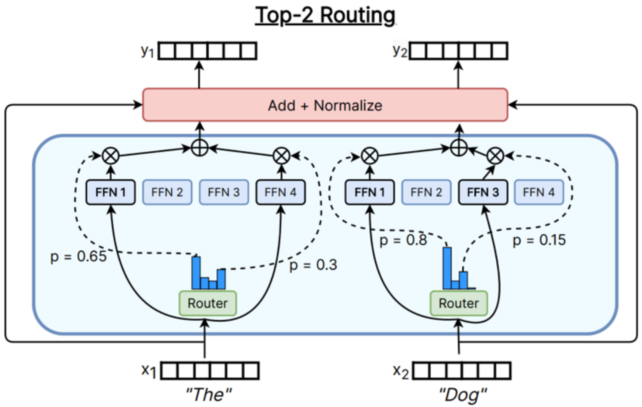
Routing type

Almost all the MoEs do a standard 'token choice topk' routing. Some recent ablations



Common routing variants in detail

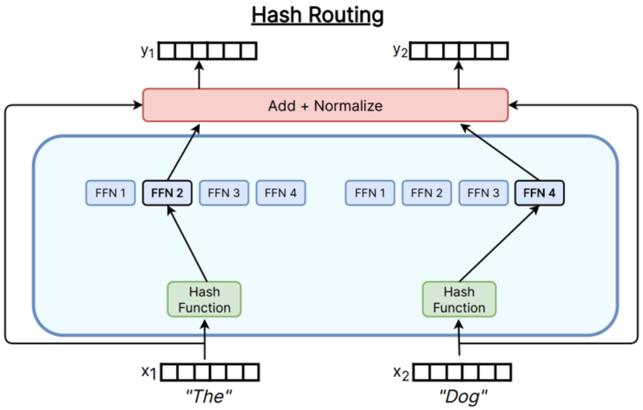
Top-k



Used in *most* MoEs

Switch Transformer (k=1)
Gshard (k=2), Grok (2), Mixtral (2), Qwen (4), DBRX (4), DeepSeek (7)

Hashing

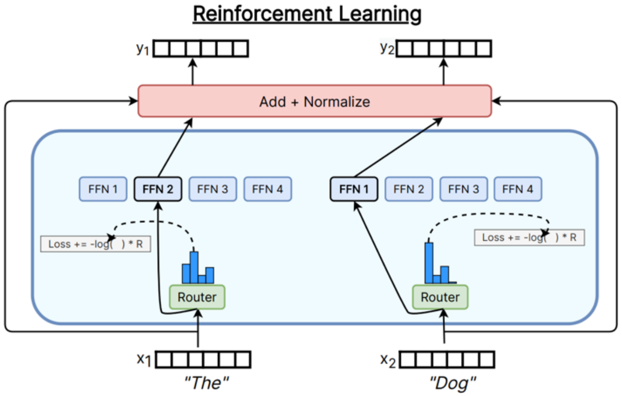


Common baseline

[Fedus et al 2022]

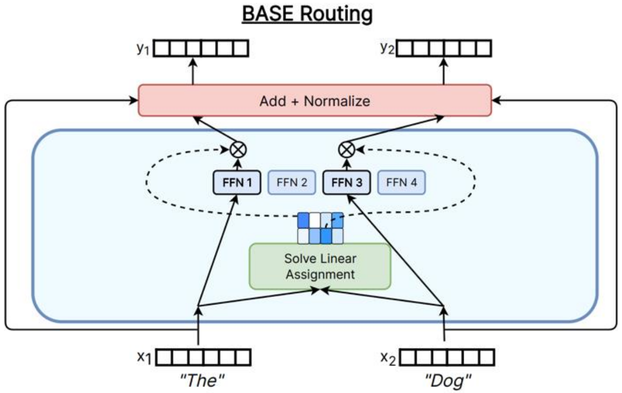
Other routing methods

RL to learn routes



Used in some of the earliest work
Bengio 2013, not common now

Solve a matching problem



Linear assignment for routing
Used in various papers like Clark '22

[Fedus et al 2022]

Top-K routing in detail.

Most papers do the old and classic top-k routing. How does this work?

Gating

$$\mathbf{h}_t^l = \sum_{i=1}^{N} \left(g_{i,t} \operatorname{FFN}_i \left(\mathbf{u}_t^l \right) \right) + \mathbf{u}_t^l,$$

$$g_{i,t} = \begin{cases} s_{i,t}, & s_{i,t} \in \operatorname{Topk}(\{s_{j,t} | 1 \leqslant j \leqslant N\}, K), \\ 0, & \text{otherwise}, \end{cases}$$

$$s_{i,t} = \operatorname{Softmax}_i \left({\mathbf{u}_t^l}^T \mathbf{e}_i^l \right),$$

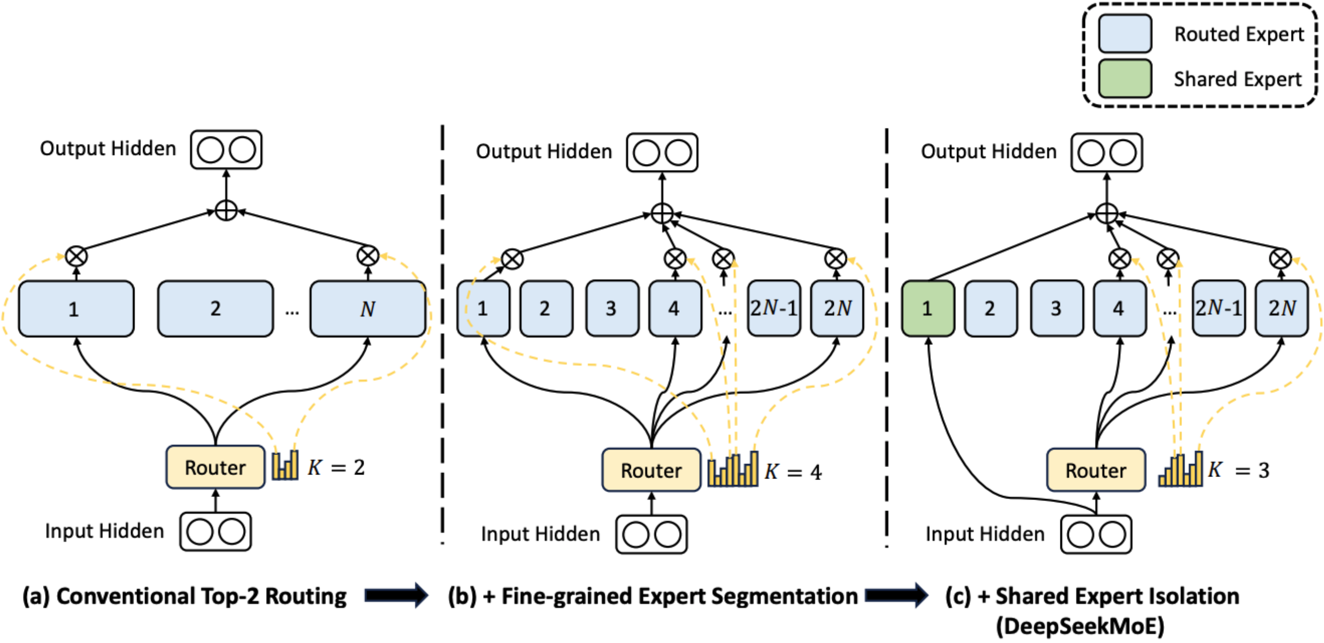
Gates selected by a logistic regressor

This is the DeepSeek (V1-2) router (Grok, Qwen do this too)

Mixtral, DBRX, DeepSeek v3 softmaxes after the TopK

[Dai et al 2024]

Recent variations from DeepSeek and other Chinese LMs



Smaller, larger number of experts + a few shared experts that are always on.

(Used in DeepSeek / Qwen, originally from DeepSpeed MoE)

Various ablations from the DeepSeek paper

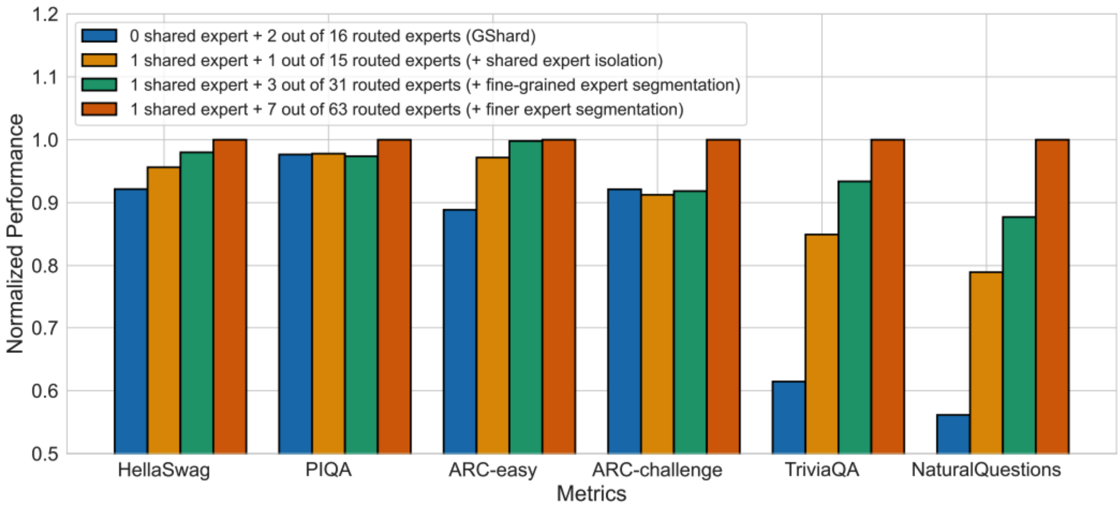
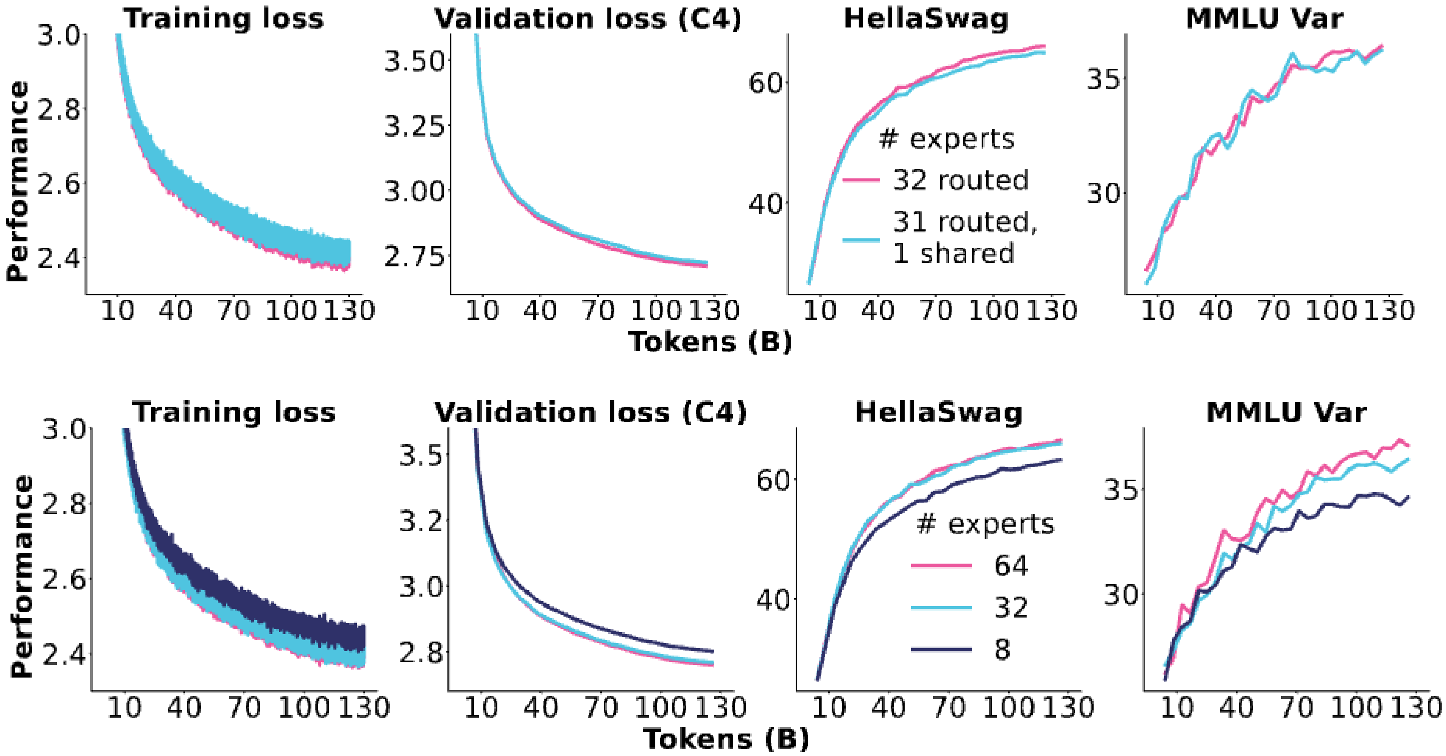


Figure 3 | Ablation studies for DeepSeekMoE. The performance is normalized by the best performance for clarity in presentation. All compared models have the same number of parameters and activated parameters. We can find that fine-grained expert segmentation and shared expert isolation both contribute to stronger overall performance.

More experts, shared experts all seem to generally help

Ablations from OlMoE

Gains from fine-grained experts, none from shared experts.



Expert routing setups for recent MoEs

Model	Routed	Active	Shared	Fine-grained ratio
GShard	2048	2	0	
Switch Transformer	64	1	0	
ST-MOE	64	2	0	
Mixtral	8	2	0	
DBRX	16	4	0	
Grok	8	2	0	
DeepSeek v1	64	6	2	1/4
Qwen 1.5	60	4	4	1/8
DeepSeek v3	256	8	1	1/14
OlMoE	64	8	0	1/8
MiniMax	32	2	0	~1/4
Llama 4 (maverick)	128	1	1	1/2

How do we train MoEs?

Major challenge: we need sparsity for training-time efficiency…

But sparse gating decisions are not differentiable!

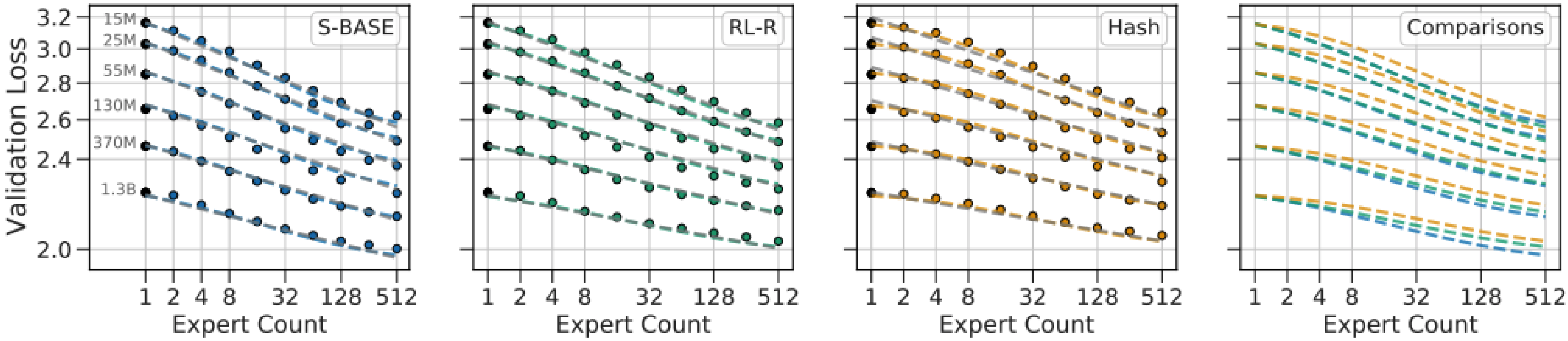
Solutions?

1. Reinforcment learning to optimize gating policies
2. Stochastic perturbations
3. Heuristic ‘balancing’ losses.

Guess which one people use in practice?

RL for MoEs

RL via REINFORCE does work, but not so much better that it's a clear win



(REINFORCE baseline approach, Clark et al 2020)

RL is the 'right solution' but gradient variances and complexity means it's not widely used

Stochastic approximations

$$G(x) = Softmax(KeepTopK(H(x), k))$$

$$H(x)_i = (x \cdot W_g)_i + StandardNormal() \cdot Softplus((x \cdot W_{noise})_i)$$

$$KeepTopK(v, k)_i = \begin{cases} v_i & \text{if } v_i \text{ is in the top } k \text{ elements of } v. \\ -\infty & \text{otherwise.} \end{cases}$$

From Shazeer et al 2017 – routing decisions are *stochastic* with gaussian perturbations.

1. This naturally leads to experts that are a bit more robust.
2. The softmax means that the model learns how to rank K experts

Stochastic approximations

```
if is_training:
    # Add noise for exploration across experts.
    router_logits += mtf.random_uniform(shape=router_logits.shape, minval=1-eps, maxval=1+eps)

# Convert input to softmax operation from bfloat16 to float32 for stability.
router_logits = mtf.to_float32(router_logits)

# Probabilities for each token of what expert it should be sent to.
router_probs = mtf.softmax(router_logits, axis=-1)
```

Stochastic jitter in Fedus et al 2022. This does a uniform multiplicative perturbation for the same goal of getting less brittle experts. This was later removed in Zoph et al 2022

Method	Fraction Stable	Quality (↑)
Baseline	4/6	**-1.755** ± 0.02
Input jitter (10^{-2})	3/3	-1.777 ± 0.03
Dropout (0.1)	3/3	-1.822 ± 0.11

Heuristic balancing losses

Another key issue – systems efficiency requires that we use experts evenly..

For each Switch layer, this auxiliary loss is added to the total model loss during training. Given N experts indexed by $i = 1$ to N and a batch $\mathcal{B}$ with T tokens, the auxiliary loss is computed as the scaled dot-product between vectors f and P,

$$\text{loss} = \alpha \cdot N \cdot \sum_{i=1}^{N} f_i \cdot P_i \tag{4}$$

where f_i is the fraction of tokens dispatched to expert i,

$$f_i = \frac{1}{T} \sum_{x \in \mathcal{B}} \mathbb{1}\{\text{argmax}\, p(x) = i\} \tag{5}$$

and P_i is the fraction of the router probability allocated for expert i, [2]

$$P_i = \frac{1}{T} \sum_{x \in \mathcal{B}} p_i(x). \tag{6}$$

From the Switch Transformer [Fedus et al 2022]

The derivative with respect to $p_i(x)$ is $\frac{\alpha N}{T^2} \sum 1_{argmax\ p(x)=i}$,
so more frequent use = stronger downweighting

Example from deepseek (v1-2)

Per-expert balancing – same as the switch transformer

$$\mathcal{L}_{\text{ExpBal}} = \alpha_1 \sum_{i=1}^{N'} f_i P_i, \tag{12}$$

$$f_i = \frac{N'}{K'T} \sum_{t=1}^{T} \mathbb{1}(\text{Token } t \text{ selects Expert } i), \tag{13}$$

$$P_i = \frac{1}{T} \sum_{t=1}^{T} s_{i,t}, \tag{14}$$

Per-device balancing – the objective above, but aggregated by device.

$$\mathcal{L}_{\text{DevBal}} = \alpha_2 \sum_{i=1}^{D} f_i' P_i', \tag{15}$$

$$f_i' = \frac{1}{|\mathcal{E}_i|} \sum_{j \in \mathcal{E}_i} f_j, \tag{16}$$

$$P_i' = \sum_{j \in \mathcal{E}_i} P_j, \tag{17}$$

DeepSeek v3 variation – per-expert biases

Set up a per-expert bias (making it more likely to get tokens) and use online learning

$$g'_{i,t} = \begin{cases} s_{i,t}, & s_{i,t} + b_i \in \operatorname{Topk}(\{s_{j,t} + b_j | 1 \leqslant j \leqslant N_r\}, K_r), \\ 0, & \text{otherwise.} \end{cases}$$

They call this '**auxiliary loss free balancing**'

Complementary Sequence-Wise Auxiliary Loss. Although DeepSeek-V3 mainly relies on the auxiliary-loss-free strategy for load balance, to prevent extreme imbalance within any single sequence, we also employ a complementary sequence-wise balance loss:

$$\mathcal{L}_{\mathrm{Bal}} = \alpha \sum_{i=1}^{N_r} f_i P_i, \tag{17}$$

$$f_i = \frac{N_r}{K_r T} \sum_{t=1}^{T} \mathbb{1}\left(s_{i,t} \in \operatorname{Topk}(\{s_{j,t} | 1 \leqslant j \leqslant N_r\}, K_r)\right), \tag{18}$$

$$s'_{i,t} = \frac{s_{i,t}}{\sum_{j=1}^{N_r} s_{j,t}}, \tag{19}$$

$$P_i = \frac{1}{T} \sum_{t=1}^{T} s'_{i,t}, \tag{20}$$

(but the approach is not fully aux loss free..)

What happens when removing load balancing losses?

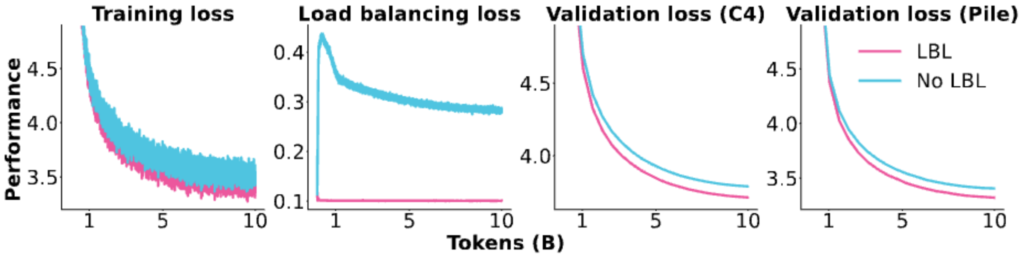


Figure 9: **Impact of applying a load balancing loss (LBL).** The training loss plot excludes the load balancing loss for both models. More results, logs, and configurations: https://wandb.ai/ai2-llm/olmoe/reports/Plot-LBL-vs-No-LBL--Vmlldzo4OTkyNDg4

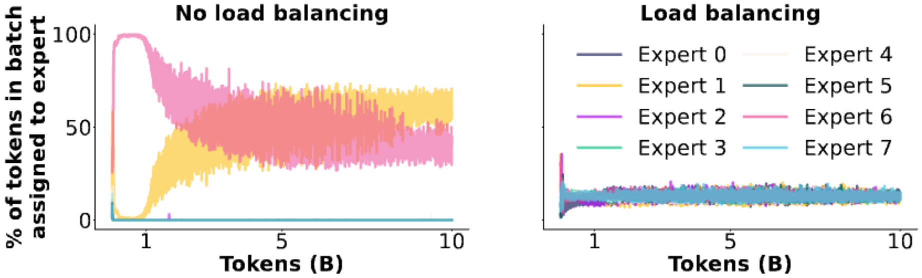
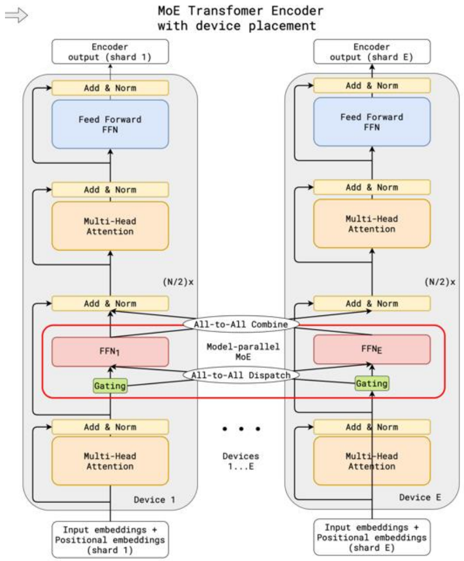


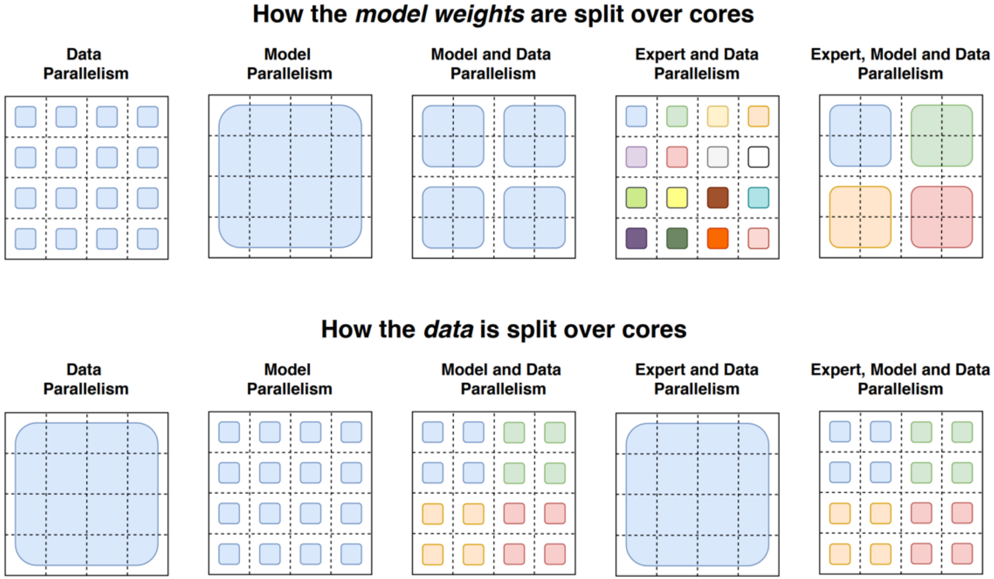
Figure 10: **Expert assignment during training when using or not using a load balancing loss for the first MoE layer.** More results, logs, and configurations: https://wandb.ai/ai2-llm/olmoe/reports/Plot-LBL-vs-No-LBL--Vmlldzo4OTkyNDg4

Training MoEs – the systems side

MoEs parallelize nicely – Each FFN can fit in a device

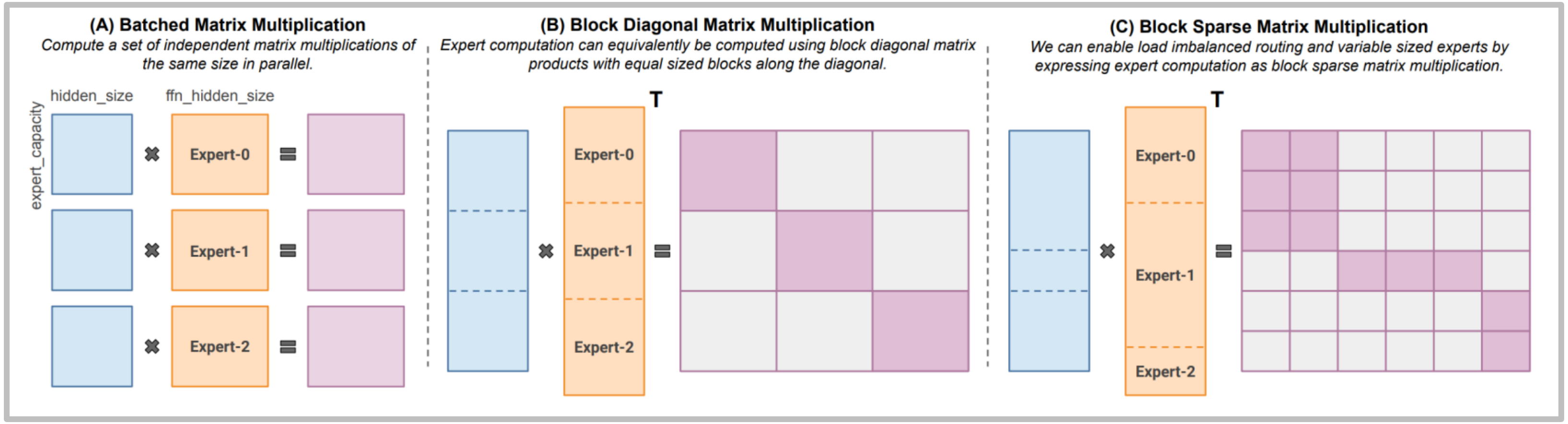


Enables additional kinds of parallelism



Training MoEs – the systems side

MoE routing allows for parallelism, but also some complexities

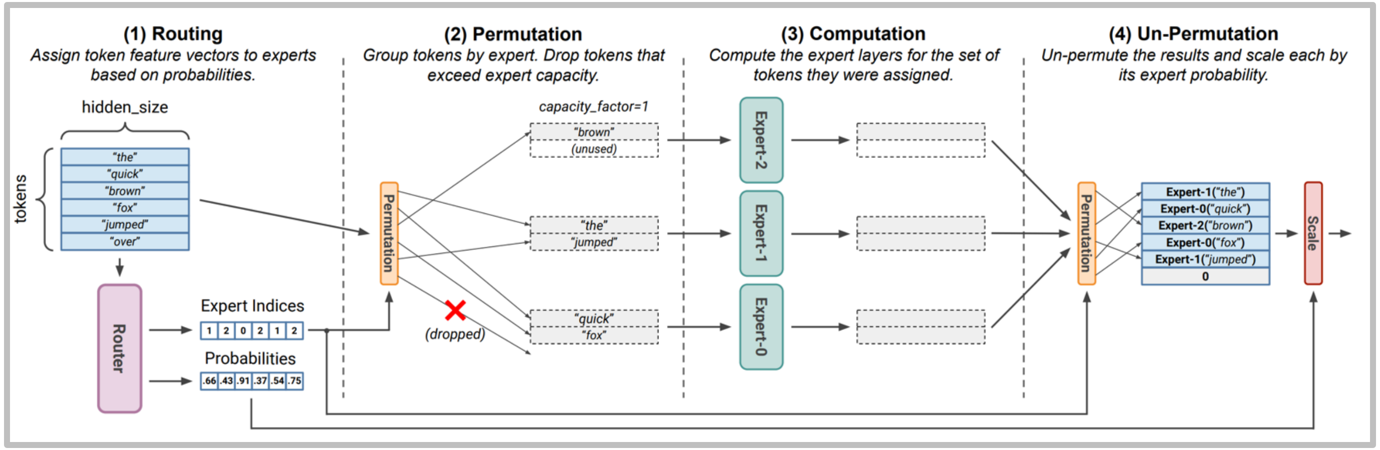


Modern libraries like MegaBlocks (used in many open MoEs) use smarter sparse MMs

Fun side issue – stochasticity of MoE models

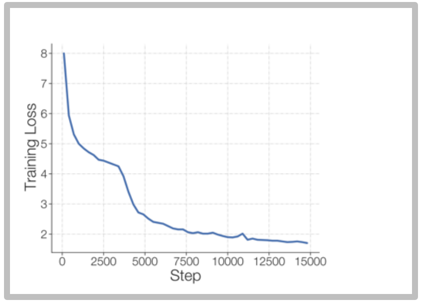
There was speculation that GPT-4's stochasticity was due to MoE..

Why would a MoE have additional randomness?



Token dropping from routing happens at a *batch* level – this means that other people's queries can drop your token!

Issues with MoEs - stability



[7] Exponential functions have the property that a small input perturbation can lead to a large difference in the output. As an example, consider inputting 10 logits to a softmax function with values of 128 and one logit with a value 128.5. A roundoff error of 0.5 in `bfloat16` will alter the softmax output by 36% and incorrectly make all logits equal. The calculation goes from $\frac{\exp(0)}{\exp(0)+10\cdot\exp(-0.5)} \approx 0.142$ to $\frac{\exp(0)}{\exp(0)+10\cdot\exp(0)} \approx 0.091$. This occurs because the max is subtracted from all logits (for numerical stability) in softmax operations and the roundoff error changes the number from 128.5 to 128. This example was in `bfloat16`, but analogous situations occur in `float32` with larger logit values.

[Zoph 2022]

Solution: Use Float 32 just for the expert router (sometimes with an aux z-loss)

$$L_z(x) = \frac{1}{B}\sum_{i=1}^{B}\left(\log\sum_{j=1}^{N} e^{x_j^{(i)}}\right)^2 \qquad (5)$$

Z-loss stability for the router

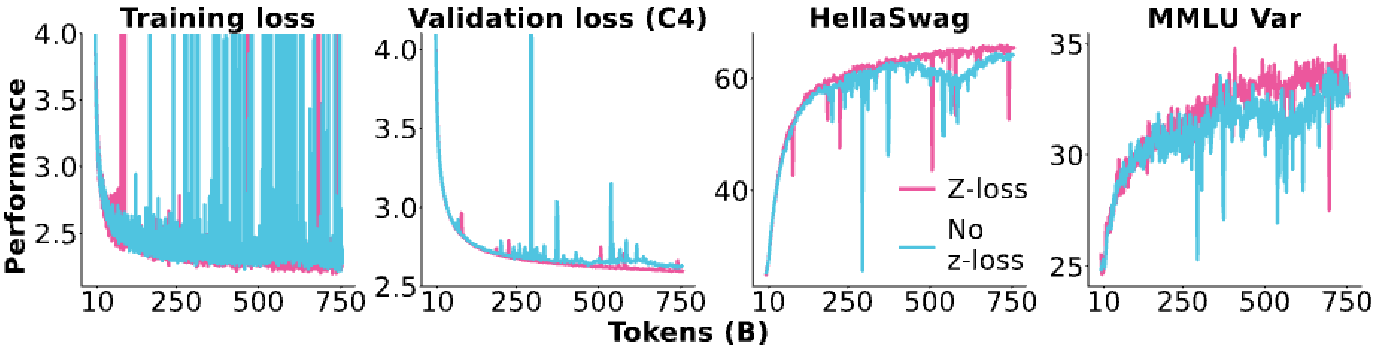
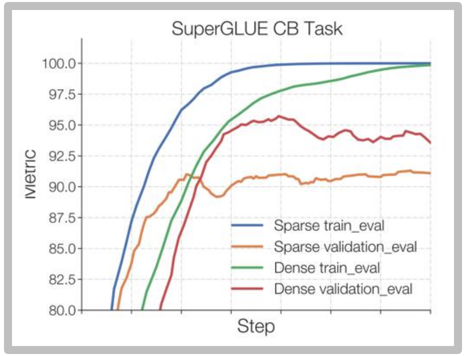


Figure 11: **Router z-loss.** We compare adding router z-loss with a loss weight of 0.001 versus no additional z-loss. More results, logs, and configurations: https://wandb.ai/ai2-llm/olmoe/reports/Plot-Zloss-vs-none--Vmlldzo4NDM4NjUz

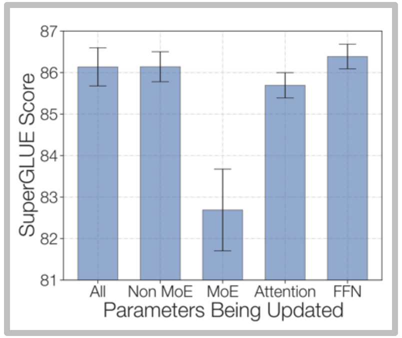
What happens when we remove the z-loss?

Issues with MoEs – fine-tuning

Sparse MoEs can overfit on smaller fine-tuning data



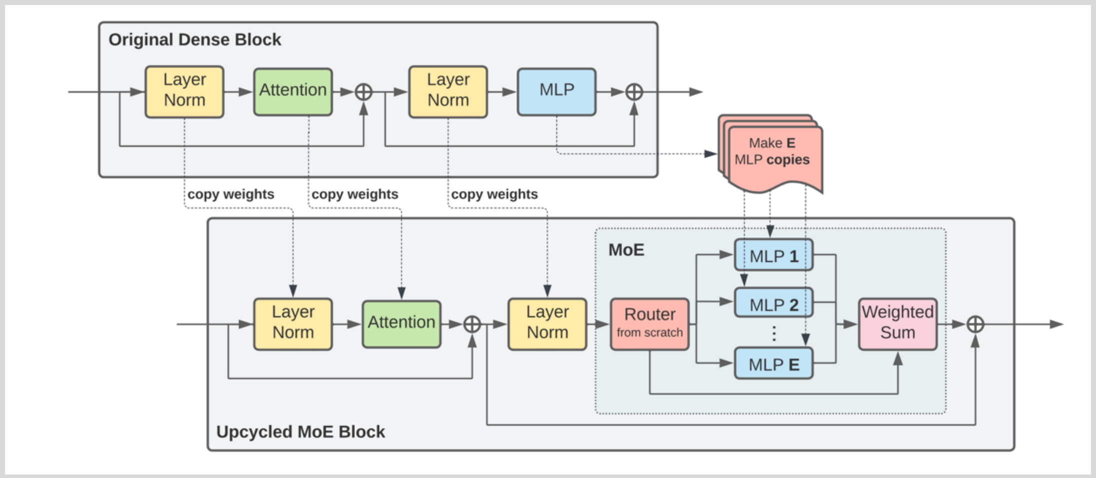
Zoph et al solution – finetune non-MoE MLPs

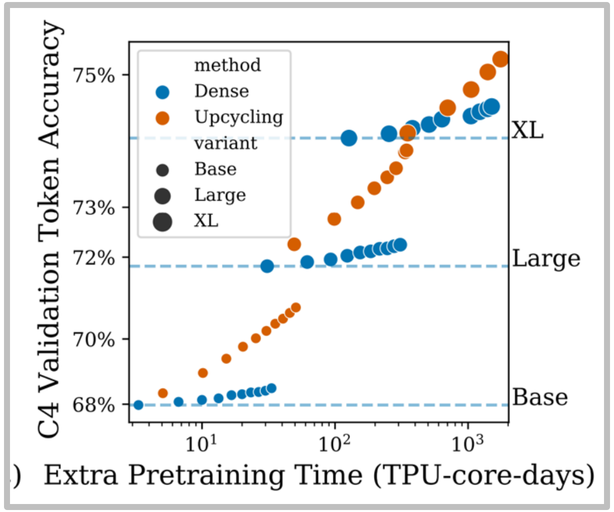


DeepSeek solution – use lots of data 1.4M SFT

Training Data. For training the chat model, we conduct supervised fine-tuning (SFT) on our in-house curated data, comprising 1.4M training examples. This dataset spans a broad range of categories including math, code, writing, question answering, reasoning, summarization, and more. The majority of our SFT training data is in English and Chinese, rendering the chat model versatile and applicable in bilingual scenarios.

Other training methods - upcycling





Can we use a pre-trained LM to initialize a MoE?

Upcycling example - MiniCPM

Uses the MiniCPM model (topk=2, 8 experts, ~ 4B active params).

Model	C-Eval	CMMLU	MMLU	HumanEval	MBPP	GSM8K	MATH	BBH
Llama2-34B	-	-	62.6	22.6	33.0†	42.2	6.24	**44.1**
Deepseek-MoE (16B)	40.6	42.5	45.0	26.8	39.2	18.8	4.3	-
Mistral-7B	46.12	42.96	**62.69**	27.44	45.20	33.13	5.0	41.06
Gemma-7B	42.57	44.20	60.83	38.41	50.12	47.31	6.18	39.19
MiniCPM-2.4B	51.13	51.07	53.46	50.00	47.31	53.83	10.24	36.87
MiniCPM-MoE (13.6B)	**58.11**	**58.80**	58.90	**56.71**	**51.05**	**61.56**	**10.52**	39.22

Table 6: Benchmark results of MiniCPM-MoE. † means evaluation results on the full set of MBPP, instead of the hand-verified set (Austin et al., 2021). The evaluation results of Llama2-34B and Qwen1.5-7B are taken from their technical reports.

Simple MoE, shows gains from the base model with ~ 520B tokens for training

Upcycling example – Qwen MoE

Qwen MoE – Initialized from the Qwen 1.8B model top-k=4, 60 experts w/ 4 shared.

Model	#Parameters	#(Activated) Parameters	MMLU	GSM8K	HumanEval	Multilingual	MT-Bench
Mistral-7B	7.2	7.2	64.1	47.5	27.4	40.0	7.60
Qwen1.5-7B	7.7	7.7	64.6	50.9	32.3	-	-
Gemma-7B	8.5	7.8	61.0	62.5	36.0	45.2	7.60
DeepSeekMoE 16B	16.4	2.8	45.0	18.8	26.8	-	6.93
Qwen1.5-MoE-A2.7B	14.3	2.7	62.5	61.5	34.2	40.8	7.17

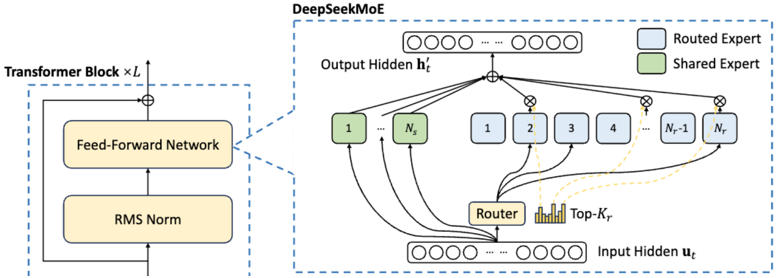
Similar architecture / setup to DeepSeekMoE, but one of the first (confirmed) upcycling successes

DeepSeek MoE v1-v2-v3

To wrap up, we'll walk through the DeepSeek MoE architecture.

V1 (16B – 2.8 active):

Shared (2) + Fine-grained (64/4) experts



Standard, top-k routing

$$\mathbf{h}_t' = \mathbf{u}_t + \sum_{i=1}^{N_s} \mathrm{FFN}_i^{(s)}(\mathbf{u}_t) + \sum_{i=1}^{N_r} g_{i,t}\, \mathrm{FFN}_i^{(r)}(\mathbf{u}_t),$$

$$g_{i,t} = \begin{cases} s_{i,t}, & s_{i,t} \in \mathrm{Topk}(\{s_{j,t} | 1 \leqslant j \leqslant N_r\}, K_r), \\ 0, & \text{otherwise}, \end{cases}$$

$$s_{i,t} = \mathrm{Softmax}_i\left({\mathbf{u}_t}^T \mathbf{e}_i\right),$$

Standard Aux-loss balancing (Expert + Device)

$$\mathcal{L}_{\mathrm{ExpBal}} = \alpha_1 \sum_{i=1}^{N_r} f_i P_i,$$

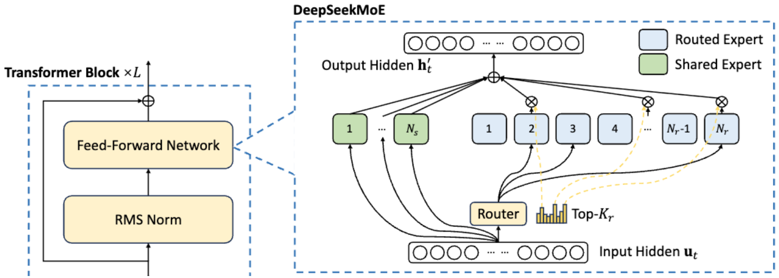
$$f_i = \frac{N_r}{K_r T} \sum_{t=1}^{T} \mathbb{1}(\text{Token } t \text{ selects Expert } i),$$

$$P_i = \frac{1}{T} \sum_{t=1}^{T} s_{i,t},$$

DeepSeek MoE v2

V2 (236B – 21 active):

Shared (2) + Fine-grained (160/10) experts, 6 active



New things:

Top-M device routing

For DeepSeek-V2, beyond the naive top-K selection of routed experts, we additionally ensure that the target experts of each token will be distributed on at most M devices. To be specific, for each token, we first select M devices that have experts with the highest affinity scores in them. Then, we perform top-K selection among experts on these M devices. In practice, we find that when $M \geqslant 3$, the device-limited routing can achieve a good performance roughly aligned with the unrestricted top-K routing.

Communication balancing loss – balancing both communication in *and* out

$$\mathcal{L}_{\text{CommBal}} = \alpha_3 \sum_{i=1}^{D} f_i'' P_i'', \tag{29}$$

$$f_i'' = \frac{D}{MT} \sum_{t=1}^{T} \mathbb{1}(\text{Token } t \text{ is sent to Device } i), \tag{30}$$

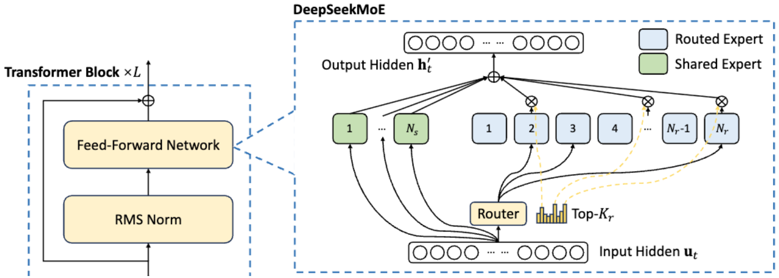
$$P_i'' = \sum_{j \in \mathcal{E}_i} P_j, \tag{31}$$

where α_3 is a hyper-parameter called communication balance factor. The device-limited routing mechanism operates on the principle of ensuring that each device transmits at most MT hidden states to other devices. Simultaneously, the communication balance loss is employed to encourage each device to receive around MT hidden states from other devices. The communication balance loss guarantees a balanced exchange of information among devices, promoting efficient communications.

DeepSeek MoE v3

V2 (671B – 37 active):

Shared (1) + Fine-grained (258) experts, 8 active



New things

Sigmoid+Softmax topK + topM

$$\mathbf{h}_t' = \mathbf{u}_t + \sum_{i=1}^{N_s} \mathrm{FFN}_i^{(s)}(\mathbf{u}_t) + \sum_{i=1}^{N_r} g_{i,t}\,\mathrm{FFN}_i^{(r)}(\mathbf{u}_t),$$

$$g_{i,t} = \frac{g_{i,t}'}{\sum_{j=1}^{N_r} g_{j,t}'},$$

$$g_{i,t}' = \begin{cases} s_{i,t}, & s_{i,t} \in \mathrm{Topk}(\{s_{j,t}|1 \leqslant j \leqslant N_r\}, K_r), \\ 0, & \text{otherwise}, \end{cases}$$

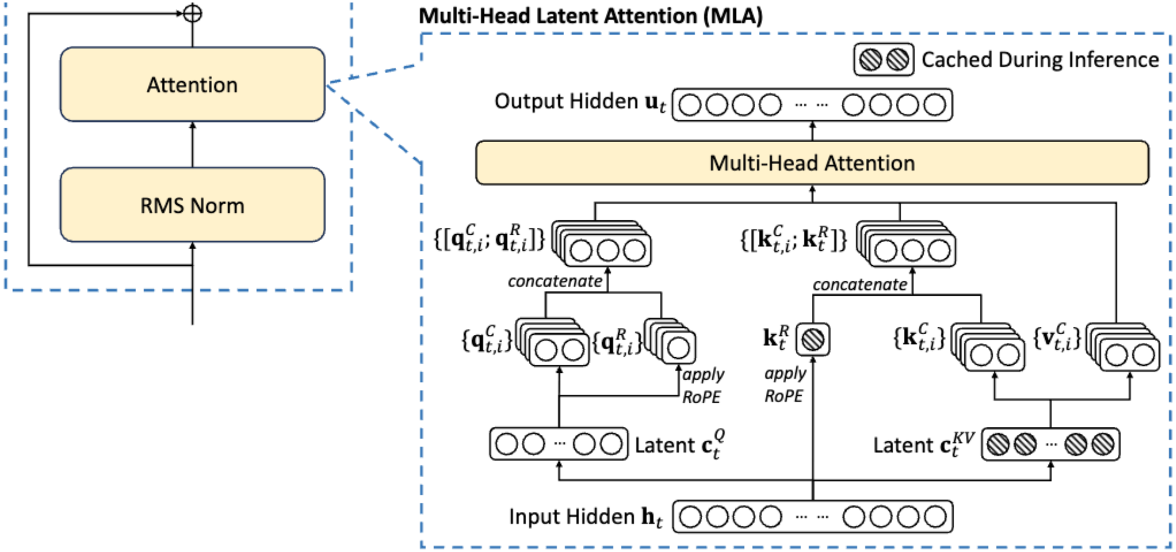
$$s_{i,t} = \mathrm{Sigmoid}\left(\mathbf{u}_t{}^T \mathbf{e}_i\right),$$

Aux-loss-free + seq-wise aux

$$g_{i,t}' = \begin{cases} s_{i,t}, & s_{i,t} + b_i \in \mathrm{Topk}(\{s_{j,t} + b_j|1 \leqslant j \leqslant N_r\}, K_r), \\ 0, & \text{otherwise}. \end{cases}$$

Bonus: What else do you need to make DeepSeek MoE v3?

MLA : Multihead, latent attention

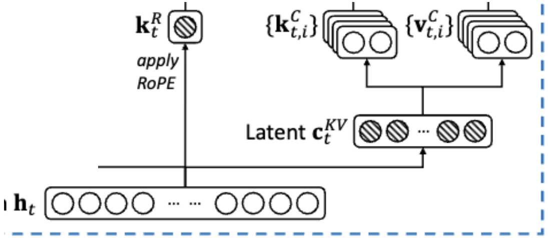


Basic idea: express the Q, K, V as functions of a lower-dim, 'latent' activation

What else do you need to make DeepSeek MoE v3?

Basic idea: express the Q, K, V as functions of a lower-dim, 'latent' activation

$$\mathbf{c}_t^{KV} = W^{DKV}\mathbf{h}_t,$$
$$\mathbf{k}_t^C = W^{UK}\mathbf{c}_t^{KV},$$
$$\mathbf{v}_t^C = W^{UV}\mathbf{c}_t^{KV},$$



Benefits: when KV-caching, we only need to store c_t^{KV}, which can be much smaller.
W^{UK} can be merged into the Q projection

(they also compress queries, for memory savings during training)

$$\mathbf{c}_t^Q = W^{DQ}\mathbf{h}_t,$$
$$\mathbf{q}_t^C = W^{UQ}\mathbf{c}_t^Q,$$

Complexity: rope conflicts with MLA-style caching.

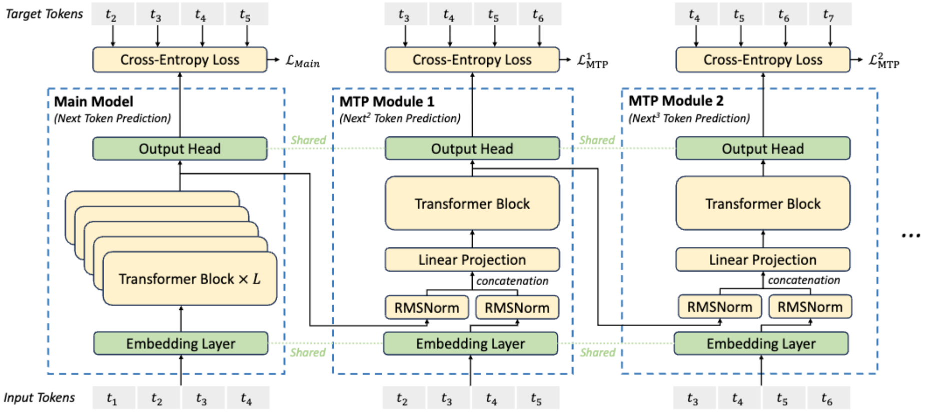
Without RoPE – $\langle Q, K\rangle = \langle hW^Q, W^{UK}c_t^{KV}\rangle = \langle h\, W^Q W^{UK}, c_t^{KV}\rangle$

With RoPE - $\langle QR_q, R_k K\rangle = \langle hW^Q R_q, R_k W^{UK} c_t^{KV}\rangle = \langle h\, W^Q R_q R_k W^{UK}, c_t^{KV}\rangle$

The solution – Have a few non-latent key dimensions that can be rotated

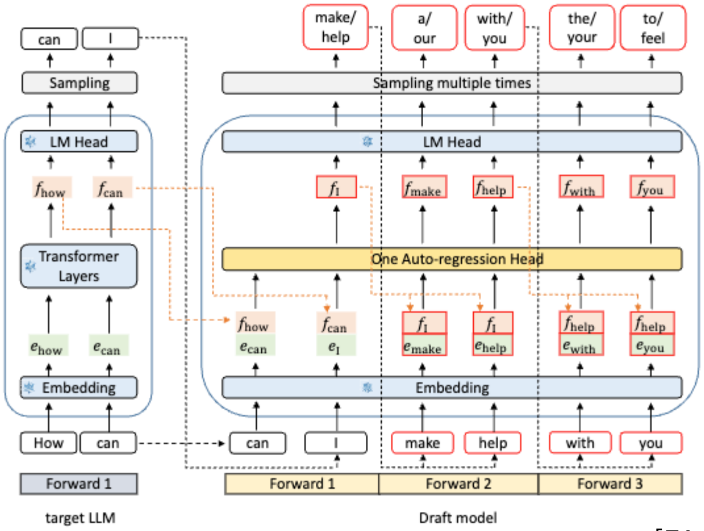
What else do you need to make DeepSeek MoE v3?

MTP: Have small, lightweight models that predict multiple steps ahead



(But they only do MTP with one token ahead)

[Deepseek v3]



[EAGLE]

$$\mathbf{h}_i'^k = M_k[\mathrm{RMSNorm}(\mathbf{h}_i^{k-1}); \mathrm{RMSNorm}(\mathrm{Emb}(t_{i+k}))],$$

$$\mathbf{h}_{1:T-k}^k = \mathrm{TRM}_k(\mathbf{h}_{1:T-k}'^k),$$

$$P_{i+k+1}^k = \mathrm{OutHead}(\mathbf{h}_i^k).$$

(See paper for ablations)

MoE summary

- MoEs take advantage of sparsity – not all inputs need the full model
- Discrete routing is hard, but top-k heuristics seem to work
- Lots of empirical evidence now that MoEs work, and are cost-effective