Economic forecasting with an agent-based model
Discussion on DSGE model in the ETRI meeting (Feb. 21, 2023)

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Introduction

- Smets and Wouters (2007): One of the most famous Representative Agent New Keynesian (RANK) model
- Main features of Smets and Wouters (2007)
  1. Almost all kinds of shocks (at that time) & Frictions
  2. Evaluate which shocks matter for explaining U.S. Business-Cycle
- HANK version of Smets and Wouters
  - Smets and Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach"
  - Bayer, Born and Luetticke (2023): "Shocks, Frictions and Inequality in US Business Cycles"

Environment

- Households (HH)
  - Income: Labor income (Wages) & Interest income (Savings)
  - Choice: Choose consumption, labor, savings and physical capital to maximize HH life-time utility
  - Individual optimality: Marginal Benefit=Marginal Cost

- (Intermediate Goods) Firms
  - Choice: Set prices (Monopolistic competitor), labor and capital
  - vs. Real Business Cycle (RBC) model: Stickiness on prices

Household's Problem

$$\max_{\{C_t, L_t, B_{H,t}, B_{F,t}, u_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} \right) \exp\left(\frac{-(1-\sigma_c)}{1+\sigma_l} L_t^{1+\sigma_l} \right) \right\}$$

subject to the BUDGET CONSTRAINT

$$P_t C_t + P_t^X X_t + \frac{1}{R_t} B_{H,t} + \frac{1}{R_t^*} B_{F,t} \leq$$

$$W_t^{hh,nom}(h) L_t dh + R_t^k u_t \bar{K}_{t-1} + a\left(u_t \bar{K}_{t-1}\right) P_t + \frac{1}{n} \int_0^n Div(h) dh + T_t$$

where $K_t = u_t \bar{K}_{t-1}$ and $C_t = \left[\gamma_c^{\frac{1}{\varepsilon}} C_{H,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma_c)^{\frac{1}{\varepsilon}} C_{F,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]$

How to solve the problem: Largrangian

$$
\begin{aligned}
\mathcal{L} = & \max_{\{C_t, L_t, B_{H,t}, B_{F,t}, u_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c} \right) \exp\left(\frac{-(1-\sigma_c)}{1+\sigma_l} L_t^{1+\sigma_l} \right) \\
& + \Lambda_t [W_t^{hh,nom}(h) L_t dh + R_t^k u_t \bar{K}_{t-1} + a\left(u_t \bar{K}_{t-1}\right) P_t + \frac{1}{n} \int_0^n Div(h) dh + T_t \\
& - \left(P_t C_t + P_t^X X_t + \frac{1}{R_t} B_{H,t} + \frac{1}{R_t^*} B_{F,t} \right)]
\end{aligned}
\tag{1}
$$

Intuition

- Given prices, each agent maximizes life-time reward/utility/profit subject to constraint
- Equilibrium makes each agent's optimality be consistent
- Inter-temporal optimization
  - Consumption - Saving: Consume today vs. Consume tomorrow
  - Basic: Consumption smoothing. Assume that you would wanna consume smoothly over time & state
  - What matters: Income, Prices
  - Income : Save (Borrow) if you are rich today (tomorrow)
  - Prices: Consume more today if it is cheaper than tomorrow
  - Higher inflation expectations: Make consume more today
  - Higher interest rate: Borrow less/Save more $\rightarrow$ Consume less $\rightarrow$ Inflation $\downarrow$

Firm's problem: Map

- vs. Classical RBC model
  - Frictions on goods prices or wages
  - What we need(not sufficient condition): Imperfect competition $\rightarrow$ Each firm could have power to set price
  - What we need more: Frictions on price setting: Random timing for price adjustment(Calvo - Yun), Adjustment cost(Rotermberg) or Menu cost(Mankiw)
- with Classical RBC
  - Equilibrium or Market clearing

What could we buy from equilibrium?

- Consistency
  - If not in equilibrium, it usually implies that someone could be better off
  - Example: If prices are lower than equilibrium, firms can be better of by posting higher prices (+HHs would not be suffered from excess demand)

- Solving the model
  - In the General Equilibrium (GE) model: Cannot solve the model due to the rank condition (coming soon)
  - In the Partial Equilibrium (PE) model: Okay but should be carefully calibrated. But in the New Keynesian (NK) model, it is not usual as its spirit depends on sticky prices

- Why we cannot solve the model (why we need rank condition)?
  - Require on identification: One set of parameter should give us only one set of solutions
  - (Similar concept) In the simulation, the result should be independent of initial condition
  - Fixed point: In the business cycle model, we usually consider the infinite horizon model → The model needs to be closed for each period → For this perspective, DRL could be one resolution

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Domestic Firm's problem

- Each firm h's production function: Cobb-Douglas

$$Y_t(h) = F\left(K_t(h), L_t(h)\right) = A_t K_t(h)^{\alpha}\left[Z_t L_t(h)\right]^{1-\alpha} - \Phi Z_t$$

where A_t: Total factor productivity (TFP), Z_t: Long-run labour - augmenting productivity factor where $Z_t = (1+\gamma)Z_{t-1}$ and Φ: fixed cost of production in relation to labour-augmenting productivity factor

- Each firm h's problem
  - In a goods market: Set prices as a monopolistic competitor
  - In factor markets: Demand $L_t(h)$ and $K_t(h)$

- How to solve
  - Cost minimization: Choose production factors
  - Profit maximization: Choose price $p_t(h)$

Cost Minimization: Capital-Labour Ratio

Each firm h solve the following cost minimization problem optimally:

$$\min_{L_t(h),K_t(h)} \left\{ \begin{array}{c} W_t^{nom} L_t(h) + R_t^{k,nom} K_t(h) + \\ MC_t^{nom}(h) \left[Y_t(h) - A_t K_t(h)^{\alpha} \left[Z_t L_t(h) \right]^{1-\alpha} + \Phi Z_t \right] \end{array} \right\}$$

Solutions:

$$L_t \ \& K_t : \frac{1-\alpha}{\alpha} R_t^{k,nom} K_t(h) = W_t^{nom} L_t(h)$$

$$MC_t^{nom} = \frac{1}{A_t} \frac{\left(R_t^{k,nom} \right)^{\alpha} \left(W_t^{nom} \right)^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \frac{1}{Z_t^{1-\alpha}}$$

Profit Maximization: Price setting

Calvo − Yun fashion: Each firm h can reset price with probability ξ_p.
Given $\Omega_{t,t+k} = \beta \frac{\Lambda_{t,t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}}$: Household's stochastic discount factor, the firm h solves the following problem optimally:

$$\max_{p_t^o, S_t, p_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Omega_{t,t+k} \left\{ \begin{array}{l} \left[p_t(h) Ind_{t,k}^p - MC_t^{nom}(h) \right] \left[\frac{1}{n} \left(\frac{p_t(h)}{P_{H,t+k}} Ind_{t,k}^p \right)^{-\theta} (A_{H,t+k}) \right] + \\ \left[S_t p_t(h)^* - MC_{t+k}^{nom}(h) \right] \left[\frac{1}{n} \left(\frac{S_t p_t(h)^*}{S_{t+k} P_{H,t+k}^*} Ind_{t,k}^p \right)^{-\theta} \left(A_{H,t+k}^* \right) \right] \end{array} \right\} \tag{2}$$

Assumption: Law of one price $\rightarrow S_t p_t^*(h) = p_t^o(h)$

Fiscal & Monetary Authority

Fiscal authority: Government spending is financed by the lump-sum tax

$$G_t + T_t = 0$$

Monetary authority: Taylor rule

$$\frac{R_t}{R_t} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\rho_Y}\right] \varepsilon_t^R$$

Equilibrium

Goods market clearing

$$Y_t(h) = A_t K_t^{\alpha} \left[Z_t L_t \right]^{1-\alpha} - \Phi Z_t =$$

$$\left[\begin{array}{c} \left[\gamma_c \left(\frac{P_{H,t}}{P_t} \right)^{-\epsilon} C_t + \gamma_x \left(\frac{P_{H,t}}{P_t^X} \right)^{-\epsilon} X_t + G_t \right] + \\ \frac{1-n}{n} \left[\gamma_c^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\epsilon} C_t^* + \gamma_x^* \left(\frac{P_{H,t}^*}{P_t^{X,*}} \right) \right] \end{array} \right]$$

Bonds markets clearing (Zero net supply):

$$\int_0^n B_{H,t}\left(j, s_{t+1}\right) dj = 0$$

$$\int_0^n B_{F,t}\left(j, s_{t+1}\right) dj + \int_n^1 B_{F,t}^*\left(j, s_{t+1}\right) dj = 0$$

Equilibrium: Continued

Labour markets clearing

$$L_t = \int_0^n L_t(h)dh = \int_0^n \int_0^n l_t(h,j)dhdj$$
$$L_t^* = \int_0^n L_t^*(f)df = \int_0^n \int_0^n l_t^*(f,j^*)dfdj^*$$

Capital markets clearing

$$K_t = \int_0^n K_t(h)dh$$
$$K_t^* = \int_0^n K_t^*(f)df$$

Labour Market

- Sticky wages
  - Seller's market power to set wages $\rightarrow$ Intermediate labour union
  - Friction on setting prices: Calvo fashion in this paper

- Labour union:
  Households supply homogeneous labour to an intermediate labour union, which differentiates the labour services from labour varieties of type l and set wages in a Calvo fashion, selling the labour varieties of type l to labour packers

$$\max_{W_t^{nom}(l)} \mathbb{E}_t \sum_{t=0}^{\infty} (\beta \xi_w)^k \frac{\Lambda_{t+k}}{\Lambda_t} \frac{P_t}{P_{t+k}} \left\{ W_{t+k}^{nom,ind} - W_{t+k}^{hh,nom} \right\} L_{t+k}(l)$$

where $1 - \xi_w$: the union can reset the wage in the current period

Labour Market (continued)

subject to

$$L_{t+k}(l) = \frac{1}{n}\left(\frac{W^{nom,ind}_{t+k}(l)}{W^{nom}_{t+k}}\right) L_{t+k}$$

$$W^{nom,ind}_{t+k}(l) = W^{nom}_{t}(l) Ind^{w}_{t,k}$$

$$W^{hh,nom}_{t+k} = \frac{P_{t+k}\left[\frac{(C_{t+k}-hC_{t+k-1})^{1-\sigma_c}}{1-\sigma_c}\right] \exp\left(\frac{-(1-\sigma_c)}{1+\sigma_l} L_{t+k}(l)^{1+\sigma_l}\right)(\sigma_c - 1) L_{t+k}(l)^{\sigma_l}}{-\Lambda_{t+k}}$$

where $Ind^{w}_{t,k}$ denotes the rule for wage indexation, which is given by

$$Ind^{w}_{t,k} = \left\{ \begin{matrix} 1 \text{ for } k = 0 \\ \left(\Pi^{k}_{l=1} \gamma \pi^{\iota_w}_{t+l-1} \pi^{1-\iota_w}\right) \end{matrix} \right\}$$

where ι_w: a parameter governing the degree of this wage indexation

Sticky Wages (ξ_w)

- Microfoundations
  - Staggered contract: Annual wage contract
  - Newly hired vs. Existing workers
  - Stayer vs. Switcher
- Estimates/Calibration
  - This paper: 0.766 (mean)
  - Smets and Wouters: 0.70 (mean)/0.73 (mode)
  - Other paper: 3−4 quarters on average
- Korean calibration/estimates
  - Claim: Limited due to the lack of business cycle frequency wage data
  - Park and Shin (2014): Give qualitative evidence but needs improvements on quantitative aspects

vs. ABM

- Labour market structure
  - ABM: Search frictions $\rightarrow$ Probability depends on $\frac{V}{U}$
  - DSGE in this paper: No unemployment but fluctuations in total margin

- Wage determination
  - ABM:

$$w_i(t) = \bar{w}_i \min\left\{1.5, \frac{\min\{Q_i^S(t), \beta_i M_i(t-1), \kappa_i K_i(t-1)\}}{N_i(t)\bar{\alpha}_i}\right\}$$

Search and Matching in DSGE

- Matching function (Constant Return to Scale, CRS)

$$M_t = M(U_t, V_t) \leq \min\{U_t, V_t\}$$

$$\text{Job seeker's job finding rate: } f(\theta) = \frac{1}{U}M(U,V) = M\left(1, \frac{V}{U}\right)$$

$$\text{Firm's vacancy filling rate: } q(\theta) = \frac{1}{V}M(U,V) = M\left(\left(\frac{V}{U}\right)^{-1}, 1\right)$$

- Value of each state
  - Employed workers: Wage income, expected value depends on probabilities of job separation, job switching, wage changes and etc.
  - Unemployed job seekers: Unemployment insurance benefit, expected value depends on probabilities of job finding, eligibility of U.I. benefit and etc.
  - Firms posting vacancies: Expected value of filling jobs
  - Jobs filled vacancies: Production - Wage cost + expected value

Value Functions (Basic model)

- Employed workers

$$W(S) = \max\left\{u(c_e) + \beta\left[\int (1-\sigma)W(s') + \sigma U(S')dF(S'|S)\right]\right\}$$

- Unemployed workers

$$U(S) = \max\left\{u(c_u) + \beta\left[\int f(\theta)W(s') + (1-f(\theta))U(S')dF(S'|S)\right]\right\}$$

- Value of posting jobs

$$V = -\kappa + q(\theta)J$$

- Value of filling jobs

$$J(S) = y - w + \beta(1-\sigma)\int J(S')dF(S'|S)$$

- Free entry condition

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Comments

- Labor market in the HANK literature
  - Sticky wages with labour union: Bayer, Born and Luetticke (forthcoming)
  - Search and Matching: Ravn and Sterk (2017) − Not quantitative
  - Income Tax Progressivity :
    $T(y) = y - \lambda y^{1-\tau} \rightarrow D(y) = y - T(y) = \lambda y^{1-\tau}$

- Issues on data for external calibration
  - Sticky wages: High frequency wage/earnings data
  - Search and Matching: Poor quality on vacancy data $\rightarrow$ Estimating matching function

- If we do not consider the search friction, then we would need to have an accurate labor supply elasticity (in DSGE)
  - With matching function, we have *few* estimates using Korean data.

With Gabriel

- Production function $\rightarrow$ Have discussed
- Human capital accumulation in DSGE
  - Ben - Porath (Learning or Doing, BP): $h_{i,t+1} = (1-\delta)h_{i,t} + l_i s_{i,t}^{\alpha_1} x_{i,t}^{\alpha_2} h_{i,t}^{\alpha_3}$
  - Learning by Doing (LBD): $h_{i,t+1} = (1-\delta)h_{i,t} + l_i n_{i,t}^{\alpha_1} h_{i,t}^{\alpha_2}$
  - Korean literature: Kim (2020), Kang (2022)
- Labor productivity/Income process
  - Guvenen, McKay and Ryan (2022): "A Tractable Income Process for Business Cycle Analysis"
    $y_{i,t} = \gamma_i + z_{i,t} + \tilde{\zeta}_{i,t} + [1 + f(\gamma_i + z_{i,t})w_t + \kappa_i(t - h_i)]$
  - Issue again: Data availability