Duration Dependence, Adverse Selection, and Information Acquisition

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Abstract

This paper studies a dynamic adverse selection model where firms post wages and acquire information about workers' productivity with costs. The equilibrium fully separates types without firms' screening if the productivity differential is small. When the differential is large, the equilibrium consists of a high-wage/screening job and a low-wage/non-screening job. The average re-employment wage increases over the unemployment duration in a fully separating equilibrium, while it may either increase or decrease in an upward pooling equilibrium. I characterize the precise condition under which duration dependence becomes negative.

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1 Introduction

The duration a good remains on a market conveys information about its quality, yet the interpretation of this information varies across market structures. In the labor market, the longer unemployment duration is regarded as evidence of a lower quality, justifying the wage and job-finding rate declines over duration (Addison and Portugal, 1989; Vishwanath, 1989; Lockwood, 1991; Gregory and Jukes, 2001; Gonzalez and Shi, 2010; Kroft et al., 2013; Schmieder et al., 2016). In the financial market, on the other hand, sellers with better assets stay on the market longer as they are less eager to sell their goods at lower prices (DeMarzo and Duffie, 1999; Camargo and Lester, 2014; Guerrieri and Shimer, 2014; Chang, 2017; Li, 2022). As such, interpreting duration information is nuanced and depends on several factors including price determination mechanisms and information accessibility to buyers. To offer a unified framework for understanding these interactions, this paper proposes a dynamic adverse selection model where uninformed buyers post prices and acquire information at a cost.

Examining a model that incorporates both price posting and information acquisition is of importance in economic analysis. Theoretically, the deterioration in quality over time, as observed in the labor market, is often a consequence of buyers' information acquisition, while the improvement in quality arises from pricing mechanisms, as in the lemon market argument. Then, it is of interest how the equilibrium allocation and dynamics are determined when both informational instruments coexist. Practically, price posting and screening are common across various markets, with notable significance in the labor market context where firms set wages and conduct interviews. This paper can enrich our understanding of when and how the stigma effect of unemployment duration explains the negative association between duration and labor market outcomes.

This paper proposes a dynamic labor market model that consists of privately-informed unemployed workers and uninformed firms with screening capacity, although the model's applicability extends to broader market contexts. In the model, firms post a wage conditional upon hiring, and after observing all posted wages, unemployed workers choose which firms they apply to. Each worker is either H(igh)- or L(ow)-productivity type, which is a private information. Although firms are unable to observe the type of job candidates, they can acquire information after matching but before making hiring decisions (Menzio and Shi, 2011). In doing so, firms choose the precision of information acquisition subject to costs proportional to the expected entropy reduction (Sims, 2003; Matějka and McKay, 2015).

Firms' incentive to acquire information varies based on post-employment wages and initial beliefs about job candidates. Intuitively, firms conduct more intensive screening when they have to pay higher wages, suggesting that information strategies differ across wage levels. When workers choose wages they apply to, they rationally anticipate how wages influence information acquisition incentives and, therefore, their likelihood of being hired. These equilibrium search decisions feed back into firms' screening decisions, as the optimal information acquisition also depends on initial beliefs, which are results of the workers' application strategies. Therefore, in equilibrium, workers' search strategies and firms' information policies interact and are jointly determined.

This paper contributes to existing literature by introducing an equilibrium concept of an economy that integrates directed search and endogenous information acquisition. This task is challenging due to the interplay between firms' screening incentives and workers' search decisions. This complexity goes beyond technical aspects, addressing a real-world concern faced by firms: evaluating a job candidate's suitability conditional upon application. It particularly complicates the determination of off-the-path beliefs, as firms' beliefs about which type will respond to a deviation indeed influence which type has a higher incentive to deviate. To address this challenge, I extend the off-the-path belief refinement proposed in previous literature (Guerrieri et al., 2010) using the divinity argument (Banks and Sobel, 1987; Gale, 1996). The resulting equilibrium belief is increasing in post-employment wages, implying that firms can attract better workers by committing higher wages.

By integrating this off-the-path belief and firms' optimal information strategies (Matějka and McKay, 2015; Kim et al., 2021), I characterize the equilibrium type and its properties. The equilibrium is either fully separating or upward pooling, contingent on the productivity differential between the types. In the case of a small differential, the equilibrium is fully separating, with H-type individuals applying for higher-wage jobs with lower job-finding probabilities. In the full separation equilibrium, firms don't screen workers, as self-selection by workers reveals their types, eliminating the need for additional information. This equilibrium is sustained only when the benefits of mimicking the H-type are relatively small for the L-type, making a small productivity differential a necessity. If the productivity differential exceeds a threshold, some screening becomes necessary. This screening is rationalized only when both types indeed apply for the same job. Consequently, the equilibrium is upward pooling, which consists of a high-wage job with screening for which both types apply, and a low-wage jobs without screening for which only the L-type applies.

The equilibrium characterization provides a precise description of the relationship between wages, job-finding rates, and duration. Concerning the job-finding rate and duration, the relationship is always negative simply because the type with a lower job-finding rate must remain in the market longer. However, it does not mean that the L-type remains in the market longer. Notably, the impact of duration on wages varies across equilibrium types and parameter values. In the full separation equilibrium, the H-type remains in the market longer, thereby the average quality and wage increase over time, aligning with the findings in Guerrieri and Shimer (2014). On the other hand, in the upward pooling equilibrium, the average wage and quality may rise or fall. If the job-finding rate of the H-type in the screening job is lower than that of the L-type in the non-screening job, and the initial fraction of the H-type is sufficiently low, then average quality and wage increase over duration. However, if either of these conditions is violated, both average quality and wage decrease over duration, generating the negative duration effects on re-employment wages observed in the labor market (Addison and Portugal, 1989; Gregory and Jukes, 2001; Cooper, 2014; Schmieder et al., 2016).

These implications underscore the distinction between the effects of duration on trading probabilities and prices. Quality deterioration over time is associated with price decline but not trading probability. Furthermore, these findings suggest that the negative duration effects in the labor market is not an immediate result from asymmetric information. Instead, it is evidence of significant heterogeneity among seemingly identical workers, prompting firms' need for screening.[1] Generically, the results underscore that duration information yields markedly different implications contingent upon the extent of heterogeneity among sellers, offering a unified perspective for understanding why and how a longer duration might be perceived as either a positive or negative signal across different markets.

Beyond the relationship between duration and equilibrium objects, the model highlights two general implications for markets with adverse selection and information acquisition. Firstly, buyers screen sellers only when the quality differential is substantial; otherwise, sellers' self-selection suffices. This implies that in markets equipped with two informational instruments, sellers' self-selection always operates, whereas buyers' screening might not, despite their capability. This explains why buyers' screening is common in some markets, such as labor markets, while less common in other goods market.

Secondly, if the quality differential is larger when buyers possess goods than when sellers do, the equilibrium differs from the allocation preferred by the social planner who face search frictions but not asymmetric information. This quality condition likely holds in the labor market, where workers' quality pertains to their productivity when employed, but it may not hold in financial and housing markets. This suggests that the costs of asymmetric information vary depending on the nature of heterogeneity. In certain cases, the equilibrium can be constrained efficient even with asymmetric information and costly information acquisition.

Related literature: It has long been recognized by economists that a longer duration of unemployment is associated with negative labor market outcomes, such as a lower job-finding rate and reduced re-employment wages (Heckman and Borjas, 1980; Heckman and Singer, 1984; Addison

[1] Gregory and Jukes (2001) documents that the wage penalties due to the unemployment duration are larger for prime-age and high-paid men compared to low-paid and young men. Ortego-Marti (2017) documents that the earnings loss associated with a longer unemployment duration is greater for more skill-required occupations and industries. These empirical findings are consistent with this paper's findings if workers' unobserved quality matters more in jobs that require more experience and skills.

and Portugal, 1989; Kroft et al., 2013; Cooper, 2014; Schmieder et al., 2016; Jarosch and Pilossoph, 2019; Kospentaris, 2021). This paper provides precise conditions under which a negative duration dependence arises under adverse selection and endogenous screening. Additionally, this paper points out that the wage decline is associated with quality deterioration, while the job-finding rate decline is not necessarily.

A strand of literature has studied the negative duration dependence through the lens of labor market learning (Vishwanath, 1989; Lockwood, 1991; Gonzalez and Shi, 2010). While the decline in the job-finding rate itself does not necessarily imply a decline in worker quality over time, previous literature often conflates these aspects due to the assumption of symmetric information or random search (Doppelt, 2016; Fernandez-Blanco and Preugschat, 2018; Fawcett and Shi, 2018; Kospentaris, 2021). These assumptions do not allow workers to adjust their job search strategies in response to firms' screening, which constitutes a central economic tension in this paper. Fernandez-Blanco and Gomes (2017) and Feng et al. (2019) study an adverse selection model, but they overlook firms' endogenous screening decisions. Jang and Kang (2021) considers information acquisition, but in a static signaling environment. This paper contributes to this strand of literature by incorporating both workers' search behavior and endogenous information acquisition in a dynamic economy. By doing so, this paper characterizes the conditions under which wages decline over the duration of unemployment.

This paper is also closely related to previous literature that examines competitive search models with asymmetric information (Guerrieri et al., 2010; Guerrieri and Shimer, 2014; Chang, 2017; Li, 2022). These papers suggest that better sellers tend to remain in the market longer, which contrasts with the negative duration dependence observed in the labor market. What makes a difference is regarded as the existence of screening device (Kaya and Kim, 2018). This paper confirms this idea in a model with endogenous information acquisition and price posting. This paper also demonstrates that whether superior or inferior sellers remain in the market longer depends on the productivity differential and the initial distribution.

There has been a large strand of literature examining costly information acquisition (Sims, 2003; Matějka and McKay, 2015). This paper contributes to this literature by demonstrating how endogenous information choice and adverse selection can be integrated within a directed search framework. In doing so, this paper proposes an off-the-path belief restriction that extends the belief refinement of Guerrieri et al. (2010), which is applicable for a general model of directed search with information acquisition. In a closely related study, Kim et al. (2021) also explore a model involving endogenous information acquisition and asymmetric information, but they do not consider workers' endogenous search decisions

The rest of the paper is organized as follows. In Section 2, I introduce the model environment and belief refinement. Then, I characterize the equilibrium and analyze its allocation and dynamics in Section 3. Section 4 concludes this paper. All the proofs for the propositions are in the appendix.

2 Model

2.1 Environment

Time is continuous. All agents in the economy are risk-neutral, and discount future at a rate of $r > 0$. For convenience, I multiply r to all flow variables such as output and wage. For instance, a job with a wage of w pays a flow wage rw, resulting in a worker's lifetime valuation of w.

At $t = 0$, there exists a measure one of unemployed workers who are searching for a job. Because the main focus of this paper is the analysis of learning and transition dynamics over the duration of unemployment, I only focus on this initial cohort. Therefore, t equivalently represents the unemployment duration. Each worker is either H(high) or L(low) type, which remains constant over time and is private information. A worker of type $j \in H, L$ generates an output flow of ry_j at each instance, where the condition $0 < y_L < y_H$ is satisfied. Note that the output level depends on the type, but not the timing of hiring, implying that there is no direct skill loss from unemployment. I discuss how the direct skill loss can be incorporated into the model in Section 3.5.2.

The worker type also influences the value of unemployment. The flow value of unemployment for a worker of type L is normalized to zero, while for a worker of type H, it is set to a positive value $rb > 0$. This assumption reflects the possibility that H-type workers may receive higher unemployment insurance benefits or access better social connections. The difference in the flow value of unemployment is assumed to be smaller than the productivity differential, $b < y_H - y_L$, resulting in greater heterogeneity when employed than when unemployed.

The fraction of H-type among the unemployed is denoted by $f(t)$, with an initial fraction $f(0) \in (0, 1)$ given exogenously. The other side of labor market consists of homogeneous firms with free entry. I assume that once a worker is hired, they stay in the job indefinitely, implying no exogenous separations or on-the-job search by workers.

2.2 Search and matching

The unemployed workers' search is directed by wages. Firms post a vacancy with a flow cost $rk > 0$, where each vacancy specifies a flow wage rw and target unemployment duration t.[2] Each worker observes the wage distribution given t and then applies to a vacancy. This formulation implies that the unemployment duration is publicly observable and contractable. Market tightness

[2]Alternatively, one can assume that a vacancy specifies a wage profile contingent on unemployment duration.

$\theta(w,t)$, representing the vacancy-worker ratio for each submarket (w,t), determines the meeting rate between workers and firms. I denote the meeting rate for a worker as $p(\theta(w,t))$, and use a functional form $p(\theta) = \theta/(1+\theta)$ for analytical convenience. All qualitative results still hold when standard assumptions on $p(\theta)$ are imposed. Similarly, the meeting rate for a firm is denoted as $q(\theta(w,t))$, where $q(\theta) = p(\theta)/\theta$. It is important to note that these rates, $p(\theta(w,t))$ and $q(\theta(w,t))$, indicate the frequency of worker-firm "meetings" in a submarket and may differ from the rate of actual match creation, as firms might choose not to hire workers based on their information acquisition results.

2.3 Information acquisition and firms' payoffs

After a firm and a worker meets in a submarket (w,t), the firm can conduct an interview to gather information about the worker's type before deciding to hire. To model this information acquisition process, I follow the rational inattention framework (Sims, 2003; Matějka and McKay, 2015).

To be specific, consider that a prior belief $\mu(w,t) \equiv P(\text{A job candidate is of H-type}|w,t)$ is given. At the interview stage, the firm selects a joint distribution $G(j,s) : \{L,H\} \times S \to [0,1]$, where s represents the signal observed during the interview, and S is an arbitrary set of signals. The chosen distribution is required to be Bayes-consistent with the prior $\mu(w,t)$.

$$\mu(w,t) = \int G(H,s)ds \tag{1}$$

The selection of a joint distribution for the interview can be understood as the choice of conditional signal distributions, denoted as $G_j(s) : S \to [0,1]$, and each conditional signal distribution can be seen as a test.[3]

To illustrate this, let's consider the example of a perfectly informative test, where a worker passes the test if and only if the worker belongs to the H-type. This is the case when the set of signals S is $\{0,1\}$, and the corresponding conditional signal distributions are given by $G_L(0) = 1$ and $G_H(1) = 1$. On the other hand, the firm also has an option to choose a completely uninformative test by selecting a singleton set $S = \{0\}$.

Regarding the joint distribution for the interview, the firm can choose any distribution, from perfect information to no information. However, there are costs involved in making the interview more informative. These costs are directly proportional to the informational value of the interview, which is measured by the expected reduction in entropy (Sims, 2003; Matějka and McKay, 2015).

$$C(G,\mu) = \lambda\left[H(\mu) - E_G(H(\mu_s))\right], \quad H(x) \equiv -x\log(x) - (1-x)\log(1-x) \tag{2}$$

[3] The relationship $G_j(s) = G(j,s)/\int G(j,s)ds$ links $G(j,s)$ and $G_j(s)$.

where $H(x)$ is the entropy of binomial distribution with probability x, $\lambda > 0$ is information cost, and μ_s is the posterior belief given a signal realization s. Therefore, the firm faces a trade-off between costs and desired information.

Following the interview stage, a signal is realized, and based on this signal, a posterior belief μ_s is determined. The firm's *ex post* payoff associated with μ_s is the following.

$$\hat{J}(w, \mu_s) = \max\{0, [\mu_s y_H + (1 - \mu_s) y_L - w]\} \tag{3}$$

The firm will hire the worker if and only if the posterior belief μ_s surpasses a threshold level $\mu_0(w) \equiv \frac{w - y_L}{y_H - y_L}$. Intuitively, as the wage increases, the threshold belief also increases.

The firm's payoff before the interview stage is then described by the following optimization problem,

$$J(w, \mu) = \max_{G \in \Delta(\mu)} E_G \left[\hat{J}(w, \mu_s) - C(G, \mu) \right] \tag{4}$$

where $\Delta(\mu)$ is the set of distributions that are Bayes-consistent with μ. Denote the firm's optimal interview signal choice that solves Equation (4) as $G^*(w, \mu)$. This optimal interview choice determines the probability of being hired for a worker conditional on meeting, and this probability depends on the worker's type. I will denote this hiring probability as $\pi_j(w, \mu)$ for $j = L, H$. The unconditional hiring probability, which is the probability of hiring a worker from a firm's perspective, is then $\mu \pi_H(w, \mu) + (1 - \mu) \pi_L(w, \mu)$.

Note that the firm value function does not directly depend on the unemployment duration t due to the absence of direct skill loss over the unemployment period. Instead, the unemployment duration indirectly affects the firm value and information acquisition decision through the equilibrium prior belief $\mu(w, t)$. This feature is realistic, as the pathway through which unemployment duration affects labor market outcomes must involve firms' assessments of job candidates based on duration information. The determination of this equilibrium belief $\mu(w, t)$ within the model is discussed in Section 2.5.

Given the belief $\mu(w, t)$, the market tightness for each submarket (w, t) is determined by the free entry condition.

$$q(\theta(w, t)) J(w, \mu(w, t)) \leq rk, \quad \theta(w, t) \geq 0 \tag{5}$$

with complementary slackness condition. The free entry condition clearly indicates that the equilibrium belief influences the number of vacancies created. Intuitively, more firms enter the market when firms expect a higher quality of job candidates, resulting in higher $\theta(w, t)$.

2.4 Workers' value function and law of motions

Denote the value of j-type unemployed workers whose unemployment duration is t as $U_j(t)$. With $\pi_j(w,t) \equiv \pi_j(w,\mu(w,t))$ and $p(w,t) \equiv p(w,\mu(w,t))$, the value function satisfies the following equation.

$$rU_j(t) = rbI_{j=H} + \max_{(w,t)\in\mathcal{M}} \{p(w,t)\pi_j(w,t)\,[w - U_j(t)]\} + \dot{U_j}(t) \tag{6}$$

Here, $I_{j=H}$ is the indicator function that takes one if $j = H$, and $\dot{U_j}(t)$ is the time-derivative of $U_j(t)$. Also, define the equilibrium return to search $R_j(t), j = L, H$ as the following.

$$R_j(t) \equiv \max_{(w,t)\in\mathcal{M}} \{p(w,t)\pi_j(w,t)[w - U_j(t)]\} \tag{7}$$

At each point in time, every worker decides which wage w to search for to maximize their search return $R_j(t)$. The resulting rate of finding a job is a combination of the meeting rate $p(w,t)$ and the hiring probability $\pi_j(w,t)$.[4] When calculating the hiring probability $\pi_j(w,t)$, the worker rationally anticipates the firm's optimal information acquisition policy $G^*(w,\mu(w,t))$ based on the equilibrium belief $\mu(w,t)$ and their wage choice w. This means workers' search decisions respond to firms' information acquisition.

The only aggregate variable in this economy is the fraction of H-type among the unemployed $f(t)$. The evolution of $f(t)$ depends on the average job-finding rate of each type, denoted by $Q_j(t)$.

$$\dot{f(t)} = -f(t)(1-f(t))(Q_H(t) - Q_L(t)) \tag{8}$$

Equation (8) implies that the fraction of H-type falls over duration $(\dot{f(t)} < 0)$ if and only if H-type workers find a job at a faster rate on average $(Q_H(t) > Q_L(t))$. The average job-finding rate $Q_j(t)$ is completely determined by (7) if the maximizer of the search problem is unique. When the maximizer is not unique, the optimal search policy along with the relative application distribution across the optimal wages jointly determine the average job-finding rate of each type.

2.5 Belief formation

This section explains how the equilibrium belief $\mu(w,t)$ is determined. Denote the set of submarkets that exist in the equilibrium path as $\mathcal{M}$. Then, for any $(w,t) \in \mathcal{M}$, $\mu(w,t)$ must be consistent with the actual ratio of H-type among the unemployed workers.

[4] $p(w,t)$ is the Poisson rate at which a meeting occurs, and $\pi(w,t)$ is the probability that a Poisson event turns into a match. As a result, $p(w,t)\pi(w,t)$ is the Poisson rate of match creation.

For the off-the-path submarket $(w, t) \notin \mathcal{M}$, an appropriate belief refinement is necessary; otherwise, arbitrary equilibria can be supported. For example, without any off-the-path belief restriction, a continuum of equilibria exists where all workers apply to a single wage level w_0 because firms do not post any vacancy except at w_0, assuming that only L-type workers will apply to any $w \neq w_0$.

In a similar environment, directed search with adverse selection, Guerrieri et al. (2010) introduces an off-the-path belief refinement, which has been adopted by other research (Guerrieri and Shimer, 2014; Chang, 2017; Li, 2022). The rationale behind this refinement is assigning weight to the type willing to apply to a deviation at a lower market tightness, aligning with the divinity argument (Banks and Sobel, 1987; Gale, 1996). However, this refinement is not directly applicable to this model because the type more willing to deviate depends on firms' beliefs about which type will deviate through firms' screening decisions.

To illustrate the challenge, consider a deviation wage w_d larger than y_L and $2(R_H(t)+U_H(t))$.[5] Applying the belief restriction of Guerrieri et al. (2010) requires a thought experiment, gradually increasing market tightness from zero until one type of worker becomes indifferent between applying to w_d and the equilibrium payoff. However, it is straightforward that L-type workers become indifferent at a lower market tightness under uninformative screening, while H-type workers become indifferent first under precise enough screening. Therefore, it is impossible to determine which type deviates first before fixing the screening intensity. This presents a challenge because the optimal screening strategy is also contingent upon a prior belief.

While the refinement of Guerrieri et al. (2010) is not directly applicable, the interdependence between belief and screening suggests a fixed-point as a natural extension applicable to this model. Therefore, I require that the off-the-path belief must satisfy the divinity argument, given that screening is optimally conducted based on that off-the-path belief. Specifically, given the search return $R_j(t)$, I require that $\mu(w, t) > 0$ implies the existence of $\theta > 0$ satisfying either (9) or (10) for all $w > U_L(t)$.[6]

$$\begin{aligned} &p(\theta)\pi_H(w, \mu(w,t))[w - U_H(t)] \geq R_H(t), \quad \text{and} \\ &p(\theta)\pi_L(w, \mu(w,t))[w - U_L(t)] \leq R_L(t) \end{aligned} \tag{9}$$

or

$$\begin{aligned} &p(\theta)[w - U_H(t)] \geq R_H(t), \quad \text{and} \\ &p(\theta) \cdot \sup_{\mu' : \pi_H(w,\mu')>0} \left\{ \frac{\pi_L(w, \mu')}{\pi_H(w, \mu')} \right\} [w - U_L(t)] \leq R_L(t) \end{aligned} \tag{10}$$

[5] $w_d > y_L$ implies that L-type workers never find it optimal to apply to w_d if screening is precise enough. $w_d > 2(R_H(t) + U_H(t))$ implies that both types have an incentive to apply to w_d if market tightness is high enough without screening.

[6] It implicitly assumes $U_H(t) > U_L(t)$, which holds in any equilibrium.

Similarly, $\mu(w,t) < 1$ must imply the existence of $\theta > 0$ satisfying either (11) and (12) for all $w > U_L(t)$.

$$\begin{aligned} &p(\theta)\pi_L(w,\mu(w,t))[w - U_L(t)] \geq R_L(t), \quad \text{and} \\ &p(\theta)\pi_H(w,\mu(w,t))[w - U_H(t)] \leq R_H(t) \end{aligned} \tag{11}$$

or

$$\begin{aligned} &p(\theta)[w - U_L(t)] \geq R_L(t), \quad \text{and} \\ &p(\theta) \cdot \sup_{\mu':\pi_L(w,\mu')>0} \left\{ \frac{\pi_H(w,\mu')}{\pi_L(w,\mu')} \right\} [w - U_H(t)] \leq R_H(t) \end{aligned} \tag{12}$$

I impose $\mu(w,t) = 0$ for all $w \leq U_L(t)$, which corresponds to the region where no applicant will apply. Note that the above conditions are automatically satisfied if (w,t) is chosen by a strictly positive measure of workers in equilibrium.

To intuitively understand the refinement, consider Equation (9) and (10) as examples. The rational belief assigns a strictly positive probability to the H-type in two distinct scenarios. The first scenario occurs when a specific level of market tightness makes searching for (w,t) weakly optimal for the H-type, given the belief $\mu(w,t)$, while it is not the case for the L-type. Essentially, in this scenario, the H-type workers are the ones who appear at a lower market tightness, based on the belief $\mu(w,t)$. The second scenario happens when a job-finding rate makes the H-type weakly optimal to search for (w,t), but that job-finding rate is never optimal for L-type, regardless of the prior belief level. In other words, H-type workers can tolerate a lower job-finding rate, even if the hiring probability ratio between the two types is most favorable for the L-type.[7] In both cases, H-type workers are more likely to apply, thereby the firm puts a positive weight on the H-type.

Note that the equilibrium refinement explicitly depends on the optimal screening strategy through $\pi_j(w,\mu)$. Thus, the properties of the equilibrium belief, such as existence, continuity, and monotonicity, cannot be discussed at this stage. I will show that the equilibrium belief is well-defined and monotone in w in Section 3.3, after establishing the firm's information acquisition solution in Section 3.2

2.6 Equilibrium

Definition 1. An equilibrium consists of the firm value function $J(w,\mu)$ and $\hat{J}(w,\mu_s)$, information strategy $G^*(w,\mu)$, hiring probability function $\pi_j(w,t)$, unemployed workers' value function $U_j(t)$, return to search $R_j(t)$, equilibrium belief $\mu(w,t)$, market tightness $\theta(w,t)$, active market

[7] The second scenario rules out the following possibility: L-type workers are more willing to apply when $\mu = 1$, but firms do not hire anyone when $\mu = 0$.

$\mathcal{M}$, average job-finding rate $Q_j(t)$, and the fraction of H-type unemployed $f(t)$ such that

1. (Optimal information acquisition) For all (w, μ), $G^*(w, \mu)$ solves Equation (4), and $J(w, \mu)$ is the firm value associated with $G^*(w, \mu)$.

2. (Optimal search) For all $j = L, H$ and t, $U_j(t)$ is the workers' value functions satisfying (6) given the market tightness $\theta(w, t)$, equilibrium belief $\mu(w, t)$, and the associated hiring probability function $\pi_j(w, t)$ consistent with $G^*(w, \mu)$.

3. (Equilibrium belief) For all $(w, t) \in \mathcal{M}$, $\mu(w, t)$ is the actual fraction of H-type among the candidates. In addition, for all (w, t), $\mu(w, t)$ satisfies the belief refinement (9) - (12) given the return to search $R_j(t)$ and hiring probability $\pi_j(w, t)$.

4. (Free entry) The market tightness $\theta(w, t)$ satisfies the free entry condition (5) given the firm value $J(w, \mu)$ and equilibrium belief $\mu(w, t)$.

5. (Consistency of labor supply and law of motion) $f(t)$ evolves as (8) given $Q_j(t)$, and there exists a positive function $l(w, t)$ on active markets $\mathcal{M}$ such that $\int_{\mathcal{M}} l(w, t)\mu(w, t)dw = f(t)$, $\int_{\mathcal{M}} l(w, t)(1 - \mu(w, t))dw = 1 - f(t)$, $\int_{\mathcal{M}} l(w, t)\mu(w, t)\pi_H(w, \mu(w, t))p(\theta(w, t))dw = Q_H(t)$, and $\int_{\mathcal{M}} l(w, t)(1 - \mu(w, t))\pi_L(w, \mu(w, t))p(\theta(w, t))dw = Q_L(t)$

The equilibrium definition is similar to previous literature on the competitive search with asymmetric information (Guerrieri et al., 2010; Guerrieri and Shimer, 2014; Chang, 2017). The first condition states that the firms' information acquisition must be optimal for all (w, μ). The second condition describes the workers' optimal search decisions. The third condition involves the off-the-path belief refinement, as explained earlier, and the fourth condition is the free entry condition. The fifth condition is the market clearing condition and the aggregate law of motion, which requires the existence of a measure of unemployed workers that ensures the belief $\mu(w, t)$ across the active market $\mathcal{M}$ is consistent with the total fraction of H-type workers $f(t)$, and the implied evolution of $f(t)$ over t.

3 Analysis

3.1 Benchmark: Social planner's problem

This section examines the social planner's problem for the benchmark comparison. In this analysis, I assume that the social planner can observe workers' types and chooses the number of firms allocated to each type of workers freely, while subjecting to search frictions. Because the type is observable, the social planner does not incur any cost for gathering information. I denote the number of j-type workers in labor market states $l \in e, u$ at time t as $n_j^l(t)$, where $\sum_{j,l} n_j^l(t) = 1$ holds for all $t \geq 0$. The social welfare, denoted as W, becomes a function of $n \equiv (n_H^e, n_H^u, n_L^e, n_L^u)$ and t.

$$
\begin{aligned}
rW(n,t) \quad = \quad & \max_{\theta_H, \theta_L} r[y_H n_H^e + y_L n_L^e + b n_H^u - k(\theta_H n_H^u + \theta_L n_L^u)] \\
& + W_1 \dot{n_H^e} + W_2 \dot{n_H^u} + W_3 \dot{n_L^e} + W_4 \dot{n_L^u} + W_t
\end{aligned}
\tag{13}
$$

where W_j is the partial derivative of W with respect to j-th component of n for $j \in \{1, 2, 3, 4\}$, W_t is the partial derivative of W with respect to t, and the law of motion for n_j^l is given by

$$
\dot{n_j^e} = p(\theta_j) n_j^u = -\dot{n_j^u}, \quad j \in \{L, H\} \tag{14}
$$

By guessing $W(n,t) = y_H n_H^e + U_H^* n_H^u + y_L n_L^e + U_L^* n_L^u$, one can characterize the social planner's problem as the following.

$$
p'(\theta_H^*) = \frac{rk}{y_H - U_H^*}, \quad p'(\theta_L^*) = \frac{rk}{y_L - U_L^*} \tag{15}
$$

$$
U_H^* = \frac{rb + p(\theta_H^*) y_H}{r + p(\theta_H^*)}, \quad U_L^* = \frac{p(\theta_L^*) y_L}{r + p(\theta_L^*)} \tag{16}
$$

The above two equations imply that $\theta_H > \theta_L$ in the social planner's choice because $y_H - b > y_L$. Because the net production of H-type exceeds that of L-type, the social planner assigns more firms to H-type workers to elevate their matching rate. In this efficient allocation, unemployed H-type workers transition to employment at a higher rate, resulting in a decrease in the average quality over the duration of unemployment. While the concept of wages is absent in the social planner's allocation, one can check that this efficient allocation can be decentralized in a directed search market if the type is publicly observable. In that equilibrium allocation, H-type workers earn a higher wage, implying that the social planner's allocation exhibits a decline in wages over duration.

3.2 Optimal information acquisition

This subsection first examines the firms' optimal information acquisition decision for each (w, μ). After that, I will discuss how this information strategy interacts with other components of equilibrium, and characterize the equilibria of the model based on the results in this section.

The optimal information choice (4) can be expressed as the following form.

$$J(w,\mu) \quad = \quad \max_{G\in\Delta(\mu)} E_G\left[\hat{J}(w,\mu_s)+\lambda H(\mu_s)\right]-\lambda H(\mu) \tag{17}$$

Equation (17) shows that the optimal information strategy can be found by the concavification of the function $\hat{J}(w,\mu_s)+\lambda H(\mu_s)$ (Kamenica and Gentzkow, 2011; Kim et al., 2021). To intuitively

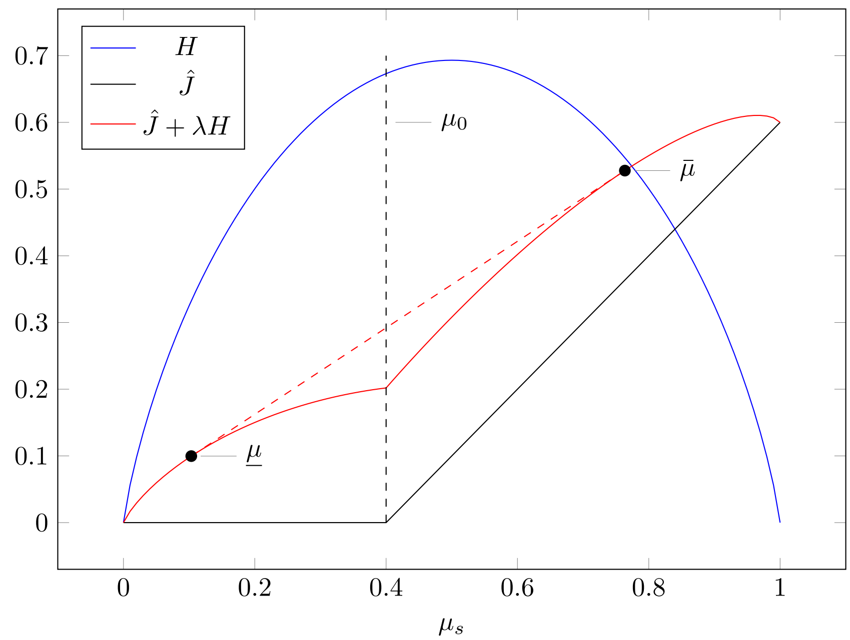


Figure 1: Entropy and firm value

understand the concavification, Figure 1 illustrates the entropy function $H(\mu_s)$, the ex-post firm value $\hat{J}(w,\mu_s)$, and the objective of function $\hat{J}(w,\mu_s)+\lambda H(\mu_s)$ for a given w. Note that the entropy function $H(\mu_s)$ is strictly concave (blue), and the ex-post firm value $\hat{J}(w,\mu_s)$ is piecewise linear (black). Therefore, non-concavity of the objective function (red) must arise around the point $\mu_0(w)=\frac{w-y_L}{y_H-y_L}$, which is the belief level that makes the firm indifferent. When $\mu_0(w)$ is between 0 and 1, the objective function has a non-concave region, and the optimal information strategy requires to split the posterior to two points $\underline{\mu}$ and $\overline{\mu}$ when μ is in between these two points. The below proposition formally establishes the optimal information strategy.

Proposition 1. *Given (w,μ), the following is an optimal information acquisition and hiring strategies.*

- *If $w\le y_L$, then the firm hires the worker without gathering any information.*

- *If $y_L < w \le y_H$, define $\underline{\mu}(w)$ and $\overline{\mu}(w)$.*

$$\underline{\mu}(w) = \frac{\exp\left(\frac{w-y_L}{\lambda}\right) - 1}{\exp\left(\frac{y_H-y_L}{\lambda}\right) - 1}, \quad \overline{\mu}(w) = \frac{\exp\left(\frac{y_H-y_L}{\lambda}\right) - \exp\left(\frac{y_H-w}{\lambda}\right)}{\exp\left(\frac{y_H-y_L}{\lambda}\right) - 1} \tag{18}$$

  - *If $\mu \le \underline{\mu}(w)$, then the firm does not hire the worker without gathering any information.*
  - *If $\mu \ge \overline{\mu}(w)$, then the firm hires the worker without gathering any information.*
  - *If $\mu \in (\underline{\mu}(w), \overline{\mu}(w))$, the firm conducts a screening that splits the posterior to either $\underline{\mu}(w)$ or $\overline{\mu}(w)$. The firm hires the worker if and only if the posterior becomes $\overline{\mu}(w)$.*

Under the optimal information strategy, the firm value function $J(w, \mu)$ is strictly increasing in μ if $\mu \ge \underline{\mu}(w)$.

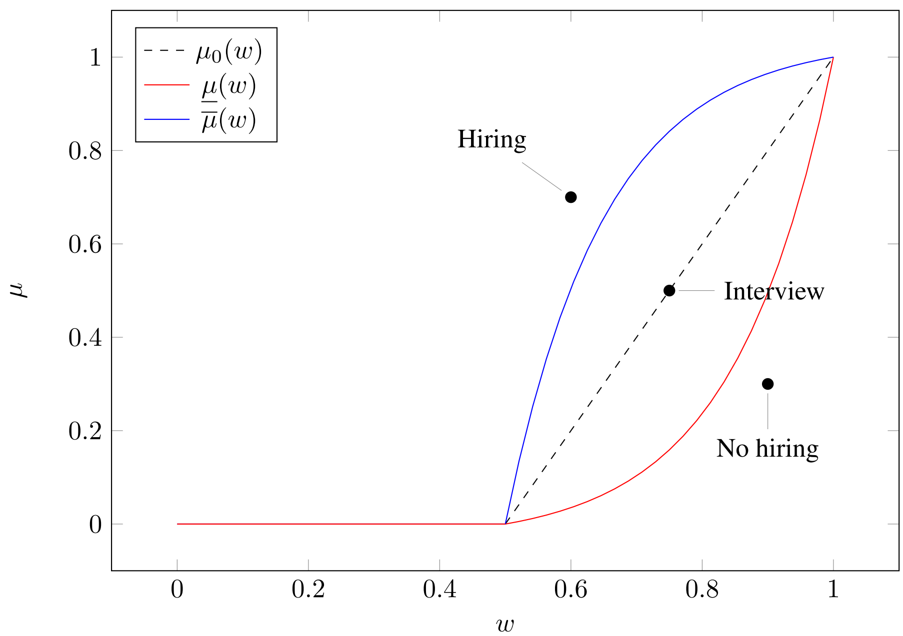


Figure 2: Firms' optimal information acquisition

Figure 2 graphically illustrates the optimal information strategy. The firm hires the worker without interview if the prior μ is sufficiently large given w, which is the region above the $\overline{\mu}(w)$ line (blue). The firm never hires the worker if the prior μ is lower than $\underline{\mu}(w)$ (red). The firm conducts an informative interview and screen workers if μ is between $\underline{\mu}(w)$ and $\overline{\mu}(w)$, and the interval always contains the indifference belief $\mu_0(w)$.

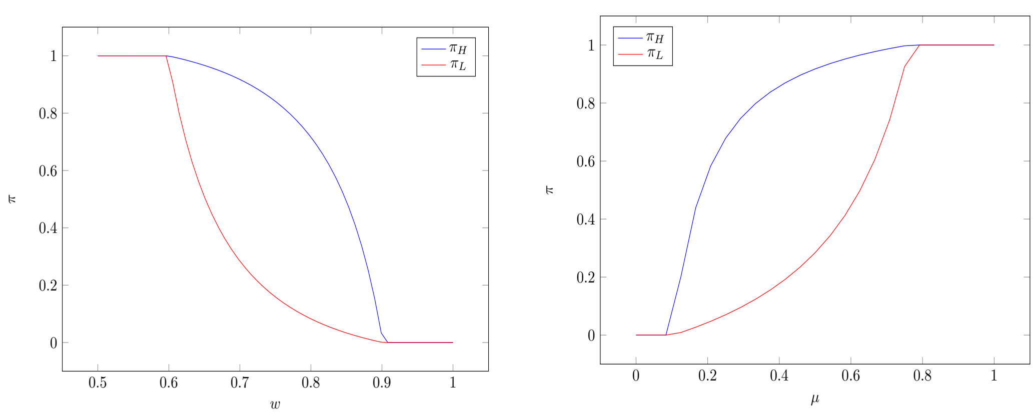
The optimal information acquisition in Proposition 1 is identical to the result of Kim et al. (2021) which assumes an exogenous wage. A distinguished point is that, in this model, wages are endogenously determined by workers' search. Therefore, it is necessary to characterize how the optimal information strategy varies across wages, which affects the trade-off that workers face when they search for a job. Proposition 2 describes how the probability of being hired for each type varies across w and μ.

Proposition 2. *When the firm conducts an informative interview, the followings hold.*

- $\pi_H(w, \mu)$ *and* $\pi_L(w, \mu)$ *are decreasing in* w*, but* $\pi_H(w, \mu)/\pi_L(w, \mu)$ *is increasing in* w*.*
- $\pi_H(w, \mu)$ *and* $\pi_L(w, \mu)$ *are increasing in* μ*, but* $\pi_H(w, \mu)/\pi_L(w, \mu)$ *is decreasing in* μ*.*
- $\pi_H(w, \mu)/\pi_L(\mu, w) \to 1$ *as* $\mu \to \overline{\mu}(w)$*, and* $\pi_H(w, \mu)/\pi_L(\mu, w) \to \exp\left(\frac{y_H - y_L}{\lambda}\right)$ *as* $\mu \to \underline{\mu}(w)$*.*

Intuitively, in order for the firm to justify hiring a worker at a higher wage, it requires a better posterior belief regarding the worker's ability. As a result, high-wage jobs are associated with more intensive screenings, which decrease the job-finding probability for both types of workers, given a prior belief. Similarly, when the prior belief is lower, firms demand more credible evidence to hire a worker. Consequently, the worker's job-finding probability is lower when the prior belief is lower, given the wage.

The intensive screening has a disproportionate impact on workers of type L, which is evident from the fact that perfectly informative screening is the most intensive form of screening. Consequently, the ratio of the hiring probabilities π_H/π_L is increasing in w and decreasing in μ, inversely related to the job-finding probability itself. In other words, jobs with intensive interviews are more challenging to obtain for both types of workers, but the difficulty is more pronounced for workers of type L. Figure 3 graphically confirms this intuition.



(a) $\pi_j(w, 0.5)$ (b) $pi_j(0.7, \mu)$

Figure 3: Conditional job-finding probability π_j

3.3 Characterization of the equilibria

This section characterizes the equilibria of the model based on the results in the previous section. In any equilibrium, $U_H(t) > U_L(t)$ must hold for all t, because H-type workers can simply mimic the search decisions of L-type. It is then immediate that the social planner's choice cannot be an equilibrium outcome.

Proposition 3. *An equilibrium allocation does not coincide with the social planner's allocation.*

The rationale behind this inefficiency is clear. To maintain a separating equilibrium, a type receiving a higher wage must encounter a lower job-finding probability; otherwise, the other type finds it profitable to deviate. Because H-type workers search for a higher wage due to the higher outside option, in equilibrium, they must face a lower job-finding probability, which is against the social planner's desired allocation. Firms' screening does not help in type separation in this scenario, as firms endogenously opt not to screen workers in a separating equilibrium. The critical factor here is endogenous screening. If firms can commit to their screening policy, full separation with $p(\theta_H) > p(\theta_L)$ is achievable through screening at w_H as in Feng et al. (2019). However, conducting screenings in such a case is not *ex post* incentive compatible, given that firms are already aware of workers' types.

Note that if $y_H - b < y_L$, a fully separating equilibrium could be efficient, as the social planner chooses for a lower market tightness for H-type workers. While this assumption is counter-intuitive in the labor market context in that high-ability workers are more productive in the work-place than at home, it may appeal to other market contexts such as assets and housing markets.

For the remaining equilibrium characterization, I focus on equilibria where the value of unemployment, optimal wages, beliefs, and market tightness remain constant over t given type. This does not imply that the average wage and job-finding rate remain constant, as the proportion of H-type workers and the endogenous division across optimal wages within each type vary over time. I show that such time-invariant equilibria exist unless the initial proportion of H-type workers is high, providing a convenient tool for analyzing duration dependence. I will discuss time-dependent equilibria in Section 3.3.3.

To begin the analysis, it is convenient to express the equilibrium belief as a function of the "required" job-finding rate. Specifically, let U_j and R_j be given, and define $\phi_j(w) \equiv R_j/(w - U_j)$, which is the job-finding rate that makes j-type workers indifferent between applying to wage w and receiving the equilibrium payoff. Note that this object is well-defined for all w, independent of $\mu(w)$. These required job-finding rates and equilibrium belief are related in the following way.

Lemma 3.1. *An equilibrium belief $\mu(w)$ satisfies the following for all $w \leq y_L$,*

$$\mu(w) = 0 \quad \textit{if} \quad \frac{\phi_H(w)}{\phi_L(w)} > 1, \qquad \mu(w) = 1 \quad \textit{if} \quad \frac{\phi_H(w)}{\phi_L(w)} \leq 1 \tag{19}$$

and $\mu(w)$ satisfies the following for all $w > y_L$.

$$\mu(w) = 0 \quad \textit{if} \quad \frac{\phi_H(w)}{\phi_L(w)} \geq \exp\left(\frac{y_H - y_L}{\lambda}\right) \tag{20}$$

$$\mu(w) = 1 \quad \textit{if} \quad \frac{\phi_H(w)}{\phi_L(w)} \leq 1 \tag{21}$$

$$\mu(w) = \mu \in (0,1) \quad \textit{s.t} \quad \frac{\pi_H(w,\mu)}{\pi_L(w,\mu)} = \frac{\phi_H(w)}{\phi_L(w)} \quad \textit{if} \quad \frac{\phi_H(w)}{\phi_L(w)} \in \left(1, \exp\left(\frac{y_H - y_L}{\lambda}\right)\right) \tag{22}$$

Therefore, $\mu(w)$ is weakly increasing in w.

When $w \leq y_L$, firms do not gather information. Therefore, the off-the-path belief is solely determined by which type requires a higher job-finding rate. If the required job-finding rate ratio between two types $\phi_H(w)/\phi_L(w)$ exceeds 1, L-type workers are more willing to apply for w, and vice versa. When the ratio is exactly 1, $\mu(w)$ must be 1 to support an equilibrium, although the belief refinement itself does not pin down the belief.

When $w > y_L$, firms have an incentive to conduct informative screening, which affects the equilibrium belief determination. To understand how it affects the belief, suppose the required job-finding rate ratio between two types $\phi_H(w)/\phi_L(w)$ exceeds $\exp\left(\frac{y_H-y_L}{\lambda}\right)$, which is the upper bound of π_H/π_L among the optimal screenings. In this scenario, compared to L-type workers, H-type workers demand impossibly higher job-finding rates, which cannot be fulfilled even by the most informative screening. Consequently, H-type workers never find it optimal to search for wage w if L-type workers are indifferent. This implies that the equilibrium belief at wage w should be 0. Conversely, if the ratio falls below one, L-type workers do not find it worthwhile to search for wage w when H-type workers are indifferent, even under uninformative screening. In this scenario, the equilibrium belief must assign full weight to H-type.

The equilibrium belief can take a value between 0 and 1 only if the hiring probability ratio $\pi_H(w,\mu)/\pi_L(w,\mu)$ coincides with the required job-finding rate ratio $\phi_H(w)/\phi_L(w)$, otherwise, only one type finds it optimal to search. This indifference condition uniquely determines the equilibrium belief $\mu(w)$ because $\pi_H(w,\mu)/\pi_L(w,\mu)$ is strictly decreasing in μ on the interval $[\underline{\mu}(w), \overline{\mu}(w)]$, where it takes the value of $\exp\left(\frac{y_H-y_L}{\lambda}\right)$ at $\underline{\mu}(w)$ and the value of 1 at $\overline{\mu}(w)$.

The required rate ratio $\phi_H(w)/\phi_L(w)$ is decreasing in w because H-type workers prefer higher wages relatively more than L-type workers do. It means that the equilibrium belief must be weakly increasing in w. Also, it is continuous in w on the interval where $\mu(w)$ is interior. Additionally, if one type finds it optimal to search for w given θ with $\mu(w) \in (0,1)$, the other type also finds it optimal to do so, as the required job-finding rate ratio between them coincides with the actual job-finding ratio as $p(\theta)$ is common for both types.

3.3.1 Case 1: when $y_H \approx y_L$

With the equilibrium belief characterization Lemma 3.1, I will characterize the equilibrium. I first analyze the case when the productivity differential $y_H - y_L$ is below a certain threshold level. In this case, there is a time-invariant equilibrium with full separation.

Proposition 4. *If $y_H - y_L \leq M$, there is no time-invariant equilibrium with pooling. Instead, a time-invariant full-separation equilibrium with following features exists:*[8]

- *All L-type workers apply to w_L that solves (23), and are hired without screening.*

$$w_L = \arg\max_w p(w)(w - U_L) \quad \textit{subject to} \quad q(w)(y_L - w) = rk \tag{23}$$

- *All H-type workers apply to w_H that solves (24), and are hired without screening.*

$$w_H = \arg\max_w p(w)(w - U_H) \quad \textit{subject to} \quad q(w)(y_H - w) = rk \tag{24}$$

$$p(w)(w - U_L) \leq R_L \quad \& \quad w > w_L \tag{25}$$

- *$\mu(w)$ is a step-function that equals 0 for all $w < w_H$, and 1 for all $w \geq w_H$.*
- *The threshold M depends on parameters related to L-type, but not on parameters of H-type.*

$$M = \frac{rk(y_L - U_L)}{y_L - (1+r)U_L} \tag{26}$$

The intuition behind full separation shares similarities with previous research studying competitive search with adverse selection, such as Guerrieri et al. (2010), Guerrieri and Shimer (2014), and Chang (2017). Given that the value of unemployment is higher for H-type workers, they have a stronger preference for high-wage jobs. Consequently, a fully separating equilibrium may emerge where H-type workers select sufficiently high wages to discourage mimicking by L-type workers. In the equilibrium, only H-type workers' choices are distorted, and the incentive compatibility condition for L-type workers is binding, implying that L-type workers are indifferent between w_H and w_L. Such indifference conditions imply $p_H < p_L$, where p_j denotes the job-finding rate for each type. This is in contrast to the social planner's allocation, where $p(\theta_H^*) > p(\theta_L^*)$. This discrepancy illustrates the distortion caused by the asymmetric information problem. The existence of fully separating equilibrium is independent of the underlying distribution of types, thereby it exists for any $f(0) \in (0, 1)$.

[8] I omit t in all equilibrium objects as they do not vary over t.

Figure 4 graphically illustrates the equilibrium allocation. The two downward-sloping curves represent the firm's iso-profit curve associated with two types, while the blue and red curves depict each worker type's indifference curve. The iso-profit curve and indifference curve are tangent to each other for the L-type, but not for the H-type, highlighting the inefficiency resulting from asymmetric information.

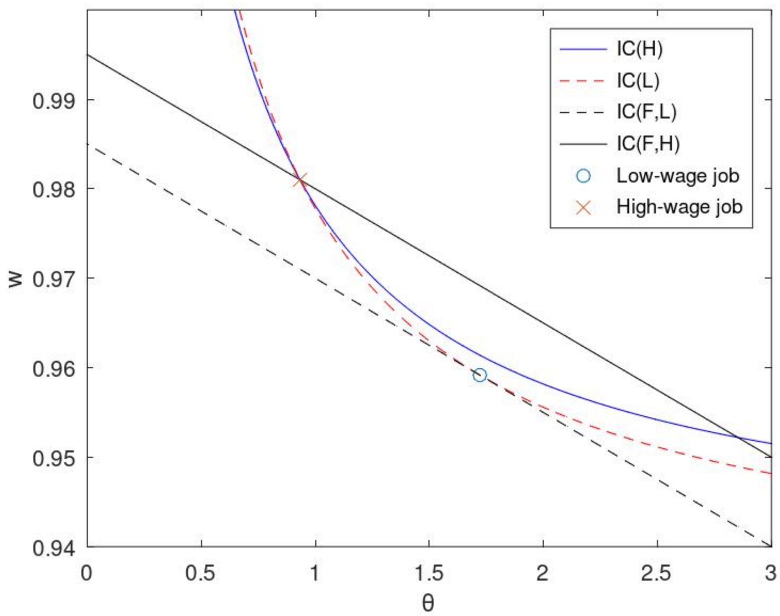


Figure 4: Full separation equilibrium. (x-axis: θ, y-axis: w)

Firms lack any incentive to gather information in a fully separating equilibrium, raising the question of why a small productivity differential is necessary for the existence of this equilibrium type. This requirement stems from firms' screening incentives in off-the-path wages, not in active markets. A crucial property supporting full separation is that even wages only slightly below w_H should lead firms to believe they will attract only L-type workers. This belief discontinuity is justified only when the separating wage w_H is lower than y_L. If $w_H > y_L$, even slightly informative screening is sufficient to equalize the hiring probability ratio and required job-finding rate ratio at $w_H - \epsilon$, as $\phi_H(w_H)/\phi_L(w_H) = 1$, implying that $\mu(w_H - \epsilon)$ should be close to 1, which is far from 0. Therefore, $w_H < y_L$ is necessary for the fully separating equilibrium, which holds only when y_H is sufficiently close to y_L. The threshold productivity differential M depends on model parameters but is independent of any H-type specific characteristics, such as the fraction of H-type $f(t)$ or the flow value of unemployed b. This independence arises because the value of unemployed for the L-type, U_L, is completely determined by L-type specific parameters only.

How are the labor market outcomes related to the unemployment duration? To answer the question, denote the average wage of the unemployed who are hired after duration t by $W(t)$, and the average job-finding rate of the unemployed after t as $Q(t)$. In a time-invariant separating equilibrium, these variables remain constant within type, but their population average changes over time as the proportion of each type varies. Proposition 5 establishes the relationship between labor market outcomes and the duration of unemployment.

Proposition 5. *(Duration dependence) In a separating equilibrium, $f(t)$ and $W(t)$ are increasing in t, but $Q(t)$ is decreasing in t.*

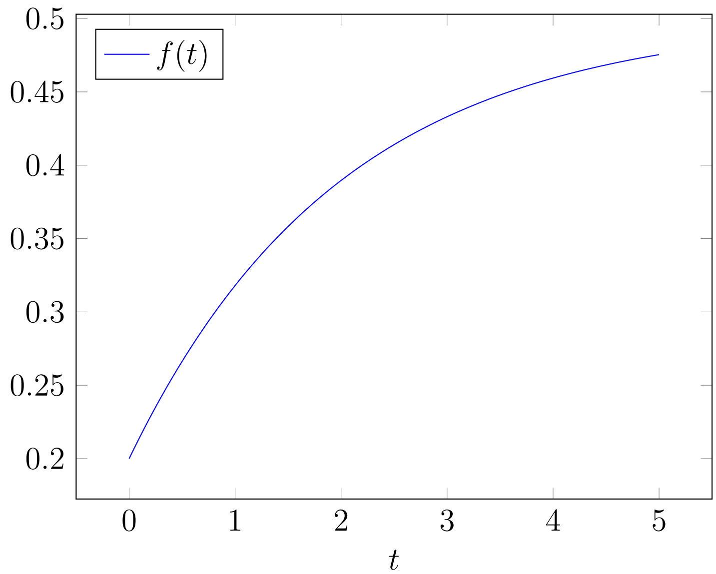


Figure 5: The average quality among the unemployed over t in a fully separating equilibrium

In this equilibrium, H-type workers experience a lower job-finding rate than L-type workers, resulting in an increase in average quality (measured by the fraction of H-type), and average wage, while decreasing the job-finding rate over the unemployment duration. Consequently, positive duration dependence emerges in terms of re-employment wages.

3.3.2 Case 2: when $y_H >> y_L$

This subsection analyzes the case when $y_H - y_L > M$. When the productivity differential is large enough, an equilibrium cannot fully separate types.

Proposition 6. *If $y_H - y_L > M$, there is no time-invariant full-separation equilibrium. Instead, a time-invariant upward-pooling equilibrium with following features exists, provided that $f(0)$ is not too high:*

- *A fraction $z(t)$ of L-type workers applies to w_L that solves (27), and are hired without screening.*

$$w_L = \arg\max_w p(w)(w - U_L) \quad \textit{subject to} \quad q(w)(y_L - w) = rk \tag{27}$$

- *The other $1-z(t)$ fraction of L-type workers and all H-type workers apply to w_M that solves (28) for both j, and are hired through screening implied by $G(w_M, \mu(w_M))$.*

$$w_M = \arg\max_w \pi_j(w, \mu(w))p(w)(w - U_j) \quad \textit{subject to} \quad q(w)J(w, \mu(w)) = rk, w \geq w_0 \tag{28}$$

where

$$w_0 = \inf \left\{ w \in [y_L, y_H] : \exp\left(\frac{y_H - y_L}{\lambda}\right) \frac{w - U_H}{w - U_L} \geq \frac{U_H - b}{U_L} \right\} \tag{29}$$

- *$\mu(w)$ is a weakly increasing function such that*

$$\mu(w) = \begin{cases} 0 \quad \textit{if} \quad w \leq w_0 \\ 1/\left\{1 + \frac{1-\overline{\mu}(w)}{\overline{\mu}(w)} \frac{(w-U_L)(U_H-b)}{(w-U_H)U_L}\right\} \quad \textit{if} \quad w > w_0 \end{cases} \tag{30}$$

- *The fraction $z(t)$ satisfies $\mu(w_M) = \frac{f(t)}{f(t)+(1-z(t))(1-f(t))}$.*

When the productivity differential is large enough, no fully separating equilibrium can exist. As explained in the previous section, firms' belief that putting $\mu = 0$ at $w_H - \epsilon$ is unreasonable because even very little informative screening induces H-type workers to prefer $w_H - \epsilon$ more than L-type workers do, which breaks the fully separating equilibrium. This pooling result distinguishes this model from previous literature studying a dynamic adverse selection model (Guerrieri and Shimer, 2014; Chang, 2017).

While complete separation is unattainable, it does not imply that pooling can emerge in equilibrium because it necessitates the existence of a wage level for which both types find it optimal to search. The existence of such a wage is guaranteed because the actual hiring probability ratio always coincides with the required job-finding rate ratio when $\mu(w)$ is interior.

Before delving into the equilibrium allocation, it is helpful to consider the shape of the equilibrium belief. Figure 6 illustrates the equilibrium belief $\mu(w)$, where $y_L = 1$ and $y_H = 1.2$ are assumed. The equilibrium belief remains zero for all $w \leq w_0$, and then strictly increases as w increases.[9] This represents the firm's initial assessment of a job candidate's quality, contingent upon the candidate having applied to a specific wage w in equilibrium.

[9]The equilibrium belief is not necessarily continuous at $w = w_0$, although it is so in this example.

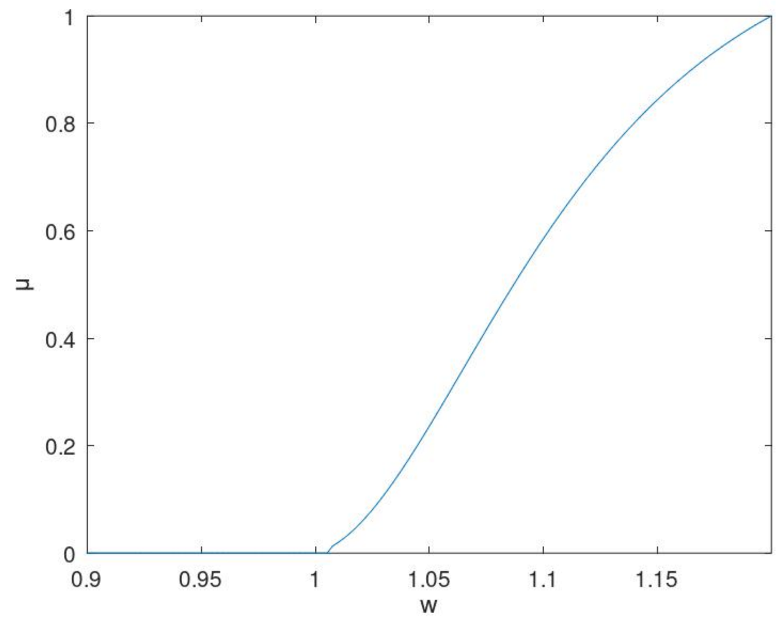


Figure 6: Equilibrium belief

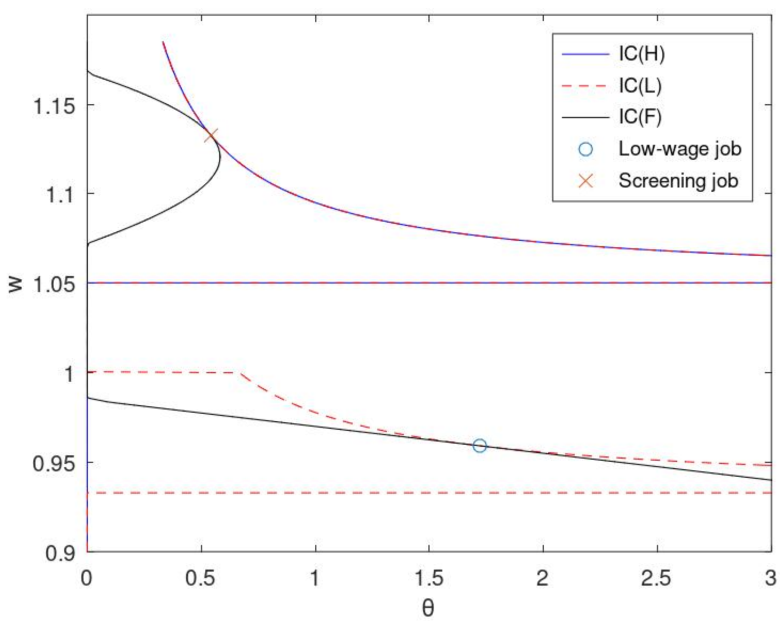


Figure 7: Upward pooling equilibrium

Figure 7 graphically illustrates the upward pooling equilibrium. The black line represents the pair of (θ, w) that provides the same profit for firms based on the equilibrium belief $\mu(w)$, which is expressed as $q(\theta)J(w, \mu(w)) = rk$. The blue and red-dashed lines are indifference curves for each worker type, taking into accounts the job-finding probability $\pi_j(w, \mu(w))$.

The firm's free entry condition consists of two distinct regions: the low-wage region, approximately $w < 1$, and the high-wage region, approximately $w > 1.05$. The free entry condition for the low-wage region follows a standard pattern, showing a negative relationship between market tightness and wage because the belief $\mu(w)$ is constant at 0. The L-type workers' indifference curve has a unique point of tangency in this low-wage region, indicating the non-screening wage w_L.

In the high-wage region, the free entry condition exhibits non-standard shape, establishes a connection between higher market tightness and a higher wage when w is not too high. This pattern arises because the equilibrium belief $\mu(w)$ increases in this region, which outweighs the cost of higher wages. Of course, an equilibrium wage cannot reside on this upward-sloping section of the free entry curve, as both workers and firms benefit from higher wages. Consequently, a point of tangency exists between the indifference curve and the descending segment of the free entry condition, representing the screening wage w_M.

It is noteworthy that, within the high-wage region, the two indifference curves, one for each type, exactly coincide. This coincidence is a consequence of the equilibrium belief adjustment, which ensures that both worker types necessitate the same market tightness. Additionally, it is worth highlighting that the indifference curve for H-type workers only presents in the high-wage region, whereas that for L-type workers is present in both high-wage and low-wage regions. This observation implies that pooling in the equilibrium is upward; there exists a high-wage job that accommodates both types, while a low-wage job is exclusively reserved for L-type workers. Endogenous information acquisition is crucial in achieving this outcome. If there were exogenously distinct types of jobs with screening and non-screening mechanisms, the possibility of downward pooling in the equilibrium could also emerge, as in Feng et al. (2019).

In this upward pooling equilibrium, the fraction of H-type in the screening wage is $\mu(w_M)$. This fraction $\mu(w_M)$ depends on parameters that govern productivity and cost, such as y_j and k, but it is independent of the fraction of H-type in the population $f(t)$. This suggests that the time-invariant upward-pooling equilibrium in Proposition 6 can exist only when $f(t) \leq \mu(w_M)$ holds for all $t \geq 0$. While $f(t)$ is endogenous, this condition can be guaranteed if the initial fraction satisfies the condition $f(0) \leq \mu(w_M)$.

To understand why the initial condition is enough, denote the job-finding rate for j-type in the screening wage w_M by p_j^M, while the job-finding rate for L-type in the non-screening wage by p_L. Then, the average job-finding rate for each type at time t is given by the following expressions.

$$Q_H(t) = p_H^M, \quad Q_L(t) = z(t)p_L + (1 - z(t))p_L^M \tag{31}$$

In the upward pooling equilibrium, H-type workers find a job at a constant rate of p_H^M, while L-type workers' job-finding rate varies over t. Although p_H^M and p_L are not directly comparable, $p_H^M > p_L^M$ and $p_L > p_L^M$ hold as a result of the firms' information acquisition and workers' indifference conditions. This implies that when the fraction $f(t)$ is close to $\mu(w_M)$, $f(t)$ must decline over time because $Q_L(t)$ is close to p_L^M, which is lower than p_H^M. Consequently, if the initial fraction $f(0)$ is below $\mu(w_M)$, it remains below $\mu(w_M)$ for all subsequent time periods, thereby supporting the existence of the upward pooling equilibrium.

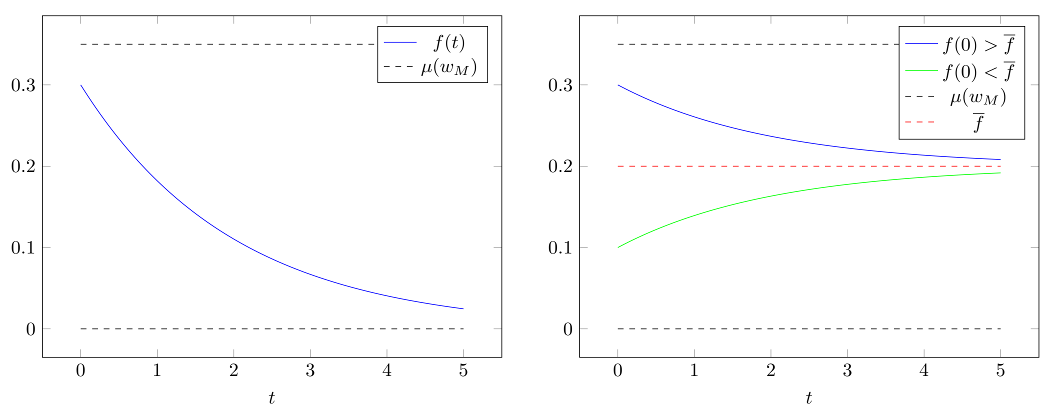
The sign of the duration dependence relies on which type escapes the unemployment faster, and it depends on the relative size of p_H^M and p_L.

Proposition 7. *(Duration dependence) Let $\bar{f}$ be the fraction of H-type that makes $Q_H(t) = Q_L(t)$, which is implicitly defined by*

$$\mu(w_M) = \frac{\bar{f}}{\bar{f} + (1-\bar{z})(1-\bar{f})}, \quad p_H^M = \bar{z}p_L + (1-\bar{z})p_L^M \tag{32}$$

Then, in an upward-pooling equilibrium,

- *If $p_H^M \geq p_L$, then $f(t), W(t)$, and $Q(t)$ are decreasing in t.*
- *If $p_H^M < p_L$ and $f(0) > \bar{f}$, then $f(t), W(t)$, and $Q(t)$ are decreasing in t.*
- *If $p_H^M < p_L$ and $f(0) < \bar{f}$, $f(t), W(t)$ are increasing in t, while $Q(t)$ is decreasing in t.*



(a) Case1: $p_H^M > p_L$

(b) Case2: $p_H^M < p_L$

Figure 8: The average quality among the unemployed over t in an upward pooling equilibrium

In the upward pooling equilibrium, the average quality and wage increase or decrease depending on two factors: whether p_H^M is greater than p_L, and whether $f(0)$ is greater than $\bar{f}$. If $p_H^M > p_L$, then H-type workers find a job even faster than L-type workers applying to the non-screening job.

Therefore, H-type workers quit the unemployed pool faster, reducing the average quality and wage over duration. On the other hand, when $p_H^M < p_L$, then the magnitude of $Q_H(t)$ and $Q_L(t)$ depends on $f(0)$. If $f(0)$ is relatively large, then majority of L-type workers must apply to screening job as the fraction at the screening job $\mu(w_M)$ is given independently of $f(t)$. As L-type workers find a job at a slower rate at w_M, the average quality and wage decline over duration in this case. However, if $f(0)$ is relatively small, then majority of L-type workers apply to the non-screening job, resulting in a higher job-finding rate. Then, the average quality and wage increase over duration. In any case, the average job-finding rate of the unemployed falls over duration because the type with a lower job-finding rate remains in the market longer. Figure 8 graphically summarizes the duration dependence in an upward pooling equilibrium.

As p_H^M and p_L are equilibrium values, it is not *ex ante* clear whether both $p_H^M > p_L$ and $p_H^M < p_L$ can occur. Figure 9 demonstrates that both scenarios can happen in equilibrium. In the exercise, I set $y_H = 1.5, y_L = 1, r = 0.05, b = 0.1, k = 0.5$, and conduct comparative statics on λ. The comparative statics reveal that p_H^M exceeds p_L if and only if λ is low, or equivalently, when information is cheaper. This result is intuitive as the H-type benefits from less costly information, while the L-type is adversely affected. Further implications of λ are discussed in more detail in Section 3.4.2.

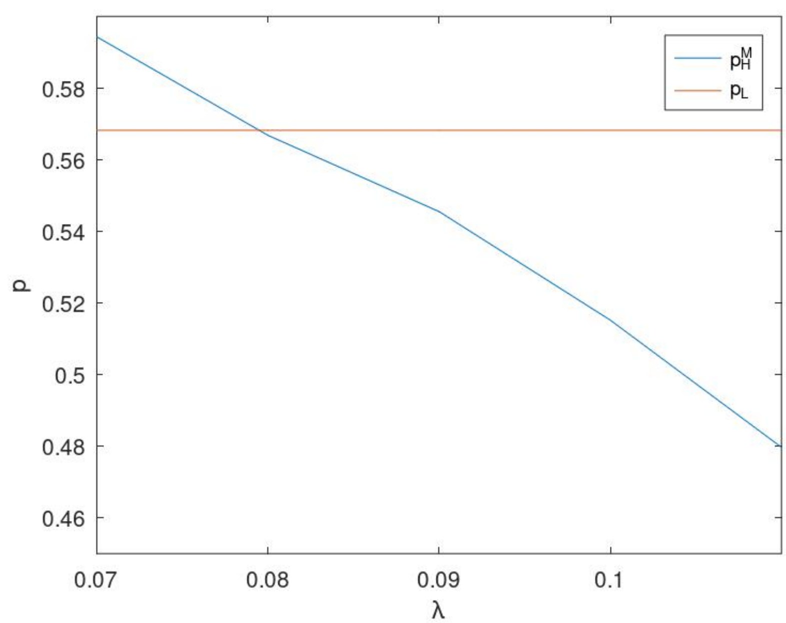


Figure 9: Comparison between p_H^M and p_L

3.3.3 When $y_H - y_L > M$ and $f(0) > \mu(w_M)$

The analysis so far has focused on time-invariant equilibria. I view this approach as natural because most dynamic programming problems in economics are interested in finding a solution that does not directly depend on t. However, such a time-invariant equilibrium does not exist when $y_H - y_L > M$, but $f(0) > \mu(w_M)$. This implies that $U_j(t)$ and all the equilibrium objects relying on $U_j(t)$ explicitly depend on t.

The intuition behind this time dependency is clear. When both types significantly differ in their productivities, and the majority of workers are of the H-type, then L-type workers can free-ride on H-type workers in the high-wage job. In this scenario, firms' screening at the high-wage job is generous enough for L-type workers to the extent that they choose not to apply for the non-screening job, even if the prior belief at the high-wage job takes its lowest possible value $f(t)$.

While I cannot provide a complete proof, I conjecture that an equilibrium with the following feature exists: initially, all workers apply to a screening wage $w_M(t)$ until t_0 such that $f(t_0) = \mu(w_M)$. After $t = t_0$, the equilibrium becomes a time-invariant upward-pooling equilibrium. In this equilibrium, $\dot{U_H}(t), \dot{U_L}(t) < 0$ for $t < t_0$, and $\dot{f}(t) < 0$ for all t. Intuitively, if an equilibrium exists, it cannot be upward pooling, implying that there exists an equilibrium in which all workers apply to the same wage. At that wage level, L-type workers must receive a strictly higher payoff than U_L in the upward-pooling equilibrium, which is guaranteed for them at any t. However, this complete pooling must end within a finite time as H-type workers find a job faster when both types apply to the same job, reducing $f(t)$ eventually below $\mu(w_M)$. During the time interval where the complete pooling occurs, the belief at pooling wage $\mu(w_M(t))$ must coincide with $f(t)$, which is larger than $\mu(w_M)$ and declining. Thus, $U_H(t)$ and $U_L(t)$ would exceed U_H and U_L, and decline over t and reach the upward-pooling equilibrium level at $t = t_0$ due to the fall in $f(t)$.

3.4 Implications

3.4.1 How do quality, wage, and job-finding rate vary over duration?

Proposition 5 and Proposition 7 demonstrate that the duration effects on the average quality and wage differ across the equilibrium type. When the heterogeneity is relatively small across types, the equilibrium is fully separating in that workers' self-selection reveals the type without screening. In this case, the average quality and wage must increase over duration due to the incentive condition for L-type, consistent with previous literature that studies adverse selection without screening (Guerrieri and Shimer, 2014; Chang, 2017). This result implies that the lessons from the previous research are applicable to more general environments where buyers can gather information, provided that their incentives to do so are not that large.

On the other hand, when the productivity differential exceeds a threshold, the equilibrium exhibits upward pooling where the negative duration dependence, defined as the decline in wages over duration, can arise. The equilibrium must involve firms' active screening, which confirms the idea that the negative duration dependence requires screening by uninformed buyers. However, the necessity of a large productivity differential emphasizes that the mere presence of screening devices is insufficient to generate negative duration dependence when screening and search are endogenous.

Regarding the effects of duration on wages, previous literature emphasizing learning and screening typically finds the negative effects (Gonzalez and Shi, 2010; Doppelt, 2016; Fawcett and Shi, 2018), while research focusing on adverse selection implies the opposite (Guerrieri and Shimer, 2014; Chang, 2017). This paper proposes a unified framework that encompasses both and highlights that the sign of duration dependence varies mainly by the degree of heterogeneity.

The results of this paper are intuitively appealing, especially considering the labor market where firms actively screen and longer duration significantly influence wages negatively. This paper explains that these two phenomena must be related. Specifically, large unobserved heterogeneity drives both widespread screening and the adverse effects of unemployment duration. Moreover, the model aligns with empirical findings that these negative duration effects are more pronounced for jobs requiring higher skills, which are likely subject to greater unobserved heterogeneity among job candidates (Gregory and Jukes, 2001; Ortego-Marti, 2017).

3.4.2 The role of information cost

This subsection examines how firms' information acquisition cost λ influences the equilibrium outcomes. When the productivity differential is small enough to result in a fully separating equilibrium, all the equilibrium components, including the threshold productivity, remain independent of λ. Therefore, λ does not play a role in this case.

When the productivity differential is large enough to result in an upward pooling equilibrium, λ influences the equilibrium allocations through various channels. To concretely understand the equilibrium dynamics, suppose λ falls. In this case, $\underline{\mu}(w)$ decreases, and $\overline{\mu}(w)$ increases, implying two effects. First, the set of (w, μ) that induces screening expands. Second, when screening occurs, it becomes more precise. These two effects make the equilibrium belief more sensitive to wages given (U_H, U_L), which is confirmed in Figure 10, representing a steeper $\mu(w)$ for a lower λ. Intuitively, wages become more informative about workers' choices when λ is lower because workers anticipate more informative screening during the interview stage, reinforcing the self-selection motive.

In addition to these partial equilibrium effects, λ affects the equilibrium value of unemployed.

Specifically, U_L remains constant as it is determined by the non-screening job, while U_H increases as λ falls because the H-type benefits from more informative screening. This equilibrium effect shifts the wage level at which the equilibrium belief becomes positive, creating a single crossing, as depicted in Figure 11.

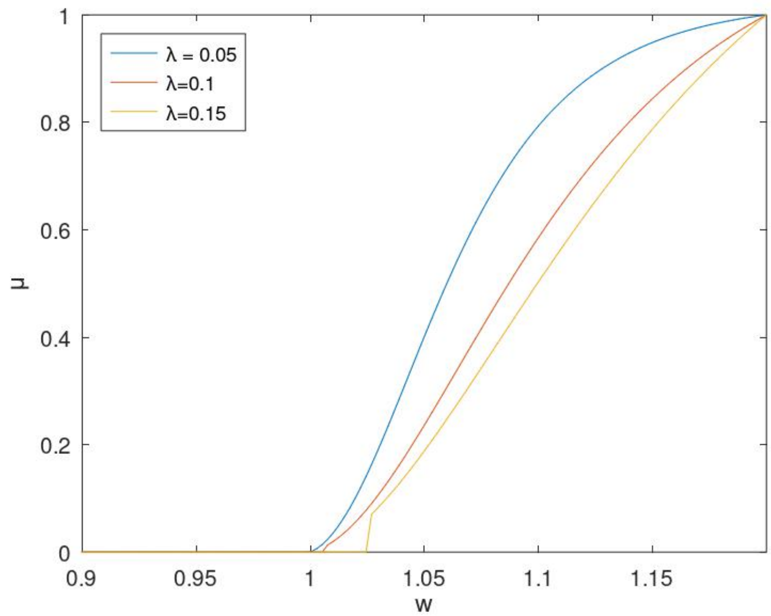


Figure 10: Partial equilibrium effect of λ

The overall equilibrium effects of the fall in λ increase w_M and $\mu(w_M)$, as confirmed by Figure 12. Intuitively, less costly information benefits the H-type, widening the value gap between the types. This, in turn, stretches out the wage differential between the screening and non-screening job.

3.4.3 Price posting vs Information acquisition

In the model economy, there are two informational instruments: informed workers' self-selection and uninformed firms' costly information acquisition. The equilibrium analysis suggests that there exists an order. The self-selection always reveals some information, regardless of the equilibrium types, while the costly information acquisition provides information only when the unobserved heterogeneity exceeds a threshold. It means that when both informational instruments are available, the equilibrium tends to utilize the price mechanism first than information acquisition.

Note that firms do not incur fixed costs in gathering information, implying that these results are not contingent on the cost structure. Instead, it depends on the commitment structure. For

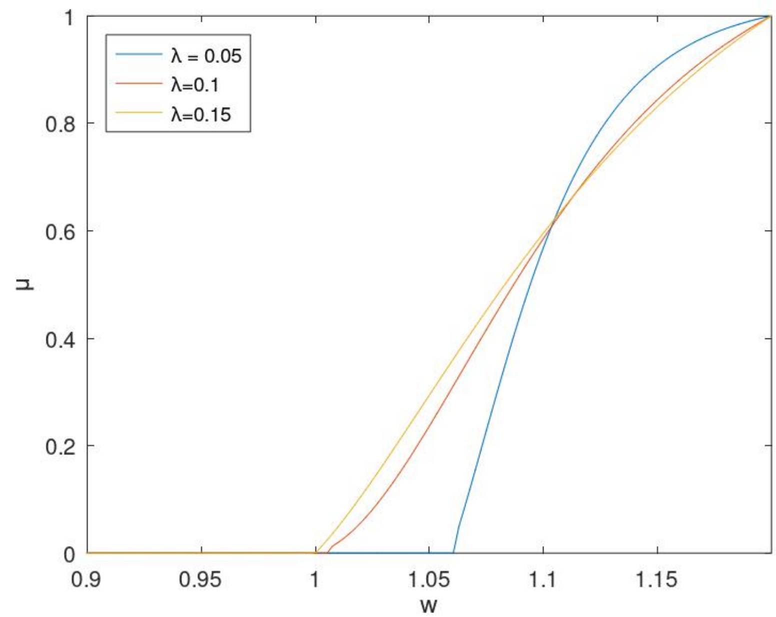


Figure 11: Equilibrium belief for different λ

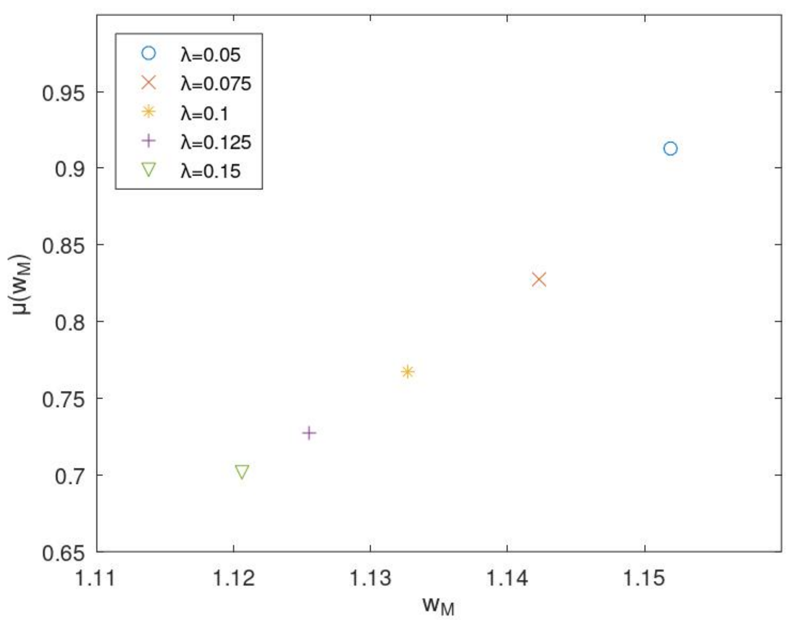


Figure 12: Screening job wage and H-type fraction for different λ

instance, consider an economy where firms commit to screening intensities as discussed in Section 3.5.1. In such an economy, a pooling equilibrium can exist where firms' information acquisition alone provides information.

3.5 Discussions

3.5.1 Commitment on screening

This paper assumes that firms commit to wages but not their screening intensities. There are two merits to this assumption. First, it accurately reflects how the hiring process operates in reality. For example, in the academic job market, candidates typically have an idea of the salary range. They also rationally infer the level of difficulty involved in giving a successful job talk, even though universities do not commit to specific questions their faculties will ask during the job talk in advance. The overall atmosphere of the job talk largely depends on the hiring committees' initial assessment of the candidate, which corresponds to the prior belief in this paper.

Second, the commitment structure assumed in this paper eliminates multiple equilibria as observed in Feng et al. (2019). Intuitively, when firms do not commit their screening intensities, the only possible semi pooling equilibrium is the upward pooling where L-type applies to both screening and non-screening jobs. However, when firms commit to their screening intensities, there might be a downward pooling equilibrium where H-type applies to both screening and non-screening jobs, and these two equilibrium types can coexist. Firms' lack of commitment on screening eliminates the issue of multiple equilibria and helps refine the model's predictions regarding duration and prices.

3.5.2 Direct skill loss over duration

This model can incorporate direct skill loss by incorporating a Poisson shock that transforms an H-type individual into an L-type individual. Given that individuals are aware of the skill depreciation shock realization, this modification does not significantly alter the results. Specifically, suppose an H-type individual faces a skill depreciation at a Poisson rate of $\delta > 0$. Then, the value function of H-type follows the below modified equation.

$$rU_H(t) = rb + \max_{w,t \in \mathcal{M}} \{p(w,t)(w - U_H(t))\} + \dot{U_H}(t) - \delta(U_H(t) - U_L(t)) \tag{33}$$

Upon the presence of the skill loss, $U_H(t)$ is lower than the equilibrium value without the skill loss, but the same set of time-invariant equilibria still exists. The aggregate fraction $f(t)$ reflects both the skill depreciation and the job-finding rate differential.

$$\dot{f(t)} = -f(t)\left\{(1 - f(t))(Q_H(t) - Q_L(t)) + \delta\right\} \tag{34}$$

The last term δ captures the skill loss effect, which reduces $f(t)$ over t. The average wage may decrease over duration even when $Q_H(t) < Q_L(t)$ if the skill depreciation rate δ is large enough.

3.5.3 Non-contractable duration

This paper assumes that firms not only observe t but also offer a contingent contract depending on t. The observable duration appears reasonable since firms often base their hiring strategies on workers' unemployment duration.[10] While the ability of firms to fully contingent their contracts on the duration of unemployment may not precisely mirror reality, the wage scarring effects suggest that firms have a means to differentiate between workers with varying durations.

Note that relaxing this assumption does not alter any results when focusing exclusively on time-invariant equilibria. This is because in any time-invariant equilibria, the unemployment duration itself does not provide additional information conditional upon workers' search decisions. Still, relaxing this assumption *ex ante* complicates the problem substantially. Specifically, if this assumption is relaxed, firms have to infer which workers with different durations of unemployment will apply to their vacancies in addition to discerning their types. This inference is both relevant to payoff and complex, unless one exclusively focuses on time-invariant equilibria even before defining the equilibrium concept.

4 Conclusion

This paper studies a model with dynamic adverse selection and information acquisition. I demonstrate that the equilibrium can be one of two types: full-separation or upward-pooling, contingent on the quality differential between sellers' types. In the full separation equilibrium, sellers' self-selection completely discloses their types, rendering buyer screening unnecessary. Consequently, both quality and price escalate over time. By contrast, the upward pooling equilibrium comprises a high-wage/screening job and a low-wage/non-screening job, implying both pricing and screening offer valuable information. In the upward pooling equilibrium, the quality and price increase over duration if the initial fraction of the H-type is low enough, and the job-finding rate for the H-type in the screening job is lower than the job-finding rate in the non-screening job. By contrast, the quality and price fall over duration if either one of these conditions is violated. This characterization confirms the insight that the direction of duration dependence relies on the choice of informational instrument. The paper further uncovers that the utilization of these instruments is

[10]For instance, Kroft et al. (2013) found that the interview callback rate decreases as the duration of unemployment increases from a field experiment.

contingent upon the level of heterogeneity among sellers. Consequently, this paper provides a unified framework that elucidate how equilibrium allocation and dynamics unfold under asymmetric information.

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A Proofs

Proof of Proposition 1

Proof. The optimal information problem can be solved by finding the concave envelope of the function $\hat{J}(\mu_s)+\lambda H(\mu_s)$. It is equivalent to solve the following maximization problem, pretending that the solution is interior.

$$\begin{aligned} J(w,\mu) &= \max_{s_i,\mu_i} s_1\hat{J}(w,\mu_1) + \lambda\sum_i s_i H(\mu_i) - \lambda H(\mu) \quad \text{subject to} && (35)\\ \sum s_i &= 1 && (36)\\ \sum s_i\mu_i &= \mu && (37)\\ \mu_1 &\geq \frac{w-y_L}{y_H-y_L} && (38)\\ \mu_2 &\leq \frac{w-y_L}{y_H-y_L} && (39)\end{aligned}$$

Note that the last two inequalities cannot bind at optimal, because if they bind, then it is optimal to adopt the uninformative interview given the linearity of J on $J>0$ region. Then, the first-order conditions are

$$\begin{aligned} \partial s_1 &: \hat{J}(w,\mu_1) - \lambda\left[\mu_1\log\left(\frac{\mu_1}{\mu}\right) + (1-\mu_1)\log\left(\frac{1-\mu_1}{1-\mu}\right)\right] - \rho_1 - \rho_2\mu_1 = 0 && (40)\\ \partial s_2 &: -\lambda\left[\mu_2\log\left(\frac{\mu_2}{\mu}\right) + (1-\mu_2)\log\left(\frac{1-\mu_2}{1-\mu}\right)\right] - \rho_1 - \rho_2\mu_2 = 0 && (41)\\ \partial \mu_1 &: s_1\hat{J}_2(w,\mu_1) - \lambda s_1\left[\log\left(\frac{\mu_1}{\mu}\right) - \log\left(\frac{1-\mu_1}{1-\mu}\right)\right] - \rho_2 s_1 = 0 && (42)\\ \partial \mu_2 &: -\lambda s_2\left[\log\left(\frac{\mu_2}{\mu}\right) - \log\left(\frac{1-\mu_2}{1-\mu}\right)\right] - \rho_2 s_2 = 0 && (43)\end{aligned}$$

The last two equations imply the following.

$$y_H - y_L = \lambda\left[\log\left(\frac{\mu_1}{1-\mu_1}\right) - \log\left(\frac{\mu_2}{1-\mu_2}\right)\right] \quad (44)$$

The above suggests that it is optimal for the firm not to interview either μ is high or low. Also, by using the second and fourth equations,

$$\rho_1 = -\lambda\log\left(\frac{1-\mu_2}{1-\mu}\right), \quad \rho_2 = -\lambda\left[\log\left(\frac{\mu_2}{\mu}\right) - \log\left(\frac{1-\mu_2}{1-\mu}\right)\right] \quad (45)$$

Therefore,

$$\mu_1 y_H + (1-\mu_1)y_L - w = \lambda\left[\mu_1 \log\left(\frac{\mu_1}{\mu_2}\right) + (1-\mu_1)\log\left(\frac{1-\mu_1}{1-\mu_2}\right)\right] \tag{46}$$

Equation (44) and (46) determine μ_1 and μ_2. Then, s_1 is defined by $s_1\mu_1 + (1-s_1)\mu_2 = \mu$. Combining these two, one can get

$$y_H - w = \lambda \log\left(\frac{\mu_1}{\mu_2}\right), \quad w - y_L = -\lambda\log\left(\frac{1-\mu_1}{1-\mu_2}\right) \tag{47}$$

$$\Rightarrow \mu_2 = \frac{\exp\left(\frac{w-y_L}{\lambda}\right)-1}{\exp\left(\frac{y_H-y_L}{\lambda}\right)-1}, \quad \mu_1 = \mu_2\exp\left(\frac{y_H-w}{\lambda}\right) = \frac{\exp\left(\frac{y_H-y_L}{\lambda}\right)-\exp\left(\frac{y_H-w}{\lambda}\right)}{\exp\left(\frac{y_H-y_L}{\lambda}\right)-1} \tag{48}$$

The signal realization probability s_1 is given by $(\mu-\mu_2)/(\mu_1-\mu_2)$. The derived μ_1,μ_2 automatically satisfy $\mu_2 < \mu^0(=\frac{w-y_l}{y_h-y_l}) < \mu_1$. when $w\in(y_l,y_h)$. Note that the derived μ_1,μ_2 are independent of μ. If either $\mu_2 > \mu$ or $\mu_1 < \mu$, then it means the information problem is binding so that either $\mu_2 = \mu$ or $\mu_1 = \mu$. Then, it becomes the uninformative case. □

Proof of Proposition 2

Proof. Denote the hiring probability for a firm as s. By the Bayes' rule,

$$s = \frac{\mu - \underline{\mu}}{\overline{\mu}-\underline{\mu}} \tag{49}$$

Also, given the definition of conditional hiring probability $\pi_j(w,\mu)$, $s = \mu\pi_H(w,\mu) + (1-\mu)\pi_L(w,\mu)$. Because the belief about the worker's type conditional upon hiring is $\bar{\mu}$,

$$\pi_H(w,\mu) = s\frac{\overline{\mu}}{\mu}, \quad \pi_L(w,\mu) = s\frac{1-\overline{\mu}}{1-\mu} \tag{50}$$

It implies $\pi_L(w,\mu) < \pi_H(w,\mu)$ as $\mu\in(\underline{\mu},\overline{\mu})$. Also,

$$\frac{\pi_H(w,\mu)}{\pi_L(w,\mu)} = \frac{1-\mu}{\mu}\frac{\overline{\mu}(w)}{1-\overline{\mu}(w)} \Rightarrow \lim_{\mu\to\overline{\mu}(w)}\frac{\pi_H(w,\mu)}{\pi_L(w,\mu)} = 1, \quad \lim_{\mu\to\underline{\mu}(w)}\frac{\pi_H(w,\mu)}{\pi_L(w,\mu)} = \exp\left(\frac{y_H-y_L}{\lambda}\right) \tag{51}$$

from the definition of $\overline{\mu}(w)$ and $\underline{\mu}(w)$, which proves the third claim.

By calculating the derivative of π_H with respect to w.

$$\frac{\partial\pi_H(w,\mu)}{\partial w} = \frac{\overline{\mu}(\overline{\mu}-\mu)\exp\left(\frac{w-y_L}{\lambda}\right) - \underline{\mu}(\mu-\underline{\mu})\exp\left(\frac{y_H-w}{\lambda}\right)}{\lambda\mu(\overline{\mu}-\underline{\mu})^2\left[\exp\left(\frac{y_H-y_L}{\lambda}\right)-1\right]} < 0 \tag{52}$$

Therefore, $\pi_H(w,\mu)$ is decreasing in w. Note that $\overline{\mu}$ is an increasing function of π_H/π_L. Because

$\overline{\mu}$ is increasing in w while $\pi_H(w,\mu)$ is decreasing in w, $\pi_L(w,\mu)$ is also decreasing in w.

Similarly, one can directly take the derivative of π_H and π_L with respect to μ,

$$\frac{\partial \pi_H(w,\mu)}{\partial \mu} = \frac{\overline{\mu}\underline{\mu}}{(\overline{\mu}-\underline{\mu})\mu^2} > 0, \quad \frac{\partial \pi_L(w,\mu)}{\partial \mu} = \frac{(1-\overline{\mu})(1-\underline{\mu})}{(\overline{\mu}-\underline{\mu})(1-\mu)^2} > 0 \tag{53}$$

Lastly, by taking the derivative of $\log(\pi_H/\pi_L)$,

$$\frac{\partial \log\left(\pi_H(w,\mu)/\pi_L(w,\mu)\right)}{\partial \mu} = \frac{1}{\mu-\underline{\mu}}\left[\frac{\underline{\mu}}{\mu} - \frac{1-\underline{\mu}}{1-\mu}\right] < 0 \tag{54}$$

□

Proof of Proposition 3

Proof. Note that the social planner's allocation requires the full separation with $\theta_H^* > \theta_L^*$ independent of t. Suppose an equilibrium is fully separating, and market tightness for each type is constant for all t. In such an equilibrium, there is no information acquisition by firms. Then, the incentive compatibility condition for each unemployed workers implies

$$p(\theta_H)(w_H - U_H) \geq p(\theta_L)(w_L - U_H) \tag{55}$$

$$p(\theta_L)(w_L - U_L) \geq p(\theta_H)(w_H - U_L) \tag{56}$$

where θ_j is short for $\theta(w_j, \mu^*(w_j,\cdot))$. Because $U_H > U_L$, the above two equations require $p(\theta_H) \leq p(\theta_L)$, which is against $\theta_H^* > \theta_L^*$ in the social planner's allocation. □

Proof of Proposition 4

Proof. Throughout the proof, the following single-crossing property is used: given (θ, w) that satisfies the firm's free entry condition $q(\theta)(y-w) = rk$ for some y, if the L-type is indifferent between (θ_0, w_0) and (θ_1, w_1) for $w_0 < w_1$ when both w_0 and w_1 do not screen, then the H-type strictly prefers (θ_1, w_1) to (θ_0, w_0). This property is due to $U_H > U_L$.

Suppose $y_H \leq \underline{y_H}$. Then, I will first show that the candidate full-separating wage w_H, which is the solution of Equation (24), is weakly smaller than y_L. To show this is the case, observe that (U_L, R_L, w_L, p_L) is completely determined by Equation (23), independent of (U_H, R_H, w_H, p_H). Note that this U_L is guaranteed for the L-type in any equilibrium, as it is the equilibrium payoff when L-type reveals the type.

Given (U_L, R_L, w_L, p_L), the L-type's incentive condition must bind at the solution of Equation (24) due to Proposition 3. Combining the L-type's incentive condition and free entry of firms, the following must hold at optimal w_H.

$$\frac{y_H - w_H - rk}{y_H - w_H}(w_H - U_L) = rU_L \tag{57}$$

On the free entry frontier of firms, the L-type prefers a lower wage on $w > w_L$ region, implying that $w_H \leq y_L$ is equivalent to the following condition.

$$\frac{y_H - y_L - rk}{y_H - y_L}(y_L - U_L) \leq rU_L \iff y_H = y_L + \frac{rk(y_L - U_L)}{y_L - (1+r)U_L} \tag{58}$$

For derivation, $rU_L = R_L$ is used. It means that the firm does not gather any information for all $w \leq w_H$, implying that the step-function $\mu(w)$ satisfies the equilibrium belief refinement due to the single-crossing condition, following the same logic as in previous literature without screening (Guerrieri and Shimer, 2014). Given the step-function belief $\mu(w)$, it is straightforward that both types do not have any incentive to deviate. Therefore, if there exists a pair (U_H, U_L) satisfying Equations (23) and (24), then it constitutes an equilibrium. The existence of such a pair (U_H, U_L) is guaranteed because Equation (23) has a solution U_L, while Equation (57) determines U_H along with w_H, given U_L.

The non-existence of a pooling equilibrium is derived from the observation that the L-type's incentive compatibility condition binds at w that is lower than y_L. Thus, there cannot be a pooling market at $w > y_L$, as it is never optimal for the L-type to apply to such high wages. If both types apply to some $w_0 < y_L$, then the equilibrium belief at $w_0 + \epsilon$ must be 1 due to the single-crossing, making everyone profitable to deviate to $w_0 + \epsilon$ from w_0. □

Proof of Proposition 5

Proof. $\dot{f(t)} = -f(t)(1 - f(t))(p_H - p_L) > 0$ as $p_H > p_L$. The average wage of the unemployed hired at t is $W(t) = f(t)w_H + (1 - f(t))w_L$. Therefore, $\dot{W(t)} = \dot{f(t)}(w_H - w_L) > 0$. Lastly, the average job-finding rate is $Q(t) = f(t)p_H + (1 - f(t))p_L$, thereby $\dot{Q(t)} = \dot{f(t)}(p_H - p_L) < 0$. □

Proof of Proposition 6

Proof. (Claim 1) There is no equilibrium with $\mu(w) \in \{0, 1\}$ for all w.

Suppose by way of contradiction, assume that such an equilibrium exists. Then, the equilibrium must be full-separation. Denote the minimum w chosen by the H-type by w_H. Because $y_H > \underline{y_H}$, this $w_H > y_L$ otherwise L-type has an incentive to apply to w_H. Then, take an off-the-path wage $w_H - \epsilon > y_L$. The equilibrium belief at $w_H - \epsilon$ cannot be 1, because if so, the H-type has an incentive to deviate. On the other hand, the equilibrium belief cannot be 0 either, because if so, no firm would post a vacancy at $w_H - \epsilon$. Thus, $\mu(w_H - \epsilon) = 0$ requires that the L-type is more likely to apply even under the worst screening for them.

$$\frac{\phi_H(w_H - \epsilon)}{\phi_L(w_L - \epsilon)} \geq \exp\left(\frac{y_H - y_L}{\lambda}\right) \tag{59}$$

However, $\mu(w_H) = 1$ implies that the above fraction ϕ_H/ϕ_L is one at w_H, which contradicts to the continuity of ϕ_H/ϕ_L. Therefore, there is no equilibrium with $\mu(w) \in \{0, 1\}$. Note that Claim 1 rules out the existence of the full-separation equilibrium in Proposition 4 as the equilibrium belief there is a step-function.

(Claim 2) A time-invariant upward-pooling equilibrium exists if $f(0)$ is not too high.
I will prove the existence by construction. Similar to the fully separating equilibrium case, (U_L, R_L, w_L, p_L) is determined independent of parameters or equilibrium objects related to H-type. Also, observe that the equilibrium belief is a function of (U_H, U_L), but nothing else. Specifically, the equilibrium belief is 0 for any $w \leq y_L$ because $\pi_H(w, \mu) = \pi_L(w, \mu) = 1$ independent of μ. For $w > y_L$, $\mu(w)$ is strictly positive if and only if $\frac{\phi_H(w)}{\phi_L(w)} < \exp\left(\frac{y_H - y_L}{\lambda}\right)$, implying that $\mu(w)$ is positive if and only if $w > w_0$. For this region, the job-finding rate ratio $\pi_H(w, \mu)/\pi_L(w, \mu)$ is $\frac{1-\mu}{\mu}\frac{\overline{\mu}}{1-\overline{\mu}}$, which leads to the expression in Equation (30) by the equilibrium requirement $\phi_H/\phi_L = \pi_H/\pi_L$. The resulting $\mu(w)$ is strictly increasing in w, and $\mu(w) \to 1$ as $w \to y_H$. Therefore, if there exists an optimal search policy w_M for H-type, $\mu(w_M)$ is in $(0, 1)$, meaning that the equilibrium must be upward-pooling.

It means that it is enough to find a pair (U_H, w_M) that satisfies Equation (28) along with the following definition of U_H, given the dependency of $\mu(w)$ on U_H. To find such a pair, one can use the property that it must be also optimal for L-type workers to search for w_M. To use it, recall the L-type's search problem for pooling markets.

$$\max_{w} \pi_L(w, \mu(w))p(w)(w - U_L) \quad \text{subject to} \quad q(w)J(w, \mu(w)) = rk, w \geq w_0 \tag{60}$$

The maximized value of the above problem is strictly and continuously decreasing in U_H. This is because $\mu(w)$ is strictly and continuously decreasing in U_H. Also, when U_H is sufficiently low, the above maximized value exceeds U_L, because it is so when $\mu(w) = 1$ for all $w \geq w_0$. It means that there exists a unique U_H that makes the above maximum to be U_L. At such U_H, due to the equilibrium refinement, H-type workers find it optimal to search for w_M as L-type workers do so. It means the following holds.

$$rU_H = rb + \pi_H(w_M, \mu(w_M))p(w_M)(w_M - U_H) \tag{61}$$

Therefore, such a pair (U_H, w_M) constitutes an upward-pooling equilibrium as long as $f(0) \leq$

$\mu(w_M)$. $\square$

Proof of Proposition 7

Proof. From $\dot{f(t)} = -f(t)(1-f(t))(Q_H(t) - Q_L(t))$, $\dot{f(t)} < 0$ if and only if $Q_H(t) > Q_L(t)$. It holds either when $p_H^M > p_L$ or $p_H^M < p_L$ with $f(0) > \bar{f}$. The average wage in an upward-pooling equilibrium is given by

$$W(t) = f(t)w_M + (1-f(t))\left((1-z(t))w_M + z(t)w_L\right) = \frac{f(t)}{\mu(w_M)}w_M + \left(1 - \frac{f(t)}{\mu(w_M)}\right)w_L \quad (62)$$

which implies that $\dot{W(t)} < 0 \iff \dot{f(t)} < 0$ as $w_M > w_L$. Lastly, the average job-finding rate $Q(t)$ is given by

$$Q(t) = \frac{f(t)}{\mu(w_M)}Q_H(t) + \left(1 - \frac{f(t)}{\mu(w_M)}\right)Q_L(t) \quad (63)$$

which implies that $\dot{Q(t)} < 0 \iff \dot{f(t)}(Q_H(t) - Q_L(t)) < 0$. There are three cases.

- If $p_H^M > p_L$, then $\dot{f(t)} < 0$ and $Q_H(t) > Q_L(t)$. Therefore, $\dot{Q(t)} < 0$.
- If $p_H^M < p_L$ and $f(0) > \bar{f}$, then $\dot{f(t)} < 0$ and $Q_H(t) > Q_L(t)$. Therefore, $\dot{Q(t)} < 0$
- If $p_H^M < p_L$ and $f(0) < \bar{f}$, then $\dot{f(t)} > 0$ and $Q_H(t) < Q_L(t)$. Therefore, $\dot{Q(t)} < 0$.

$\square$