A Trajectories-Based Approach to Measuring Intergenerational Mobility

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2022 한국노동연구원
저출산 시대와 노동 포럼

July 8, 2022

Introduction

This Paper

- Despite of the renaissance of research on inequality, the questions of appropriate metrics for mobility have received relatively little attention
- Links the study of intergenerational mobility to the dynamics of family influences (parental income, family structure) that children and adolescents experience
- Formulates the measurement of childhood and adolescence experiences as representing a trajectory of family influences
  - Mobility measurement involves calculating the sensitivity of adult outcomes to the values of the trajectories

Conventional Mobility Measure: IGE of Income

- Links permanent income of children to that of parents
- Classic intergenerational income elasticity (IGE) regression

$$y_{c,i} = \alpha + \beta y_{p,i} + \varepsilon_i$$

  - $y_{c,i}$ $(y_{p,i})$: log of child's (parents') permanent income
  - β: intergenerational income elasticity (IGE)
- Theorectical background: 2-stage overlapping generation model
  - Becker and Tomes (1979), Loury (1981)
  - One period of childhood
  - Parental investment at different timings is perfect substitute

Our Mobility Measure: IGE of Income Trajectory

- Links permanent income of children to a trajectory of parental influences that are experienced by children through their childhood and adolescence
- Functional regression

$$y_i = \alpha + \int_p^q \beta(r) f_i(r) dr + \varepsilon_i$$

  - y_i: Child's log permanent income
  - $f_i(r)$: Trajectory of parental influences from child's age p to q
    - Parental income, family structure, father's occupation
  - $\beta(r)$: Functional coefficient signifying marginal response of y_i on $f_i(r)$ at each r
    - Age-varying IGE (intergenerational elasticity of income), mobility curve
    - Measures the age-specific association between parental income trajectory and offspring's permanent income - key object of our analysis

Mobility Measures: Background

Theorectical background: Modern skill formation model

- Cunha and Heckman (2007), Cunha, Heckman, and Schennach (2010)
- Parental influences are key factors for development of child's human capital
- Multiple periods of childhood
- Sensitive periods:
  - Existence of stages that are more effective in producing certain skill
- Dynamic complementarity:
  - Levels of skill investments at different ages bolster each other

Summary of Results

1. Clear evidence of age-specific IGE coefficients
   - Age-varying IGE is larger in mid-to-late adolescence than earlier
   - Parental income trajectories that are rising in the later years contribute to upward mobility
   - Conventional permanent income approach therefore loses information about the mobility process.

2. Interaction effects link incomes at different ages to mobility
   - Quadratic generalization of the linear FDA mobility regressions reveals nonzero coefficients that correspond to cross-partial derivatives at different ages
   - Reveals substitutability between incomes with the periods of early childhood and those within years of middle and late adolescence
   - Complementarities, in contrast, appear for incomes between early childhood and adolescent years
   - These results are a potential puzzle relative to uniform dynamic complementarity findings in skills literature.

Summary of Results

3. Family structure dynamics play a distinct role from family income for mobility
    - Single parent households in mid teen years are relatively strongly associated with lower future income
    - Income and family structure exhibit complementarity in their mobility effects within the early childhood and middle and late adolescent years
    - Income and family structure exhibit substitutability between early and later years
4. Proximate determinants of income, education and occupation, exhibit analogous parental income and family structure influences.
    - Mid-to-late adolescence years receive greater weight than others to explain offspring's education and occupation
    - Combined with the income results, family inputs in the later years are relatively important in understanding mobility

Econometric Methodology

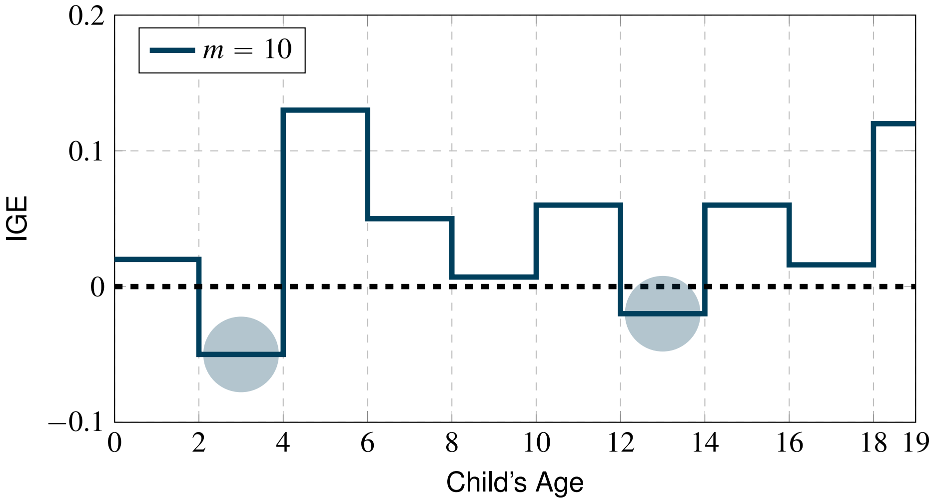
Conventional Approach

- Estimate the effects of family income at different childhood stages, simply include an income measure from each period as a regressor in the model

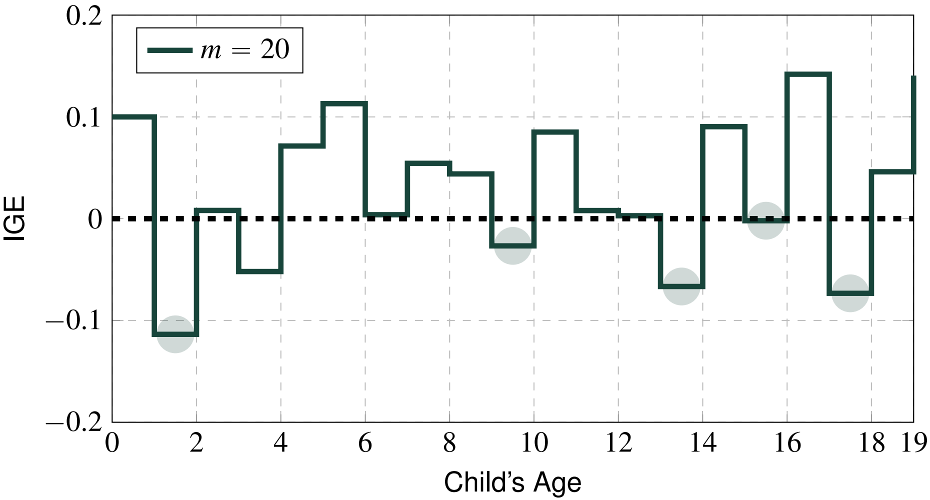
$$y_{c,i} = \alpha + \beta_1 y_{p,1,i} + \cdots + \beta_m y_{p,m,i} + \varepsilon_i$$

  - $y_{c,i}$: log of child's permanent income
  - $y_{p,m,i}$: log of parents' income at childhood stage m
  - Levy and Duncan (2000), Jenkins and Schluter (2002), Aakvik, Salvanes, and Vaage (2005), Humlum (2011)

Possible Approach



Possible Approach



Preview of Our Functional Approach

- Starting point for functional regression:
  approximate infinite-dimensional regressor (f_i) to finite dimensional object

- Our approach: approximation by functional principal components
  1. Approximates (f_i) most effectively
     - Variation of (f_i): captured by functional principal components $>$ any other approximations
  2. Provides unbiased estimator
     - Regressor: orthogonal to approximation errors
  3. More efficient than conventional regression

More on Our Functional Approach

- Part of what we want to do is to demonstrate there is a better way to estimate trajectories than the conventional panel regression approach.
- Intuitively, linear regressions project offspring permanent income on to many highly dependent variables, i.e., parental income each year.
- Our functional data approach, using data-dependent functional principal components, provides a new path to more efficient and precise estimation.

Econometric Methodology

- Comparison of regression model for measure of economic mobility
  - Conventional measure: linear regression, regressor = parental average income at each stage
  - Our measure: functional regression, regressor = trajectory of parental income
- Approach
  1. How to represent function by finite dimensional vector
  2. Choose appropriate orthonormal basis
  3. Represent it by finite number (m number) of orthonormal basis
- Functional regression with different choices of orthonormal basis
  - Conventional regression: functional regression with the scale-adjusted indicator functions over a partition of $[p, q)$ as basis
  - Our regression: functional regression with the functional principal components basis

Functional Regression

- Functional regression model

$$y_i = \int_p^q \beta(r) f_i(r) dr + \varepsilon_i = \langle \beta, f_i \rangle + \varepsilon_i$$

  - y_i: variable to be explained
  - f_i: functional covariate on $[p, q)$
  - β: $\equiv \beta(\cdot)$, functional parameter on $[p, q)$
  - WLOG, both y_i and f_i are demeaned

- Link to conventional regression

  - Partition: $\big([p_j, q_j)\big)$ is a partition of $[p, q)$ for $j = 1, \ldots, m$
  - Define

$$f_{ij} = \frac{1}{q_j - p_j} \int_{p_j}^{q_j} f_i(r) dr \text{ for } j = 1, \ldots, m$$

  - Example: $m = 1$, f_{i1} = average of f_i over $[p, q)$

Finite Dimensional Representation

- Representation and approximation:

  For any $w \in H$, and given orthonormal basis (v_j) for H,

  $$w = \sum_{j=1}^{\infty} \langle v_j, w \rangle v_j \approx \sum_{j=1}^{m} \langle v_j, w \rangle v_j$$

  for an appropriately chosen m

- Mapping: $\pi : H \to \mathbb{R}^m$,

  $$\pi : w \to \begin{pmatrix} \langle v_1, w \rangle \\ \vdots \\ \langle v_m, w \rangle \end{pmatrix}$$

- Isometry: mapping π defines an isometry between H_m and $\mathbb{R}^m$

  H_m: subspace of H spanned by the sub-basis $(v_j)_{j=1}^m$, Π_m: projection on H_m

Finite Dimensional Representation

- Functional regression

$$y_i = \langle \beta, f_i \rangle + \varepsilon_i = \langle \beta, \Pi_m f_i \rangle + \left\langle \beta, (1 - \Pi_m) f_i \right\rangle + \varepsilon_i \approx \langle \beta, \Pi_m f_i \rangle + \varepsilon_i$$

  Approximation error, incurred in representing infinite-dimensional functions by finite-dimensional vectors, $\langle \beta, (1 - \Pi_m) f_i \rangle$, is ignored

$$\langle \beta, \Pi_m f_i \rangle = \left\langle \beta, \sum_{j=1}^{m} \langle v_j, f_i \rangle v_j \right\rangle = \sum_{j=1}^{m} \langle v_j, \beta \rangle \langle v_j, f_i \rangle = (\!|\beta|\!)' (\!|f_i|\!),$$

  where $(\!|w|\!) = \pi(w)$ for all $w \in H$

- Estimation: using the least squares estimator $(\!|\widehat{\beta}|\!)$ of $(\!|\beta|\!)$, obtain an estimate for the functional coefficient $\beta(\cdot)$ by $\widehat{\beta}(\cdot) = \pi^{-1}\left((\!|\widehat{\beta}|\!)\right)$ from

$$y_i \approx (\!|\beta|\!)' (\!|f_i|\!) + \varepsilon_i,$$

Finite Dimensional Representation

- Conventional linear regression = a special case of functional regression
- Orthonormal basis

$$v_j = \frac{1}{\sqrt{q_j - p_j}} 1\{p_j \leqslant r < q_j\} \text{ where } 1\{\cdot\} = \text{indicator function}$$

- Regressors in the conventional linear regression

$$\langle v_j, f_i \rangle = \frac{1}{\sqrt{q_j - p_j}} \int_{p_j}^{q_j} f_i(r) dr,$$

$$f_{ij} = \frac{1}{\sqrt{q_j - p_j}} \langle v_j, f_i \rangle$$

Optimal Basis: Non-Asymptotic Analysis

- Functional principal component basis:
  eigenfunctions of Q corresponding to leading eigenvalues ▷ More

$$Q = \sum_{i=1}^{n} (f_i \otimes f_i)$$

- Optimal properties of functional principal components basis
  - Functional principal components basis approximates functional regressor most effectively by the squared error sense
  - Estimator of $\beta(\cdot)$ using finite number of functional principal components basis is unbiased
  - Bootstrap is asymptotically consistent (Chang, Park, and Pyun (2021))

Functional vs. Conventional Regression

Measures for comparison between $\widehat{\beta}$ in conventional and $\widehat{\beta}^*$ in functional regression

- Functional R-square (FR2): how much variation of (f_i) can be explained when (f_i) is approximated by m basis, $(v_j)_{j=1}^m$ or $(v_j^*)_{j=1}^m$

$$\rho^2 = \frac{\sum_{i=1}^n \left\|\Pi_m f_i\right\|^2}{\sum_{i=1}^n \|f_i\|^2} = \frac{\text{trace}\left(\Pi_m Q \Pi_m\right)}{\text{trace}\, Q}, \quad \rho_*^2 = \frac{\sum_{i=1}^n \left\|\Pi_m^* f_i\right\|^2}{\sum_{i=1}^n \|f_i\|^2} = \frac{\text{trace}\left(\Pi_m^* Q \Pi_m^*\right)}{\text{trace}\, Q}$$

- Integrated variance (IVAR): integrated variance of estimator when m basis, $(v_j)_{j=1}^m$ or $(v_j^*)_{j=1}^m$, are used for estimation

$$\text{IVAR}(\widehat{\beta}) = \sigma^2\, \text{trace}\left(\Pi_m Q \Pi_m\right)^+, \quad \text{IVAR}(\widehat{\beta}^*) = \sigma^2\, \text{trace}\left(\Pi_m^* Q \Pi_m^*\right)^+$$

- Integrated bias squared (IBS): integrated bias squared of estimator when m basis, $(v_j)_{j=1}^m$ or $(v_j^*)_{j=1}^m$, are used for estimation

$$\text{bias}\,(\hat{\beta}) = (\Pi_m Q \Pi_m)^+ \Pi_m Q (1 - \Pi_m)\beta, \quad \text{IBS} = \left\|\text{bias}\,(\hat{\beta})\right\|^2$$

Functional vs. Conventional Regression

	m	1	2	3	4	5	10	20
$\hat{\beta}^*$	FR2	0.61	0.70	**0.77**	0.80	0.83	0.93	1
	IVAR ($\times 10^{-4}$)	1.06	7.59	**17.21**	33.82	53.25	229.53	1133.88
$\hat{\beta}$	FR2	0.59	0.67		**0.76**	0.79	0.88	1
	IVAR ($\times 10^{-4}$)	1.01	9.73		**49.60**	75.70	337.49	1133.88
	IBS ($\times 10^{-5}$)	7.29	10.42		**2.00**	0.82	0.31	0

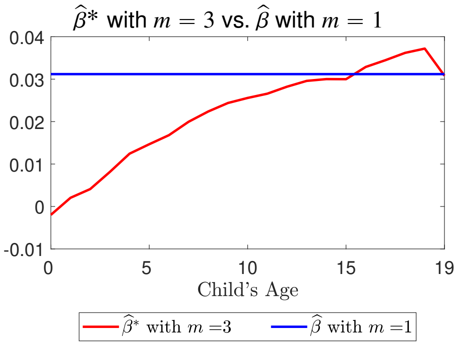
- $\hat{\beta}^*$ with $m = 3$ used as a proxy for unknown functional parameter β to calculate IBS
- $m = 3$ set by a leave-one-out cross-validation method ▷ More
- Our functional regression with $m = 3$ vs. conventional regression with $m = 4$

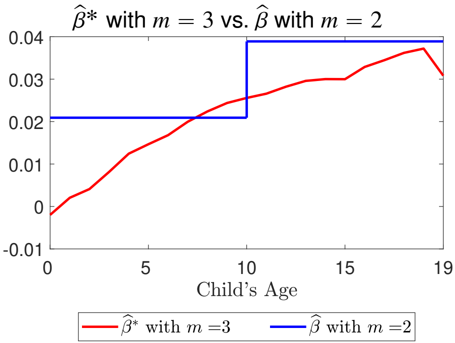
  FR2: 0.77 $\approx$ 0.76

  IVAR ($\times 10^{-4}$): 17.21 $<$ 49.60

  IBS ($\times 10^{-5}$): 0 (unbiased) $<$ 2.00

Functional vs. Conventional Regression



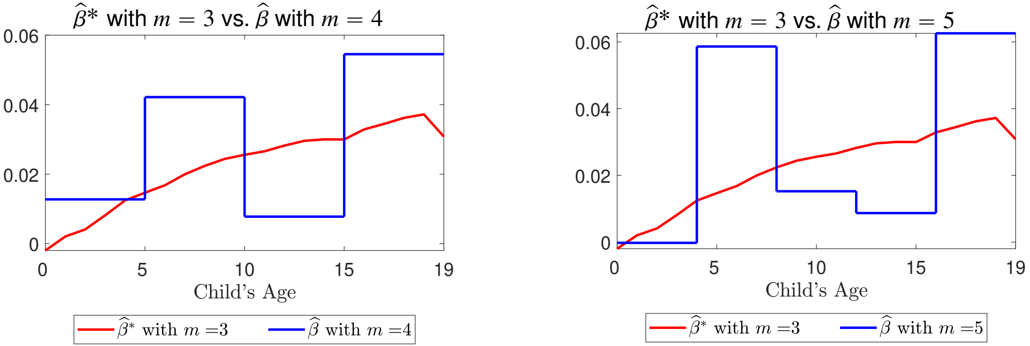


- δ: measures how distinct $\hat{\beta}$ is from $\hat{\beta}^*$

$$\delta(\hat{\beta}, \hat{\beta}^*) = \|\hat{\beta} - \hat{\beta}^*\| / \|\hat{\beta}^*\|$$

0.56 and 0.39 for $m = 1$ and 2, respectively

Functional vs. Conventional Regression

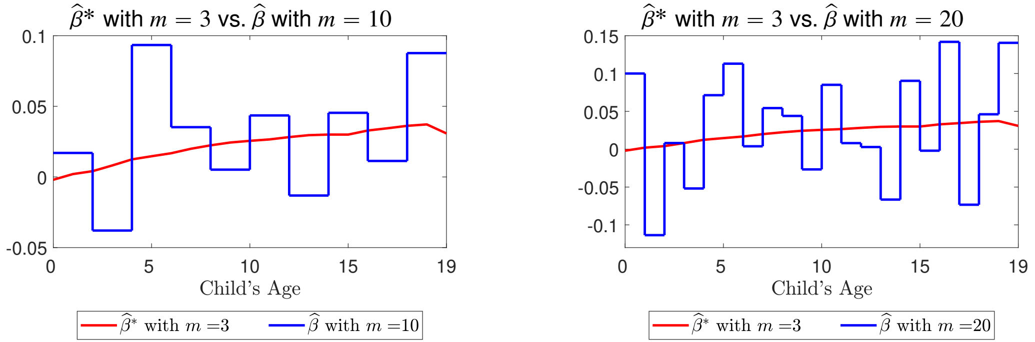


- δ: measures how distinct $\hat{\beta}$ is from $\hat{\beta}^*$

$$\delta(\hat{\beta}, \hat{\beta}^*) = \|\hat{\beta} - \hat{\beta}^*\| / \|\hat{\beta}^*\|$$

0.75 and 1.00 for $m = 4$ and 5, respectively

Functional vs. Conventional Regression



- δ: measures how distinct $\hat{\beta}$ is from $\hat{\beta}^*$

$$\delta(\hat{\beta}, \hat{\beta}^*) = \|\hat{\beta} - \hat{\beta}^*\| / \|\hat{\beta}^*\|$$

  1.54 and 2.72 for $m = 10$ and 20, respectively

- Unstable and nonsensical estimates for $m = 10$ and 20

Intergenerational Mobility: From Regressions to Functional Relations

Data

- Data source: PSID family file + cross-year individual file 1968-2015
- Our sample: 1967-1977 birth cohorts ($n = 817$ families)
- Definition of family income: head's income + spouse's income
- Method

  For each year, from 1967 to 1977,

  1. Construct a birth cohort: collect families who have a newborn baby in that year
  2. Define (f_i): track parental income for twenty years (from child's birth to age 19) ▷ More
  3. Define (y_i): track a child's adult income for six years (from child's age 30 to 35)
- Income: adjusted in 2010 USD using CPI-U
- Control variables: age, race, region, education, family structure, and occupation ▷ More

Intergenerational Mobility Curves

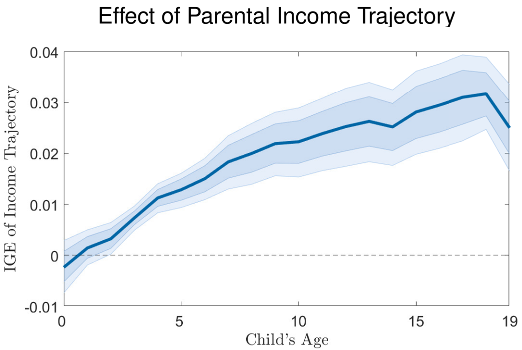
- Functional Mobility Relations: measure mobility by mapping parental income trajectory to children's permanent income

- Functional regression model

$$y_i = \alpha + \int_0^{20} \beta(r) f_i(r) dr + \epsilon_i$$

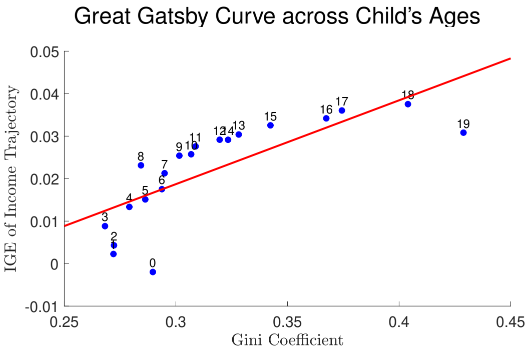
  - y_i: average log income of child
  - $f_i(r)$: a trajectory of parental log income across child's age $r \in [0, 20)$, covering the first 20 years of the child's life
  - $\beta(r)$: age-varying IGE (Intergenerational elasticity of income), i.e., mobility curve
    - Measures the age-specific association between parental income trajectory and offspring's permanent income

Intergenerational Mobility Curve



- $\widehat{\beta}$: monotonically increase up to child age 18 and then decreases slightly
- More productive and efficient in forming children's human capital in mid-to-late adolescence ▷ More
- Functional IGE $\beta(\cdot)$ is comparable to the scalar β by taking the integral, $\int_0^{20} \hat{\beta}(r)dr = 0.3653$
- Darker (lighter) shaded areas: 68% (90%) bootstrap confidence intervals

Life Cycle Great Gatsby Curve



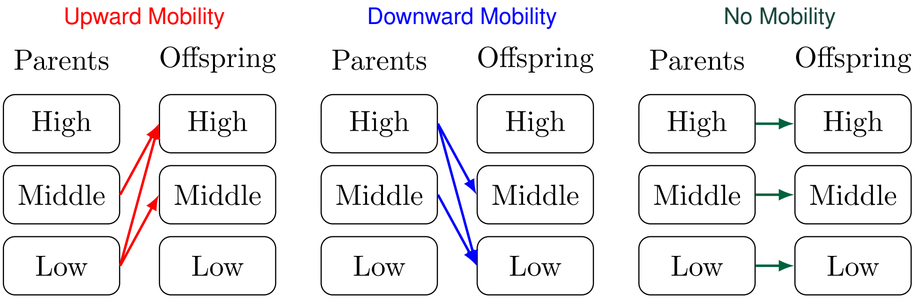
- Micro-level evidence: this type of Gatsby curve is new, not observed before. This suggests the age composition of populations can generate an aggregate Gatsby curve across different countries
- The relationship reflects rising inequality among parents at different ages
- Possible evidence of nonlinearity
- Possible evidence of mechanisms such as income segregation

Linking Parental Income Trajectories to Mobility

Characterizing Sets of Mobility Trajectories

- Essential implication of our findings: offspring outcomes depend on more than the average parental income across the life course, non-constant IGE function
- Beyond average income, which features of parental income trajectories play an important role in mobility?
- Our approach: explore common characteristics present in the shapes of parental income trajectories of the families which have experienced a generational upward economic mobility and also a downward mobility
- Any meaningful distinction between the shapes of the parental income trajectories from these two groups of families associated with upward mobility and downward mobility beyond the means of the trajectories? Which shapes make successes more likely?

Upward/Downward Mobility



To define upward mobility, downward mobility, and no mobility

- Split both parents and children into three income groups (low, middle, high) by their time average incomes.
- Track transition from each parental income group to each of their offsprings' income group
- Define Upward/Downward/No mobility if their offspring's income belongs to a higher/lower/same income group

Upward/Downward Mobility

To characterize the parental income trajectory related to upward (downward, no) mobility

- Define three sets of parental income trajectories, S_p^U, S_p^D, and S_p^N, which are associated respectively with upward, downward and no mobilities as

$$S_p^U (S_p^D, S_p^N) = \{\text{Parental income trajectory such that family (parents and child) exhibits upward (downward, no) income mobility with } p\% \text{ probability}\}$$

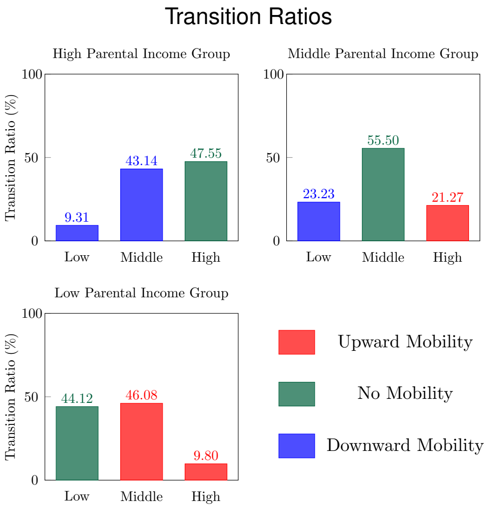
- Define three binary random variables z_i^U, z_i^D, and z_i^N as

$$z_i^U (z_i^D, z_i^N) = \begin{cases} 1 & \text{if family } i \text{ exhibits upward (downward, no) mobility} \\ 0 & \text{otherwise} \end{cases}$$

Upward/Downward Mobility

- Using each of these binary variables, fit a weighted functional logit model using the parental income trajectory for upward, downward and no mobility cases separately.
- Derive the predicted probability of the upward (downward, no) mobility for each family i.
- If the predicted probability of upward (downward, no) mobility is greater than a prescribed probability level p, then include child i's parental income trajectory in the set S_p^U (S_p^D, S_p^N).
- P is set at 50% for our baseline analysis

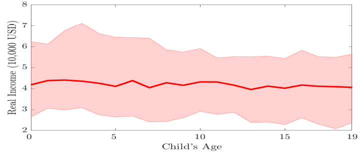
Upward/Downward Mobility



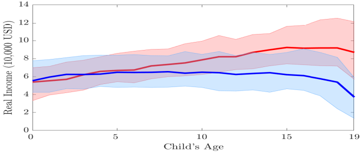
- Transition ratio from high income parents to high income children: high at 47%
- Transition ratio from low income parents to the income children: 44% of the children from the poorest parents belong to the lowest income group
- Wealthiness and poverty tends to pass on to next generation: future income of a child at top as well as bottom of the income distn is closely related to their parental incomes

Upward/Downward Mobility

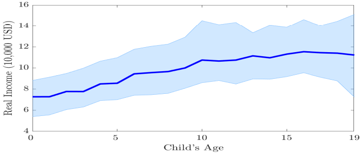
Income Profiles of Parents in Low Group



Income Profiles of Parents in Middle Group



Income Profiles of Parents in High Group



- Evidence may speak to role of aspirations in affecting mobility
- Statistics for Trajectories Producing Upward/Downward Mobility

		Parents Income Group		
		High	Middle	Low
Upward	Average ($)		78,789	43,697
	Variance ($\times 10^8$)		8.06	3.38
	Growth Rate (%)		2.91	-0.36
Downward	Average ($)	102,856	64,111	
	Variance ($\times 10^8$)	12.28	6.22	
	Growth Rate (%)	1.81	-2.83	

- For middle 50%, while mean incomes differ by about $14,000, growth rates differ even more dramatically

Measurement Error

Measurement Error in Permanent Parental Incomes

- Natural interpretation questions involve measurement error in the parental income series.
- The classic rationale for projecting offspring permanent income against parental permanent income is that transitory parental income fluctuations constitute measurement error relative to the permanent component which is the source of parental influences.
- From this perspective, the estimates of the functional regression model

$$y_i = \alpha + \int_p^q \beta(r) f_i(r) dr + \varepsilon_i,$$

  could be subject to bias.
- We take an approach to model the permanent income/transitory income distinction explicitly and evaluate the robustness of our findings.
- Permanent/transitory distinction is made by Beveridge-Nelson (BN) decomposition to extract permanent/transitory components of observed parental incomes.

BN Decomposition of Observed Parental Income

Let (w_{it}) denote the observed parental income in log for family i at time t, the BN procedure equates the permanent component of the income series to the long term forecast of the series given its history.

For the actual BN decomposition, following Morley (2002), we set

$$\Delta w_{it} - \mu_i = \phi_i(\Delta w_{i,t-1} - \mu_i) + \epsilon_{it},$$

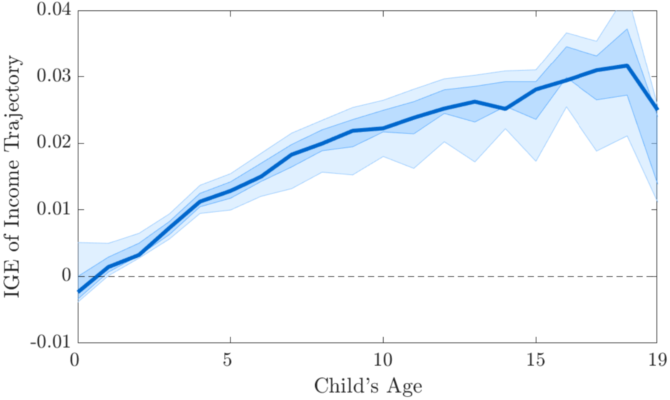
from which it follows that

$$\begin{aligned} \tau_{it} &= w_{it} + \frac{\phi_i}{1-\phi_i}(\Delta w_{it} - \mu_i) \\ c_{it} &= w_{it} - \tau_{it} = -\frac{\phi_i}{1-\phi_i}(\Delta w_{it} - \mu_i). \end{aligned}$$

We estimate the parameters μ_i and ϕ_i for each i using (w_{it}) for $t = 0, \ldots, 19$.

Our Approach and IGE Curve with Permanent Parental Income

Redefine the log parental income trajectory (f_i) by the estimated permanent component, (τ_{it}), in place of the observed income (w_{it}) for each i, and re-estimate the IGE curve β using redefined parental income trajectory. This makes little difference.



Potential Endogeneity and Functional IV Estimation

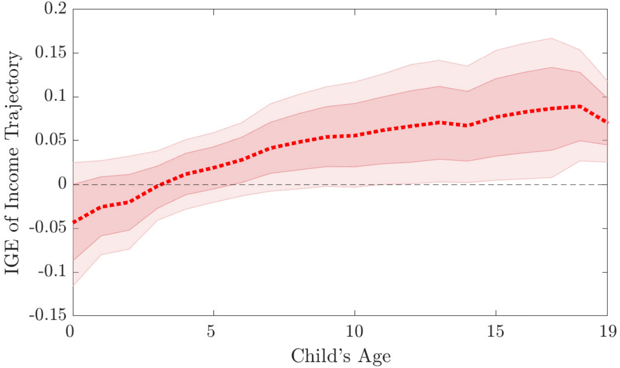
Functional IV Estimation

- Unobserved heterogeneity: may cause correlation between regressors and residuals, which leads to biased estimate
- Starategy: Lefgren, Sims and Lindquist (2012)

1. Decompose the intergenerational income elasticity into the effect of financial resources and the transmission of human capital
2. Focus only on the effect of financial resources
3. IV: Luck (shocks to employment status)
   Residuals to employment status after controlling for past education and earnings history
4. Functional instrumental variable: trajectories of employment status residuals for 19 years

Functional IV Estimates

- Same qualitative shape as its functional OLS counterpart: monotonic increases in the curve until age 18



Quadratic Mobility Models

Quadratic Effects of Parental Income Trajectory

- Dynamic complementarity: complementarities between investments in skills at different ages, one of essential finding in modern skill formation model

- Functional regression model with quadratic effects

$$y_i = \alpha + \int_0^{20} \beta(r) f_i(r) dr + \int_0^{20} \int_0^{20} \Gamma(r,s) f_i(r) f_i(s) ds dr + \varepsilon_i$$

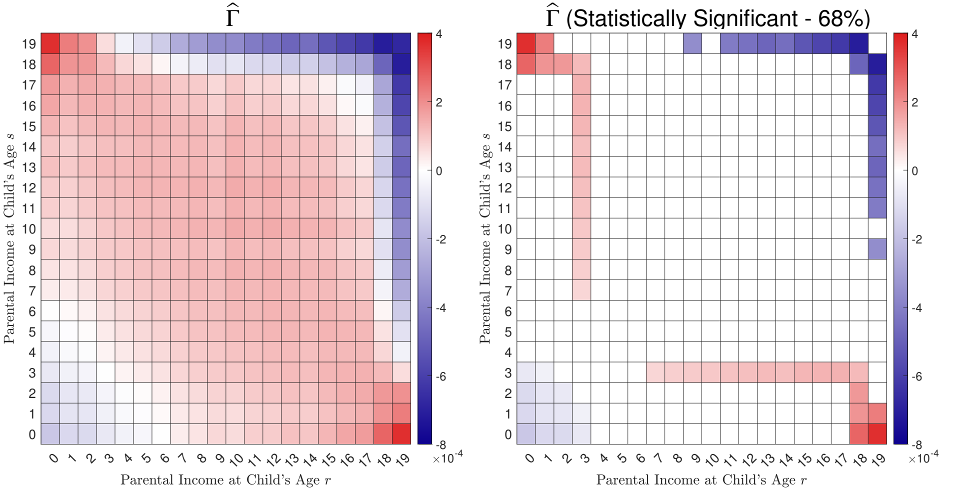
  - y_i: average log income of child
  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life

- Dynamic complementarity (substitutability)

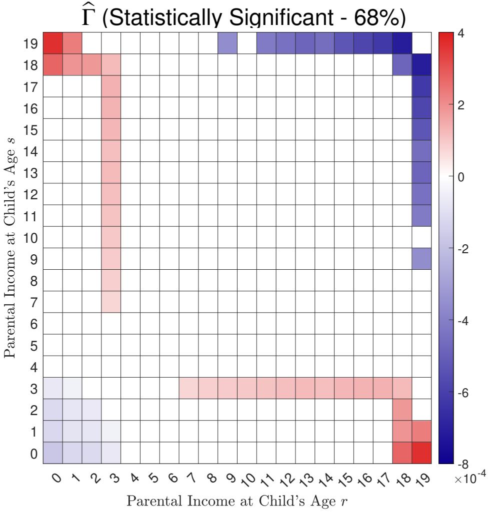
$$\frac{\partial^2 y_i}{\partial f_i(r) \partial f_i(s)} = \Gamma(r,s) > 0 \quad (\Gamma(r,s) < 0)$$

Quadratic Effects of Parental Income Trajectory

- $\widehat{\Gamma}$: $\widehat{\Gamma}(r,s) > 0$, $\widehat{\Gamma}(r,s) < 0$



Dynamic Complementarity and Substitutabilitty



- Dynamic complementarity:
  observed at top-left and bottom-right corners,
  evidence of dynamic complementarity between early and late childhood
- Dynamic substitutability:
  observed at top-right and bottom-left corners,
  evidence of dynamic substitutability within early and late childhood

Quadratic Effects of Parental Income Trajectory

- $\widehat{\beta}$: robust to quadratic effects
- IGE under the presence of quadratic effects

$$\frac{\partial y_i}{\partial f_i(r)} = \beta(r) + 2\int_p^q \Gamma(r,s) f_i(s)ds$$

depends on whole parental income trajectory f_i

- Decomposition of IGE at child's age r
  - Linear effect:

$$\beta(r)$$

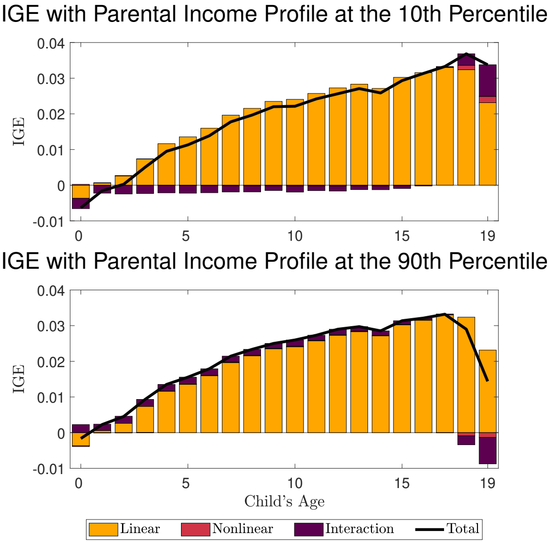
  - Nonlinear effect:

$$2\int_{r-1/2}^{r+1/2} \Gamma(r,s) f_i(s)ds$$

  - Interaction effect:

$$2\int_{[0,20)\setminus[r-1/2,r+1/2)} \Gamma(r,s) f_i(s)ds$$

Quadratic Effects of Parental Income Trajectory



- Poorer families (upper panel): parental influence on children increases during late teens
- Richer families (lower panel): influence on children rises until child's age 16, then falls sharply for age 18 and 19
- Scalar measure of IGE: $\int_0^{20} (\partial y_i / \partial f_i(r)) dr$

  poorer families=0.3810

  richer families=0.4119

Family Structure Trajectory

Joint Linear Model

- Measures mobility by mapping both parental income and family structure trajectories to children's permanent income

- Functional joint linear model

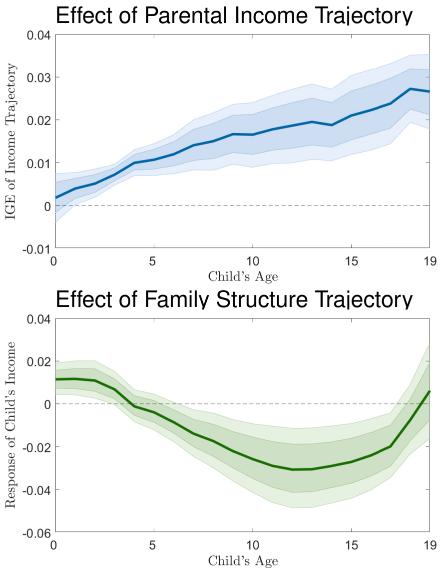
$$y_i = \alpha + \int_0^{20} \beta(r) f_i(r) dr + \int_0^{20} \gamma(r) g_i(r) dr + \varepsilon_i$$

  - y_i: average log income of child
  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $\beta(\cdot)$: IGE over the first 20 years of the child's life
  - $g_i(\cdot)$: trajectory of family structure over the first 20 years of the child's life

$$g_i(r) = \begin{cases} 1 & \text{if child } i \text{ lives in a single-parent family at age } r \\ 0 & \text{if child } i \text{ lives in a two-parent family at age } r \end{cases} \quad \text{for } r \in [0, 20)$$

  - $\gamma(\cdot)$: measures the association between a trajectory of family structure and the child's income

Trajectories of Family Influences and Economic Mobility



- Common causes of single-parent families: divorce or separation
- $\widehat{\gamma}$ (lower panel): negative for most of the ages, which implies that living in a single-parent family impedes child's human capital formation and accumulation
- Strong negative effect when the child is in the teens: a child in this period tends to be sensitive, so may be more affected a lot by the changes to family structure

Interaction Between Parental Income Trajectory and Family Structure Trajectory

Joint Quadratic Model

- Dynamic complementarity: complementarities between investments in skills at different ages, one of essential finding in modern skill formation model

- Functional joint quadratic model with interaction effects

$$y_i = \alpha + \int_0^{20} \beta(r) f_i(r) dr + \int_0^{20} \gamma(r) g_i(r) dr + \int_0^{20} \int_0^{20} \Delta(r, s) f_i(r) g_i(s) ds dr + \varepsilon_i$$

  - y_i: average log income of child
  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $g_i(\cdot)$: trajectory of family structure over the first 20 years of the child's life

$$g_i(r) = \begin{cases} 1 & \text{if child } i \text{ lives in a single-parent family at age } r \\ 0 & \text{if child } i \text{ lives in a two-parent family at age } r \end{cases} \quad \text{for } r \in [0, 20)$$

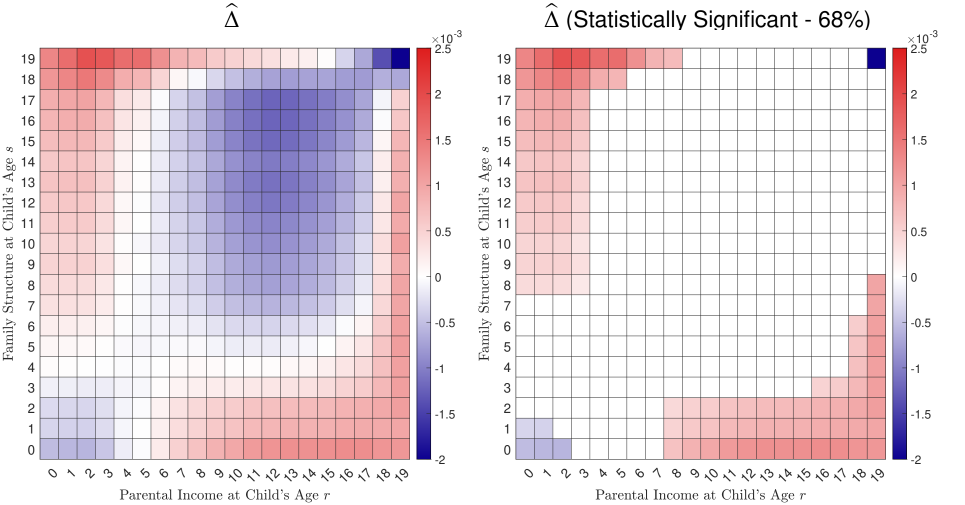
Interaction of Parental Income and Family Structure Trajectories

- Characteristics of modern skill formation model (Heckman and Mosso (2014)): multiple periods of childhood, multiple forms of investment, multiple skills
- Time investment: family structure can be viewed as proxy for parental time investment amount of time children live with both parents is important for mobility (Bloome (2017))
- Interaction effects measure dynamic association between different forms of parental investment
- $g_i = 1$: more time investment (two-parent family)
  $g_i = 0$: less time investment (single-parent family)
- Dynamic complementarity (substitutability)

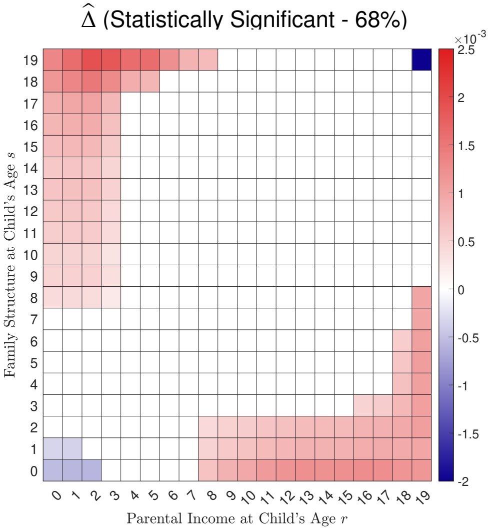
$$\frac{\partial^2 y_i}{\partial f_i(r) \partial g_i(s)} = \Delta(r,s) > 0 \quad (\Delta(r,s) < 0)$$

Interaction of Parental Income and Family Structure Trajectories

- $\widehat{\Delta}$: $\widehat{\Delta}(r,s) > 0$, $\widehat{\Delta}(r,s) < 0$



Dynamic Complementarity and Substitutability



- Dynamic complementarity:
  observed at top-left and bottom-right corners,
  evidence of dynamic complementarity **between early and mid-to-late** childhood
- Dynamic substitutability:
  observed at top-right and bottom-left corners,
  evidence of dynamic substitutability **within early and late** childhood

Interaction of Parental Income and Family Structure Trajectories

- $\widehat{\beta}$ and $\widehat{\gamma}$: robust to interaction effects

- IGE under the presence of interaction effects

$$\frac{\partial y_i}{\partial f_i(r)} = \beta(r) + \int_0^{20} \Delta(r,s) g_i(s) ds$$

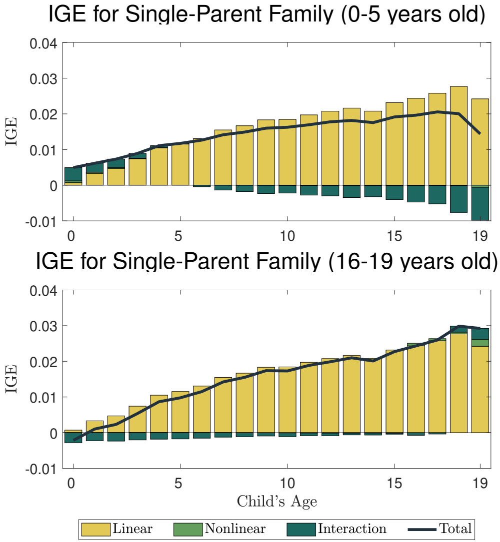
depends on whole family structure trajectory g_i

- Effect of family structure under the presence of interaction effects

$$\frac{\partial y_i}{\partial g_i(s)} = \gamma(s) + \int_0^{20} \Delta(r,s) f_i(r) dr$$

depends on whole parental income trajectory f_i

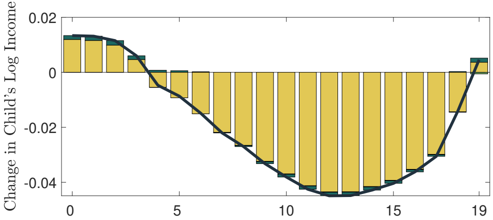
Interaction of Parental Income and Family Structure Trajectories



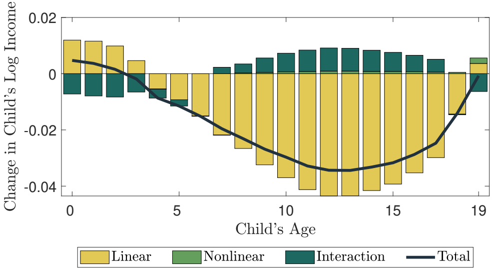
- Under the period of single-parent (two-parent) family, IGE tends to increase (decrease)
- Single-parent family: investment in this period is more efficient, additional investment may effective to resilience of children

Interaction of Parental Income and Family Structure Trajectories

Effect of Single-Parent Family with Parental Income at 10th Percentile



Effect of Single-Parent Family with Parental Income at 90th Percentile



- No change (upper panel) in the effect of single-parent family on permanent income
- Children from richer family (lower panel) less affected by single-parent family structure

Proximate Mechanisms

Mechanisms Underlying Intergenerational Economic Mobility

- Our results: mid-to-late adolescence is crucial for forming human capital and determining income levels
- What drives mid-to-late adolescence as sensitive periods?
- Our viewpoint:
  1. One's income is a realization of accumulated human capital
  2. Educational attainment and occupation are intermediate outcomes of accumulated human capital
- Possible mechanism:

  parental influence in mid-to-late adolescence $\Rightarrow$ offspring's education, occupation $\Rightarrow$ offspring's income

Education

Parental Income and Offspring's Educational Attainment

- Measures association between parental income trajectory and children's college attendance

- Functional logit model

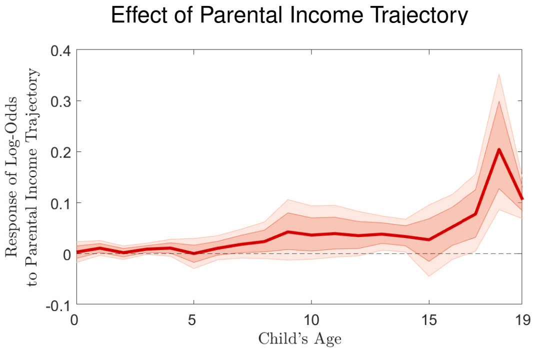
$$\log \frac{p_i}{1-p_i} = \omega + \int_0^{20} \mu(r) f_i(r)\, dr$$

  - p_i: $\mathbb{P}(z_i = 1)$ where

$$z_i = \begin{cases} 1 & \text{if the child } i \text{ attends college} \\ 0 & \text{otherwise} \end{cases}$$

  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $\mu(\cdot)$: measures the association between a trajectory of parental income and log-odds of college attendance
    Positive μ: positive relationship between parental income and the probability of offspring's college attendance

Parental Income and Offspring's Educational Attainment



- $\widehat{\mu}$: positive at all child's ages
- Positive relationship between parental income and the probability of offspring attending college at all ages
- Parental income in mid-to-late adolescence is more critical for the offspring's college attendance
- Darker (lighter) shaded areas: 68% (90%) bootstrap confidence intervals

Trajectories of Family Influences and Offspring's Educational Attainment

- Measures associations between parental income, family structure trajectories and children's college attendance
- Extended Functional logit model

$$\log \frac{p_i}{1-p_i} = \omega + \int_0^{20} \mu(r) f_i(r) dr + \int_0^{20} \nu(r) g_i(r) dr + \varepsilon_i$$

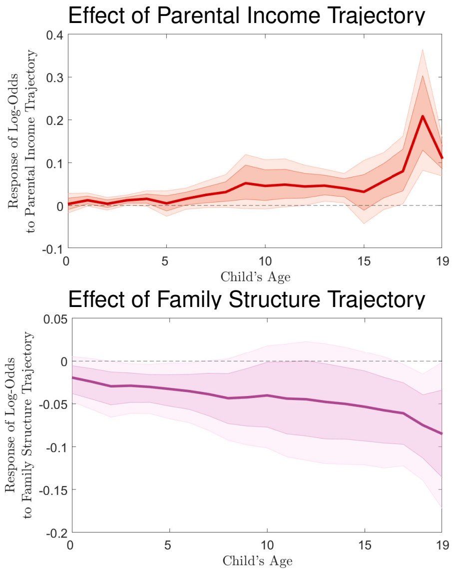
  - p_i: $\mathbb{P}(z_i = 1)$ where

$$z_i = \begin{cases} 1 & \text{if the child } i \text{ attends college} \\ 0 & \text{otherwise} \end{cases}$$

  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $g_i(\cdot)$: trajectory of family structure over the first 20 years of the child's life

$$g_i(r) = \begin{cases} 1 & \text{if child } i \text{ lives in a single-parent family at age } r \\ 0 & \text{if child } i \text{ lives in a two-parent family at age } r \end{cases} \quad \text{for } r \in [0, 20)$$

Trajectories of Family Influences and Offspring's Educational Attainment



- Common causes of single-parent families: divorce or separation
- $\hat{\nu}$ (lower panel): negative at all ages, which implies that living in a single-parent family impedes child's college attendance
- Negative effect is stronger when the child is in the teens: a child in this period tends to be sensitive, so may be more affected a lot by the changes to family structure
- Darker (lighter) shaded areas: 68% (90%) bootstrap confidence intervals

Occupation

Parental Income and Offspring's Occupational Choices

- Measures association between parental income trajectory and children's occupational choices
- Occupational types (Long and Ferrie (2013))

White Collar (Type 1)	Professional, technical and kindred workers; Managers, officials and proprietors; Clerical and sales workers
Skilled/Semiskilled (Type 2)	Craftsmen, foremen, and kindred workers; Operatives and kindred workers
Unskilled (Type 3)	Laborers and service workers; Farm laborers

- Limit our focus on pairs of father and son, following the standard approach in intergenerational occupational mobility

Parental Income and Offspring's Occupational Choices

- Measures association between parental income trajectory and children's occupational choices

- Functional multinomial logit model

$$\log\left(\frac{\mathbb{P}\{z_i = k\}}{\mathbb{P}\{z_i = 3\}}\right) = \tau_k + \int_0^{20} \mu_k(r) f_i(r) dr \text{ for } k = 1, 2$$

  - k: son's occupational type, $k = 1, 2, 3$
  - z_i: qualitative variable of son's occupational type

$$z_i = \begin{cases} 1 & \text{if son } i\text{'s job belongs to Type 1 (White Collar)} \\ 2 & \text{if son } i\text{'s job belongs to Type 2 (Skilled/Semiskilled)} \\ 3 & \text{if son } i\text{'s job belongs to Type 3 (Unskilled)} \end{cases}$$

  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life

Parental Income and Offspring's Occupational Choices

- Interpretation of μ_k not straightforward, so present the marginal effects instead.
- Computing the marginal effects of parental income at child's age r:
  1. Calculate the son's occupational choice probabilities

     $$\mathbb{P}(z_i = k) = \frac{\phi_k}{\phi_1 + \phi_2 + \phi_3} \text{ for } k = 1, 2, 3$$

     where

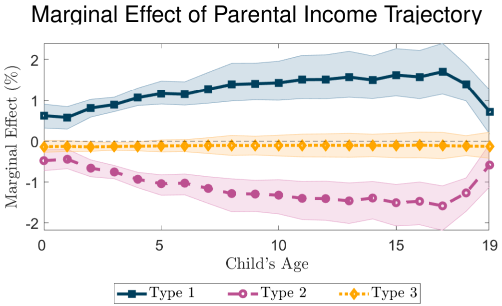
     $$\phi_k = \begin{cases} \exp\left(\tau_k + \int_0^{20} \mu_k(r) f_i(r) dr\right) & \text{for } k = 1, 2 \\ 1 & \text{for } k = 3. \end{cases}$$

  2. Obtain the marginal effect by taking the derivative of $\mathbb{P}(z_i = k)$ with respect to $f_i(r)$

     $$\frac{\partial \mathbb{P}(z_i = k)}{\partial f_i(s)} = \mathbb{P}(z_i = k)\left(\mu_k(s) - \sum_{j=1}^{3} \mathbb{P}(z_i = j)\mu_j(s)\right) \text{ for } k = 1, 2, 3$$

     where $\mu_3 = 0$.

Parental Income and Offspring's Occupational Choices



- Marginal effects: similar for all ages, but the marginal effect at the teens is more substantial than the effect during early childhood
- Parental income and the probability of having a Type 1 (Type 2) occupation are positively (negatively) related
- Son from a wealthy family is more likely to get a job in Type 1 which includes mostly professional occupations
- Shaded areas: 68% bootstrap confidence intervals

Trajectories of Family Influences and Offspring's Occupational Choices

- Measures association of children's occupational choices with parental income trajectory and family structure trajectory
- Functional multinomial logit model

$$\log\left(\frac{\mathbb{P}\{z_i = k\}}{\mathbb{P}\{z_i = 3\}}\right) = \tau_k + \int_0^{20} \mu_k(r) f_i(r) dr + \int_0^{20} \nu_k(r) g_i(r) dr \text{ for } k = 1, 2$$

  - k: son's occupational type, $k = 1, 2, 3$
  - z_i: qualitative variable of son's occupational type

$$z_i = \begin{cases} 1 & \text{if son } i\text{'s job belongs to Type 1 (White Collar)} \\ 2 & \text{if son } i\text{'s job belongs to Type 2 (Skilled/Semiskilled)} \\ 3 & \text{if son } i\text{'s job belongs to Type 3 (Unskilled)} \end{cases}$$

  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $g_i(\cdot)$: trajectory of family structure over the first 20 years of the child's life

$$g_i(r) = \begin{cases} 1 & \text{if child } i \text{ lives in a single-parent family at age } r \\ 0 & \text{if child } i \text{ lives in a two-parent family at age } r \end{cases} \text{ for } r \in [0, 20)$$

Effects of Family Influences on Offspring's Occupational Choices

1. Calculate the son's occupational choice probabilities

$$\mathbb{P}(z_i = k) = \frac{\varphi_k}{\varphi_1 + \varphi_2 + \varphi_3} \text{ for } k = 1, 2, 3$$

   where

$$\varphi_k = \begin{cases} \exp\left(\tau_k + \int_0^{20} \mu_k(r) f_i(r) dr + \int_0^{20} \nu_k(r) g_i(r) dr\right) & \text{for } k = 1, 2 \\ 1 & \text{for } k = 3 \end{cases}$$

2. Define the marginal effect of family structure at child's age s on the probability of son having job in occupation type k

$$\mathbb{P}\left\{z_i = k \middle| f_i = \bar{f}, g_i = \delta_s\right\} - \mathbb{P}\left\{z_i = k \middle| f_i = \bar{f}, g_i = 0\right\} \text{ for } k = 1, 2, 3$$

   $\delta_s(\cdot)$ represents a hypothetical family structure trajectory

$$\delta_s(r) = \begin{cases} 1 & \text{if } r = s \text{ (single-parent family at child's age } s\text{)} \\ 0 & \text{otherwise} \end{cases}$$

Duration of Family Structure and Offspring's Occupational Choices

We also consider how duration of family structure is associated with son's occupational choice probabilities.

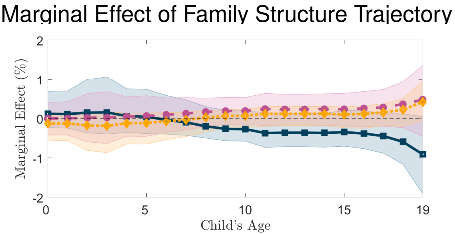
- For this, we calculate the following probabilities

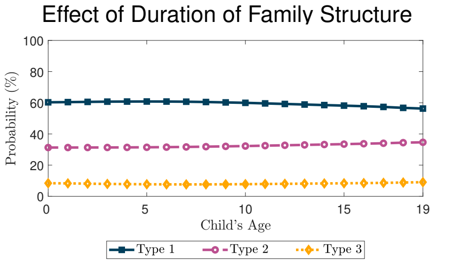
$$\mathbb{P}\left\{z_i = k \,\middle|\, f_i = \bar{f}, g_i = 1_{r \leqslant s}\right\}$$

for son's job type $k = 1, 2, 3$ and age $s = 0, \ldots, 19$, where

$$1_{r \leqslant s}(r) = \begin{cases} 1 & \text{if } r \leqslant s \text{ (single-parent family up to child's age } s\text{)} \\ 0 & \text{otherwise} \end{cases}$$

Duration of Family Structure and Offspring's Occupational Choices





- Common causes of single-parent families: divorce or separation
- Marginal effect: negative relationship between living with only one parent and a son's probability of having a Type 1 job, stronger (but not huge) effect during his teens
- Occupational choice probabilities: 0 year → 20 years single-parent
  Type 1: 60.31% → 56.28% (-4.03%)
  Type 2: 31.28% → 34.65% (+3.37%)
  Type 3: 8.41% → 9.07% (+0.66%)
- Shaded areas: 68% bootstrap confidence intervals

Intergenerational Occupational Mobility

- Measures association of children's occupational choices with parental income trajectory and father's occupational type trajectories
- Functional multinomial logit model

$$\log\left(\frac{\mathbb{P}\{z_i = k\}}{\mathbb{P}\{z_i = 3\}}\right) = \tau_k^j + \int_0^{20} \mu_k^j(r) f_i(r) dr + \int_0^{20} \nu_k^j(r) g_i^j(r) dr \text{ for } j = 1,2,3, k = 1,2$$

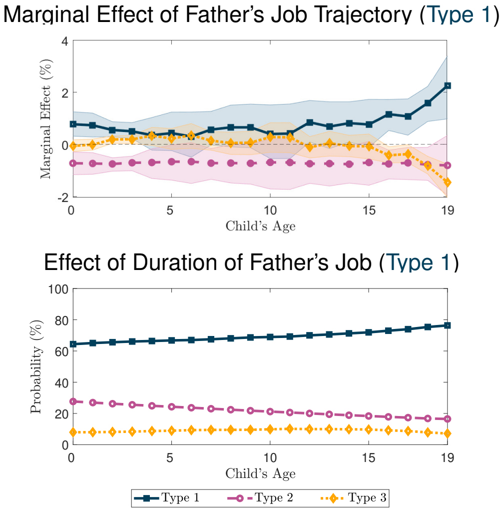
  - j (k): father's (son's) occupational type, $j, k = 1, 2, 3$
  - z_i: qualitative variable of son's occupational type

$$z_i = \begin{cases} 1 & \text{if son } i\text{'s job belongs to Type 1 (White Collar)} \\ 2 & \text{if son } i\text{'s job belongs to Type 2 (Skilled/Semiskilled)} \\ 3 & \text{if son } i\text{'s job belongs to Type 3 (Unskilled)} \end{cases}$$

  - $f_i(\cdot)$: trajectory of parental log income over the first 20 years of the child's life
  - $g_i^j(\cdot)$: trajectory of whether father's belongs to Type j over the first 20 years of the child's life

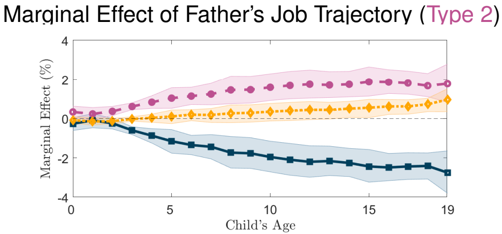
$$g_i^j(r) = \begin{cases} 1 & \text{if occupation of } i\text{'s father pertains to Type } j \text{ at child's age } r \\ 0 & \text{otherwise} \end{cases}$$

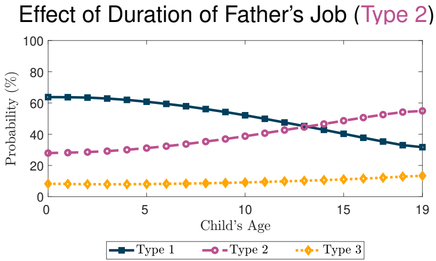
Intergenerational Occupational Mobility - Type 1



- Marginal effect: positive relationship between father's job in Type 1 and a son's probability of having a Type 1 job, more prominent effect after the mid-teens
- Occupational choice probabilities:
  0 year → 20 years Type 1 father
  Type 1: 63.94% → 75.85% (+11.91%)
  Type 2: 27.71% → 16.53% (-11.18%)
  Type 3: 8.35% → 7.62% (-0.73%)
- Shaded areas: 68% bootstrap confidence intervals

Intergenerational Occupational Mobility - Type 2

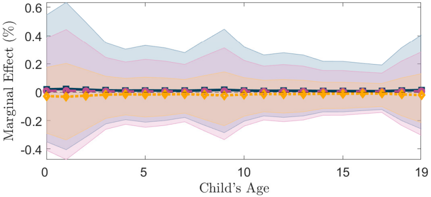




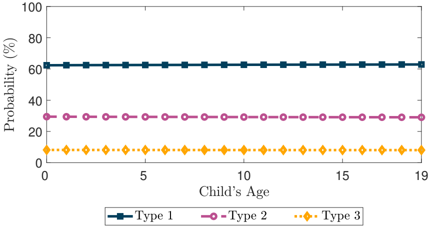
- Marginal effect: positive relationship between father's job in Type 2 and a son's probability of having a Type 2 job, more prominent effect at the teens
- Occupational choice probabilities:
  0 year → 20 years Type 2 father
  Type 1: 63.31% → 31.05% (-32.26%)
  Type 2: 28.03% → 55.19% (+27.16%)
  Type 3: 8.66% → 13.76% (+5.10%)
- Shaded areas: 68% bootstrap confidence intervals

Intergenerational Occupational Mobility - Type 3

Marginal Effect of Father's Job Trajectory (Type 3)



Effect of Duration of Father's Job (Type 3)



- Marginal effect: no distinct relationship between father's job in Type 3 and a son's probability of having a Type 3 job
- Occupational choice probabilities:
  0 year → 20 years Type 3 father
  Type 1: 61.88% → 62.09% (+0.21%)
  Type 2: 29.57% → 29.66% (+0.09%)
  Type 3: 8.55% → 8.25% (-0.30%)
- Shaded areas: 68% bootstrap confidence intervals

Conclusion

Conclusion

1. The conventional permanent income approach loses information about mobility
   - Age-varying IGE assigns larger weights to middle and late adolescent incomes than earlier
   - Parental income trajectories that are rising in the later years become to the fore in upward mobility
2. Family structure dynamics play a distinct role in income mobility
   - Single parent households in mid teen years are associated with lower future income
   - Inadequacy of treating incomes as sufficient statistic for family influences
3. Investigate how education and occupation are affected by family income and family structure
   - Mid to late adolescent years receive greater weight than others to explain offspring's education and occupation
   - Combined with the income results, family inputs in the later years are relatively important for mobility
4. Further discussion
   - What parents do with income?
   - Why are parental influences in the later years are more important?

Appendix

Leave-One-Out Cross-Validation

- Model

$$y_i = \int_p^q \beta(r) f_i(r) dr + \varepsilon_i$$

- LOOCV error: for $i = 1 \cdots, n$, do the followings:
  1. Fit the model without i-th observation, and obtain estimates $\widehat{\beta}_{-i}$.
  2. Calculate the fitted value of i-th observation, $\widehat{y}_i$,

$$\widehat{y}_i = \int_p^q \widehat{\beta}_{-i}(r) f_i(r) dr$$

  3. Measure the squared difference between actual value y_i and fitted value $\widehat{y}_i$

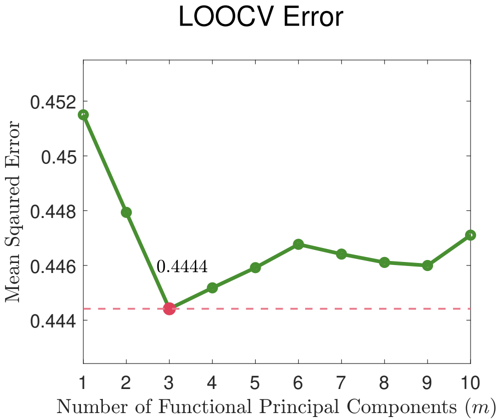
$$L(y_i, \widehat{y}_i) = (y_i - \widehat{y}_i)^2$$

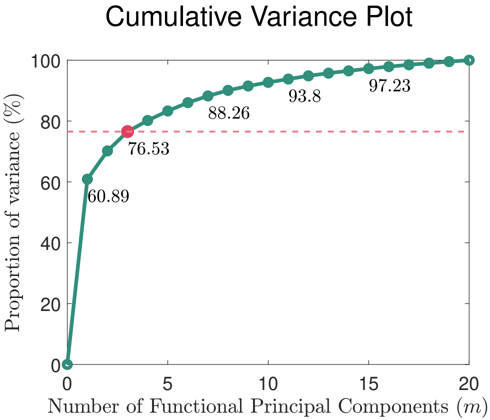
  4. Obtain the average of $L(y_i, \widehat{y}_i)$.

$$L_m = \frac{1}{n} \sum_{i=1}^{n} L(y_i, \widehat{y}_i)$$

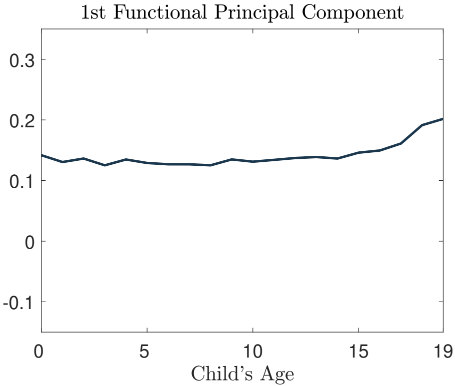
- Calculate LOOCV error for each number of functional principal component $m = 1, \ldots, M$, and find m^* via minimization of LOOCV error

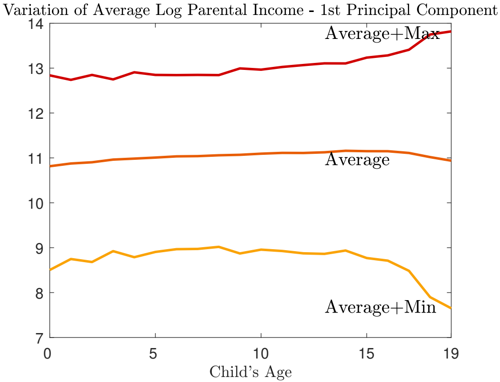
Leave-One-Out Cross-Validation



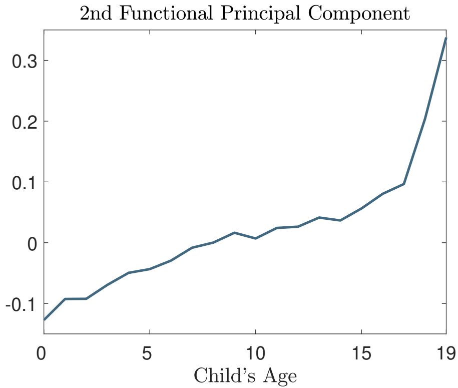


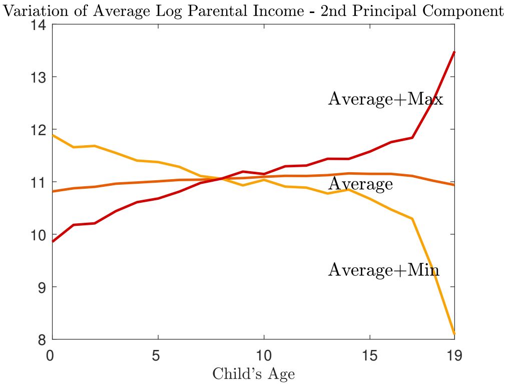
1st Functional Principal Components



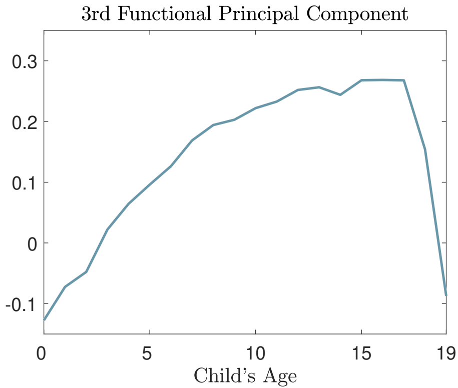


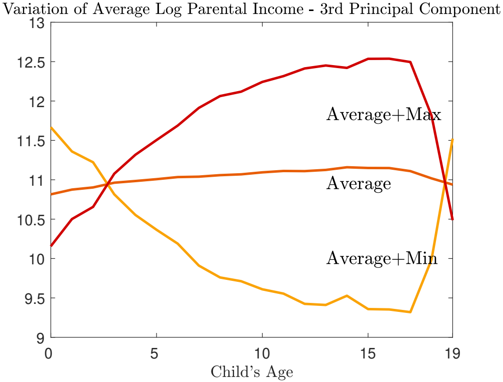
2nd Functional Principal Components





3rd Functional Principal Components





Connection to Karhunen-Loeve Expansion

If $(f_i \otimes f_i)$ has a common expectation for $i = 1, 2, \ldots$, we would expect

$$\frac{1}{n} Q = \frac{1}{n} \sum_{i=1}^{n} (f_i \otimes f_i) \to_p \mathbb{E}(f_i \otimes f_i)$$

under suitable regularity conditions, where the limit becomes the variance operator of (f_i) when (f_i) is of mean zero. Therefore, Q may be viewed as the (unnormalized) sample variance.

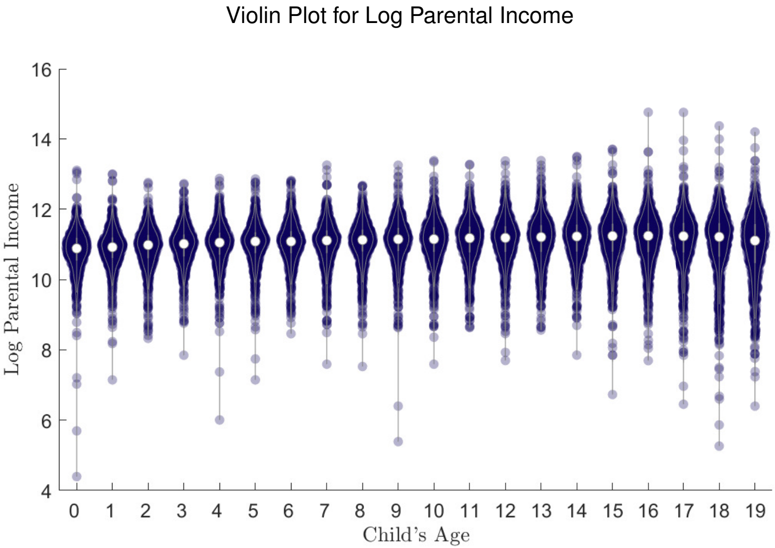
If the variance operator of (f_i) is known, we may use its eigenfunctions as a basis, in which case our representation

$$f = \sum_{j=1}^{\infty} \langle v_j, f \rangle v_j,$$

is commonly referred to as the Karhunen-Loeve expansion.

Note that $\mathbb{E}\langle v_j, f_i \rangle \langle v_k, f_i \rangle = \big\langle v_j, [\mathbb{E}(f_i \otimes f_i)] v_k \big\rangle = 0$ for all $j \neq k$, and therefore, $\big(\langle v_j, f_i \rangle\big)_{j=1}^{\infty}$ is an uncorrelated sequence of random variables.

Log Parental Income



Control Variables

	Mean	Stdev
A. Age at Birth		
Average Age of Parents at Birth	27.43	6.00
B. Race of Head		
Caucasian	0.93	0.26
African American	0.05	0.23
Others	0.02	0.14
C. Region		
East	0.18	0.38
Midwest	0.35	0.48
South	0.30	0.46
West	0.17	0.37

	Mean	Stdev
D. Education Level		
College Educated Father	0.25	0.44
College Educated Mother	0.14	0.35
E. Family Structure		
Single-Parent	0.36	0.48
Working Mother	0.48	0.50
F. Occupation Type		
White Collar	0.45	0.50
Skilled/Semiskilled	0.49	0.50
Unskilled	0.05	0.22
Farmer	0.01	0.08

Statistics to Summarize Age-specific Heterogeneity in Income Effects

- Studentized range of $\hat{\beta}(\cdot)$:

$$\hat{\beta}_{range} = \frac{\max_{0 \leqslant r < 20} \hat{\beta}(r) - \min_{0 \leqslant r < 20} \hat{\beta}(r)}{\sqrt{\frac{1}{20} \int_0^{20} \left(\hat{\beta}(r) - \bar{\beta}\right)^2 dr}} = 3.38$$

  where $\bar{\beta} = (1/20) \int_0^{20} \hat{\beta}(r) dr = 0.0183$, implying the age-specific heterogeneity in IGE of income trajectory is more than three times bigger than its overall age variation.

- Scale adjusted average area between two curves $\hat{\beta}(r)$ and $\bar{\beta}$:

$$\widehat{\beta}_{dev} = \frac{\frac{1}{20} \int_0^{20} \left|\hat{\beta}(r) - \bar{\beta}\right| dr}{\bar{\beta}} = 0.44$$

  which means the estimated IGE at each child's age is deviated from its mean value by 44% of its mean value on average. This measure also supports evidence of age-specific heterogeneity in IGE of income trajectory.