Aggregate and Intergenerational Implications of School Closures: A Quantitative Assessment

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August 2022

Motivation

- Governments around the world unprecedentedly closed schools in response to the COVID-19 pandemic.
    - Key to such decisions is understanding benefits & costs of closures
- Growing lit. on short-run effects (e.g., Alon et al. 20, Isphording et al. 21)
- In this paper, we explore medium- and long-term implications.
- **Questions**: How would school closures affect
    - aggregate economy in the longer run via intergenerational channel?
    - intergenerational mobility & lifetime income of children directly affected by school closures?

What we do

- Use a macro model of intergenerational mobility to study these questions.
  - model building on incomplete-markets GE framework (Aiyagari 94), augmented with endogenous child human capital (HK) formation.
- Calibrate stationary equilibrium of model to US economy in normal times.
- School closures : unexpected temporary $\searrow$ in public investment productivity
  - Analysis based on full transition equilibrium paths following the shock
- Our model framework enables us to analyze:
  - how do the effects differ by child cohorts with different age?
  - the role of substitutability between public and private investments.

Preview of main findings

Aggregate consequences:

- Long-lasting negative effects on aggregate output
  - up to -0.8% for many decades following 1-year closure
  - accumulated output loss relative to annual output: 35% for 1Y closure
- General equilibrium plays a quantitatively significantly role
  - w/o GE, decline in output 50% larger
  - and decreases in college-educated labor input 2-3 times larger

Preview of main findings

Intergenerational consequences:

- Intergenerational mobility declines quite notably: 1Y closure $\Rightarrow$
  - IGE, rank cor increase by up to 4%; upward mobility falls by up to 7%.
  - effects are particularly stronger for older children.
- Sizable loss in lifetime income of affected children: 1Y closure $\Rightarrow$
  - -0.3% (pre-school-aged) to -3% (older children)
- Virtual school disproportionately benefiting kids from more educated parents
  - **mitigates** avg income loss, at the expense of **lower** intergen. mobility.

Preview of main findings

Role of substitutability between public & parental investments:

- Consider a model with a lower ES between public & parental investments
  - potentially different across countries
- Lower substitutability between public and private education investments $\Rightarrow$
  - a **larger** fall in aggregate output + lifetime income loss.
  - a **lower** decline in intergenerational mobility.

Related literature

- Our mechanisms in line with real-time evidence on COVID-19
  - Chetty et al. (20): students from lower income areas reduced online learning participation as the pandemic progressed
  - Engzell et al. (21), Grewenig et al. (21): details later
- Few structural work on school closure highlighting intergenerational channel
- An exception: Fuchs-Schündeln, Krueger, Ludwig, and Popova (20)
  - Both study implications for distributions, HK investments, college along transitions.
  - Our paper: intergenerational mobility in OLG; aggregate effects (e.g., output) in a production economy in GE
- Structural GE model of intergenerational mobility
  - Restuccia & Urrutia (04), Lee & Seshadri (19), Daruich (20), Yum (21) among others.

Model Economy

Overview of the model economy

- Households:
  - Overlapping generations
  - Heterogeneity in HK, assets, age (+ child HK, learning ability)
  - Multi-stage HK production from parental & public investments
  - Other choices: college education, inter-vivos transfers
  - Aggregate state variable: entire distribution of households $\boldsymbol{\pi}_t$
- Representative firm:
  - Cobb-Douglas on capital and labor
  - CES aggregation of skilled- and unskilled- labor
- Government:
  - collects labor + capital taxes
  - provides transfers + public education investments (school)
- Incomplete markets; competitive general equilibrium

Timeline for a parent-child pair

- one model period = five years
- People live 12 periods as adult.

	Parent												
Age	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*	*50-54*	*55-59*	*60-64*	*65-69*	*70-74*	*75-79*	
$j =$	1	2	3	4	5	6	7	8	9	10	11	12	
	← – – – – Consumption-savings – – – – →												
	← – – – – Labor supply – – – – →									← Retirement →			
	College		← – Parental – → investments			Inter-vivos							
	Child												
Age			*0-4*	*5-9*	*10-14*	*15-19*	*20-24*	*25-29*	*30-34*	*35-39*	*40-44*	*45-49*	...
$j =$			← – – Childhood – – →				1	2	3	4	5	6	...
							← – – – – Consumption-savings – –						...
							← – – – – Labor supply – –						...
							College		← – Parental – → investments			Inter-vivos	

Model age 1: college choice

Value at the beginning of $j = 1$

$$V_1(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \mathbb{E}_\xi \max\big(\underbrace{N(h_t, a_t, \phi; \boldsymbol{\pi}_t)}_{\substack{\text{value of}\\ \text{not going}\\ \text{to college}}}, \underbrace{C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t)}_{\substack{\text{value of}\\ \text{completing}\\ \text{college}}}\big)$$

$$N(h_t, a_t, \phi; \boldsymbol{\pi}_t) = \max_{\substack{c \geq 0;\ a' \geq \underline{a}\\ n \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi, \boldsymbol{\pi}_{t+1}) \right\}$$

s.t.

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\ \kappa &= 1 \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t), \end{aligned}$$

- $z \sim G(z) = \mathcal{N}(0, \sigma_z^2)$: period-by-period idiosyncratic shocks to h

Model age 1: college choice

Value of completing college

$$C(h_t, a_t, \phi, \xi; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_2(h_{t+1}, a_{t+1}, \kappa, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

s.t.

$$\begin{aligned} c_t + a_{t+1} + \xi &\leq \lambda_1 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_1} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{1,\kappa} h_t \\ \kappa &= 2 \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t) \end{aligned} \tag{1}$$

- $\xi \sim \log \mathcal{N}(\mu_\xi, \sigma_\xi^2)$: stochastic college costs

Model age 2: Young adults without children

Standard life-cycle problem:

$$
\begin{aligned}
& V_2(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) \\
= & \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}, \phi' | \phi} V_3(h_{t+1}, a_{t+1}, \kappa, \phi'; \boldsymbol{\pi}_{t+1}) \right\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
c_t + a_{t+1} &\leq \lambda_2 \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_2} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
&\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} &= \exp(z_{t+1}) \gamma_{2,\kappa} h_t \\
\boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t).
\end{aligned}
$$

- taking expectation on ϕ' (to-be-born children's ability) which follows

$$\log \phi' = \rho_\phi \log \phi + \epsilon_\phi$$

  where $\epsilon_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$.

Model ages 3-5: Parental investment

- Children's human capital production featuring
  - dynamic complementarity over multiple periods & self-productivity (Cunha & Heckman 07)
  - time & monetary inputs from parents & public inputs (schools) (Lee & Seshadri 19, Fuchs-Schündeln et al. 20)
- Let I_j denote total aggregated investment

$$I_j = \left\{ \theta_j^p \left(\theta_j^x \left(\varsigma^x \frac{x_j}{\bar{x}} \right)^{\zeta_j} + \left(1 - \theta_j^x\right) \left(\varsigma^e \frac{e_j}{\bar{e}} \right)^{\zeta_j} \right)^{\frac{\psi}{\zeta_j}} + \left(1 - \theta_j^p\right) \left(\varsigma^g \frac{g_j}{\bar{g}} \right)^{\psi} \right\}^{\frac{1}{\psi}}$$

where $\theta_j^x \in (0, 1)$ and $\zeta_j, \psi \leq 1$.

  - x_j : parental time investment in j
  - e_j : monetary investment in j
  - g_j : public investments in j
  - $\frac{1}{1-\zeta_j}$: elasticity of substitution between time & money
  - $\frac{1}{1-\psi}$: elasticity of substitution between public & parental investments

Model ages 3-5: Parental investment

- The aggregated inputs in $j = 3, 4, 5$ shape the child human capital at the end of childhood
$$h_{c,6} = \phi f(I_3, I_4, I_5)$$
  - $\frac{\partial^2 f}{\partial I_i \partial I_j} > 0$: **dynamic complementarity** (Cunha & Heckman 07, Caucutt & Lochner 20)
- With unit elasticity of substitution across periods (Lee & Seshadri 19, Fuchs-Schündeln et al. 20, Yum 21), we can write recursively,

$$\begin{aligned} h_{c,j+1} &= \phi I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 5 \\ &= I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 3, 4 \end{aligned} \tag{2}$$

where $\theta_j^I \in (0, 1)$.
  - **Self-productivity**: Higher $h_{c,j}$ raises $h_{c,j+1}$ (Cunha & Heckman 07)

Model details

Production side

A representative, competitive firm produces output with CRS technology.

$$Y_t = z_t K_t^{\alpha} H_t^{1-\alpha}$$

- Capital depreciates at the rate of δ.
- Aggregate labor input:

$$H_t = \left[\nu H_{1,t}^{\rho} + (1-\nu) H_{2,t}^{\rho}\right]^{\frac{1}{\rho}}$$

  where $\rho < 1$.
  - $\frac{1}{1-\rho}$: elasticity of substitution between skilled and unskilled workers

Government and Equilibrium

- Given an initial distribution $\boldsymbol{\pi}_{-T} \equiv (\pi_{j,-T})_{j=1}^{12}$, a **competitive general equilibrium** is a sequence of factor prices, decisions, value functions, government policies, and distributions such that
  - Households solves the problems described above.
  - Firm maximizes profit.
  - Markets clear.
  - Govt budget balances.

$$\begin{aligned} & \text{transfers} + \text{social security} + \text{public education} + \text{G} \\ = \; & \text{labor} + \text{capital income tax revenues} \end{aligned}$$

  - Consistency: $\boldsymbol{\pi}_{t+1} = \Gamma(\boldsymbol{\pi}_t)$ consistent with individual decisions.
- This competitive equilibrium nests **stationary equilibrium** where market-clearing prices and aggregate quantities are constant over time.

Calibration

Calibrating the model in stationary equilibrium

- We consider several economies:
  - $\psi = 2/3$: ES $= 3$ **Baseline**
    similar to Fuchs-Schündeln et al. (20); Kotera & Seshadri (17)
  - $\psi = 1/3$ or $4/5$: ES $= 1.5$ or 5
- Common parameters:
  - Progressive taxation from Holter et al. (19)
  - Lifecycle wage growths by skill groups: Rupert & Zanella (15)
  - Public investments (following Restuccia & Urrutia 04, Holter 15)

OECD Education at Glance 2016	g_j
$j = 3$; Child age $0 - 4$	0.060
$j = 4$; Child age $5 - 9$	0.098
$j = 5$; Child age $10 - 14$	0.111

in line with Lee & Seshadri (19) as well.

Parameter		Target statistics	Data	Model
β	.948	Equilibrium real interest rate (annualized)	.04	.04
b	6.54	Mean hours of work in $j = 3, ..., 9$	.287	.306
φ	.820	Mean hours of work in $j = 3, 4, 5$	.299	.296
η	.259	Ratio of inter-vivos transfers over total savings	.30	.302
θ_3^x	.545	Mean parental time investments in $j = 3$	.061	.061
θ_4^x	.245	Mean parental time investments in $j = 4$	.036	.036
θ_5^x	.178	Mean parental time investments in $j = 5$	.020	.020
θ_3^p	.667	Rank corr. of parental income & child earnings	.282	.282
θ_3^I	.833	Mean parental monetary investments in $j = 3$	.098	.099
θ_4^I	.605	Mean parental monetary investments in $j = 4$	.113	.111
θ_5^I	.352	Mean parental monetary investments in $j = 5$	.128	.125
ζ_3	-1.53	Educational gradients in parental time in $j = 3$ (%)	20.9	20.2
ζ_4	0.25	Educational gradients in parental time in $j = 4$ (%)	14.8	15.6
ζ_5	0.24	Educational gradients in parental time in $j = 5$ (%)	20.2	21.6
ν	.536	Fraction with a college degree (%)	34.2	34.7
μ_ξ	.209	Average college expenses/GDP per-capita	.140	.135
δ_ξ	.587	Observed college wage gap (%)	75.0	76.6
ρ_ϕ	.053	Intergenerational corr. of percentile-rank income	.341	.367
σ_ϕ	.508	Gini wage	.37	.351
σ_z	.143	Slope of variance of log wage from $j = 2$ to $j = 8$	.18	.181

Slope estimates of income persistence

	U.S. data	Model	
	Chetty et al. (2014)	Proxy income	Lifetime income
IGE: log-log slope	0.344	.331	.391
Rank corr: rank-rank slope	0.341	.367	.372

- IGE (log-log slope)

$$\log y_c = \rho_0 + \rho_1 \log y_p + \varepsilon$$

- Percentile rank correlation (rank-rank slope)

$$y_{c,pct} = \rho_0 + \rho_1 y_{p,pct} + \varepsilon$$

  y is income (labor earnings + capital income), consistent with Chetty et al. (14)

- Lifetime income: discounted with interest rate (Haider & Solon 06)

Income quintile transition matrix

(%)	U.S. data					Model				
	Chetty et al. (2014a)					Proxy income				
Parent	Child quintile					Child quintile				
quint.	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	35.7	24.6	18.3	14.0	7.5
2nd	24.2	24.2	21.7	17.6	12.3	25.8	22.4	21.3	17.8	12.7
3rd	17.8	19.8	22.1	22.0	18.3	19.6	20.5	21.8	20.5	17.6
4th	13.4	16.0	20.9	24.4	25.4	13.6	18.1	20.7	23.2	24.5
5th	10.9	11.9	17.0	23.6	36.5	5.3	14.5	17.9	24.6	37.7

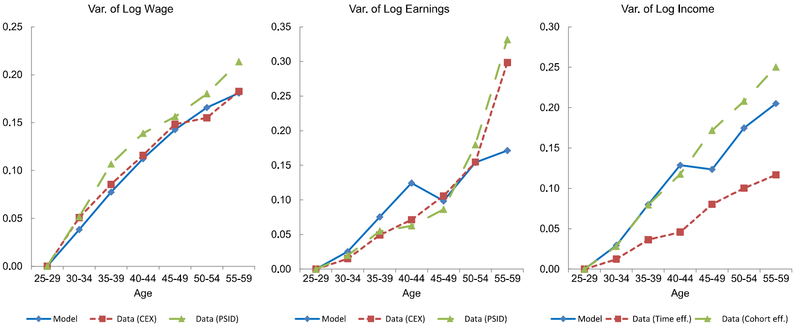
- Upward mobility (Chetty et al. 14): 7.5% (data) vs. 7.2% (model)

Income quintile transition matrix

(%)	Model				
	Lifetime income (discounted)				
Parent	Child quintile				
quint.	1st	2nd	3rd	4th	5th
1st	35.9	25.2	17.9	13.9	**7.1**
2nd	25.6	22.7	21.0	18.2	12.6
3rd	19.6	19.7	22.6	20.5	17.6
4th	13.5	17.5	21.2	23.0	24.9
5th	5.5	14.9	17.4	24.5	37.8

- In the following numerical exercises, we will focus on IGE, rank corr, and upward mobility rate based on lifetime income .
    - In the end, this is what matters (not proxy income).

Lifecycle inequality



- Cross-sectional inequality in labor market variables tends to increase over lifecycle in the data (Heathcote et al. 2010).
- These features are important because
  - income dispersion among parents ⇒ income gradients of parental responses following school closure shocks.

Quantitative Analysis of School Closures

Computational experiment design

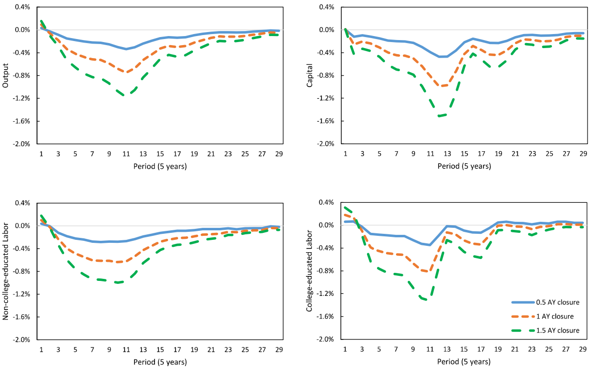
- Technical details:
  - Overlapping-generations $N = 500,000 \times 12$ each period t.
  - Simulate 50 periods with steady state decision rules (and discard)
  - Simulate 5 more periods (steady state) $t = -4, -3, ..., 0$.
  - **School closures** take place unexpectedly at the beginning of $t = 1$
  - No further shocks and transition back to original steady state.
- We consider three different lengths: 0.5AY, 1AY, 1.5AY
  - Note: unit is academic year, so pure lengths of closures are smaller (excluding vacations etc.)
- We report results by matched cohort
  - **Cohort 1 (C1)**: School closed when kid is in $j_c = 1$ (age 0-4)
  - **Cohort 2 (C2)**: School closed when kid is in $j_c = 2$ (age 5-9)
  - **Cohort 3 (C3)**: School closed when kid is in $j_c = 3$ (age 10-14)

Consistency with short-run evidence on school closures

- Limited empirical evidence on the effects of direct school closures on general child performance even in the short run
  - mostly suggestive evidence (e.g., Chetty et al. 20)
  - might infer indirectly via learning loss from summer vacations, teacher strikes (Cooper et al. 96, Atteberry & McEachin 20, Kuhfeld et al. 20)
- An exception: Engzell et al. (21)
  - rich nationally representative data set in the Netherlands
  - academic progress observed twice within a year not only in the regular year but also during the pandemic period
  - DiD estimation: 2–2.5 months ⇒ **3.2 percentile points** or **0.08 s.d.**
  - In our model with 0.25-AY closure, child human capital falls by **2.6 percentile points** or **0.07 s.d.**
- Grewenig et al. (21): detailed time use in Germany
  - learnings reduction was not statistically different by parental education.
  - in our model, positive income gradients in parental responses only in money not in time.

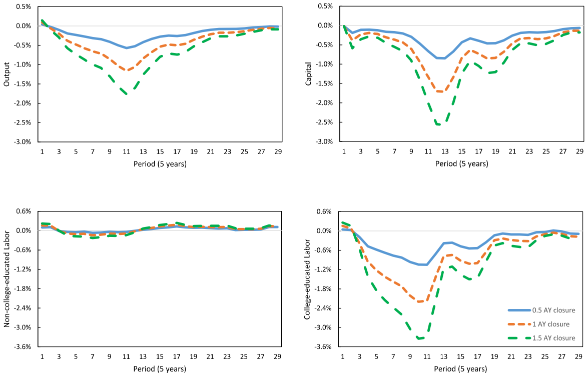
Aggregate Implications of School Closures

Evolution of macroeconomic aggregates



- Persistent output effects over many decades.
  1. initially due to compensatory financial investment
  2. then due to lower HK stocks of new cohorts
  3. later due to intergenerational investment deficiency
- **accumulated output loss** over next century relative to annual output: 12% for 0.5Y, 35% for 1Y, 60% for 1.5Y

Role of GE price adjustments



- Fix factor prices at stationary equilibrium levels. Without GE,
  - declines in college fraction are much over-stated (2–3 times).
  - capital falls much stronger, output fall is over-stated by 50%.

Distributional changes over time

		Time (1 period: 5 years)				
		1	2	3	4	5
	Steady state	**% change** rel. to no school closure				
Closure length: 0.5 AY						
Gini income	.344	0.0	0.0	0.1	0.2	0.1
Bottom 20% inc (%)	7.9	-0.0	0.0	-0.1	-0.1	0.0
Share of college (%)	34.7	0.1	0.1	0.0	-0.1	-0.0
Closure length: 1 AY						
Gini income	.344	0.0	-0.0	0.2	0.3	0.2
Bottom 20% inc (%)	7.9	-0.0	0.0	-0.2	-0.2	-0.0
Share of college (%)	34.7	0.1	0.2	-0.0	-0.2	-0.1
Closure length: 1.5 AY						
Gini income	.344	0.0	-0.1	0.3	0.6	0.3
Bottom 20% inc (%)	7.9	-0.0	0.1	-0.3	-0.4	-0.0
Share of college (%)	34.7	0.1	0.3	-0.1	-0.4	-0.3

- School closures have relatively small adverse effects on cross-sectional inequality.

Intergenerational Implications of School Closures

Effects on intergenerational mobility of lifetime income

	IGE			Rank cor.			Upward Mobility		
Steady state	.391			.372			7.1%		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.2	1.9	2.2	0.2	1.8	2.0	-0.4	-3.0	-3.4
1.0 AY	0.5	4.0	4.5	0.4	3.7	4.2	-0.8	-6.1	-6.6
1.5 AY	0.7	6.2	6.9	0.6	5.7	6.4	-1.3	-9.5	-10.5

- School closures reduce **intergenerational mobility** quite substantially.
  - In particular, these effects are larger for older children (C3 and C2).

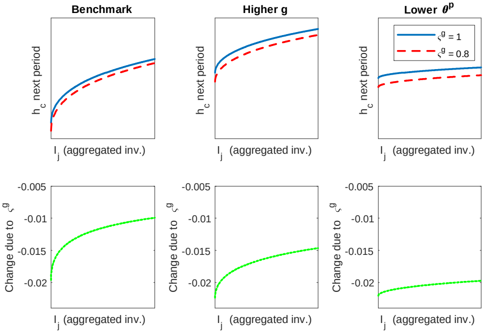
Effects on (within-cohort) inequality & level of lifetime income

	Lifetime income						Fraction of		
	Gini index			Average/Y			College-educated		
Steady state	.284			4.29			.347		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.2	0.2	-0.1	-1.5	-1.5	0.4	-1.2	-1.3
1.0 AY	0.1	0.5	0.5	-0.3	-3.1	-3.0	1.0	-2.3	-2.7
1.5 AY	0.1	0.7	0.7	-0.4	-4.6	-4.6	1.5	-3.5	-4.1

- Inequality of lifetime income within cohort increases.
- Sizable loss in mean lifetime income especially for school-aged children.

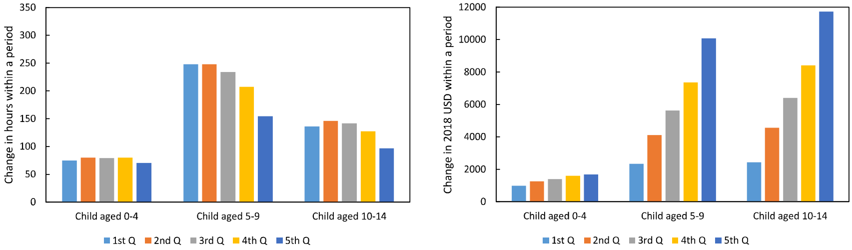
School closures: direct & indirect effects.

- Direct effects of g_j on child human capital production



  - Within cohort: larger fall in HK for children from low SES families
  - Across cohorts: larger fall in HK for older children (higher g_j, lower θ_j^p)

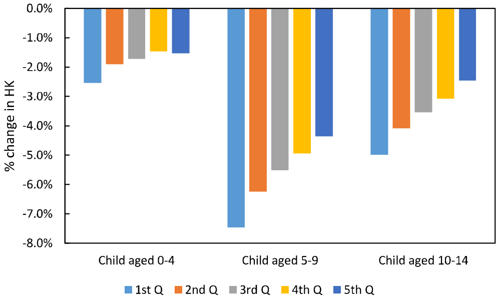
- Indirect effects: endogenous parental responses



<Parental responses by permanent income>

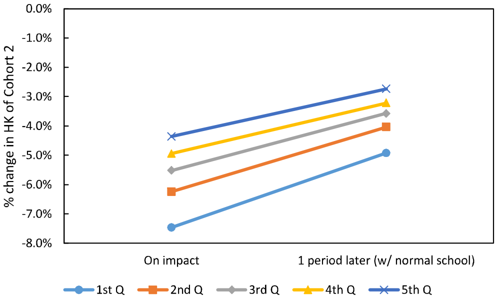
- Substantial income gradients in parental monetary investmentresponses.
    - not much gradients (slightly negative) in time (Grewenig et al. 20)
- Importance of monetary inv increases with child age (Del Boca et al. 14)
    - stronger impacts on mobility among older children

Equilibrium change in child HK on impact



- Children from richer parents tend to suffer less, in line with summer learning loss literature (e.g., Cooper et al. 96).
- These declines are largest among young school-aged children.
    - may seem inconsistent with mobility results..?

Dynamic evolution of child HK



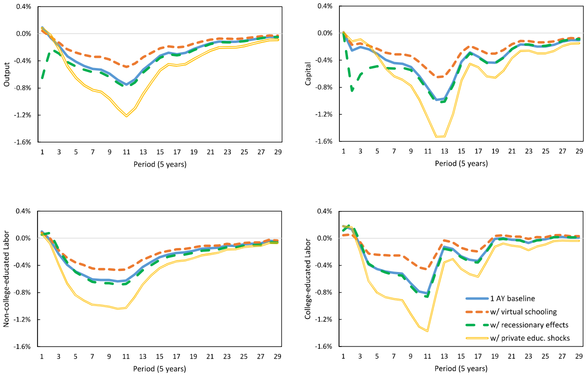
<Effects of school closures on child HK (initially aged 0-4) over time>

- If we follow the initial changes over time,
  - all children tend to recover (higher MP due to lower HK)
  - children who lost more ground experience faster growth, in line empirical evidence by Kuhfeld et al. (2020).
- Older children do not have enough time before labor market entry.

Results in extended models

- We now explore how our benchmark results (shock to $\varsigma^{g}_{t=1}$) could change when we also incorporate
  1. Virtual schooling: unequal mitigation
  2. Recessionary effects of COVID-19: shock to $z_{t=1}$
  3. Shocks to private monetary investments: shock to $\varsigma^{e}_{t=1}$
- School closure length: 1Y

Macroeconomic aggregates



1. Virtual schooling: mitigate aggregate losses
2. Recessionary effects of COVID-19: similar longer-run aggregate losses
3. Shocks to private monetary investments: stronger & earlier aggregate losses

Lifetime income effects

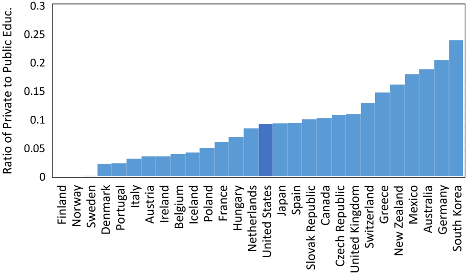
	IGE			Rank cor.			Upward Mobility		
Steady state	.391			.372			7.1%		
	% change rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
Baseline	0.5	4.0	4.5	0.4	3.7	4.2	-0.8	-6.1	-6.6
+ Virtual sch.	0.7	5.4	5.6	0.6	4.7	4.9	-1.1	-8.3	-8.6
+ Recession	1.0	2.7	3.0	0.8	2.6	2.9	-1.7	-3.8	-4.1
+ Private educ.	1.5	1.2	1.3	1.3	1.4	1.6	-2.4	-1.4	-1.1

	Lifetime income						Fraction of		
	Gini index			Average/Y			College-educated		
Steady state	.284			4.29			.347		
	% change rel. to no school closure, by cohort								
	C1	C2	C3	C1	C2	C3	C1	C2	C3
Baseline	0.1	0.5	0.5	-0.3	-3.1	-3.0	1.0	-2.3	-2.7
+ Virtual sch.	0.1	0.9	0.9	-0.2	-2.0	-2.0	0.7	-1.4	-1.7
+ Recession	0.2	0.2	0.2	-1.2	-4.5	-4.5	0.6	-2.7	-3.2
+ Private educ.	0.3	-0.1	-0.2	-2.2	-5.9	-5.9	0.1	-3.3	-3.9

Substitutability between public and parental investments

Substitutability between public & private investment

- Limited empirical evidence on ψ (ES); could potentially differ by countries.

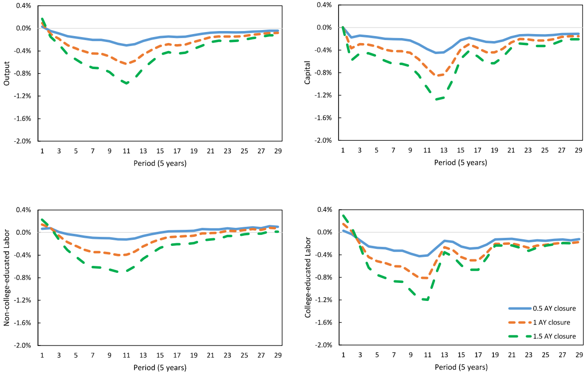


- The ratio of private to public education spending varies quite substantially across OECD countries
- A simple model: demand for private education relative to public education $\uparrow$ with higher ES between them
  - in the presence of various subsidies (or tax credits) for child investment.
- East Asian countries, such as South Korea, with larger private education market size, could have a higher ES.

Alternative model economy

- We now consider a model with a higher elasticity of substitution between public & parental investments.
  - $\psi = 4/5 \Rightarrow \text{ES} = \mathbf{5}$ (vs. 3 in the baseline model).
- Recalibrate the model to match the same target statistics.
  - Overall model fit in terms of the target statistics is equally good.

Evolution of macroeconomic aggregates



- weaker declines in macroeconomic aggregates.
  - e.g., output fall following 1AY closure: -0.7% vs. -0.8% (baseline)
- private more substitutable to public ⇒ dampened damage.

Effects on (within-cohort) inequality and loss of lifetime income

	Lifetime income Gini			Lifetime income Average/Y			Fraction of College-educated		
Steady state	.286			4.49			.339		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.0	0.2	0.3	-0.1	-1.2	-1.3	0.5	-1.0	-1.4
1.0 AY	0.1	0.5	0.6	-0.2	-2.4	-2.6	1.0	-2.0	-2.7
1.5 AY	0.1	0.7	0.9	-0.3	-3.6	-3.9	1.6	-2.9	-4.1

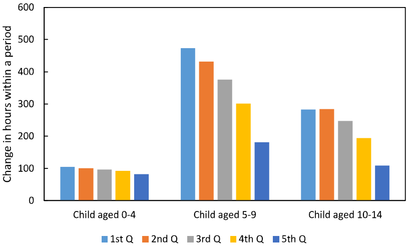
- By the same logic, smaller losses in lifetime income.
    - lifetime income fall (1AY baseline): -0.3 (C1), -3.1 (C2), -3.0 (C3)

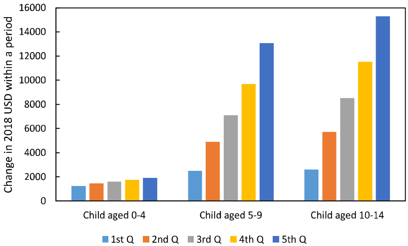
Effects on intergenerational mobility of lifetime income

	IGE			Rank cor.			Upward Mobility		
Steady state	.390			.370			7.4%		
	% change rel. to no school closure, by cohort								
Closure length	C1	C2	C3	C1	C2	C3	C1	C2	C3
0.5 AY	0.2	2.4	2.8	0.2	2.3	2.7	-0.3	-3.9	-4.4
1.0 AY	0.4	4.7	5.7	0.3	4.4	5.4	-0.8	-7.2	-8.8
1.5 AY	0.6	6.9	8.5	0.5	6.6	8.0	-1.1	-11.0	-13.4

- Greater impact on mobility
  - e.g., 1Y baseline (IGE): 0.5 (C1), 4.0 (C2), 4.5 (C3)
- Answer can be found in parental responses

Parental responses





- Generally, much stronger parental responses both in time and money
- School closures worsen intergen. mobility & inequality to larger extent.

Concluding remarks

- Explore how school closures affect macroeconomy and distributions.
- School closures adversely affect
  - aggregates such as output for many decades
  - intergenerational mobility & lifetime income of the affected children.
- With higher ES between public & private investments school closures induce
  - a **smaller** fall in aggregate output + lifetime income loss
  - a **larger** decline in intergenerational mobility + inequality.
- Policy implications:
  - sizeable economic costs of school closures in the longer term
  - welfare costs of school closures may differ across countries depending on govt.'s relative preference on macro aggregates vs. mobility (inequality).

Model age 3: with very young children

$$
\begin{aligned}
& V_3(h_t, a_t, \kappa, \phi; \boldsymbol{\pi}_t) \\
= & \max_{\substack{c_t, e_t \geq 0; \\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{\left(\frac{c_t}{q}\right)^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_4(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
c_t + a_{t+1} + e_t &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
&\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} &= \exp(z_{t+1}) \gamma_{3,\kappa} h_t \\
\boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t)
\end{aligned}
$$

$$
\begin{aligned}
& h_{c,t+1} \\
= & \left\{ \theta_3^p \left(\theta_3^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_3} + (1-\theta_3^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_3} \right)^{\frac{\psi}{\zeta_3}} + (1-\theta_3^p) \left(\frac{\varsigma_t^g g_3}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_3^l}{\psi}} h_{c,t}^{1-\theta_3^l}
\end{aligned}
$$

Model age 4: with young school-aged children

$$
\begin{aligned}
& V_4(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) \\
= \quad & \max_{\substack{c_t, e_t \geq 0; \\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{\left(\frac{c_t}{q}\right)^{1-\sigma}}{1-\sigma} - b\frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_5(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
c_t + a_{t+1} + e_t \quad & \leq \quad \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
& \quad\quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} \quad & = \quad \exp(z_{t+1}) \gamma_{4,\kappa} h_t \\
\boldsymbol{\pi}_{t+1} \quad & = \quad \Gamma(\boldsymbol{\pi}_t)
\end{aligned}
$$

$$
\begin{aligned}
& h_{c,t+1} \\
= \quad & \left\{ \theta_4^p \left(\theta_4^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_4} + (1 - \theta_4^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_4} \right)^{\frac{\psi}{\zeta_4}} + (1 - \theta_4^p) \left(\frac{\varsigma_t^g g_4}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_4^I}{\psi}} h_{c,t}^{1-\theta_4^I}
\end{aligned}
$$

Model age 5: with old school-aged children

$$
\begin{aligned}
& V_5(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) \\
= \quad & \max_{\substack{c_t, e_t \geq 0; \\ a_{t+1} \geq \underline{a} \\ x_t, n_t \in [0,1]}} \left\{ \frac{\left(\frac{c_t}{q}\right)^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} - \varphi x_t + \beta \mathbb{E}_{z_{t+1}} V_6(h_{t+1}, a_{t+1}, \kappa, h_{c,t+1}, \phi; \boldsymbol{\pi}_{t+1}) \right\}
\end{aligned}
$$

s.t.

$$
\begin{aligned}
c_t + a_{t+1} + e_t & \leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y}\right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\
& \quad + (1 + r_t(\boldsymbol{\pi}_t))\, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\
h_{t+1} & = \exp(z_{t+1}) \gamma_{5,\kappa} h_t \\
\boldsymbol{\pi}_{t+1} & = \Gamma(\boldsymbol{\pi}_t)
\end{aligned}
$$

$$
\begin{aligned}
& h_{c,t+1} \\
= \quad & \phi \left\{ \theta_5^p \left(\theta_5^x \left(\frac{\varsigma_t^x x_t}{\bar{x}} \right)^{\zeta_5} + (1 - \theta_5^x) \left(\frac{\varsigma_t^e e_t}{\bar{e}} \right)^{\zeta_5} \right)^{\frac{\psi}{\zeta_5}} + (1 - \theta_5^p) \left(\frac{\varsigma_t^g g_5}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_5^l}{\psi}} h_{c,t}^{1-\theta_5^l}
\end{aligned}
$$

Model age 6: Inter-vivos transfers

At the end of $j = 6$, child becomes independent.

$$V_6(h_t, a_t, \kappa, h_{c,t}, \phi; \boldsymbol{\pi}_t) = \max_{a'_c} \left\{ \tilde{V}_6(h_t, a_t - a'_c, \kappa; \boldsymbol{\pi}_t) + \eta \beta V_1(h'_c, a'_c, \phi; \boldsymbol{\pi}_{t+1}) \right\}$$

$$\begin{aligned} a'_c &\in [0, a_t] \\ h'_c &= \gamma_c h_{c,t} \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t) \end{aligned}$$

- Dynastic utility: continuation value includes lifetime value of child.

$$\tilde{V}_6(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{(c_t/q)^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_7(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\}$$

s.t.

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t)) \, a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{6,\kappa} h_t \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

- $j = 7, 8, 9$ similar to $j = 2$

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{\substack{c_t \geq 0;\ a_{t+1} \geq \underline{a} \\ n_t \in [0,1]}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - b \frac{n_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z_{t+1}} V_{j+1}(h_{t+1}, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\}$$

s.t.

$$\begin{aligned} c_t + a_{t+1} &\leq \lambda_j \left(w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t / \bar{y} \right)^{-\tau_j} w_{\kappa,t}(\boldsymbol{\pi}_t) h_t n_t \\ &\quad + (1 + r_t(\boldsymbol{\pi}_t)) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T \\ h_{t+1} &= \exp(z_{t+1}) \gamma_{j,\kappa} h_t \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

- $j = 10, 11, 12$: retirement

$$V_j(h_t, a_t, \kappa; \boldsymbol{\pi}_t) = \max_{c_t \geq 0;\ a_{t+1} \geq \underline{a}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V_{j+1}(h_t, a_{t+1}, \kappa; \boldsymbol{\pi}_{t+1}) \right\}$$

s.t.

$$\begin{aligned} c_t + a_{t+1} &\leq (1 + r_t(\boldsymbol{\pi}_t)) a_t - \tau_k r_t(\boldsymbol{\pi}_t) \max\{a_t, 0\} + T_t + \Omega_t \\ \boldsymbol{\pi}_{t+1} &= \Gamma(\boldsymbol{\pi}_t). \end{aligned}$$

Return