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1	Geometry	9	1.1 PS	
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	4.2 Convex Hull	11	#Include \bits/stuc++.11/	
	4.3 Poiont in Polygon		using namespace std;	
	4.4 Polygon Cut			
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			<pre>#define for1j(s, e) for(int j = s; j < e; j++) #define for5ab(b) for(auto i = b)</pre>	
	4.6 Seperating Axis Theorem		<pre>#define forEach(k) for(auto i : k) #define forEachj(k) for(auto j : k)</pre>	
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5		13	<pre>#define uniq(vct) sort(all(vct));vct.erase(unique(all(vct)), vct.end())</pre>	
5	5.1 Dijkstra		#define fi first	
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5	5.1 Dijkstra 5.2 Bellman-Ford 5.2 Spfa 5.3 Spfa 5.4 Topological Sort 5.4 Topological Sort	13 14 14 15	<pre>#define fi first #define se second #define INF (111 << 6011) typedef unsigned long long ull; typedef long long l1;</pre>	
5	5.1 Dijkstra	13 14 14 15 15	<pre>#define fi first #define se second #define INF (111 << 6011) typedef unsigned long long ull; typedef long long l1; typedef 11 llint;</pre>	
5	5.1 Dijkstra 5.2 Bellman-Ford 5.2 Spfa 5.3 Spfa 5.4 Topological Sort 5.4 Topological Sort	13 14 14 15 15 15	<pre>#define fi first #define se second #define INF (111 << 6011) typedef unsigned long long ull; typedef long long l1;</pre>	

1

```
typedef ull ullint;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair<double, double> pdd;
typedef pair<double, int> pdi;
typedef pair<string, string> pss;
typedef vector<int> iv1;
typedef vector<iv1> iv2;
typedef vector<ll> llv1;
typedef vector<llv1> 11v2;
typedef vector<pii> piiv1;
typedef vector<piiv1> piiv2;
typedef vector<pll> pllv1;
typedef vector<pllv1> pllv2;
typedef vector<pdd> pddv1;
typedef vector<pddv1> pddv2;
const double EPS = 1e-8;
const double PI = acos(-1);
template<typename T>
T sq(T x) \{ return x * x; \}
int sign(ll x) { return x < 0 ? -1 : x > 0 ? 1 : 0; }
int sign(int x) { return x < 0 ? -1 : x > 0 ? 1 : 0; }
int sign(double x) { return abs(x) < EPS ? 0 : x < 0 ? -1 : 1; }
void solve() {
}
int main() {
 ios::sync_with_stdio(0);
 cin.tie(NULL);cout.tie(NULL);
 int tc = 1; // cin >> tc;
 while(tc--) solve();
}
    Math
2.1 Basic Arithmetics
```

```
typedef long long ll;
typedef unsigned long long ull;

// calculate lg2(a)
inline int lg2(ll a) {
    return 63 - __builtin_clzll(a);
}

// calculate the number of 1-bits
```

```
inline int bitcount(ll a) {
    return __builtin_popcountll(a);
}
// calculate ceil(a/b)
// |a|, |b| \le (2^63)-1  (does not dover -2^63)
ll ceildiv(ll a, ll b) {
    if (b < 0) return ceildiv(-a, -b);</pre>
    if (a < 0) return (-a) / b;
    return ((ull)a + (ull)b - 1ull) / b;
}
// calculate floor(a/b)
// |a|, |b| <= (2^63)-1 (does not cover -2^63)
11 floordiv(ll a, ll b) {
    if (b < 0) return floordiv(-a, -b);</pre>
    if (a >= 0) return a / b;
    return -(11)(((ull)(-a) + b - 1) / b);
}
// calculate a*b % m
// x86-64 only
11 large_mod_mul(l1 a, l1 b, l1 m) {
    return 11((__int128)a*(__int128)b%m);
// calculate a*b % m
// |m| < 2^62, x86 available
// O(Logb)
11 large mod mul(ll a, ll b, ll m) {
    a \% = m; b \% = m; 11 r = 0, v = a;
    while (b) {
        if (b\&1) r = (r + v) \% m;
        b >>= 1;
        v = (v << 1) \% m;
    return r;
}
// calculate n^k % m
11 modpow(11 n, 11 k, 11 m) {
    ll ret = 1;
    n \% = m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large mod mul(n, n, m);
        k /= 2;
    }
    return ret;
}
// calculate qcd(a, b)
11 gcd(ll a, ll b) {
    return b == 0 ? a : gcd(b, a % b);
}
```

```
// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<11, 11> extended_gcd(11 a, 11 b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
    return { t.second, t.first - t.second * (a / b) };
// find x in [0,m) s.t. ax === qcd(a, m) \pmod{m}
11 modinverse(ll a, ll m) {
    return (extended gcd(a, m).first % m + m) % m;
// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)</pre>
        ret[i] = (11)(mod - mod/i) * ret[mod%i] % mod;
}
// p is prime
// calculate a^b % p
ll pow(ll a, ll b){
   if(b == 0) return 1;
   11 n = pow(a, b/2) \% p;
    11 temp = (n * n) % p;
    if(b%2==0) return temp;
    return (a * temp) % p;
}
// p is prime
// calculate a/b % p
11 fermat(11 a, 11 b){
    return a % p * pow(b, p-2) % p;
}
      Convex Hull Trick
struct CHTLinear {
    struct Line {
        long long a, b;
        long long y(long long x) const { return a * x + b; }
    };
    vector<Line> stk;
    int qpt;
    CHTLinear() : qpt(0) { }
    // when you need maximum : (previous L).a < (now L).a
    // when you need minimum : (previous l).a > (now l).a
    void pushLine(const Line& 1) {
        while (stk.size() > 1) {
            Line& 10 = stk[stk.size() - 1];
            Line& 11 = stk[stk.size() - 2];
            if ((10.b - 1.b) * (10.a - 11.a) > (11.b - 10.b) * (1.a - 10.a))
```

```
break;
            stk.pop_back();
        stk.push back(1);
    // (previous x) <= (current x)</pre>
    // it calculates max/min at x
    long long query(long long x) {
        while (qpt + 1 < stk.size()) {</pre>
            Line& 10 = stk[qpt];
            Line& 11 = stk[qpt + 1];
            if (11.a - 10.a > 0 \& (10.b - 11.b) > x * (11.a - 10.a)) break;
            if (l1.a - l0.a < 0 && (l0.b - l1.b) < x * (l1.a - l0.a)) break;
            ++apt;
        return stk[qpt].y(x);
};
     Luca Theorem
// calculate nCm % p when p is prime
int lucas theorem(const char *n, const char *m, int p) {
    vector<int> np, mp;
    int i;
    for (i = 0; n[i]; i++) {
        if (n[i] == '0' && np.empty()) continue;
        np.push_back(n[i] - '0');
    for (i = 0; m[i]; i++) {
        if (m[i] == '0' && mp.empty()) continue;
        mp.push_back(m[i] - '0');
    }
    int ret = 1;
    int ni = 0, mi = 0;
    while (ni < np.size() || mi < mp.size()) {</pre>
        int nmod = 0, mmod = 0;
        for (i = ni; i < np.size(); i++) {</pre>
            if (i + 1 < np.size())</pre>
                 np[i + 1] += (np[i] \% p) * 10;
            else
                 nmod = np[i] \% p;
            np[i] /= p;
        for (i = mi; i < mp.size(); i++) {</pre>
            if (i + 1 < mp.size())</pre>
                mp[i + 1] += (mp[i] \% p) * 10;
            else
                 mmod = mp[i] \% p;
            mp[i] /= p;
        while (ni < np.size() && np[ni] == 0) ni++;</pre>
        while (mi < mp.size() && mp[mi] == 0) mi++;</pre>
        // implement binomial. binomial(m,n) = 0 if m < n
```

```
ret = (ret * binomial(nmod, mmod)) % p;
    return ret;
}
2.4 Pollard Rho
11 pollard rho(11 n) {
    random_device rd;
    mt19937 gen(rd());
    uniform int distribution<ll> dis(1, n - 1);
   11 x = dis(gen);
   11 y = x;
   11 c = dis(gen);
   11 g = 1;
    while (g == 1) {
        x = (modmul(x, x, n) + c) \% n;
        y = (modmul(y, y, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        g = gcd(abs(x - y), n);
    return g;
}
// integer factorization
// O(n^0.25 * Logn)
void factorize(ll n, vector<ll>& fl) {
    if (n == 1) {
        return;
    if (n % 2 == 0) {
        fl.push back(2);
        factorize(n / 2, fl);
    else if (is prime(n)) {
        fl.push_back(n);
    }
    else {
        11 f = pollard rho(n);
        factorize(f, fl);
        factorize(n / f, fl);
}
     Primality Test
bool test witness(ull a, ull n, ull s) {
    if (a >= n) a %= n;
    if (a <= 1) return true;</pre>
    ull d = n \gg s;
    ull x = modpow(a, d, n);
    if (x == 1 \mid | x == n-1) return true;
    while (s-- > 1) {
        x = large mod mul(x, x, n);
```

```
if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}
// test whether n is prime
// based on miller-rabin test
// O(logn*logn)
bool is_prime(ull n) {
    if (n == 2) return true;
    if (n < 2 || n % 2 == 0) return false;</pre>
    ull d = n \gg 1, s = 1;
    for(; (d&1) == 0; s++) d >>= 1;
#define T(a) test witness(a##ull, n, s)
    if (n < 4759123141ull) return T(2) && T(7) && T(61);</pre>
    return T(2) && T(325) && T(9375) && T(28178)
        && T(450775) && T(9780504) && T(1795265022);
#undef T
     Matrix Operations
const int MATSZ = 100;
inline bool is_zero(double a) { return fabs(a) < 1e-9; }</pre>
// out = A^{(-1)}, returns det(A)
// A becomes invalid after call this
// O(n^3)
double inverse_and_det(int n, double A[][MATSZ], double out[][MATSZ]) {
    double det = 1;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) out[i][j] = 0;</pre>
        out[i][i] = 1;
    for (int i = 0; i < n; i++) {
        if (is zero(A[i][i])) {
            double maxv = 0;
            int maxid = -1;
            for (int j = i + 1; j < n; j++) {
                auto cur = fabs(A[j][i]);
                if (maxv < cur) {</pre>
                     maxv = cur;
                     maxid = j;
                }
            if (maxid == -1 || is zero(A[maxid][i])) return 0;
            for (int k = 0; k < n; k++) {
                A[i][k] += A[maxid][k];
                out[i][k] += out[maxid][k];
```

```
det *= A[i][i];
                                                                                              for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);</pre>
        double coeff = 1.0 / A[i][i];
        for (int j = 0; j < n; j++) A[i][j] *= coeff;</pre>
                                                                                          return true;
                                                                                      }
        for (int j = 0; j < n; j++) out[i][j] *= coeff;</pre>
        for (int j = 0; j < n; j++) if (j != i) {
            double mp = A[j][i];
                                                                                      2.8
                                                                                           Sieve
            for (int k = 0; k < n; k++) A[j][k] -= A[i][k] * mp;
            for (int k = 0; k < n; k++) out[j][k] -= out[i][k] * mp;
                                                                                      // find prime numbers in 1 ~ n
        }
                                                                                      // ret[x] = false -> x is prime
    }
                                                                                      // O(n*loglogn)
    return det;
                                                                                      void sieve(int n, bool ret[]) {
                                                                                          for (int i = 2; i * i <= n; ++i)
                                                                                              if (!ret[i])
      Gaussian Elimination
                                                                                                  for (int j = i * i; j <= n; j += i)
                                                                                                      ret[i] = true;
const double EPS = 1e-10;
                                                                                      }
typedef vector<vector<double>> VVD;
                                                                                      // calculate number of divisors for 1 ~ n
// Gauss-Jordan elimination with full pivoting.
                                                                                      // when you need to calculate sum, change += 1 to += i
// solving systems of linear equations (AX=B)
                                                                                      // O(n*logn)
// INPUT:
             a[][] = an n*n matrix
                                                                                      void num of divisors(int n, int ret[]) {
             b[][] = an n*m matrix
                                                                                          for (int i = 1; i <= n; ++i)
// OUTPUT:
                   = an n*m matrix (stored in b[][])
                                                                                              for (int j = i; j <= n; j += i)
//
             A^{-1} = an n*n matrix (stored in a[][])
                                                                                                  ret[j] += 1;
// O(n^3)
                                                                                      }
bool gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
                                                                                      // calculate euler totient function for 1 ~ n
    const int m = b[0].size();
                                                                                      // phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
    vector<int> irow(n), icol(n), ipiv(n);
                                                                                      // O(n*loglogn)
                                                                                      void euler_phi(int n, int ret[]) {
    for (int i = 0; i < n; i++) {
                                                                                          for (int i = 1; i <= n; ++i) ret[i] = i;
        int pj = -1, pk = -1;
                                                                                          for (int i = 2; i <= n; ++i)
        for (int j = 0; j < n; j++) if (!ipiv[j])
                                                                                              if (ret[i] == i)
            for (int k = 0; k < n; k++) if (!ipiv[k])
                                                                                                  for (int j = i; j <= n; j += i)
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk =
                                                                                                      ret[j] -= ret[j] / i;
                  k; }
                                                                                      }
        if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular</pre>
        ipiv[pk]++;
                                                                                           \mathbf{FFT}
        swap(a[pj], a[pk]);
                                                                                      2.9
        swap(b[pj], b[pk]);
        irow[i] = pj;
                                                                                      void fft(int sign, int n, double *real, double *imag) {
        icol[i] = pk;
                                                                                          double theta = sign * 2 * pi / n;
                                                                                          for (int m = n; m >= 2; m >>= 1, theta *= 2) {
        double c = 1.0 / a[pk][pk];
                                                                                              double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
        a[pk][pk] = 1.0;
                                                                                              for (int i = 0, mh = m >> 1; i < mh; ++i) {
        for (int p = 0; p < n; p++) a[pk][p] *= c;
                                                                                                  for (int j = i; j < n; j += m) {
        for (int p = 0; p < m; p++) b[pk][p] *= c;
                                                                                                      int k = j + mh;
        for (int p = 0; p < n; p++) if (p != pk) {
                                                                                                      double xr = real[j] - real[k], xi = imag[j] - imag[k];
            c = a[p][pk];
                                                                                                      real[j] += real[k], imag[j] += imag[k];
            a[p][pk] = 0;
                                                                                                      real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
                                                                                                  double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
        }
                                                                                                  wr = \_wr, wi = \_wi;
    for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
                                                                                          }
```

```
for (int i = 1, j = 0; i < n; ++i) {
        for (int k = n >> 1; k > (j ^= k); k >>= 1);
        if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);</pre>
}
// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*Loan)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 100;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
   int fn = 1;
    while (fn < n + m) fn <<= 1; // n + m: interested Length
   for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
   for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;</pre>
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(1, fn, ra, ia);
   fft(1, fn, rb, ib);
   for (int i = 0; i < fn; ++i) {
        double real = ra[i] * rb[i] - ia[i] * ib[i];
        double imag = ra[i] * ib[i] + rb[i] * ia[i];
        ra[i] = real, ia[i] = imag;
   fft(-1, fn, ra, ia);
    for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);</pre>
    return fn;
}
2.10 Chinese Remainder
// find x s.t. x === a[0] \pmod{n[0]}
                  === a[1] \pmod{n[1]}
//
// assumption: qcd(n[i], n[j]) = 1
11 chinese_remainder(ll* a, ll* n, int size) {
   if (size == 1) return *a;
   ll tmp = modinverse(n[0], n[1]);
   ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
   ll ora = a[1];
   11 tgcd = gcd(n[0], n[1]);
   a[1] = a[0] + n[0] / tgcd * tmp2;
   n[1] *= n[0] / tgcd;
   ll ret = chinese remainder(a + 1, n + 1, size - 1);
   n[1] /= n[0] / tgcd;
   a[1] = ora;
    return ret;
}
    Data Structure
```

3.1 Order Statistic Tree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <functional>
#include <iostream>
using namespace gnu pbds;
using namespace std;
// tree<key type, value type(set if null), comparator, ...>
using ordered_set = tree<int, null_type, less<int>, rb_tree_tag,
           tree order statistics node update>;
int main()
           ordered set X;
           for (int i = 1; i < 10; i += 2) X.insert(i); // 1 3 5 7 9
           cout << *X.find_by_order(4) << endl; // 9</pre>
           cout << X.order_of_key(5) << endl; // 2; 보다5 작은수의갯수
           X.erase(3);
           for (int t : X) printf("%d<sub>\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\underline{\under</sub>
}
3.2 Fenwick Tree
const int TSIZE = 100000;
int tree[TSIZE + 1];
// Returns the sum from index 1 to p, inclusive
int query(int p) {
           int ret = 0;
           for (; p > 0; p -= p & -p) ret += tree[p];
           return ret;
// Adds val to element with index pos
void add(int p, int val) {
           for (; p <= TSIZE; p += p & -p) tree[p] += val;</pre>
3.3 Fenwick Tree 2D
struct FenwickTree2D {
           ll size:
           11v2 data;
           FenwickTree2D(11 N) {
                       size = N;
                       data = llv2(size+1, llv1(size+1));
           }
           void update(int x, int y, ll val) {
                       11 dv = val - sum(x, y, x, y);
                       while(x <= size) {</pre>
                                 int y2 = y;
                                  while(y2 <= size) {</pre>
                                             data[x][y2] += dv;
```

```
y2 += y2 & -y2;
                                                                                        // example implementation of sum tree
                                                                                        const int TSIZE = 131072; // always 2^k form && n <= TSIZE</pre>
            x += x \& -x;
                                                                                        int segtree[TSIZE * 2], prop[TSIZE * 2];
                                                                                        void seg init(int nod, int 1, int r) {
   }
                                                                                            if (1 == r) segtree[nod] = dat[1];
                                                                                            else {
    11 sum(int x, int y) {
                                                                                                 int m = (1 + r) >> 1;
        11 \text{ ret} = 0;
                                                                                                 seg_init(nod << 1, 1, m);</pre>
                                                                                                 seg_init(nod << 1 | 1, m + 1, r);
        while(x) {
            int y2 = y;
                                                                                                 segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
            while(y2) {
                                                                                            }
                ret += data[x][y2];
                y2 -= y2 \& -y2;
                                                                                        void seg_relax(int nod, int 1, int r) {
                                                                                            if (prop[nod] == 0) return;
            x -= x \& -x;
                                                                                            if (1 < r) {
                                                                                                 int m = (1 + r) >> 1;
                                                                                                 segtree[nod \langle\langle 1] += (m - 1 + 1) * prop[nod];
        return ret;
   }
                                                                                                 prop[nod << 1] += prop[nod];</pre>
                                                                                                 segtree[nod << 1 | 1] += (r - m) * prop[nod];
                                                                                                 prop[nod << 1 | 1] += prop[nod];</pre>
   11 sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) + sum(x1 - 1, y1 - 1) - sum(x1 - 1, y2) - sum(x2, y1 - 1);
                                                                                            prop[nod] = 0;
};
                                                                                        int seg_query(int nod, int 1, int r, int s, int e) {
                                                                                            if (r < s || e < 1) return 0;
3.4 Merge Sort Tree
                                                                                            if (s <= 1 && r <= e) return segtree[nod];</pre>
                                                                                            seg_relax(nod, 1, r);
llv1 a;
                                                                                            int m = (1 + r) >> 1;
llv1 mTree[Mx];
                                                                                            return seg_query(nod << 1, 1, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s
void makeTree(ll idx, ll ss, ll se) {
 if (ss == se) {
    mTree[idx].push_back(a[ss]);
                                                                                        void seg_update(int nod, int 1, int r, int s, int e, int val) {
    return;
                                                                                            if (r < s \mid | e < 1) return;
                                                                                            if (s <= 1 && r <= e) {
 11 \text{ mid} = (ss + se) / 2;
                                                                                                 segtree[nod] += (r - l + 1) * val;
  makeTree(2 * idx + 1, ss, mid);
                                                                                                 prop[nod] += val;
  makeTree(2 * idx + 2, mid + 1, se);
                                                                                                 return;
 merge(mTree[2 * idx + 1].begin(), mTree[2 * idx + 1].end(), mTree[2 * idx + 1].end()
   2].begin(), mTree[2 * idx + 2].end(), back_inserter(mTree[idx]));
                                                                                            seg_relax(nod, 1, r);
                                                                                            int m = (1 + r) >> 1;
11 query(11 node, 11 start, 11 end, 11 q_s, 11 q_e, 11 k) {
                                                                                            seg_update(nod << 1, 1, m, s, e, val);</pre>
 // i j k: Ai, Ai+1, ..., 로Aj 이루어진부분수열중에서보다 k 큰원소의개수를출력한
                                                                                             seg update(nod \langle\langle 1 \mid 1, m + 1, r, s, e, val)\rangle;
                                                                                            segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];</pre>
 if (q_s > end || start > q_e) return 0;
 if (q_s <= start && q_e >= end) {
                                                                                        // usage:
    return mTree[node].size() - (upper bound(mTree[node].begin(), mTree[node].
                                                                                        // seg_update(1, 0, n - 1, qs, qe, val);
      end(), k) - mTree[node].begin());
                                                                                        // seg_query(1, 0, n - 1, qs, qe);
 11 \text{ mid} = (\text{start} + \text{end}) / 2;
 ll p1 = query(2 * node + 1, start, mid, q_s, q_e, k);
                                                                                        3.6 Treap
 11 p2 = query(2 * node + 2, mid + 1, end, q s, q e, k);
  return p1 + p2;
                                                                                        // Treap* root = NULL;
}
                                                                                        // root = insert(root, new Treap(3));
                                                                                        typedef int type;
                                                                                        struct Treap {
      SegmentTree Lazy Propagation
                                                                                          Treap* left = NULL, * right = NULL;
```

```
int size = 1, prio = rand();
 type key;
 Treap(type key) : key(key) { }
 void calcSize() {
   size = 1;
   if (left != NULL) size += left->size;
   if (right != NULL) size += right->size;
 void setLeft(Treap* 1) { left = 1, calcSize(); }
 void setRight(Treap* r) { right = r, calcSize(); }
};
typedef pair<Treap*, Treap*> TPair;
TPair split(Treap* root, type key) {
 if (root == NULL) return TPair(NULL, NULL);
 if (root->key < key) {</pre>
   TPair rs = split(root->right, key);
    root->setRight(rs.first);
    return TPair(root, rs.second);
 TPair ls = split(root->left, key);
 root->setLeft(ls.second);
 return TPair(ls.first, root);
Treap* insert(Treap* root, Treap* node) {
 if (root == NULL) return node;
 if (root->prio < node->prio) {
   TPair s = split(root, node->key);
    node->setLeft(s.first);
   node->setRight(s.second);
   return node;
 else if (node->key < root->key)
    root->setLeft(insert(root->left, node));
    root->setRight(insert(root->right, node));
 return root:
Treap* merge(Treap* a, Treap* b) {
 if (a == NULL) return b;
 if (b == NULL) return a;
 if (a->prio < b->prio) {
   b->setLeft(merge(a, b->left));
    return b;
 a->setRight(merge(a->right, b));
 return a;
Treap* erase(Treap* root, type key) {
 if (root == NULL) return root;
 if (root->key == key) {
   Treap* ret = merge(root->left, root->right);
   delete root;
    return ret;
 if (key < root->key)
```

```
root->setLeft(erase(root->left, key));
     root->setRight(erase(root->right, key));
   return root;
Treap* kth(Treap* root, int k) { // kth key
  int l size = 0:
  if (root->left != NULL) l_size += root->left->size;
  if (k <= l size) return kth(root->left, k);
  if (k == l_size + 1) return root;
  return kth(root->right, k - 1 size - 1);
int countLess(Treap* root, type key) { // count less than key
  if (root == NULL) return 0;
  if (root->key >= key)
     return countLess(root->left, key);
  int ls = (root->left ? root->left->size : 0);
  return ls + 1 + countLess(root->right, key);
}
3.7 Splay Tree
// example : https://www.acmicpc.net/problem/13159
struct node {
     node* 1, * r, * p;
     int cnt, min, max, val;
     long long sum;
     bool inv;
     node(int val) :
          cnt(1), sum(_val), min(_val), max(_val), val(_val), inv(false),
          l(nullptr), r(nullptr), p(nullptr) {
     }
};
node* root;
void update(node* x) {
     x \rightarrow cnt = 1:
     x \rightarrow sum = x \rightarrow min = x \rightarrow max = x \rightarrow val;
     if (x->1) {
          x \rightarrow cnt += x \rightarrow 1 \rightarrow cnt;
          x \rightarrow sum += x \rightarrow 1 \rightarrow sum;
          x - \min = \min(x - \min, x - > 1 - > \min);
          x \rightarrow max = max(x \rightarrow max, x \rightarrow 1 \rightarrow max);
     if (x->r) {
          x \rightarrow cnt += x \rightarrow r \rightarrow cnt;
          x \rightarrow sum += x \rightarrow r \rightarrow sum;
          x - \min = \min(x - \min, x - r - \min);
          x -> max = max(x -> max, x -> r -> max);
}
void rotate(node* x) {
     node* p = x-p;
     node* b = nullptr;
```

```
if (x == p->1) {
          p->1 = b = x->r;
          x \rightarrow r = p;
     else {
          p->r = b = x->1;
          x \rightarrow 1 = p;
    x->p = p->p;
     p \rightarrow p = x;
     if (b) b \rightarrow p = p;
     x \rightarrow p? (p == x \rightarrow p \rightarrow l ? x \rightarrow p \rightarrow l : x \rightarrow p \rightarrow r) = x : (root = x);
     update(p);
     update(x);
// make x into root
void splay(node* x) {
    while (x->p) {
          node* p = x->p;
          node* g = p - p;
          if (g) rotate((x == p \rightarrow 1) == (p == g \rightarrow 1) ? p : x);
          rotate(x);
    }
}
void relax_lazy(node* x) {
     if (!x->inv) return;
     swap(x->1, x->r);
    x->inv = false;
     if (x\rightarrow 1) x\rightarrow 1\rightarrow inv = !x\rightarrow 1\rightarrow inv;
     if (x->r) x->r->inv = !x->r->inv;
}
// find kth node in splay tree
void find_kth(int k) {
    node* x = root;
     relax lazy(x);
     while (true) {
          while (x->1 && x->1->cnt > k) {
               x = x \rightarrow 1;
               relax_lazy(x);
          if (x->1) k -= x->1->cnt;
          if (!k--) break;
          x = x - r;
          relax_lazy(x);
     splay(x);
}
// collect [l, r] nodes into one subtree and return its root
node* interval(int 1, int r) {
     find kth(l - 1);
     node* x = root;
```

```
root = x->r;
    root->p = nullptr;
    find kth(r - l + 1);
    x \rightarrow r = root;
    root -> p = x;
    root = x;
    return root->r->l;
}
void traverse(node* x) {
    relax lazy(x);
    if (x\rightarrow 1) {
         traverse(x->1);
    // do something
    if (x->r) {
         traverse(x->r);
}
void uptree(node* x) {
    if (x->p) {
         uptree(x->p);
    relax_lazy(x);
}
```

4 Geometry

4.1 Basic Operations

```
const double eps = 1e-9;
inline int diff(double lhs, double rhs) {
    if (lhs - eps < rhs && rhs < lhs + eps) return 0;</pre>
    return (lhs < rhs) ? -1 : 1;</pre>
}
inline bool is between(double check, double a, double b) {
    if(a < b)
        return (a - eps < check && check < b + eps);</pre>
    else
        return (b - eps < check && check < a + eps);</pre>
}
struct Point {
    double x, y;
    bool operator==(const Point& rhs) const {
        return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
    Point operator+(const Point& rhs) const {
        return Point{ x + rhs.x, y + rhs.y };
    Point operator-(const Point& rhs) const {
```

```
return Point{ x - rhs.x, y - rhs.y };
                                                                                     bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
    Point operator*(double t) const {
                                                                                         double mdet = outer(b.dir, a.dir);
        return Point{ x * t, y * t };
                                                                                         if (diff(mdet, 0) == 0) return false;
                                                                                         double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
};
                                                                                         double t2 = outer(a.dir, b.pos - a.pos) / mdet;
                                                                                         if (!is between(t1, 0, 1) || !is between(t2, 0, 1)) return false;
                                                                                         ret = b.pos + b.dir * t2;
struct Circle {
   Point center;
                                                                                         return true;
    double r;
};
                                                                                     Point inner center(const Point &a, const Point &b, const Point &c) {
struct Line {
                                                                                         double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
    Point pos, dir;
                                                                                         double w = wa + wb + wc;
                                                                                         return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y +
};
                                                                                           wc * c.v) / w };
inline double inner(const Point& a, const Point& b) {
                                                                                     }
    return a.x * b.x + a.v * b.v;
                                                                                     Point outer center(const Point &a, const Point &b, const Point &c) {
                                                                                         Point d1 = b - a, d2 = c - a;
inline double outer(const Point& a, const Point& b) {
                                                                                         double area = outer(d1, d2);
    return a.x * b.y - a.y * b.x;
                                                                                         double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
}
                                                                                             + d1.y * d2.y * (d1.y - d2.y);
                                                                                         double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
inline int ccw line(const Line& line, const Point& point) {
                                                                                             + d1.x * d2.x * (d1.x - d2.y);
    return diff(outer(line.dir, point - line.pos), 0);
                                                                                         return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
}
                                                                                     }
inline int ccw(const Point& a, const Point& b, const Point& c) {
                                                                                     vector<Point> circle line(const Circle& circle, const Line& line) {
    return diff(outer(b - a, c - a), 0);
                                                                                         vector<Point> result;
                                                                                         double a = 2 * inner(line.dir, line.dir);
}
                                                                                         double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
                                                                                             + line.dir.y * (line.pos.y - circle.center.y));
inline double dist(const Point& a, const Point& b) {
    return sqrt(inner(a - b, a - b));
                                                                                         double c = inner(line.pos - circle.center, line.pos - circle.center)
                                                                                             - circle.r * circle.r;
                                                                                         double det = b * b - 2 * a * c;
                                                                                         int pred = diff(det, 0);
inline double dist2(const Point &a, const Point &b) {
    return inner(a - b, a - b);
                                                                                         if (pred == 0)
}
                                                                                             result.push_back(line.pos + line.dir * (-b / a));
                                                                                         else if (pred > 0) {
inline double dist(const Line& line, const Point& point, bool segment = false) {
                                                                                             det = sart(det);
    double c1 = inner(point - line.pos, line.dir);
                                                                                             result.push back(line.pos + line.dir * ((-b + det) / a));
                                                                                             result.push back(line.pos + line.dir * ((-b - det) / a));
   if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);</pre>
    double c2 = inner(line.dir, line.dir);
   if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);</pre>
                                                                                         return result;
   return dist(line.pos + line.dir * (c1 / c2), point);
                                                                                     }
}
                                                                                     vector<Point> circle_circle(const Circle& a, const Circle& b) {
bool get cross(const Line& a, const Line& b, Point& ret) {
                                                                                         vector<Point> result:
    double mdet = outer(b.dir, a.dir);
                                                                                         int pred = diff(dist(a.center, b.center), a.r + b.r);
                                                                                         if (pred > 0) return result;
   if (diff(mdet, 0) == 0) return false;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
                                                                                         if (pred == 0) {
    ret = b.pos + b.dir * t2;
                                                                                             result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
    return true;
                                                                                             return result:
}
                                                                                         }
```

```
double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
                                                                                             lower.emplace back(p);
    double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
    double tmp = (bb - aa) / 2.0;
                                                                                         upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
    Point cdiff = b.center - a.center;
                                                                                         return upper;
   if (diff(cdiff.x, 0) == 0) {
                                                                                     }
        if (diff(cdiff.y, 0) == 0)
            return result; // if (diff(a.r, b.r) == 0): same circle
                                                                                     4.3 Poiont in Polygon
        return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } });
   }
                                                                                     typedef double coord_t;
    return circle_line(a,
        Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
                                                                                     inline coord_t is_left(Point p0, Point p1, Point p2) {
}
                                                                                         return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
                                                                                     }
Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
    Point ba = b - a, cb = c - b;
                                                                                     // point in polygon test
    Line p\{(a + b) * 0.5, Point\{ba.y, -ba.x\}\};
                                                                                     // http://geomalgorithms.com/a03- inclusion.html
    Line q\{(b + c) * 0.5, Point\{cb.y, -cb.x\}\};
                                                                                     bool is in polygon(Point p, vector<Point>& poly) {
    Circle circle;
                                                                                         int wn = 0;
   if (!get_cross(p, q, circle.center))
                                                                                         for (int i = 0; i < poly.size(); ++i) {</pre>
        circle.r = -1:
                                                                                             int ni = (i + 1 == poly.size()) ? 0 : i + 1;
    else
                                                                                             if (poly[i].y <= p.y) {</pre>
        circle.r = dist(circle.center, a);
                                                                                                 if (poly[ni].y > p.y) {
    return circle;
                                                                                                     if (is_left(poly[i], poly[ni], p) > 0) {
}
Circle circle_from_2pts_rad(const Point& a, const Point& b, double r) {
                                                                                                 }
    double det = r * r / dist2(a, b) - 0.25;
    Circle circle;
                                                                                             else {
   if (det < 0)
                                                                                                 if (poly[ni].y <= p.y) {</pre>
        circle.r = -1;
                                                                                                     if (is_left(poly[i], poly[ni], p) < 0) {</pre>
    else {
        double h = sqrt(det);
        // center is to the left of a->b
                                                                                                 }
        circle.center = (a + b) * 0.5 + Point{a.y - b.y, b.x - a.x} * h;
        circle.r = r;
                                                                                         }
   }
                                                                                         return wn != 0;
    return circle;
                                                                                     }
}
                                                                                     4.4 Polygon Cut
      Convex Hull
// find convex hull
                                                                                     // Left side of a->b
// O(n*Logn)
                                                                                     vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
vector<Point> convex_hull(vector<Point>& dat) {
                                                                                         if (!polygon.size()) return polygon;
    if (dat.size() <= 3) return dat;</pre>
                                                                                         typedef vector<Point>::const iterator piter;
    vector<Point> upper, lower;
                                                                                         piter la, lan, fi, fip, i, j;
    sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
                                                                                         la = lan = fi = fip = polygon.end();
        return (a.x == b.x)? a.y < b.y: a.x < b.x;
                                                                                         i = polygon.end() - 1;
                                                                                         bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
   });
                                                                                         for (j = polygon.begin(); j != polygon.end(); j++) {
    for (const auto& p : dat) {
        while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
                                                                                             bool thisin = diff(ccw_line(line, *j), 0) > 0;
                                                                                             if (lastin && !thisin) {
          >= 0) upper.pop_back();
        while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
                                                                                                 la = i;
          <= 0) lower.pop_back();
                                                                                                 lan = j;
        upper.emplace back(p);
```

}

```
if (!lastin && thisin) {
                                                                                                     > (long long)(up[i + 1].x - up[i].x) * (lo[j].y - lo[j - 1].y))
           fi = j;
            fip = i;
                                                                                                 ++i;
       i = j;
                                                                                             else {
        lastin = thisin;
                                                                                                 --j;
                                                                                         }
   if (fi == polygon.end()) {
                                                                                     }
        if (!lastin) return vector<Point>();
        return polygon;
   }
    vector<Point> result;
                                                                                           Separating Axis Theorem
    for (i = fi ; i != lan ; i++) {
        if (i == polygon.end()) {
                                                                                     pair<double, double> get_projection(vector<Vector2> &points, Vector2 &axis) {
            i = polygon.begin();
                                                                                       double min_val = axis.dot(points[0]);
            if (i == lan) break;
                                                                                       double max_val = min_val;
                                                                                       for (int i = 1; i < points.size(); i++) {</pre>
        result.push_back(*i);
                                                                                         double projected = axis.dot(points[i]);
                                                                                         min_val = min(min_val, projected);
   Point lc, fc;
                                                                                         max_val = max(max_val, projected);
    get_cross(Line{ *la, *lan - *la }, line, lc);
    get cross(Line{ *fip, *fi - *fip }, line, fc);
                                                                                       return {min_val, max_val};
    result.push_back(lc);
   if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
    return result;
                                                                                     vector<Vector2> get_normals(vector<Vector2> &points) {
                                                                                       vector<Vector2> ret;
                                                                                       if (points.size() == 1)
                                                                                         return ret;
     Rotating Calipers
                                                                                       for (int i = 0; i < points.size(); i++) {</pre>
                                                                                         Vector2 &a = points[i];
// get all antipodal pairs
                                                                                         Vector2 &b = points[(i + 1) % points.size()];
// O(n)
                                                                                         ret.push_back(Vector2((b - a).y, -(b - a).x));
void antipodal pairs(vector<Point>& pt) {
   // calculate convex hull
                                                                                       return ret;
    sort(pt.begin(), pt.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x)? a.y < b.y: a.x < b.x;
                                                                                     bool can_separate(vector<Vector2> &A, vector<Vector2> &B) {
   });
                                                                                       if (A.size() == 1 && B.size() == 1)
    vector<Point> up, lo;
                                                                                         return true;
    for (const auto& p : pt) {
                                                                                       auto c_a = get_convex_hull(A);
        while (up.size() >= 2 \& ccw(*++up.rbegin(), *up.rbegin(), p) >= 0) up.
                                                                                       auto c_b = get_convex_hull(B);
                                                                                       auto n_a = get_normals(c_a);
        while (lo.size() >= 2 \& ccw(*++lo.rbegin(), *lo.rbegin(), p) <= 0) lo.
                                                                                       auto n b = get normals(c b);
          pop_back();
        up.emplace_back(p);
                                                                                       n_a.insert(n_a.end(), n_b.begin(), n_b.end());
        lo.emplace_back(p);
                                                                                       if (c_a.size() > 1) n_a.push_back(Vector2(c_a[1] - c_a[0]));
   }
                                                                                       if (c_b.size() > 1) n_a.push_back(Vector2(c_b[1] - c_b[0]));
    for (int i = 0, j = (int)lo.size() - 1; i + 1 < up.size() || j > 0; ) {
                                                                                       for (Vector2 &axis : n a) {
        get_pair(up[i], lo[j]); // DO WHAT YOU WANT
                                                                                         auto p_a = get_projection(c_a, axis);
        if (i + 1 == up.size()) {
                                                                                         auto p_b = get_projection(c_b, axis);
            --j;
                                                                                         if (!((p_a.second >= p_b.first) && (p_b.second >= p_a.first))) return true;
        else if (j == 0) {
                                                                                       return false;
            ++i:
```

else if ((long long)(up[i + 1].y - up[i].y) * (lo[j].x - lo[j - 1].x)

4.7 Two Far Point

```
int left = 0, right = max_element(points.begin(), points.end()) - points.begin
   ();
 int ret1 = left, ret2 = right;
 double max_len = (points[right] - points[left]).norm();
 int end = right;
 Vector2 left_dir = Vector2(0, -1.0);
 vector<Vector2> edges;
 for (int i = 0; i < points.size(); i++)</pre>
    edges.push_back((points[(i + 1) % points.size()] - points[i]).normalized());
 while (right != 0 || left != end) {
    double next1 = left_dir.dot(edges[left]);
    double next2 = -left dir.dot(edges[right]);
    if (left != end && (right == 0 || next1 > next2)) {
     left_dir = edges[left];
     left = (left + 1) % points.size();
   } else {
     left_dir = -edges[right];
     right = (right + 1) % points.size();
    double len = (points[right] - points[left]).norm();
   if (len > max len) {
     ret1 = left;
     ret2 = right;
     max_len = len;
 return {points[ret1], points[ret2]};
     Two Nearest Point
int dist(Point &p, Point &q) {
 return (p.x - q.x) * (p.x - q.x) + (p.y - q.y) * (p.y - q.y);
struct Comp {
 bool comp_in_x;
 Comp(bool b) : comp_in_x(b) {}
 bool operator()(Point &p, Point &q) {
    return (this->comp_in_x ? p.x < q.x : p.y < q.y);</pre>
 }
};
int nearest(vector<Point>::iterator it, int n) {
 if (n == 2) return dist(it[0], it[1]);
 if (n == 3) return min({dist(it[0], it[1]), dist(it[1], it[2]), dist(it[2], it
   [0])});
 int line = (it[n / 2 - 1].x + it[n / 2].x) / 2;
 int d = min(nearest(it, n / 2), nearest(it + n / 2, n - n / 2));
 vector<Point> mid;
 for (int i = 0; i < n; i++) {
   int t = line - it[i].x;
   if (t * t < d) mid.push_back(it[i]);</pre>
```

pair<Vector2, Vector2> get_max_points(vector<Vector2> &points) {

```
sort(mid.begin(), mid.end(), Comp(false));
int mid_sz = mid.size();
for (int i = 0; i < mid_sz - 1; i++)
   for (int j = i + 1; j < mid_sz && (mid[j].y - mid[i].y) * (mid[j].y - mid[i
            ].y) < d; j++)
            d = min(d, dist(mid[i], mid[j]));
return d;
}</pre>
```

5 Graph

5.1 Dijkstra

```
template<typename T> struct Dijkstra {
   T: 간선가중치타입
  struct Edge {
   ll node;
   T cost;
   bool operator<(const Edge &to) const {</pre>
     return cost > to.cost;
 };
 vector<vector<Edge>> adj;
 vector<ll> prev;
 Dijkstra(ll\ n): n\{n\}, adj(n+1) {}
 void addEdge(ll s, ll e, T cost) {
   adj[s].push back(Edge(e, cost));
 void addUndirectedEdge(ll s, ll e, T cost) {
   addEdge(s, e, cost);
   addEdge(e, s, cost);
 vector <ll> dijkstra(ll s) {
   vector <ll> dist(n+1, INF);
   prev.resize(n+1, -1);
   priority queue<edge> pq;
   pq.push({ s, 011 });
   dist[s] = 0;
   while (!pq.empty()) {
      edge cur = pq.top();
      pq.pop();
     if (cur.cost > dist[cur.node]) continue;
      for (auto &nxt : adj[cur.node])
        if (dist[cur.node] + nxt.cost < dist[nxt.node]) {</pre>
          prev[nxt.node] = cur.node;
          dist[nxt.node] = dist[cur.node] + nxt.cost;
```

```
pq.push({ nxt.node, dist[nxt.node] });
                                                                                                 if (i == N) isCycle = true;
   }
                                                                                             }
                                                                                           }
    return dist;
                                                                                         return isCycle;
  vector<ll> getPath(ll s, ll e) {
   vector<ll> ret;
   11 current = e;
                                                                                       llv1 getPath(ll s, ll e) {
    while(current != -1) {
                                                                                         vector<ll> ret;
      ret.push back(current);
                                                                                         11 current = e;
      current = prev[current];
                                                                                         while(current != -1) {
                                                                                           ret.push_back(current);
   reverse(ret.begin(), ret.end());
                                                                                           current = prev[current];
   return ret;
                                                                                         reverse(ret.begin(), ret.end());
};
                                                                                         return ret;
                                                                                     };
      Bellman-Ford
struct BellmanFord {
                                                                                     5.3 Spfa
 struct BellmanEdge {
   ll to, cost;
                                                                                     // shortest path faster algorithm
   BellmanEdge(ll to, ll cost) : to(to), cost(cost) {}
                                                                                     // average for random graph : O(E) , worst : O(VE)
 };
                                                                                     const int MAXN = 20001;
 11 N;
                                                                                     const int INF = 100000000;
 vector<vector <BellmanEdge> > adj;
                                                                                     int n, m;
 11v1 D;
                                                                                     vector<pii> graph[MAXN];
 vector<ll> prev;
                                                                                     bool inqueue[MAXN];
 BellmanFord(ll N) : N(N) {
                                                                                     int dist[MAXN];
    adj.resize(N + 1);
                                                                                     void spfa(int start) {
                                                                                         for (int i = 0; i < n; ++i) dist[i] = INF;</pre>
 void addEdge(ll s, ll e, ll cost) {
                                                                                         dist[start] = 0;
    adj[s].push back(BellmanEdge(e, cost));
                                                                                         queue<int> q;
                                                                                         q.push(start);
  bool run(ll start point) {
                                                                                         inqueue[start] = true;
   // 음수간선 cycle 유무를반환합니다 . 거리정보는 D 벡터에저장됩니다 . O(V * E)
   D.resize(N + 1, INF);
                                                                                         while (!q.empty()) {
                                                                                             int here = q.front();
   prev.resize(N + 1, -1);
   D[start_point] = 0;
                                                                                             q.pop();
    bool isCycle = false;
                                                                                             inqueue[here] = false;
   for1(1, N + 1) {
                                                                                             for (auto& nxt : graph[here]) {
     for1j(1, N + 1) {
                                                                                                 if (dist[here] + nxt.second < dist[nxt.first]) {</pre>
        for(int k=0; k < sz(adj[j]); k++) {</pre>
                                                                                                     dist[nxt.first] = dist[here] + nxt.second;
          BellmanEdge p = adj[j][k];
                                                                                                     if (!inqueue[nxt.first]) {
          int end = p.to;
                                                                                                         q.push(nxt.first);
          ll \ dist = D[j] + p.cost;
                                                                                                         inqueue[nxt.first] = true;
          if (D[j] != INF && D[end] > dist) {
                                                                                                 }
            D[end] = dist;
```

```
}
     Topological Sort
struct TopologicalSort {
 // 1-index
 int n;
 iv1 in_degree;
 iv2 graph;
 iv1 result;
 TopologicalSort(int n) : n(n) {
   in_degree.resize(n + 1, 0);
   graph.resize(n + 1);
 void addEdge(int s, int e) {
    graph[s].push_back(e);
   in degree[e]++;
 void run() {
    queue<int> q;
    for1(1, n+1) {
     if(in_degree[i] == 0) q.push(i);
    while(!q.empty()) {
     int here = q.front(); q.pop();
     result.push back(here);
     for1(0, sz(graph[here])) {
       int there = graph[here][i];
       if(--in_degree[there]==0) q.push(there);
};
     Strongly Connected Component
struct SCC {
 // 1-index
 // run() 후에에 components 결과가담김 .
 11 V;
 11v2 edges, reversed_edges, components;
 vector<bool> visited;
 stack<ll> visit_log;
 SCC(11 V): V(V) {
```

```
edges.resize(V + 1);
    reversed_edges.resize(V + 1);
  void addEdge(int s, int e) {
    edges[s].push back(e);
    reversed_edges[e].push_back(s);
  void dfs(int node) {
    visited[node] = true;
    for (int next : edges[node])
      if (!visited[next]) dfs(next);
    visit_log.push(node);
  void dfs2(int node) {
    visited[node] = true;
    for (int next:reversed_edges[node])
      if (!visited[next]) dfs2(next);
    components.back().push_back(node);
  void run() {
    visited = vector<bool>(V + 1, false);
    for (int node = 1; node <= V; node++)</pre>
      if (!visited[node]) dfs(node);
    visited = vector<bool>(V + 1, false);
    while (!visit_log.empty()) {
      11 node = visit_log.top(); visit_log.pop();
      if (!visited[node]) {
        components.push_back(llv1());
        dfs2(node);
      }
 }
};
5.6 2-SAT
struct Graph {
  int V;
  vector<bool> visited;
  stack<int> visit stack;
  vector<int> component_number, source_components;
  vector<vector<int>> edges, reversed_edges, components, components_edges;
  Graph(int V) : V(V) {
    edges.resize(V);
    reversed_edges.resize(V);
  void append(int u, int v) {
    edges[u].push_back(v);
    reversed_edges[v].push_back(u);
```

```
void dfs(int node) {
    visited[node] = true;
    for (int next : edges[node])
     if (!visited[next])
        dfs(next);
    visit_stack.push(node);
  void scc(int node) {
    visited[node] = true;
    for (int next : reversed edges[node])
      if (!visited[next])
        scc(next);
    components.back().push back(node);
 void build_scc() {
    visited = vector<bool>(V, false);
    for (int node = 0; node < V; node++)</pre>
     if (!visited[node]) dfs(node);
    visited = vector<bool>(V, false);
    while (!visit_stack.empty()) {
      int node = visit stack.top();
      visit_stack.pop();
      if (!visited[node]) {
        components.push back(vector<int>());
        scc(node);
    component_number.resize(V);
    for (int i = 0; i < components.size(); i++)</pre>
      for (int node : components[i]) component_number[node] = i;
    vector<bool> is_source = vector<bool>(components.size(), true);
    components edges.resize(components.size());
    for (int u = 0; u < V; u++)
      for (int v : edges[u]) {
        int cu = component number[u];
        int cv = component_number[v];
        if (cu == cv) continue;
        components_edges[cu].push_back(cv);
        is_source[cv] = false;
    for (int component = 0; component < components.size(); component++) {</pre>
      if (is_source[component]) source_components.push_back(component);
 }
};
int main(void) {
 int V, E;
 cin >> V >> E;
 Graph graph(2 * V + 1);
 for (int i = 0; i < E; i++) {
   int u, v;
    cin >> u >> v;
    graph.append(-u + V, v + V);
```

```
graph.append(-v + V, u + V);
  graph.build_scc();
  vector<int> last component(2 * V + 1, -1);
  bool is_answer = true;
  for (int i = 0; i < graph.components.size(); i++) {</pre>
    for (int node : graph.components[i]) {
      int negation = -(node - V) + V;
      if (last component[negation] == i) is answer = false;
      last_component[node] = i;
  if (is_answer) {
    vector<int> result(V);
    for (int i = 1; i <= V; i++) {
      int val = i + V;
      int negation = -i + V;
      result[i - 1] = graph.component_number[val] > graph.component_number[
        negation];
    for (int val : result) cout << val << "";
    cout << "\n";</pre>
}
     Union Find
struct UnionFind {
  int n;
  vector<int> u;
  UnionFind(int n) : n(n) {
    u.resize(n + 1);
    for(int i = 1; i <= n; i++) {
      u[i] = i;
    }
  int find(int k) {
    if(u[u[k]] == u[k]) return u[k];
    else return u[k]=find(u[k]);
  void uni(int a, int b) {
    a = find(a);
    b = find(b);
   if(a < b) u[b] = a;
    else u[a] = b;
};
5.8
     MST Prim
struct edge {
```

```
11 crt:
 11 node, cost;
};
struct WGraph {
 11 V;
 vector<edge> adj[MAX];
 vector<ll> prev;
 WGraph(ll\ V): V\{V\} \{\}
 void addEdge(ll s, ll e, ll cost) {
   adj[s].push_back({s, e, cost});
    adj[e].push back({e, s, cost});
 ll prim(vector<edge> &selected) { // 에selected 선택된간선정보 vector 담김
   selected.clear();
    vector<bool> added(V, false);
   llv1 minWeight(V, INF), parent(V, -1);
   ll ret = 0:
    minWeight[0] = parent[0] = 0;
    for (int iter = 0; iter < V; iter++) {</pre>
     int u = -1;
     for (int v = 0; v < V; v++) {
       if (!added[v] && (u == -1 || minWeight[u]>minWeight[v]))
          u = v;
     }
     if (parent[u] != u)
        selected.push_back({parent[u], u, minWeight[u]});
     ret += minWeight[u];
     added[u] = true;
     for1(0, sz(adj[u])) {
       int v = adj[u][i].node, weight = adj[u][i].cost;
       if (!added[v] && minWeight[v]>weight) {
          parent[v] = u;
          minWeight[v] = weight;
       }
     }
    return ret;
};
     Lowest Common Ancestor
#define MAX_DEGREE 20
struct LCA {
 // root: 트리의루트설정 , n: 트리의노드개수
 // addEdge -> init -> query(O(log(n))
 ll root, n;
```

```
llv1 depth:
llv2 adj;
11v2 parent; // n X MAX_DEGREE
LCA(ll root, ll n) : root(root), n(n) {
  depth.resize(n + 1);
  adj.resize(n + 1);
  parent.resize(n + 1, llv1(MAX_DEGREE, 0));
void addEdge(ll a, ll b) {
  adj[a].push back(b);
  adj[b].push_back(a);
void init() {
  dfs(root, 0, 1);
  for(int i = 1; i < MAX_DEGREE; i++) {</pre>
    for(int j = 1; j <= n; j++) {
      parent[j][i] = parent[parent[j][i-1]][i-1];
}
void dfs(int here, int par, int d) {
  depth[here] = d;
  parent[here][0] = par;
  for(int there : adj[here]) {
    if(depth[there] > 0) continue;
    dfs(there, here, d + 1);
}
int query(int a, int b) {
  if(depth[a] > depth[b]) {
    swap(a, b);
  for(int i = MAX DEGREE - 1; i >= 0; i--) {
    if (depth[b] - depth[a] >= (1 << i)) {</pre>
      b = parent[b][i];
   }
  }
  if(a == b) {
    return a;
  for(int i = MAX DEGREE - 1; i >= 0; i--) {
    if(parent[a][i] != parent[b][i]) {
      a = parent[a][i];
      b = parent[b][i];
```

```
return parent[a][0];
};
       Maxflow dinic
5.10
// usage:
// MaxFlowDinic::init(n);
// MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
// MaxFlowDinic::add edge(1, 2, 100); // directional edge
// result = MaxFlowDinic::solve(0, 2); // source -> sink
// graph[i][edgeIndex].res -> residual
//
// in order to find out the minimum cut, use `l'.
// if l[i] == 0, i is unrechable.
//
// O(V*V*E)
// with unit capacities, O(\min(V^{(2/3)}, E^{(1/2)}) * E)
struct MaxFlowDinic {
    typedef int flow t;
    struct Edge {
        int next;
        size_t inv; /* inverse edge index */
        flow_t res; /* residual */
    };
    int n;
    vector<vector<Edge>> graph;
    vector<int> q, 1, start;
    void init(int _n) {
        n = n;
        graph.resize(n);
        for (int i = 0; i < n; i++) graph[i].clear();</pre>
    void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
        Edge forward{ e, graph[e].size(), cap };
        Edge reverse{ s, graph[s].size(), caprev };
        graph[s].push back(forward);
        graph[e].push back(reverse);
    bool assign_level(int source, int sink) {
        int t = 0;
        memset(&l[0], 0, sizeof(l[0]) * l.size());
        1[source] = 1;
        q[t++] = source;
        for (int h = 0; h < t && !1[sink]; h++) {</pre>
            int cur = q[h];
            for (const auto& e : graph[cur]) {
                if (l[e.next] || e.res == 0) continue;
                l[e.next] = l[cur] + 1;
                q[t++] = e.next;
            }
```

```
return 1[sink] != 0;
    flow t block flow(int cur, int sink, flow t current) {
        if (cur == sink) return current;
        for (int& i = start[cur]; i < graph[cur].size(); i++) {</pre>
            auto& e = graph[cur][i];
            if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
            if (flow t res = block flow(e.next, sink, min(e.res, current))) {
                e.res -= res;
                graph[e.next][e.inv].res += res;
                return res;
            }
        return 0;
    flow t solve(int source, int sink) {
        a.resize(n);
        1.resize(n);
        start.resize(n);
        flow_t ans = 0;
        while (assign level(source, sink)) {
            memset(&start[0], 0, sizeof(start[0]) * n);
            while (flow t flow = block flow(source, sink, numeric limits<flow t
             >::max()))
                ans += flow;
        }
        return ans;
};
5.11 Maxflow Edmonds-Karp
struct MaxFlowEdgeDemands
    MaxFlowDinic mf;
    using flow t = MaxFlowDinic::flow t;
    vector<flow t> ind, outd;
    flow t D; int n;
    void init(int n) {
        n = _n; D = 0; mf.init(n + 2);
        ind.clear(); outd.clear();
        ind.resize(n, 0); outd.resize(n, 0);
    }
    void add_edge(int s, int e, flow_t cap, flow_t demands = 0) {
        mf.add edge(s, e, cap - demands);
        D += demands; ind[e] += demands; outd[s] += demands;
    }
    // returns { false, 0 } if infeasible
    // { true, maxflow } if feasible
    pair<bool, flow t> solve(int source, int sink) {
```

```
mf.add edge(sink, source, numeric limits<flow t>::max());
                                                                                        graph[u].push_back(ori);
        for (int i = 0; i < n; i++) {
                                                                                        graph[v].push_back(rev);
            if (ind[i]) mf.add edge(n, i, ind[i]);
           if (outd[i]) mf.add_edge(i, n + 1, outd[i]);
        }
                                                                                      void addEdgeFromSrc(ll v, ll cap, ll cost) {
                                                                                        // 출발지점에서출발하는간선추가
       if (mf.solve(n, n + 1) != D) return{ false, 0 };
                                                                                        addEdge(source, v, cap, cost);
        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.graph[i].pop back();
                                                                                      void addEdgeToSink(ll u, ll cap, ll cost) {
                                                                                        // 도착지점으로가는간선추가
            if (outd[i]) mf.graph[i].pop_back();
        }
                                                                                        addEdge(u, sink, cap, cost);
        return{ true, mf.solve(source, sink) };
                                                                                      bool spfa(ll &total_flow, ll &total_cost) {
};
                                                                                        // spfa 기반의 MCMF
                                                                                        fill(check.begin(), check.end(), false);
       MCMF SPFA
5.12
                                                                                        fill(distance.begin(), distance.end(), INF);
                                                                                        fill(from.begin(), from.end(), make_pair(-1, -1));
struct MCMF {
  struct Edge {
                                                                                        distance[source] = 0;
   11 to;
                                                                                        queue <ll> q;
   11 capacity;
                                                                                        q.push(source);
   ll cost;
                                                                                        while(!q.empty()) {
    Edge* rev;
                                                                                          11 x = q.front(); q.pop();
   Edge(ll to, ll capacity, ll cost) : to(to), capacity(capacity), cost(cost)
     {}
                                                                                          check[x] = false;
 };
                                                                                          for(ll i = 0; i < graph[x].size(); i++) {</pre>
 11 n;
                                                                                            Edge* e = graph[x][i];
 11 source, sink;
                                                                                            11 y = e \rightarrow to;
 vector<vector<Edge *>> graph;
 vector<bool> check;
                                                                                            if(e->capacity > 0 && distance[x] + e->cost < distance[y]) {</pre>
 vector<ll> distance;
                                                                                              distance[y] = distance[x] + e->cost;
 vector<pair<ll, ll>> from;
                                                                                              from[y] = make pair(x, i);
 MCMF(ll n, ll source, ll sink): n(n), source(source), sink(sink) {
                                                                                              if(!check[y]) {
                                                                                                check[y] = true;
   // source: 시작점
                                                                                                q.push(y);
   // sink: 도착점
   // n: 모델링한그래프의정점개수
    graph.resize(n + 1);
    check.resize(n + 1);
    from.resize(n + 1, make_pair(-1, -1));
                                                                                        }
    distance.resize(n + 1);
 };
                                                                                        if(distance[sink] == INF) return false;
 void addEdge(ll u, ll v, ll cap, ll cost) {
                                                                                        // 간선을에서부터 sink 역추적하여경로를만든다 .
    Edge *ori = new Edge(v, cap, cost);
                                                                                        11 x = sink;
    Edge *rev = new Edge(u, 0, -cost);
                                                                                        11 c = graph[from[x].first][from[x].second]->capacity;
   ori->rev = rev;
                                                                                        while(from[x].first != -1) {
   rev->rev = ori;
```

```
if(c > graph[from[x].first][from[x].second]->capacity) {
        c = graph[from[x].first][from[x].second]->capacity;
     x = from[x].first;
    // 만든경로를따라유량을흘린다
   x = sink;
    while(from[x].first != -1) {
     Edge* e = graph[from[x].first][from[x].second];
     e->capacity -= c;
     e->rev->capacity += c;
     x = from[x].first;
    total flow += c;
    total cost += c * distance[sink];
    return true;
 }
 pair <11, 11> flow() {
   11 total flow = 0;
   11 total cost = 0;
   while(spfa(total_flow, total_cost));
    return make pair(total flow, total cost);
};
5.13 MCMF
// precondition: there is no negative cycle.
// usage: MinCostFlow mcf(n);
// for(each edges) mcf.addEdge(from, to, cost, capacity);
// mcf.solve(source, sink); // min cost max flow
// mcf.solve(source, sink, 0); // min cost flow
// mcf.solve(source, sink, goal flow); // min cost flow with total flow >=
 aoal flow if possible
struct MinCostFlow {
    typedef int cap t;
    typedef int cost_t;
   bool iszerocap(cap t cap) { return cap == 0; }
    struct edge {
        int target;
        cost t cost;
        cap t residual capacity;
        cap_t orig_capacity;
        size t revid;
   };
   int n;
```

```
vector<vector<edge>> graph;
MinCostFlow(int n) : graph(n), n(n) {}
void addEdge(int s, int e, cost_t cost, cap_t cap) {
    if (s == e) return;
    edge forward{ e, cost, cap, cap, graph[e].size() };
    edge backward{ s, -cost, 0, 0, graph[s].size() };
    graph[s].emplace back(forward);
    graph[e].emplace_back(backward);
}
pair<cost t, cap t> augmentShortest(int s, int e, cap t flow limit) {
    auto infinite cost = numeric limits<cost t>::max();
    auto infinite_flow = numeric_limits<cap_t>::max();
    vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
    vector<int> from(n, -1), v(n);
    dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
    aueue<int> a:
    v[s] = 1; q.push(s);
    while(!q.empty()) {
        int cur = q.front();
        v[cur] = 0; q.pop();
        for (const auto& e : graph[cur]) {
            if (iszerocap(e.residual_capacity)) continue;
            auto next = e.target;
            auto ncost = dist[cur].first + e.cost;
            auto nflow = min(dist[cur].second, e.residual capacity);
            if (dist[next].first > ncost) {
                dist[next] = make pair(ncost, nflow);
                from[next] = e.revid;
                if (v[next]) continue;
                v[next] = 1; q.push(next);
        }
    }
    auto p = e;
    auto pathcost = dist[p].first;
    auto flow = dist[p].second;
    if (iszerocap(flow)|| (flow limit <= 0 && pathcost >= 0)) return pair
      cost_t, cap_t>(0, 0);
    if (flow limit > 0) flow = min(flow, flow limit);
    while (from[p] != -1) {
        auto nedge = from[p];
        auto np = graph[p][nedge].target;
        auto fedge = graph[p][nedge].revid;
        graph[p][nedge].residual_capacity += flow;
        graph[np][fedge].residual capacity -= flow;
        p = np;
    return make pair(pathcost * flow, flow);
```

```
pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits
     cap_t>::max()) {
        cost t total cost = 0;
        cap_t total_flow = 0;
        for(;;) {
            auto res = augmentShortest(s, e, flow minimum - total flow);
            if (res.second <= 0) break;</pre>
            total cost += res.first;
            total_flow += res.second;
        return make_pair(total_cost, total_flow);
};
5.14 BCC
const int MAXN = 100:
vector<pair<int, int>> graph[MAXN]; // { next vertex id, edge id }
int up[MAXN], visit[MAXN], vtime;
vector<int> stk;
int is cut[MAXN];
                              // v is cut vertex if is cut[v] > 0
vector<int> bridge;
                              // list of edge ids
vector<int> bcc_edges[MAXN]; // list of edge ids in a bcc
int bcc cnt;
void dfs(int nod, int par_edge) {
   up[nod] = visit[nod] = ++vtime;
    int child = 0;
    for (const auto& e : graph[nod]) {
        int next = e.first, eid = e.second;
        if (eid == par_edge) continue;
        if (visit[next] == 0) {
            stk.push_back(eid);
            ++child;
            dfs(next, eid);
            if (up[next] == visit[next]) bridge.push_back(eid);
            if (up[next] >= visit[nod]) {
                ++bcc cnt;
                do {
                    auto lasteid = stk.back();
                    stk.pop_back();
                    bcc_edges[bcc_cnt].push_back(lasteid);
                    if (lasteid == eid) break;
                } while (!stk.empty());
                is_cut[nod]++;
            }
            up[nod] = min(up[nod], up[next]);
        else if (visit[next] < visit[nod]) {</pre>
            stk.push_back(eid);
            up[nod] = min(up[nod], visit[next]);
   }
```

```
if (par edge == -1 && is cut[nod] == 1)
        is_cut[nod] = 0;
}
// find BCCs & cut vertexs & bridges in undirected graph
// O(V+E)
void get bcc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    memset(is_cut, 0, sizeof(is_cut));
    bridge.clear();
    for (int i = 0; i < n; ++i) bcc edges[i].clear();</pre>
    bcc cnt = 0;
    for (int i = 0; i < n; ++i) {
        if (visit[i] == 0)
            dfs(i, -1);
}
5.15 Bipartite Matching
// in: n, m, graph
// out: match, matched
// vertex cover: (reached[0][left node] == 0) || (reached[1][right node] == 1)
// 0(E*sqrt(V))
struct BipartiteMatching {
    int n, m;
    vector<vector<int>> graph;
    vector<int> matched, match, edgeview, level;
    vector<int> reached[2];
    BipartiteMatching(int n, int m) : n(n), m(m), graph(n), matched(m, -1),
      match(n, -1) {}
    bool assignLevel() {
        bool reachable = false;
        level.assign(n, -1);
        reached[0].assign(n, 0);
        reached[1].assign(m, 0);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (match[i] == -1) {
                level[i] = 0;
                reached[0][i] = 1;
                q.push(i);
            }
        while (!q.empty()) {
            auto cur = q.front(); q.pop();
            for (auto adj : graph[cur]) {
                reached[1][adj] = 1;
                auto next = matched[adj];
                if (next == -1) {
                    reachable = true;
                else if (level[next] == -1) {
```

```
level[next] = level[cur] + 1;
                    reached[0][next] = 1;
                    q.push(next);
        return reachable;
    }
    int findpath(int nod) {
        for (int &i = edgeview[nod]; i < graph[nod].size(); i++) {</pre>
            int adj = graph[nod][i];
            int next = matched[adj];
            if (next >= 0 && level[next] != level[nod] + 1) continue;
            if (next == -1 || findpath(next)) {
                match[nod] = adj;
                matched[adj] = nod;
                return 1;
            }
        return 0;
   }
    int solve() {
        int ans = 0;
        while (assignLevel()) {
            edgeview.assign(n, 0);
            for (int i = 0; i < n; i++)
                if (match[i] == -1)
                    ans += findpath(i);
        return ans;
};
```

6 String

6.1 KMP

```
struct KMP {
    /*
    s 문자열에서문자열을 o 찾습니다.매칭이시작되는인덱스목록을반환합니다
    .
    Time: O(n + m)
    */
    vector<int> result;
    int MX;
    string s, o;
    int n, m; // n : s.length(), m :o.length();
    vector<int> fail;

KMP(string s, string o) : s(s), o(o) {
    n = s.length();
    m = o.length();
```

```
MX = max(n, m) + 1;
    fail.resize(MX, 0);
    run();
  void run() {
    for(int i = 1, j = 0; i < m; i++){
      while(j > 0 && o[i] != o[j]) j = fail[j-1];
      if(o[i] == o[j]) fail[i] = ++j;
    for(int i = 0, j = 0; i < n; i++) {
      while(j > 0 && s[i] != o[j]) {
        j = fail[j - 1];
      if(s[i] == o[j]) {
        if(j == m - 1) {
          // matching OK;
          result.push_back(i - m + 1);
          j = fail[j];
        else {
          j++;
 }
};
      Manacher
// Use space to insert space between each character
// To get even length palindromes!
// 0(|str|)
vector<int> manacher(string &s) {
  int n = s.size(), R = -1, p = -1;
  vector<int> A(n);
  for (int i = 0; i < n; i++) {
    if (i \le R) A[i] = min(A[2 * p - i], R - i);
    while (i - A[i] - 1 >= 0 \& i + A[i] + 1 < n \& s[i - A[i] - 1] == s[i + A[i]
     ] + 1])
      A[i]++;
    if (i + A[i] > R)
      R = i + A[i], p = i;
  return A;
string space(string &s) {
  string t;
  for (char c : s) t += c, t += 'u';
```

t.pop_back();

return t;

```
return height;
                                                                                      }
int maxpalin(vector<int> &M, int i) {
 if (i % 2) return (M[i] + 1) / 2 * 2;
 return M[i] / 2 * 2 + 1;
                                                                                      6.4 Trie
                                                                                      int chToIdx(char ch) { return ch - 'a'; }
6.3
     Suffix Array
                                                                                      struct Trie {
                                                                                        int terminal = -1;
typedef char T;
                                                                                        Trie *fail; // fail, 은output 아호코라식에사용
                                                                                        vector<int> output;
// calculates suffix array.
                                                                                        Trie *chil[ALPHABETS];
// O(n*Logn)
                                                                                        Trie() {
vector<int> suffix_array(const vector<T>& in) {
                                                                                          for (int i = 0; i < ALPHABETS; i++)</pre>
    int n = (int)in.size(), c = 0;
                                                                                            chil[i] = NULL;
    vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
    for (int i = 0; i < n; i++) out[i] = i;
                                                                                        // number -> 문자열번호 (ith string)
    sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });</pre>
                                                                                        void insert(string &s, int number, int idx) {
    for (int i = 0; i < n; i++) {
                                                                                          if (idx == s.size()) {
        bckt[i] = c;
                                                                                            terminal = number;
        if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
                                                                                            return;
    for (int h = 1; h < n && c < n; h <<= 1) {
                                                                                          int next = chToIdx(s[idx]);
        for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];</pre>
                                                                                          if (chil[next] == NULL)
        for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
                                                                                            chil[next] = new Trie();
        for (int i = 0; i < n; i++)
                                                                                          chil[next]->insert(s, number, idx + 1);
            if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
        for (int i = 0; i < n; i++)</pre>
                                                                                        int find(string &s, int idx = 0) {
            if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
                                                                                          if (idx == s.size())
        c = 0;
                                                                                            return terminal;
        for (int i = 0; i + 1 < n; i++) {
                                                                                          int next = chToIdx(s[idx]);
            int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
                                                                                          if (chil[next] == NULL)
                    || (pos2bckt[temp[i + 1] + h] != pos2bckt[temp[i] + h]);
                                                                                            return false;
            bckt[i] = c;
                                                                                          return chil[next]->find(s, idx + 1);
            c += a;
                                                                                      };
        bckt[n - 1] = c++;
        temp.swap(out);
    }
                                                                                           Aho-Corasick
    return out;
}
                                                                                      void computeFail(Trie *root) {
                                                                                          queue<Trie *> q;
// calculates lcp array. it needs suffix array & original sequence.
                                                                                          root->fail = root;
// O(n)
                                                                                          q.push(root);
vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
                                                                                          while (!q.empty()) {
    int n = (int)in.size();
                                                                                              Trie *here = q.front();
    if (n == 0) return vector<int>();
                                                                                              q.pop();
    vector<int> rank(n), height(n - 1);
                                                                                              for (int i = 0; i < ALPHABETS; i++) {</pre>
    for (int i = 0; i < n; i++) rank[sa[i]] = i;</pre>
                                                                                                  Trie *child = here->chil[i];
    for (int i = 0, h = 0; i < n; i++) {
                                                                                                  if (!child) continue;
                                                                                                  if (here == root) child->fail = root;
        if (rank[i] == 0) continue;
        int j = sa[rank[i] - 1];
                                                                                                  else {
        while (i + h < n \&\& j + h < n \&\& in[i + h] == in[j + h]) h++;
                                                                                                      Trie *t = here->fail;
        height[rank[i] - 1] = h;
                                                                                                      while (t != root && t->chil[i] == NULL) t = t->fail;
        if (h > 0) h--;
                                                                                                      if (t->chil[i]) t = t->chil[i];
   }
                                                                                                      child->fail = t;
```

```
child->output = child->fail->output;
           if (child->terminal != -1) child->output.push_back(child->terminal);
           q.push(child);
       }
   }
}
vector<pair<int, int>> ahoCorasick(string &s, Trie *root) {
   vector<pair<int, int>> ret;
   Trie *state = root;
   for (int i = 0; i < s.size(); i++) {
        int idx = chToIdx(s[i]);
        while (state != root && state->chil[idx] == NULL) state = state->fail;
       if (state->chil[idx]) state = state->chil[idx];
       for (int j = 0; j < state->output.size(); j++) ret.push_back({i, state->
         output[j]});
   }
    return ret;
     Z Algorithm
// Z[i] : maximum common prefix length of &s[0] and &s[i]
// O(|s|)
using seq_t = string;
vector<int> z_func(const seq_t &s) {
   vector<int> z(s.size());
   z[0] = s.size();
   int 1 = 0, r = 0;
   for (int i = 1; i < s.size(); i++) {
       if (i > r) {
           int j;
           for (j = 0; i + j < s.size() && s[i + j] == s[j]; j++);
           z[i] = j; l = i; r = i + j - 1;
       \} else if (z[i-1] < r-i+1) {
           z[i] = z[i - 1];
       } else {
           for (j = 1; r + j < s.size() && s[r + j] == s[r - i + j]; j++);
           z[i] = r - i + j; l = i; r += j - 1;
   }
    return z;
}
    Dynamic Programming
7.1 LIS
```

```
struct LIS {
   llv1 ar;
```

```
llv1 v, buffer;
  llv1::iterator vv;
  vector<pair<ll, 11> > d;
  void perform() {
    v.pb(200000000011);
    11 n = sz(ar);
    for1(0, n){
      if (ar[i] > *v.rbegin()) {
        v.pb(ar[i]);
        d.push_back({ v.size() - 1, ar[i] });
      else {
        vv = lower_bound(v.begin(), v.end(), ar[i]);
        *vv = ar[i];
        d.push_back({ vv - v.begin(), ar[i] });
    for(int i = sz(d) - 1; i > -1; i--){
      if(d[i].first == sz(v)-1){
        buffer.pb(d[i].second);
        v.pop_back();
    reverse(buffer.begin(), buffer.end());
  11 length() {
    return buffer.size();
  llv1 result() {
    return buffer;
};
     KnapSack
11 N, maxWeight, ans;
ll D[2][11000];
11 weight[110], cost[110];
void knapsack() {
 for (int x = 1; x <= N; x++) {
    for (int y = 0; y \leftarrow maxWeight; y++) {
      if (y >= weight[x]) {
        D[x \% 2][y] = max(D[(x + 1) \% 2][y], D[(x + 1) \% 2][y - weight[x]] +
          cost[x]);
      } else {
        D[x \% 2][y] = D[(x + 1) \% 2][y];
      ans = max(ans, D[x \% 2][y]);
```

```
}
void input() {
 cin >> N >> maxWeight;
 for (int x = 1; x <= N; x++) {
    cin >> weight[x] >> cost[x];
 }
}
     Bit Field DP
#define MOD 9901;
int dp[1 << 14 + 1][200];</pre>
int n, m;
int solve(int pos, int check, int dep) {
 if (dp[check][pos] != 0) return dp[check][pos];
 int &ret = dp[check][pos];
 if (dep == n * m) return ret = 1;
 if ((check & 1)) return ret = solve(pos - 1, check >> 1, dep) % MOD;
 int sum = 0;
 if (!(check & 1) && (pos - 1) / m > 0)
    sum += solve(pos - 1, (check >> 1) | (1 << (m - 1)), dep + 2) % MOD;
 if (!(check & 1) && pos % m != 1 && !(check & 2) && pos >= 2 && m > 1)
    sum += solve(pos - 2, check >> 2, dep + 2) % MOD;
 return ret = sum % MOD;
int main() {
 cin >> n >> m;
 if (n * m % 2 == 1)
    cout << 0;
    cout << solve(n * m, 0, 0) % MOD;</pre>
}
     Knuth Optimization
int solve(int n) {
    for (int m = 2; m <= n; m++) {
        for (int i = 0; m + i <= n; i++) {
            int j = i + m;
            for (int k = K[i][j - 1]; k \leftarrow K[i + 1][j]; k++) {
                int now = dp[i][k] + dp[k][j] + sum[j] - sum[i];
                if (dp[i][j] > now)
                    dp[i][j] = now, K[i][j] = k;
        }
    return dp[0][n];
}
int main() {
    int n;
    cin >> n;
```

```
fill(&dp[0][0], &dp[MAX-1][MAX-1], INF);
for (int i = 1; i <= n; i++){
    cin >> arr[i];
    sum[i] = sum[i - 1] + arr[i];
    K[i - 1][i] = i;
    dp[i - 1][i] = 0;
}
cout << solve(n) << "\n";
}

/*
if
C[a][c] + C[b][d] <= C[a][d] + C[b][c] (a<=b<=c<=d)
C[b][c] <= C[a][d] (a<=b<=c<=d)

then
dp[i][j] = min(dp[i][k] + dp[k][j]) + C[i][j]
range of k: A[i, j-1] <= A[i][j]=k <= A[i+1][j]
*/</pre>
```