Contents

1 Setting

1.1 PS

```cpp
#include <bits/stdc++.h>

using namespace std;

#define for1(s, e) for(int i = s; i < e; i++)
#define for1j(s, e) for(int j = s; j < e; j++)
#define forEach(k) for(auto i : k)
#define forEachj(k) for(auto j : k)
#define sz(vct) vct.size()
#define all(vct) vct.begin(), vct.end()
#define sortv(vct) sort(vct.begin(), vct.end())
#define uniq(vct) sort(all(vct));vct.erase(unique(all(vct)), vct.end())
#define fi first
#define se second
#define INF (1ll << 60ll)

typedef unsigned long long ull;
typedef long long ll;
typedef ll llint;
typedef unsigned int uint;
typedef unsigned long long int ull;
typedef ull ullint;

typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair<double, double> pdd;
```

```
typedef pair<double, int> pdi;
typedef pair<string, string> pss;

typedef vector<int> iv1;
typedef vector<iv1> iv2;
typedef vector<ll> llv1;
typedef vector<llv1> llv2;

typedef vector<pii> piiv1;
typedef vector<piiv1> piiv2;
typedef vector<pll> pllv1;
typedef vector<pllv1> pllv2;
typedef vector<pdd> pddv1;
typedef vector<pddv1> pddv2;

const double EPS = 1e-8;
const double PI = acos(-1);

template<typename T>
T sq(T x) { return x * x; }

int sign(ll x) { return x < 0 ? -1 : x > 0 ? 1 : 0; }
int sign(int x) { return x < 0 ? -1 : x > 0 ? 1 : 0; }
int sign(double x) { return abs(x) < EPS ? 0 : x < 0 ? -1 : 1; }

void solve() {

}

int main() {
  ios::sync_with_stdio(0);
  cin.tie(NULL);cout.tie(NULL);
  int tc = 1; // cin >> tc;
  while(tc--) solve();
}
```

2 Math

2.1 Basic Arithmetics

```
typedef long long ll;
typedef unsigned long long ull;

// calculate lg2(a)
inline int lg2(ll a) {
    return 63 - __builtin_clzll(a);
}

// calculate the number of 1-bits
inline int bitcount(ll a) {
    return __builtin_popcountll(a);
}

// calculate ceil(a/b)
// |a|, |b| <= (2^63)-1 (does not dover -2^63)
ll ceildiv(ll a, ll b) {
    if (b < 0) return ceildiv(-a, -b);
    if (a < 0) return (-a) / b;
    return ((ull)a + (ull)b - 1ull) / b;
}

// calculate floor(a/b)
// |a|, |b| <= (2^63)-1 (does not cover -2^63)
ll floordiv(ll a, ll b) {
    if (b < 0) return floordiv(-a, -b);
    if (a >= 0) return a / b;
    return -(ll)(((ull)(-a) + b - 1) / b);
}

// calculate a*b % m
// x86-64 only
ll large_mod_mul(ll a, ll b, ll m) {
    return ll((__int128)a*(__int128)b%m);
}

// calculate a*b % m
// |m| < 2^62, x86 available
// O(logb)
ll large_mod_mul(ll a, ll b, ll m) {
    a %= m; b %= m; ll r = 0, v = a;
    while (b) {
        if (b&1) r = (r + v) % m;
        b >>= 1;
        v = (v << 1) % m;
    }
    return r;
}

// calculate n^k % m
ll modpow(ll n, ll k, ll m) {
    ll ret = 1;
    n %= m;
    while (k) {
        if (k & 1) ret = large_mod_mul(ret, n, m);
        n = large_mod_mul(n, n, m);
        k /= 2;
    }
    return ret;
}

// calculate gcd(a, b)
ll gcd(ll a, ll b) {
    return b == 0 ? a : gcd(b, a % b);
}

// find a pair (c, d) s.t. ac + bd = gcd(a, b)
pair<ll, ll> extended_gcd(ll a, ll b) {
    if (b == 0) return { 1, 0 };
    auto t = extended_gcd(b, a % b);
```

```
    return { t.second, t.first - t.second * (a / b) };
}

// find x in [0,m) s.t. ax === gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (extended_gcd(a, m).first % m + m) % m;
}

// calculate modular inverse for 1 ~ n
void calc_range_modinv(int n, int mod, int ret[]) {
    ret[1] = 1;
    for (int i = 2; i <= n; ++i)
        ret[i] = (ll)(mod - mod/i) * ret[mod%i] % mod;
}


// p is prime
// calculate a^b % p
ll pow(ll a, ll b){
    if(b == 0) return 1;
    ll n = pow(a, b/2) % p;
    ll temp = (n * n) % p;

    if(b%2==0) return temp;
    return (a * temp) % p;
}

// p is prime
// calculate a/b % p
ll fermat(ll a, ll b){
    return a % p * pow(b, p-2) % p;
}
```

2.2 Convex Hull Trick

```
struct CHTLinear {
    struct Line {
        long long a, b;
        long long y(long long x) const { return a * x + b; }
    };
    vector<Line> stk;
    int qpt;
    CHTLinear() : qpt(0) { }
    // when you need maximum :  (previous l).a < (now l).a
    // when you need minimum :  (previous l).a > (now l).a
    void pushLine(const Line& l) {
        while (stk.size() > 1) {
            Line& l0 = stk[stk.size() - 1];
            Line& l1 = stk[stk.size() - 2];
            if ((l0.b - l.b) * (l0.a - l1.a) > (l1.b - l0.b) * (l.a - l0.a))
              break;
            stk.pop_back();
        }
        stk.push_back(l);
    }
    // (previous x) <= (current x)
    // it calculates max/min at x
    long long query(long long x) {
        while (qpt + 1 < stk.size()) {
            Line& l0 = stk[qpt];
            Line& l1 = stk[qpt + 1];
            if (l1.a - l0.a > 0 && (l0.b - l1.b) > x * (l1.a - l0.a)) break;
            if (l1.a - l0.a < 0 && (l0.b - l1.b) < x * (l1.a - l0.a)) break;
            ++qpt;
        }
        return stk[qpt].y(x);
    }
};
```

2.3 Luca Theorem

```
// calculate nCm % p  when p is prime
int lucas_theorem(const char *n, const char *m, int p) {
    vector<int> np, mp;
    int i;
    for (i = 0; n[i]; i++) {
        if (n[i] == '0' && np.empty()) continue;
        np.push_back(n[i] - '0');
    }
    for (i = 0; m[i]; i++) {
        if (m[i] == '0' && mp.empty()) continue;
        mp.push_back(m[i] - '0');
    }

    int ret = 1;
    int ni = 0, mi = 0;
    while (ni < np.size() || mi < mp.size()) {
        int nmod = 0, mmod = 0;
        for (i = ni; i < np.size(); i++) {
            if (i + 1 < np.size())
                np[i + 1] += (np[i] % p) * 10;
            else
                nmod = np[i] % p;
            np[i] /= p;
        }
        for (i = mi; i < mp.size(); i++) {
            if (i + 1 < mp.size())
                mp[i + 1] += (mp[i] % p) * 10;
            else
                mmod = mp[i] % p;
            mp[i] /= p;
        }
        while (ni < np.size() && np[ni] == 0) ni++;
        while (mi < mp.size() && mp[mi] == 0) mi++;
        // implement binomial. binomial(m,n) = 0 if m < n
        ret = (ret * binomial(nmod, mmod)) % p;
    }
    return ret;
}
```

2.4 Pollard Rho

```
ll pollard_rho(ll n) {
    random_device rd;
    mt19937 gen(rd());
    uniform_int_distribution<ll> dis(1, n - 1);
    ll x = dis(gen);
    ll y = x;
    ll c = dis(gen);
    ll g = 1;
    while (g == 1) {
        x = (modmul(x, x, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        y = (modmul(y, y, n) + c) % n;
        g = gcd(abs(x - y), n);
    }
    return g;
}

// integer factorization
// O(n^0.25 * logn)
void factorize(ll n, vector<ll>& fl) {
    if (n == 1) {
        return;
    }
    if (n % 2 == 0) {
        fl.push_back(2);
        factorize(n / 2, fl);
    }
    else if (is_prime(n)) {
        fl.push_back(n);
    }
    else {
        ll f = pollard_rho(n);
        factorize(f, fl);
        factorize(n / f, fl);
    }
}
```

2.5 Gaussian Elimination

```
const double EPS = 1e-10;
typedef vector<vector<double>> VVD;

// Gauss-Jordan elimination with full pivoting.
// solving systems of linear equations (AX=B)
// INPUT:    a[][] = an n*n matrix
//           b[][] = an n*m matrix
// OUTPUT:   X       = an n*m matrix (stored in b[][])
//           A^{-1} = an n*n matrix (stored in a[][])
// O(n^3)
bool gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
    const int m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);
    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk =
                 k; }
        if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        irow[i] = pj;
        icol[i] = pk;

        double c = 1.0 / a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }
    for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return true;
}
```

2.6 Sieve

```
// find prime numbers in 1 ~ n
// ret[x] = false  ->  x is prime
// O(n*loglogn)
void sieve(int n, bool ret[]) {
    for (int i = 2; i * i <= n; ++i)
        if (!ret[i])
            for (int j = i * i; j <= n; j += i)
                ret[i] = true;
}

// calculate number of divisors for 1 ~ n
// when you need to calculate sum, change += 1 to += i
// O(n*logn)
void num_of_divisors(int n, int ret[]) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            ret[j] += 1;
}

// calculate euler totient function for 1 ~ n
// phi(n) = number of x s.t. 0 < x < n && gcd(n, x) = 1
// O(n*loglogn)
```

```
void euler_phi(int n, int ret[]) {
    for (int i = 1; i <= n; ++i) ret[i] = i;
    for (int i = 2; i <= n; ++i)
        if (ret[i] == i)
            for (int j = i; j <= n; j += i)
                ret[j] -= ret[j] / i;
}
```

2.7 FFT

```
void fft(int sign, int n, double *real, double *imag) {
    double theta = sign * 2 * pi / n;
    for (int m = n; m >= 2; m >>= 1, theta *= 2) {
        double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
        for (int i = 0, mh = m >> 1; i < mh; ++i) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                double xr = real[j] - real[k], xi = imag[j] - imag[k];
                real[j] += real[k], imag[j] += imag[k];
                real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
            }
            double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
            wr = _wr, wi = _wi;
        }
    }
    for (int i = 1, j = 0; i < n; ++i) {
        for (int k = n >> 1; k > (j ^= k); k >>= 1);
        if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);
    }
}
// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
    const int maxn = 100;
    static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
    int fn = 1;
    while (fn < n + m) fn <<= 1; // n + m: interested length
    for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
    for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
    for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
    for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
    fft(1, fn, ra, ia);
    fft(1, fn, rb, ib);
    for (int i = 0; i < fn; ++i) {
        double real = ra[i] * rb[i] - ia[i] * ib[i];
        double imag = ra[i] * ib[i] + rb[i] * ia[i];
        ra[i] = real, ia[i] = imag;
    }
    fft(-1, fn, ra, ia);
    for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
    return fn;
}
```

2.8 Chinese Remainder

```
// find x s.t.  x === a[0] (mod n[0])
//                === a[1] (mod n[1])
//                ...
// assumption: gcd(n[i], n[j]) = 1
ll chinese_remainder(ll* a, ll* n, int size) {
    if (size == 1) return *a;
    ll tmp = modinverse(n[0], n[1]);
    ll tmp2 = (tmp * (a[1] - a[0]) % n[1] + n[1]) % n[1];
    ll ora = a[1];
    ll tgcd = gcd(n[0], n[1]);
    a[1] = a[0] + n[0] / tgcd * tmp2;
    n[1] *= n[0] / tgcd;
    ll ret = chinese_remainder(a + 1, n + 1, size - 1);
    n[1] /= n[0] / tgcd;
    a[1] = ora;
    return ret;
}
```

3 Data Structure

3.1 Fenwick Tree

```
const int TSIZE = 100000;
int tree[TSIZE + 1];

// Returns the sum from index 1 to p, inclusive
int query(int p) {
    int ret = 0;
    for (; p > 0; p -= p & -p) ret += tree[p];
    return ret;
}

// Adds val to element with index pos
void add(int p, int val) {
    for (; p <= TSIZE; p += p & -p) tree[p] += val;
}
```

3.2 Fenwick Tree 2D

```
struct FenwickTree2D {
    ll size;
    llv2 data;

    FenwickTree2D(ll N) {
        size = N;
        data = llv2(size+1, llv1(size+1));
    }

    void update(int x, int y, ll val) {
        ll dv = val - sum(x, y, x, y);
        while(x <= size) {
            int y2 = y;
            while(y2 <= size) {
                data[x][y2] += dv;
```

```
                y2 += y2 & -y2;
            }
            x += x & -x;
        }
    }

    ll sum(int x, int y) {
        ll ret = 0;
        while(x) {
            int y2 = y;
            while(y2) {
                ret += data[x][y2];
                y2 -= y2 & -y2;
            }
            x -= x & -x;
        }
        return ret;
    }

    ll sum(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) + sum(x1 - 1, y1 -1) - sum(x1-1, y2) - sum(x2, y1-1);
    }
};
```

3.3 Merge Sort Tree

```
llv1 a;
llv1 mTree[Mx];
void makeTree(ll idx, ll ss, ll se) {
  if (ss == se) {
    mTree[idx].push_back(a[ss]);
    return;
  }
  ll mid = (ss + se) / 2;
  makeTree(2 * idx + 1, ss, mid);
  makeTree(2 * idx + 2, mid + 1, se);
  merge(mTree[2 * idx + 1].begin(), mTree[2 * idx + 1].end(), mTree[2 * idx +
    2].begin(), mTree[2 * idx + 2].end(), back_inserter(mTree[idx]));
}
ll query(ll node, ll start, ll end, ll q_s, ll q_e, ll k) {
  // i j k: Ai, Ai+1, ..., 로Aj 이루어진부분수열중에서보다    k 큰원소의개수를출력한
    다   .
  if (q_s > end || start > q_e) return 0;
  if (q_s <= start && q_e >= end) {
    return mTree[node].size() - (upper_bound(mTree[node].begin(), mTree[node].
      end(), k) - mTree[node].begin());
  }
  ll mid = (start + end) / 2;
  ll p1 = query(2 * node + 1, start, mid, q_s, q_e, k);
  ll p2 = query(2 * node + 2, mid + 1, end, q_s, q_e, k);
  return p1 + p2;
}
```

3.4 SegmentTree Lazy Propagation

```
// example implementation of sum tree
const int TSIZE = 131072; // always 2^k form && n <= TSIZE
int segtree[TSIZE * 2], prop[TSIZE * 2];
void seg_init(int nod, int l, int r) {
    if (l == r) segtree[nod] = dat[l];
    else {
        int m = (l + r) >> 1;
        seg_init(nod << 1, l, m);
        seg_init(nod << 1 | 1, m + 1, r);
        segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
    }
}
void seg_relax(int nod, int l, int r) {
    if (prop[nod] == 0) return;
    if (l < r) {
        int m = (l + r) >> 1;
        segtree[nod << 1] += (m - l + 1) * prop[nod];
        prop[nod << 1] += prop[nod];
        segtree[nod << 1 | 1] += (r - m) * prop[nod];
        prop[nod << 1 | 1] += prop[nod];
    }
    prop[nod] = 0;
}
int seg_query(int nod, int l, int r, int s, int e) {
    if (r < s || e < l) return 0;
    if (s <= l && r <= e) return segtree[nod];
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    return seg_query(nod << 1, l, m, s, e) + seg_query(nod << 1 | 1, m + 1, r, s
      , e);
}
void seg_update(int nod, int l, int r, int s, int e, int val) {
    if (r < s || e < l) return;
    if (s <= l && r <= e) {
        segtree[nod] += (r - l + 1) * val;
        prop[nod] += val;
        return;
    }
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    seg_update(nod << 1, l, m, s, e, val);
    seg_update(nod << 1 | 1, m + 1, r, s, e, val);
    segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
}
// usage:
// seg_update(1, 0, n - 1, qs, qe, val);
// seg_query(1, 0, n - 1, qs, qe);
```

3.5 Treap

```
// Treap* root = NULL;
// root = insert(root, new Treap(3));
typedef int type;
struct Treap {
  Treap* left = NULL, * right = NULL;
```

```
  int size = 1, prio = rand();
  type key;
  Treap(type key) : key(key) { }
  void calcSize() {
    size = 1;
    if (left != NULL) size += left->size;
    if (right != NULL) size += right->size;
  }
  void setLeft(Treap* l) { left = l, calcSize(); }
  void setRight(Treap* r) { right = r, calcSize(); }
};
typedef pair<Treap*, Treap*> TPair;
TPair split(Treap* root, type key) {
  if (root == NULL) return TPair(NULL, NULL);
  if (root->key < key) {
    TPair rs = split(root->right, key);
    root->setRight(rs.first);
    return TPair(root, rs.second);
  }
  TPair ls = split(root->left, key);
  root->setLeft(ls.second);
  return TPair(ls.first, root);
}
Treap* insert(Treap* root, Treap* node) {
  if (root == NULL) return node;
  if (root->prio < node->prio) {
    TPair s = split(root, node->key);
    node->setLeft(s.first);
    node->setRight(s.second);
    return node;
  }
  else if (node->key < root->key)
    root->setLeft(insert(root->left, node));
  else
    root->setRight(insert(root->right, node));
  return root;
}
Treap* merge(Treap* a, Treap* b) {
  if (a == NULL) return b;
  if (b == NULL) return a;
  if (a->prio < b->prio) {
    b->setLeft(merge(a, b->left));
    return b;
  }
  a->setRight(merge(a->right, b));
  return a;
}
Treap* erase(Treap* root, type key) {
  if (root == NULL) return root;
  if (root->key == key) {
    Treap* ret = merge(root->left, root->right);
    delete root;
    return ret;
  }
  if (key < root->key)
    root->setLeft(erase(root->left, key));
  else
    root->setRight(erase(root->right, key));
  return root;
}
Treap* kth(Treap* root, int k) { // kth key
  int l_size = 0;
  if (root->left != NULL) l_size += root->left->size;
  if (k <= l_size) return kth(root->left, k);
  if (k == l_size + 1) return root;
  return kth(root->right, k - l_size - 1);
}
int countLess(Treap* root, type key) { // count less than key
  if (root == NULL) return 0;
  if (root->key >= key)
    return countLess(root->left, key);
  int ls = (root->left ? root->left->size : 0);
  return ls + 1 + countLess(root->right, key);
}
```

4 Geometry

4.1 Basic Operations

```
const double eps = 1e-9;

inline int diff(double lhs, double rhs) {
    if (lhs - eps < rhs && rhs < lhs + eps) return 0;
    return (lhs < rhs) ? -1 : 1;
}

inline bool is_between(double check, double a, double b) {
    if (a < b)
        return (a - eps < check && check < b + eps);
    else
        return (b - eps < check && check < a + eps);
}

struct Point {
    double x, y;
    bool operator==(const Point& rhs) const {
        return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
    }
    Point operator+(const Point& rhs) const {
        return Point{ x + rhs.x, y + rhs.y };
    }
    Point operator-(const Point& rhs) const {
        return Point{ x - rhs.x, y - rhs.y };
    }
    Point operator*(double t) const {
        return Point{ x * t, y * t };
    }
};
```

```
struct Circle {
    Point center;
    double r;
};

struct Line {
    Point pos, dir;
};

inline double inner(const Point& a, const Point& b) {
    return a.x * b.x + a.y * b.y;
}

inline double outer(const Point& a, const Point& b) {
    return a.x * b.y - a.y * b.x;
}

inline int ccw_line(const Line& line, const Point& point) {
    return diff(outer(line.dir, point - line.pos), 0);
}

inline int ccw(const Point& a, const Point& b, const Point& c) {
    return diff(outer(b - a, c - a), 0);
}

inline double dist(const Point& a, const Point& b) {
    return sqrt(inner(a - b, a - b));
}

inline double dist2(const Point &a, const Point &b) {
    return inner(a - b, a - b);
}

inline double dist(const Line& line, const Point& point, bool segment = false) {
    double c1 = inner(point - line.pos, line.dir);
    if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);
    double c2 = inner(line.dir, line.dir);
    if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);
    return dist(line.pos + line.dir * (c1 / c2), point);
}

bool get_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    ret = b.pos + b.dir * t2;
    return true;
}

bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    if (!is_between(t1, 0, 1) || !is_between(t2, 0, 1)) return false;
    ret = b.pos + b.dir * t2;
    return true;
}

Point inner_center(const Point &a, const Point &b, const Point &c) {
    double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
    double w = wa + wb + wc;
    return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y +
      wc * c.y) / w };
}

Point outer_center(const Point &a, const Point &b, const Point &c) {
    Point d1 = b - a, d2 = c - a;
    double area = outer(d1, d2);
    double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
        + d1.y * d2.y * (d1.y - d2.y);
    double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
        + d1.x * d2.x * (d1.x - d2.y);
    return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
}

vector<Point> circle_line(const Circle& circle, const Line& line) {
    vector<Point> result;
    double a = 2 * inner(line.dir, line.dir);
    double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
        + line.dir.y * (line.pos.y - circle.center.y));
    double c = inner(line.pos - circle.center, line.pos - circle.center)
        - circle.r * circle.r;
    double det = b * b - 2 * a * c;
    int pred = diff(det, 0);
    if (pred == 0)
        result.push_back(line.pos + line.dir * (-b / a));
    else if (pred > 0) {
        det = sqrt(det);
        result.push_back(line.pos + line.dir * ((-b + det) / a));
        result.push_back(line.pos + line.dir * ((-b - det) / a));
    }
    return result;
}

vector<Point> circle_circle(const Circle& a, const Circle& b) {
    vector<Point> result;
    int pred = diff(dist(a.center, b.center), a.r + b.r);
    if (pred > 0) return result;
    if (pred == 0) {
        result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
        return result;
    }
    double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
    double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
    double tmp = (bb - aa) / 2.0;
    Point cdiff = b.center - a.center;
    if (diff(cdiff.x, 0) == 0) {
        if (diff(cdiff.y, 0) == 0)
            return result; // if (diff(a.r, b.r) == 0): same circle
```

```
        return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } });
    }
    return circle_line(a,
        Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
}

Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
    Point ba = b - a, cb = c - b;
    Line p{ (a + b) * 0.5, Point{ ba.y, -ba.x } };
    Line q{ (b + c) * 0.5, Point{ cb.y, -cb.x } };
    Circle circle;
    if (!get_cross(p, q, circle.center))
        circle.r = -1;
    else
        circle.r = dist(circle.center, a);
    return circle;
}

Circle circle_from_2pts_rad(const Point& a, const Point& b, double r) {
    double det = r * r / dist2(a, b) - 0.25;
    Circle circle;
    if (det < 0)
        circle.r = -1;
    else {
        double h = sqrt(det);
        // center is to the left of a->b
        circle.center = (a + b) * 0.5 + Point{ a.y - b.y, b.x - a.x } * h;
        circle.r = r;
    }
    return circle;
}
```

4.2 Convex Hull

```
// find convex hull
// O(n*logn)
vector<Point> convex_hull(vector<Point>& dat) {
    if (dat.size() <= 3) return dat;
    vector<Point> upper, lower;
    sort(dat.begin(), dat.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    for (const auto& p : dat) {
        while (upper.size() >= 2 && ccw(*++upper.rbegin(), *upper.rbegin(), p)
          >= 0) upper.pop_back();
        while (lower.size() >= 2 && ccw(*++lower.rbegin(), *lower.rbegin(), p)
          <= 0) lower.pop_back();
        upper.emplace_back(p);
        lower.emplace_back(p);
    }
    upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
    return upper;
}
```

4.3 Poiont in Polygon

```
typedef double coord_t;

inline coord_t is_left(Point p0, Point p1, Point p2) {
    return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
}

// point in polygon test
// http://geomalgorithms.com/a03-_inclusion.html
bool is_in_polygon(Point p, vector<Point>& poly) {
    int wn = 0;
    for (int i = 0; i < poly.size(); ++i) {
        int ni = (i + 1 == poly.size()) ? 0 : i + 1;
        if (poly[i].y <= p.y) {
            if (poly[ni].y > p.y) {
                if (is_left(poly[i], poly[ni], p) > 0) {
                    ++wn;
                }
            }
        }
        else {
            if (poly[ni].y <= p.y) {
                if (is_left(poly[i], poly[ni], p) < 0) {
                    --wn;
                }
            }
        }
    }
    return wn != 0;
}
```

4.4 Polygon Cut

```
// left side of a->b
vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
    if (!polygon.size()) return polygon;
    typedef vector<Point>::const_iterator piter;
    piter la, lan, fi, fip, i, j;
    la = lan = fi = fip = polygon.end();
    i = polygon.end() - 1;
    bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
    for (j = polygon.begin() ; j != polygon.end() ; j++) {
        bool thisin = diff(ccw_line(line, *j), 0) > 0;
        if (lastin && !thisin) {
            la = i;
            lan = j;
        }
        if (!lastin && thisin) {
            fi = j;
            fip = i;
        }
        i = j;
        lastin = thisin;
    }
```

```
    if (fi == polygon.end()) {
        if (!lastin) return vector<Point>();
        return polygon;
    }
    vector<Point> result;
    for (i = fi ; i != lan ; i++) {
        if (i == polygon.end()) {
            i = polygon.begin();
            if (i == lan) break;
        }
        result.push_back(*i);
    }
    Point lc, fc;
    get_cross(Line{ *la, *lan - *la }, line, lc);
    get_cross(Line{ *fip, *fi - *fip }, line, fc);
    result.push_back(lc);
    if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
    return result;
}
```

4.5 Rotating Calipers

```
// get all antipodal pairs
// O(n)
void antipodal_pairs(vector<Point>& pt) {
    // calculate convex hull
    sort(pt.begin(), pt.end(), [](const Point& a, const Point& b) {
        return (a.x == b.x) ? a.y < b.y : a.x < b.x;
    });
    vector<Point> up, lo;
    for (const auto& p : pt) {
        while (up.size() >= 2 && ccw(*++up.rbegin(), *up.rbegin(), p) >= 0) up.
          pop_back();
        while (lo.size() >= 2 && ccw(*++lo.rbegin(), *lo.rbegin(), p) <= 0) lo.
          pop_back();
        up.emplace_back(p);
        lo.emplace_back(p);
    }

    for (int i = 0, j = (int)lo.size() - 1; i + 1 < up.size() || j > 0; ) {
        get_pair(up[i], lo[j]);  // DO WHAT YOU WANT
        if (i + 1 == up.size()) {
            --j;
        }
        else if (j == 0) {
            ++i;
        }
        else if ((long long)(up[i + 1].y - up[i].y) * (lo[j].x - lo[j - 1].x)
                > (long long)(up[i + 1].x - up[i].x) * (lo[j].y - lo[j - 1].y))
                  {
            ++i;
        }
        else {
            --j;
        }
    }
}
```

4.6 Seperating Axis Theorem

```
pair<double, double> get_projection(vector<Vector2> &points, Vector2 &axis) {
  double min_val = axis.dot(points[0]);
  double max_val = min_val;
  for (int i = 1; i < points.size(); i++) {
    double projected = axis.dot(points[i]);
    min_val = min(min_val, projected);
    max_val = max(max_val, projected);
  }
  return {min_val, max_val};
}

vector<Vector2> get_normals(vector<Vector2> &points) {
  vector<Vector2> ret;
  if (points.size() == 1)
    return ret;
  for (int i = 0; i < points.size(); i++) {
    Vector2 &a = points[i];
    Vector2 &b = points[(i + 1) % points.size()];
    ret.push_back(Vector2((b - a).y, -(b - a).x));
  }
  return ret;
}
bool can_separate(vector<Vector2> &A, vector<Vector2> &B) {
  if (A.size() == 1 && B.size() == 1)
    return true;
  auto c_a = get_convex_hull(A);
  auto c_b = get_convex_hull(B);
  auto n_a = get_normals(c_a);
  auto n_b = get_normals(c_b);

  n_a.insert(n_a.end(), n_b.begin(), n_b.end());
  if (c_a.size() > 1) n_a.push_back(Vector2(c_a[1] - c_a[0]));
  if (c_b.size() > 1) n_a.push_back(Vector2(c_b[1] - c_b[0]));

  for (Vector2 &axis : n_a) {
    auto p_a = get_projection(c_a, axis);
    auto p_b = get_projection(c_b, axis);
    if (!((p_a.second >= p_b.first) && (p_b.second >= p_a.first))) return true;
  }
  return false;
}
```

4.7 Two Far Point

```
pair<Vector2, Vector2> get_max_points(vector<Vector2> &points) {
  int left = 0, right = max_element(points.begin(), points.end()) - points.begin
    ();
  int ret1 = left, ret2 = right;
  double max_len = (points[right] - points[left]).norm();
```

```
  int end = right;
  Vector2 left_dir = Vector2(0, -1.0);
  vector<Vector2> edges;
  for (int i = 0; i < points.size(); i++)
    edges.push_back((points[(i + 1) % points.size()] - points[i]).normalized());
  while (right != 0 || left != end) {
    double next1 = left_dir.dot(edges[left]);
    double next2 = -left_dir.dot(edges[right]);
    if (left != end && (right == 0 || next1 > next2)) {
      left_dir = edges[left];
      left = (left + 1) % points.size();
    } else {
      left_dir = -edges[right];
      right = (right + 1) % points.size();
    }
    double len = (points[right] - points[left]).norm();
    if (len > max_len) {
      ret1 = left;
      ret2 = right;
      max_len = len;
    }
  }
  return {points[ret1], points[ret2]};
}
```

4.8 Two Nearest Point

```
int dist(Point &p, Point &q) {
  return (p.x - q.x) * (p.x - q.x) + (p.y - q.y) * (p.y - q.y);
}
struct Comp {
  bool comp_in_x;
  Comp(bool b) : comp_in_x(b) {}
  bool operator()(Point &p, Point &q) {
    return (this->comp_in_x ? p.x < q.x : p.y < q.y);
  }
};
int nearest(vector<Point>::iterator it, int n) {
  if (n == 2) return dist(it[0], it[1]);
  if (n == 3) return min({dist(it[0], it[1]), dist(it[1], it[2]), dist(it[2], it
    [0])});
  int line = (it[n / 2 - 1].x + it[n / 2].x) / 2;
  int d = min(nearest(it, n / 2), nearest(it + n / 2, n - n / 2));
  vector<Point> mid;
  for (int i = 0; i < n; i++) {
    int t = line - it[i].x;
    if (t * t < d) mid.push_back(it[i]);
  }
  sort(mid.begin(), mid.end(), Comp(false));
  int mid_sz = mid.size();
  for (int i = 0; i < mid_sz - 1; i++)
    for (int j = i + 1; j < mid_sz && (mid[j].y - mid[i].y) * (mid[j].y - mid[i
      ].y) < d; j++)
      d = min(d, dist(mid[i], mid[j]));
  return d;
}
```

5 Graph

5.1 Dijkstra

```
template<typename T> struct Dijkstra {
  /*
    T: 간선가중치타입
  */
  struct Edge {
    ll node;
    T cost;
    bool operator<(const Edge &to) const {
      return cost > to.cost;
    }
  };

  ll n;
  vector<vector<Edge>> adj;
  vector<ll> prev;

  Dijkstra(ll n) : n{n}, adj(n+1) {}

  void addEdge(ll s, ll e, T cost) {
    adj[s].push_back(Edge(e, cost));
  }

  void addUndirectedEdge(ll s, ll e, T cost) {
    addEdge(s, e, cost);
    addEdge(e, s, cost);
  }

  vector <ll> dijkstra(ll s) {
    vector <ll> dist(n+1, INF);
    prev.resize(n+1, -1);
    priority_queue<edge> pq;
    pq.push({ s, 0ll });
    dist[s] = 0;
    while (!pq.empty()) {
      edge cur = pq.top();
      pq.pop();
      if (cur.cost > dist[cur.node]) continue;
      for (auto &nxt : adj[cur.node])
        if (dist[cur.node] + nxt.cost < dist[nxt.node]) {
          prev[nxt.node] = cur.node;
          dist[nxt.node] = dist[cur.node] + nxt.cost;
          pq.push({ nxt.node, dist[nxt.node] });
        }
    }
    return dist;
  }

  vector<ll> getPath(ll s, ll e) {
```

```
  vector<ll> ret;
  ll current = e;
  while(current != -1) {
    ret.push_back(current);
    current = prev[current];
  }
  reverse(ret.begin(), ret.end());
  return ret;
  }
};
```

5.2 Bellman-Ford

```
struct BellmanFord {
  struct BellmanEdge {
    ll to, cost;

    BellmanEdge(ll to, ll cost) : to(to), cost(cost) {}
  };

  ll N;
  vector<vector <BellmanEdge> > adj;
  llv1 D;
  vector<ll> prev;

  BellmanFord(ll N) : N(N) {
    adj.resize(N + 1);
  }

  void addEdge(ll s, ll e, ll cost) {
    adj[s].push_back(BellmanEdge(e, cost));
  }

  bool run(ll start_point) {
    // 음수간선  cycle 유무를반환합니다 .
    // 거리정보는  D 벡터에저장됩니다 .
    // O(V * E)

    D.resize(N + 1, INF);
    prev.resize(N + 1, -1);
    D[start_point] = 0;

    bool isCycle = false;

    for1(1, N + 1) {
      for1j(1, N + 1) {
        for(int k=0; k < sz(adj[j]); k++) {
          BellmanEdge p = adj[j][k];
          int end = p.to;
          ll dist = D[j] + p.cost;

          if (D[j] != INF && D[end] > dist) {
            D[end] = dist;
            if (i == N) isCycle = true;
          }
        }
      }
    }
    return isCycle;
  }

  llv1 getPath(ll s, ll e) {
    vector<ll> ret;
    ll current = e;
    while(current != -1) {
      ret.push_back(current);
      current = prev[current];
    }
    reverse(ret.begin(), ret.end());
    return ret;
  }
};
```

5.3 Spfa

```
// shortest path faster algorithm
// average for random graph : O(E) , worst : O(VE)

const int MAXN = 20001;
const int INF = 100000000;
int n, m;
vector<pii> graph[MAXN];

bool inqueue[MAXN];
int dist[MAXN];

void spfa(int start) {
    for (int i = 0; i < n; ++i) dist[i] = INF;
    dist[start] = 0;

    queue<int> q;
    q.push(start);
    inqueue[start] = true;

    while (!q.empty()) {
        int here = q.front();
        q.pop();

        inqueue[here] = false;
        for (auto& nxt : graph[here]) {
            if (dist[here] + nxt.second < dist[nxt.first]) {
                dist[nxt.first] = dist[here] + nxt.second;
                if (!inqueue[nxt.first]) {
                    q.push(nxt.first);
                    inqueue[nxt.first] = true;
                }
            }
        }
    }
}
```

5.4 Topological Sort

```
struct TopologicalSort {
  // 1-index

  int n;
  iv1 in_degree;
  iv2 graph;
  iv1 result;

  TopologicalSort(int n) : n(n) {
    in_degree.resize(n + 1, 0);
    graph.resize(n + 1);
  }

  void addEdge(int s, int e) {
    graph[s].push_back(e);
    in_degree[e]++;
  }

  void run() {
    queue<int> q;

    for1(1, n+1) {
      if(in_degree[i] == 0) q.push(i);
    }
    while(!q.empty()) {
      int here = q.front(); q.pop();
      result.push_back(here);

      for1(0, sz(graph[here])) {
        int there = graph[here][i];

        if(--in_degree[there]==0) q.push(there);
      }
    }
  }
};
```

5.5 Strongly Connected Component

```
struct SCC {
  // 1-index
  // run() 후에에 components 결과가담김 .

  ll V;
  llv2 edges, reversed_edges, components;
  vector<bool> visited;
  stack<ll> visit_log;

  SCC(ll V): V(V) {
    edges.resize(V + 1);
    reversed_edges.resize(V + 1);
  }

  void addEdge(int s, int e) {
    edges[s].push_back(e);
    reversed_edges[e].push_back(s);
  }

  void dfs(int node) {
    visited[node] = true;

    for (int next : edges[node])
      if (!visited[next]) dfs(next);
    visit_log.push(node);
  }
  void dfs2(int node) {
    visited[node] = true;
    for (int next:reversed_edges[node])
      if (!visited[next]) dfs2(next);
    components.back().push_back(node);
  }

  void run() {
    visited = vector<bool>(V + 1, false);
    for (int node = 1; node <= V; node++)
      if (!visited[node]) dfs(node);

    visited = vector<bool>(V + 1, false);
    while (!visit_log.empty()) {
      ll node = visit_log.top(); visit_log.pop();
      if (!visited[node]) {
        components.push_back(llv1());
        dfs2(node);
      }
    }
  }
};
```

5.6 2-SAT

```
struct Graph {
  int V;
  vector<bool> visited;
  stack<int> visit_stack;
  vector<int> component_number, source_components;
  vector<vector<int>> edges, reversed_edges, components, components_edges;

  Graph(int V) : V(V) {
    edges.resize(V);
    reversed_edges.resize(V);
  }
  void append(int u, int v) {
    edges[u].push_back(v);
    reversed_edges[v].push_back(u);
  }
  void dfs(int node) {
    visited[node] = true;
    for (int next : edges[node])
```

```cpp
      if (!visited[next])
        dfs(next);
    visit_stack.push(node);
  }
  void scc(int node) {
    visited[node] = true;
    for (int next : reversed_edges[node])
      if (!visited[next])
        scc(next);
    components.back().push_back(node);
  }
  void build_scc() {
    visited = vector<bool>(V, false);
    for (int node = 0; node < V; node++)
      if (!visited[node]) dfs(node);
    visited = vector<bool>(V, false);
    while (!visit_stack.empty()) {
      int node = visit_stack.top();
      visit_stack.pop();
      if (!visited[node]) {
        components.push_back(vector<int>());
        scc(node);
      }
    }
    component_number.resize(V);
    for (int i = 0; i < components.size(); i++)
      for (int node : components[i]) component_number[node] = i;
    vector<bool> is_source = vector<bool>(components.size(), true);
    components_edges.resize(components.size());
    for (int u = 0; u < V; u++)
      for (int v : edges[u]) {
        int cu = component_number[u];
        int cv = component_number[v];
        if (cu == cv) continue;
        components_edges[cu].push_back(cv);
        is_source[cv] = false;
      }
    for (int component = 0; component < components.size(); component++) {
      if (is_source[component]) source_components.push_back(component);
    }
  }
};

int main(void) {
  int V, E;
  cin >> V >> E;
  Graph graph(2 * V + 1);
  for (int i = 0; i < E; i++) {
    int u, v;
    cin >> u >> v;
    graph.append(-u + V, v + V);
    graph.append(-v + V, u + V);
  }
  graph.build_scc();
  vector<int> last_component(2 * V + 1, -1);
  bool is_answer = true;
  for (int i = 0; i < graph.components.size(); i++) {
    for (int node : graph.components[i]) {
      int negation = -(node - V) + V;
      if (last_component[negation] == i) is_answer = false;
      last_component[node] = i;
    }
  }
  if (is_answer) {
    vector<int> result(V);
    for (int i = 1; i <= V; i++) {
      int val = i + V;
      int negation = -i + V;
      result[i - 1] = graph.component_number[val] > graph.component_number[
        negation];
    }
    for (int val : result) cout << val << "␣";
    cout << "\n";
  }
}
```

5.7 Union Find

```cpp
struct UnionFind {
  int n;
  vector<int> u;

  UnionFind(int n) : n(n) {
    u.resize(n + 1);
    for(int i = 1; i <= n; i++) {
      u[i] = i;
    }
  }

  int find(int k) {
    if(u[u[k]] == u[k]) return u[k];
    else return u[k]=find(u[k]);
  }

  void uni(int a, int b) {
    a = find(a);
    b = find(b);
    if(a < b) u[b] = a;
    else u[a] = b;
  }
};
```

5.8 MST Prim

```cpp
struct edge {
  ll crt;
  ll node, cost;
};
```

```
struct WGraph {
    ll V;
    vector<edge> adj[MAX];
    vector<ll> prev;
    WGraph(ll V) : V{V} {}
    void addEdge(ll s, ll e, ll cost) {
        adj[s].push_back({s, e, cost});
        adj[e].push_back({e, s, cost});
    }

    ll prim(vector<edge> &selected) { // 에selected 선택된간선정보  vector 담김
        selected.clear();

        vector<bool> added(V, false);
        llv1 minWeight(V, INF), parent(V, -1);

        ll ret = 0;
        minWeight[0] = parent[0] = 0;
        for (int iter = 0; iter < V; iter++) {
            int u = -1;
            for (int v = 0; v < V; v++) {
                if (!added[v] && (u == -1 || minWeight[u]>minWeight[v]))
                    u = v;
            }

            if (parent[u] != u)
                selected.push_back({parent[u], u, minWeight[u]});

            ret += minWeight[u];
            added[u] = true;

            for1(0, sz(adj[u])) {
                int v = adj[u][i].node, weight = adj[u][i].cost;
                if (!added[v] && minWeight[v]>weight) {
                    parent[v] = u;
                    minWeight[v] = weight;
                }
            }
        }
        return ret;
    }
};
```

5.9 Lowest Common Ancestor

```
#define MAX_DEGREE 20

struct LCA {
  // root: 트리의루트설정  , n: 트리의노드개수
  // addEdge -> init -> query(O(log(n))

  ll root, n;
  llv1 depth;
  llv2 adj;
  llv2 parent; // n X MAX_DEGREE

  LCA(ll root, ll n) : root(root), n(n) {
    depth.resize(n + 1);
    adj.resize(n + 1);
    parent.resize(n + 1, llv1(MAX_DEGREE, 0));
  }

  void addEdge(ll a, ll b) {
    adj[a].push_back(b);
    adj[b].push_back(a);
  }

  void init() {
    dfs(root, 0, 1);

    for(int i = 1; i < MAX_DEGREE; i++) {
      for(int j = 1; j <= n; j++) {
        parent[j][i] = parent[parent[j][i-1]][i-1];
      }
    }
  }

  void dfs(int here, int par, int d) {
    depth[here] = d;
    parent[here][0] = par;

    for(int there : adj[here]) {
      if(depth[there] > 0) continue;

      dfs(there, here, d + 1);
    }
  }

  int query(int a, int b) {
    if(depth[a] > depth[b]) {
      swap(a, b);
    }

    for(int i = MAX_DEGREE - 1; i >= 0; i--) {
      if (depth[b] - depth[a] >= (1 << i)) {
        b = parent[b][i];
      }
    }

    if(a == b) {
      return a;
    }

    for(int i = MAX_DEGREE - 1; i >= 0; i--) {
      if(parent[a][i] != parent[b][i]) {
        a = parent[a][i];
        b = parent[b][i];
      }
    }
```

```
    return parent[a][0];
  }
};
```

5.10 Maxflow dinic

```
// usage:
// MaxFlowDinic::init(n);
// MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
// MaxFlowDinic::add_edge(1, 2, 100); // directional edge
// result = MaxFlowDinic::solve(0, 2); // source -> sink
// graph[i][edgeIndex].res -> residual
//
// in order to find out the minimum cut, use `l'.
// if l[i] == 0, i is unrechable.
//
// O(V*V*E)
// with unit capacities, O(min(V^(2/3), E^(1/2)) * E)
struct MaxFlowDinic {
    typedef int flow_t;
    struct Edge {
        int next;
        size_t inv; /* inverse edge index */
        flow_t res; /* residual */
    };
    int n;
    vector<vector<Edge>> graph;
    vector<int> q, l, start;

    void init(int _n) {
        n = _n;
        graph.resize(n);
        for (int i = 0; i < n; i++) graph[i].clear();
    }
    void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
        Edge forward{ e, graph[e].size(), cap };
        Edge reverse{ s, graph[s].size(), caprev };
        graph[s].push_back(forward);
        graph[e].push_back(reverse);
    }
    bool assign_level(int source, int sink) {
        int t = 0;
        memset(&l[0], 0, sizeof(l[0]) * l.size());
        l[source] = 1;
        q[t++] = source;
        for (int h = 0; h < t && !l[sink]; h++) {
            int cur = q[h];
            for (const auto& e : graph[cur]) {
                if (l[e.next] || e.res == 0) continue;
                l[e.next] = l[cur] + 1;
                q[t++] = e.next;
            }
        }
        return l[sink] != 0;
    }
    flow_t block_flow(int cur, int sink, flow_t current) {
        if (cur == sink) return current;
        for (int& i = start[cur]; i < graph[cur].size(); i++) {
            auto& e = graph[cur][i];
            if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
            if (flow_t res = block_flow(e.next, sink, min(e.res, current))) {
                e.res -= res;
                graph[e.next][e.inv].res += res;
                return res;
            }
        }
        return 0;
    }
    flow_t solve(int source, int sink) {
        q.resize(n);
        l.resize(n);
        start.resize(n);
        flow_t ans = 0;
        while (assign_level(source, sink)) {
            memset(&start[0], 0, sizeof(start[0]) * n);
            while (flow_t flow = block_flow(source, sink, numeric_limits<flow_t
              >::max()))
                ans += flow;
        }
        return ans;
    }
};
```

5.11 Maxflow Edmonds-Karp

```
struct MaxFlowEdgeDemands
{
    MaxFlowDinic mf;
    using flow_t = MaxFlowDinic::flow_t;

    vector<flow_t> ind, outd;
    flow_t D; int n;

    void init(int _n) {
        n = _n; D = 0; mf.init(n + 2);
        ind.clear(); outd.clear();
        ind.resize(n, 0); outd.resize(n, 0);
    }

    void add_edge(int s, int e, flow_t cap, flow_t demands = 0) {
        mf.add_edge(s, e, cap - demands);
        D += demands; ind[e] += demands; outd[s] += demands;
    }

    // returns { false, 0 } if infeasible
    // { true, maxflow } if feasible
    pair<bool, flow_t> solve(int source, int sink) {
        mf.add_edge(sink, source, numeric_limits<flow_t>::max());

        for (int i = 0; i < n; i++) {
```

```
            if (ind[i]) mf.add_edge(n, i, ind[i]);
            if (outd[i]) mf.add_edge(i, n + 1, outd[i]);
        }

        if (mf.solve(n, n + 1) != D) return{ false, 0 };

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.graph[i].pop_back();
            if (outd[i]) mf.graph[i].pop_back();
        }

        return{ true, mf.solve(source, sink) };
    }
};
```

5.12 MCMF SPFA

```
struct MCMF {
  struct Edge {
    ll to;
    ll capacity;
    ll cost;

    Edge* rev;
    Edge(ll to, ll capacity, ll cost) : to(to), capacity(capacity), cost(cost)
      {}
  };

  ll n;
  ll source, sink;
  vector<vector<Edge *>> graph;
  vector<bool> check;
  vector<ll> distance;
  vector<pair<ll, ll>> from;

  MCMF(ll n, ll source, ll sink): n(n), source(source), sink(sink) {

    // source: 시작점
    // sink: 도착점
    // n: 모델링한그래프의정점개수
    graph.resize(n + 1);
    check.resize(n + 1);
    from.resize(n + 1, make_pair(-1, -1));
    distance.resize(n + 1);
  };

  void addEdge(ll u, ll v, ll cap, ll cost) {
    Edge *ori = new Edge(v, cap, cost);
    Edge *rev = new Edge(u, 0, -cost);

    ori->rev = rev;
    rev->rev = ori;

    graph[u].push_back(ori);
    graph[v].push_back(rev);
  }

  void addEdgeFromSrc(ll v, ll cap, ll cost) {
    // 출발지점에서출발하는간선추가
    addEdge(source, v, cap, cost);
  }

  void addEdgeToSink(ll u, ll cap, ll cost) {
    // 도착지점으로가는간선추가
    addEdge(u, sink, cap, cost);
  }

  bool spfa(ll &total_flow, ll &total_cost) {
    // spfa 기반의 MCMF

    fill(check.begin(), check.end(), false);
    fill(distance.begin(), distance.end(), INF);
    fill(from.begin(), from.end(), make_pair(-1, -1));

    distance[source] = 0;
    queue <ll> q;
    q.push(source);

    while(!q.empty()) {
      ll x = q.front(); q.pop();

      check[x] = false;

      for(ll i = 0; i < graph[x].size(); i++) {
        Edge* e = graph[x][i];
        ll y = e->to;

        if(e->capacity > 0 && distance[x] + e->cost < distance[y]) {
          distance[y] = distance[x] + e->cost;
          from[y] = make_pair(x, i);

          if(!check[y]) {
            check[y] = true;
            q.push(y);
          }
        }

      }
    }

    if(distance[sink] == INF) return false;

    // 간선을에서부터 sink 역추적하여경로를만든다  .
    ll x = sink;
    ll c = graph[from[x].first][from[x].second]->capacity;

    while(from[x].first != -1) {
      if(c > graph[from[x].first][from[x].second]->capacity) {
        c = graph[from[x].first][from[x].second]->capacity;
      }
```

```
      x = from[x].first;
    }

    // 만든경로를따라유량을흘린다    .
    x = sink;
    while(from[x].first != -1) {
      Edge* e = graph[from[x].first][from[x].second];
      e->capacity -= c;
      e->rev->capacity += c;
      x = from[x].first;
    }

    total_flow += c;
    total_cost += c * distance[sink];

    return true;
  }

  pair <ll, ll> flow() {
    ll total_flow = 0;
    ll total_cost = 0;

    while(spfa(total_flow, total_cost));

    return make_pair(total_flow, total_cost);
  }
};
```

5.13 MCMF

```
// precondition: there is no negative cycle.
// usage:
// MinCostFlow mcf(n);
// for(each edges) mcf.addEdge(from, to, cost, capacity);
// mcf.solve(source, sink); // min cost max flow
// mcf.solve(source, sink, 0); // min cost flow
// mcf.solve(source, sink, goal_flow); // min cost flow with total_flow >=
  goal_flow if possible
struct MinCostFlow {
    typedef int cap_t;
    typedef int cost_t;

    bool iszerocap(cap_t cap) { return cap == 0; }

    struct edge {
        int target;
        cost_t cost;
        cap_t residual_capacity;
        cap_t orig_capacity;
        size_t revid;
    };

    int n;
    vector<vector<edge>> graph;

    MinCostFlow(int n) : graph(n), n(n) {}

    void addEdge(int s, int e, cost_t cost, cap_t cap) {
        if (s == e) return;
        edge forward{ e, cost, cap, cap, graph[e].size() };
        edge backward{ s, -cost, 0, 0, graph[s].size() };
        graph[s].emplace_back(forward);
        graph[e].emplace_back(backward);
    }

    pair<cost_t, cap_t> augmentShortest(int s, int e, cap_t flow_limit) {
        auto infinite_cost = numeric_limits<cost_t>::max();
        auto infinite_flow = numeric_limits<cap_t>::max();
        vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
        vector<int> from(n, -1), v(n);

        dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
        queue<int> q;
        v[s] = 1; q.push(s);
        while(!q.empty()) {
            int cur = q.front();
            v[cur] = 0; q.pop();
            for (const auto& e : graph[cur]) {
                if (iszerocap(e.residual_capacity)) continue;
                auto next = e.target;
                auto ncost = dist[cur].first + e.cost;
                auto nflow = min(dist[cur].second, e.residual_capacity);
                if (dist[next].first > ncost) {
                    dist[next] = make_pair(ncost, nflow);
                    from[next] = e.revid;
                    if (v[next]) continue;
                    v[next] = 1; q.push(next);
                }
            }
        }

        auto p = e;
        auto pathcost = dist[p].first;
        auto flow = dist[p].second;
        if (iszerocap(flow)|| (flow_limit <= 0 && pathcost >= 0)) return pair<
          cost_t, cap_t>(0, 0);
        if (flow_limit > 0) flow = min(flow, flow_limit);

        while (from[p] != -1) {
            auto nedge = from[p];
            auto np = graph[p][nedge].target;
            auto fedge = graph[p][nedge].revid;
            graph[p][nedge].residual_capacity += flow;
            graph[np][fedge].residual_capacity -= flow;
            p = np;
        }
        return make_pair(pathcost * flow, flow);
    }

    pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits<
```

```
      cap_t>::max()) {
        cost_t total_cost = 0;
        cap_t total_flow = 0;
        for(;;) {
            auto res = augmentShortest(s, e, flow_minimum - total_flow);
            if (res.second <= 0) break;
            total_cost += res.first;
            total_flow += res.second;
        }
        return make_pair(total_cost, total_flow);
    }
};
```

6 String

6.1 KMP

```
struct KMP {
  /*
    s 문자열에서문자열을 o 찾습니다.매칭이시작되는인덱스목록을반환합니다
      .
    Time: O(n + m)
  */
  vector<int> result;
  int MX;
  string s, o;
  int n, m; // n : s.length(), m :o.length();
  vector<int> fail;

  KMP(string s, string o) : s(s), o(o) {
    n = s.length();
    m = o.length();
    MX = max(n, m) + 1;
    fail.resize(MX, 0);

    run();
  }

  void run() {
    for(int i = 1, j = 0; i < m; i++){
      while(j > 0 && o[i] != o[j]) j = fail[j-1];
      if(o[i] == o[j]) fail[i] = ++j;
    }
    for(int i = 0, j = 0; i < n; i++) {
      while(j > 0 && s[i] != o[j]) {
        j = fail[j - 1];
      }
      if(s[i] == o[j]) {
        if(j == m - 1) {
          // matching OK;
          result.push_back(i - m + 1);
          j = fail[j];
        }
        else {
          j++;
        }
      }
    }
  }
};
```

6.2 Manacher

```
// Use space to insert space between each character
// To get even length palindromes!
// O(|str|)

vector<int> manacher(string &s) {
  int n = s.size(), R = -1, p = -1;
  vector<int> A(n);
  for (int i = 0; i < n; i++) {
    if (i <= R) A[i] = min(A[2 * p - i], R - i);
    while (i - A[i] - 1 >= 0 && i + A[i] + 1 < n && s[i - A[i] - 1] == s[i + A[i
      ] + 1])
      A[i]++;
    if (i + A[i] > R)
      R = i + A[i], p = i;
  }
  return A;
}

string space(string &s) {
  string t;
  for (char c : s) t += c, t += '␣';
  t.pop_back();
  return t;
}

int maxpalin(vector<int> &M, int i) {
  if (i % 2) return (M[i] + 1) / 2 * 2;
  return M[i] / 2 * 2 + 1;
}
```

6.3 Suffix Array

```
typedef char T;

// calculates suffix array.
// O(n*logn)
vector<int> suffix_array(const vector<T>& in) {
    int n = (int)in.size(), c = 0;
    vector<int> temp(n), pos2bckt(n), bckt(n), bpos(n), out(n);
    for (int i = 0; i < n; i++) out[i] = i;
    sort(out.begin(), out.end(), [&](int a, int b) { return in[a] < in[b]; });
    for (int i = 0; i < n; i++) {
        bckt[i] = c;
        if (i + 1 == n || in[out[i]] != in[out[i + 1]]) c++;
    }
```

```
    for (int h = 1; h < n && c < n; h <<= 1) {
        for (int i = 0; i < n; i++) pos2bckt[out[i]] = bckt[i];
        for (int i = n - 1; i >= 0; i--) bpos[bckt[i]] = i;
        for (int i = 0; i < n; i++)
            if (out[i] >= n - h) temp[bpos[bckt[i]]++] = out[i];
        for (int i = 0; i < n; i++)
            if (out[i] >= h) temp[bpos[pos2bckt[out[i] - h]]++] = out[i] - h;
        c = 0;
        for (int i = 0; i + 1 < n; i++) {
            int a = (bckt[i] != bckt[i + 1]) || (temp[i] >= n - h)
                    || (pos2bckt[temp[i + 1] + h] != pos2bckt[temp[i] + h]);
            bckt[i] = c;
            c += a;
        }
        bckt[n - 1] = c++;
        temp.swap(out);
    }
    return out;
}

// calculates lcp array. it needs suffix array & original sequence.
// O(n)
vector<int> lcp(const vector<T>& in, const vector<int>& sa) {
    int n = (int)in.size();
    if (n == 0) return vector<int>();
    vector<int> rank(n), height(n - 1);
    for (int i = 0; i < n; i++) rank[sa[i]] = i;
    for (int i = 0, h = 0; i < n; i++) {
        if (rank[i] == 0) continue;
        int j = sa[rank[i] - 1];
        while (i + h < n && j + h < n && in[i + h] == in[j + h]) h++;
        height[rank[i] - 1] = h;
        if (h > 0) h--;
    }
    return height;
}
```

6.4 Trie

```
int chToIdx(char ch) { return ch - 'a'; }
struct Trie {
  int terminal = -1;
  Trie *fail; // fail, 은output 아호코라식에사용
  vector<int> output;
  Trie *chil[ALPHABETS];
  Trie() {
    for (int i = 0; i < ALPHABETS; i++)
      chil[i] = NULL;
  }
  // number -> 문자열번호 (ith string)
  void insert(string &s, int number, int idx) {
    if (idx == s.size()) {
      terminal = number;
      return;
    }
    int next = chToIdx(s[idx]);
    if (chil[next] == NULL)
      chil[next] = new Trie();
    chil[next]->insert(s, number, idx + 1);
  }
  int find(string &s, int idx = 0) {
    if (idx == s.size())
      return terminal;
    int next = chToIdx(s[idx]);
    if (chil[next] == NULL)
      return false;
    return chil[next]->find(s, idx + 1);
  }
};
```

6.5 Aho-Corasick

```
void computeFail(Trie *root) {
    queue<Trie *> q;
    root->fail = root;
    q.push(root);
    while (!q.empty()) {
        Trie *here = q.front();
        q.pop();
        for (int i = 0; i < ALPHABETS; i++) {
            Trie *child = here->chil[i];
            if (!child) continue;
            if (here == root) child->fail = root;
            else {
                Trie *t = here->fail;
                while (t != root && t->chil[i] == NULL) t = t->fail;
                if (t->chil[i]) t = t->chil[i];
                child->fail = t;
            }
            child->output = child->fail->output;
            if (child->terminal != -1) child->output.push_back(child->terminal);
            q.push(child);
        }
    }
}
vector<pair<int, int>> ahoCorasick(string &s, Trie *root) {
    vector<pair<int, int>> ret;
    Trie *state = root;
    for (int i = 0; i < s.size(); i++) {
        int idx = chToIdx(s[i]);
        while (state != root && state->chil[idx] == NULL) state = state->fail;
        if (state->chil[idx]) state = state->chil[idx];
        for (int j = 0; j < state->output.size(); j++) ret.push_back({i, state->
          output[j]});
    }
    return ret;
}
```

6.6 Z Algorithm

```cpp
// Z[i] : maximum common prefix length of &s[0] and &s[i]
// O(|s|)
using seq_t = string;
vector<int> z_func(const seq_t &s) {
    vector<int> z(s.size());
    z[0] = s.size();
    int l = 0, r = 0;

    for (int i = 1; i < s.size(); i++) {
        if (i > r) {
            int j;
            for (j = 0; i + j < s.size() && s[i + j] == s[j]; j++) ;
            z[i] = j; l = i; r = i + j - 1;
        } else if (z[i - l] < r - i + 1) {
            z[i] = z[i - l];
        } else {
            int j;
            for (j = 1; r + j < s.size() && s[r + j] == s[r - i + j]; j++) ;
            z[i] = r - i + j; l = i; r += j - 1;
        }
    }

    return z;
}
```

7 Dynamic Programming

7.1 LIS

```cpp
struct LIS {
  llv1 ar;

  llv1 v, buffer;
  llv1::iterator vv;
  vector<pair<ll, ll> > d;

  void perform() {
    v.pb(2000000000ll);

    ll n = sz(ar);

    for1(0, n){
      if (ar[i] > *v.rbegin()) {
        v.pb(ar[i]);
        d.push_back({ v.size() - 1, ar[i] });
      }
      else {
        vv = lower_bound(v.begin(), v.end(), ar[i]);
        *vv = ar[i];
        d.push_back({ vv - v.begin(), ar[i] });
      }
    }

    for(int i = sz(d) - 1; i > -1; i--){
        if(d[i].first == sz(v)-1){
          buffer.pb(d[i].second);
          v.pop_back();
        }
      }

      reverse(buffer.begin(), buffer.end());
    }

    ll length() {
      return buffer.size();
    }

    llv1 result() {
      return buffer;
    }
};
```

7.2 LIS only length

```cpp
ll lis(llv1& ar) {
  llv1 v, buffer;
  llv1::iterator vv;
  v.pb(2000000000ll);

  ll n = sz(ar);

  for1(0, n){
    if(ar[i] > *v.rbegin()) {
      v.pb(ar[i]);
    }
    else{
      vv = lower_bound(v.begin(), v.end(), ar[i]);
      *vv = ar[i];
    }
  }
  return sz(v);
}
```

7.3 KnapSack

```cpp
ll N, maxWeight, ans;
ll D[2][11000];
ll weight[110], cost[110];
void knapsack() {
  for (int x = 1; x <= N; x++) {
    for (int y = 0; y <= maxWeight; y++) {
      if (y >= weight[x]) {
        D[x % 2][y] = max(D[(x + 1) % 2][y], D[(x + 1) % 2][y - weight[x]] +
          cost[x]);
      } else {
        D[x % 2][y] = D[(x + 1) % 2][y];
      }
      ans = max(ans, D[x % 2][y]);
```

```
    }
  }
}
void input() {
  cin >> N >> maxWeight;
  for (int x = 1; x <= N; x++) {
    cin >> weight[x] >> cost[x];
  }
}
```

7.4 Coin Change

```
// 경우의수
ll CC(llv1 &coin, ll money, ll MX) {
  ll D[MX];
  fill(D, D + MX, 0);
  D[0] = 1;
  for (int i = coin.size() - 1; i >= 0; i--) {
    for (int j = coin[i]; j <= money; j++) {
      D[j] += D[j - coin[i]];
      D[j] %= MOD;
    }
  }
  return D[money] % MOD;
}
```

7.5 Bit Field DP

```
#define MOD 9901;

int dp[1 << 14 + 1][200];
int n, m;

int solve(int pos, int check, int dep) {
  if (dp[check][pos] != 0) return dp[check][pos];
  int &ret = dp[check][pos];
  if (dep == n * m) return ret = 1;
  if ((check & 1)) return ret = solve(pos - 1, check >> 1, dep) % MOD;

  int sum = 0;
  if (!(check & 1) && (pos - 1) / m > 0)
    sum += solve(pos - 1, (check >> 1) | (1 << (m - 1)), dep + 2) % MOD;
  if (!(check & 1) && pos % m != 1 && !(check & 2) && pos >= 2 && m > 1)
    sum += solve(pos - 2, check >> 2, dep + 2) % MOD;
  // cout<<pos<<" "<<check<<" "<<dep<<" "<<sum<<endl;

  return ret = sum % MOD;
}
int main() {
  cin >> n >> m;

  if (n * m % 2 == 1)
    cout << 0;
  else
    cout << solve(n * m, 0, 0) % MOD;

  return 0;
}
```

7.6 Knuth Optimization

```
int solve(int n) {
    for (int m = 2; m <= n; m++) {
        for (int i = 0; m + i <= n; i++) {
            int j = i + m;
            for (int k = K[i][j - 1]; k <= K[i + 1][j]; k++) {
                int now = dp[i][k] + dp[k][j] + sum[j] - sum[i];
                if (dp[i][j] > now)
                    dp[i][j] = now, K[i][j] = k;
            }
        }
    }
    return dp[0][n];
}

int main() {
    int n;
    cin >> n;
    fill(&dp[0][0], &dp[MAX-1][MAX-1], INF);
    for (int i = 1; i <= n; i++){
        cin >> arr[i];
        sum[i] = sum[i - 1] + arr[i];
        K[i - 1][i] = i;
        dp[i - 1][i] = 0;
    }
    cout << solve(n) << "\n";
}

/*
if
C[a][c] + C[b][d] <= C[a][d] + C[b][c] (a<=b<=c<=d)
C[b][c] <= C[a][d] (a<=b<=c<=d)

then
dp[i][j] = min(dp[i][k] + dp[k][j]) + C[i][j]
range of k: A[i, j-1] <= A[i][j]=k <= A[i+1][j]
*/
```