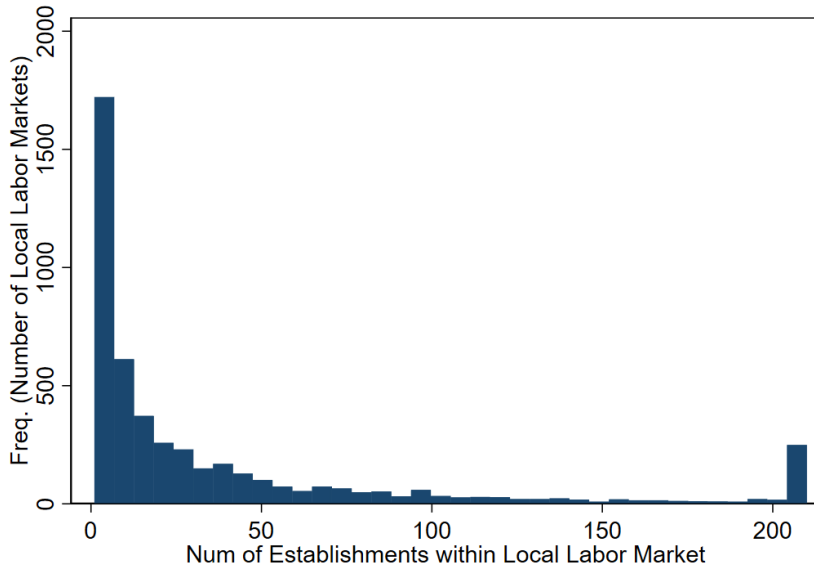


# The Granular Origins of Agglomeration

Shinnosuke Kikuchi   Daniel G. O'Connor

December 15, 2023  
UTokyo

# Median Local Labor Market has 13 Plants



Note\* Cut at top 5% percentile of 210

# Motivation

- Individual firms play a key role in local labor markets
  - Kodak in Rochester, Toyota in Toyota, Microsoft for engineers in Seattle
  - Japanese local labor market (2-digit mfg  $\times$  CZ): median of 13 plants
    - Percentiles: p25=3, p10=1
    - 3-digit  $\times$  CZ: p50=3, p25=1
- Firm-specific shocks can have a big impact on the whole labor market
  - People can end up unemployed because a single firm had a bad year
  - Firms can have a tough time finding workers to expand

# What We do

1. A new model of a local labor market with a **finite number** of firms subject to idiosyncratic shocks
  - Show that there are increasing returns to scale
  - Derive three testable empirical predictions that speak directly to the mechanism
2. Tests of the empirical predictions in Japanese administrative data
  - The variance of the log wage bill decreases in the size of the labor market
  - The variance of log firm employment increases in the size of the labor market
  - Firms with a larger employment share respond less to demand shocks
3. A quantitative model of economic geography to quantify the mechanism

# Related Literature

- **Labor Market Pooling: Theory:** Marshall (1890), Krugman (1991), Duranton and Puga (2004), Stahl and Walz (2001); **Empirics:** Overman and Puga (2010), Nakajima and Okazaki (2012), Almeida and Rocha (2018)

This paper: Stylized model for empirical predictions, direct quantification of the mechanism

- **Granularity:** Gabaix (2011), Hottman, Redding, Weinstein (2016), Gaubert and Itskhoki (2021)

This paper: Spatial implications, relevant for medium-sized cities, not just small towns

- **Job Search in Large/Thick Markets:** Moretti and Yi (WP), Andersson et al. (2014), Gan and Zhang (2006)

This paper: Similar implications, different mechanism

- **Japan:** Nakamura (1985), Tabuchi and Yoshida (2000), Nakajima et al. (2012), Miyauchi (2018)

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# Baseline Model

# The Model

- Small, open region with  $E$  establishments (firms) and a mass  $\ell$  of workers
- **Ex-ante** homogeneous firms (for now)
- In a pre-period, the state of the world  $\mathbf{s} \in \mathcal{S}$  is revealed which determines firm productivity
- Firms then choose labor to maximize profits taking wages and prices as given

$$\ell_e(\mathbf{s}) \in \operatorname{argmax}_{\ell'} a_e(\mathbf{s}) f(\ell') - w(\mathbf{s}) \ell'$$

where  $a_e(\mathbf{s})$  are iid across firms,  $f(x) = x^\eta$

- Workers inelastically supply labor

$$\ell = \sum_e \ell_e(\mathbf{s})$$



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# Characterization: Expected Production

- Labor demand is characterized by the FOC

$$a_e(s)f'(\ell_e(s)) = w(s)$$

- Wages adjust to clear the labor market in every state of the world  $s$

$$w(s) = \eta \ell^{\eta-1} \left[ \sum_{e \in \mathcal{E}} (z_e a_e(s))^{\frac{1}{1-\eta}} \right]^{1-\eta}$$

- Then expected production is

$$Y(\ell, \mathcal{E}) = \mathbb{E} \left[ \ell^\eta \left[ \sum_{e \in \mathcal{E}} (z_e a_e(s))^{\frac{1}{1-\eta}} \right]^{1-\eta} \right]$$

# Increasing returns to scale

## Proposition

*If  $\text{Var}(a_e(s)) > 0$ , then expected production has increasing returns to scale.  
In math, for any  $\ell > 0$ ,  $E \in \mathbb{N}$ , and  $\alpha > 1$  so that  $\alpha E \in \mathbb{N}$ ,*

$$Y(\alpha\ell, \alpha E) > \alpha Y(\ell, E).$$

## Comments:

- Larger markets are more productive!
- Without uncertainty, no benefit to being in a larger labor market

## Sketch of proof when $\alpha = 2$

- Two separate labor markets each with  $E$  establishments and  $\ell$  workers  $\implies$  double the production.
- The competitive equilibrium is efficient so if we can do better, the market will do better.
- Idiosyncratic firm shocks  $\implies$  sometimes the wages in labor market 1 will be higher
- Move a small number of workers from labor market 2 to labor market 1 when wages are higher. This must increase production.

## Firm Side Intuition: In response to shock to $a_e(s)$

- Recall labor demand is

$$a_e(s)f'(\ell_e(s)) = w(s)$$

## Firm Side Intuition: In response to shock to $a_e(s)$

- Recall labor demand is

$$a_e(s)f'(\ell_e(s)) = w(s)$$

- Suppose that there is one firm. It must always hire everyone even if unproductive

$$a_e(s)f'(\ell) = w(s)$$

## Firm Side Intuition: In response to shock to $a_e(s)$

- Recall labor demand is

$$a_e(s)f'(\ell_e(s)) = w(s)$$

- Suppose that there is one firm. It must always hire everyone even if unproductive

$$a_e(s)f'(\ell) = w(s)$$

- Suppose that there are many firms so that wages are constant. Firms can adjust labor as they wish

$$a_e(s)f'(\ell_e(s)) = w$$

# Disappearing Agglomeration

## Proposition

*As the labor market becomes infinitely large, production converges to constant returns to scale.*

*In math, suppose that  $\ell > 0$ ,  $E > 0$  and  $\alpha > 1$ . Then*

$$\frac{Y(\alpha\kappa\ell, \alpha\kappa E)}{\alpha Y(\kappa\ell, \kappa E)} \rightarrow 1$$

*as  $\kappa \rightarrow \infty$ .*

## Comments:

- By using models with a continuum of firms, we miss this force.
- The agglomeration force is not log-linear



# New Reason for Spatial Policy

## Proposition

*Adding new firms increases expected production more than the profits those firms would earn.*

*In math, for  $\alpha > 1$ ,*

$$\mathbb{E}\left[\sum_{e \in \alpha\mathcal{E} \setminus \mathcal{E}} \pi_e(\mathbf{s})\right] < Y(\ell, \alpha\mathcal{E}) - Y(\ell, \mathcal{E}),$$

*where  $\pi_e(\mathbf{s}) = z_e a_e(\mathbf{s}) \ell_e(\mathbf{s})^\eta - w(\mathbf{s}) \ell_e(\mathbf{s})$  are the profits earned when there are  $\alpha\mathcal{E}$  set of firms operating.*

## Comments:

- If the firm entry is somewhat elastic, under-entry
- Violates FWT because it's not Walrasian entry: firms internalize the increase in wages when they enter

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# Cross-sectional Implications of the Model

## Proposition

*To a first-order log-linear approximation around a symmetric equilibrium:*

- The variance of log wage bill is decreasing  $E$ :*

$$\text{Var}(\log w(s)\ell) \approx \frac{\sigma^2}{E};$$

- The variance of log employment for an establishment is increasing in  $E$ :*

$$\text{Var}(\log \ell_e(s)) \approx \frac{\sigma^2}{(1-\eta)^2} \left(1 - \frac{1}{E}\right)$$

*where  $\sigma^2 = \text{var}(\log a_e(s))$ .*

# Comparative Statics Implied by the Model

## Proposition

*In response to a productivity shock, firms that have a larger share of the labor market expand less.*

*In math:*

$$\Delta \log \ell_e(s) \approx \frac{1}{1 - \eta} [1 - \mu_e] \Delta \log a_e(s)$$

where  $\mu_e = \frac{\ell_e(s)}{\sum_{e'} \ell_{e'}(s)}$  is the share of labor hired by establishment  $e$

## Comments:

- In larger labor markets, firms are a smaller share of the market and so can expand without issue

# Robustness

- Imperfect mobility across establishments and labor markets [▶ Details](#)
- Monopsony power [▶ Details](#)
- Labor hoarding/employer insurance [▶ Details](#)
- Wage rigidity [▶ Details](#)

# Empirical Evidence

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# Data

- Japanese Census of Manufactures (CoM)
  - Annual survey of all manufacturing establishments with at least 4 employees
  - For 2011, 2016 (Economic Census)
  - Employment, product sales, export sales by establishment
- Sample Construction: 724,417 unique establishments
  - 1986-2016
  - Manufacturing
  - Must appear for at least 5 years consecutively
- Local Labor Market:
  - JSIC 2 digit manufacturing industry  $\times$  commuting zone
  - 25 unique 2-digit manufacturing industries
  - 256 commuting zones



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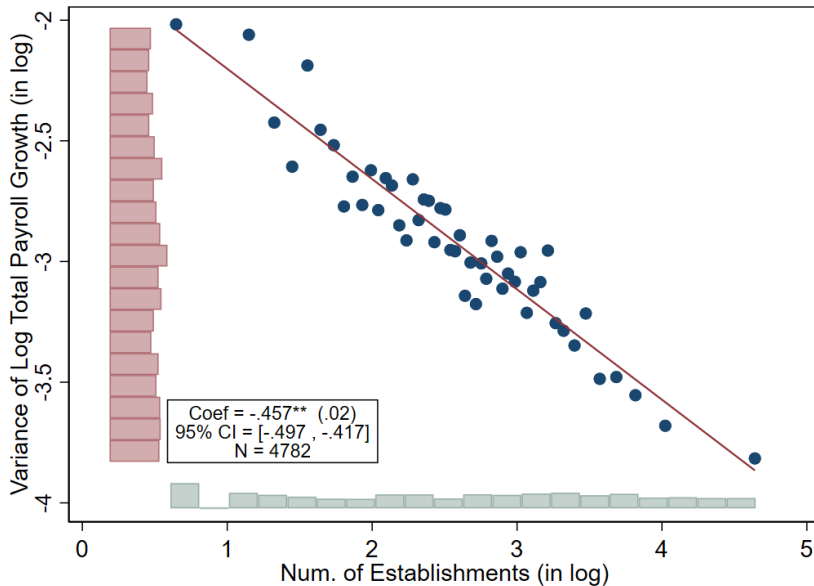
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# Fact 1: Volatility of Log Total Payment to Labor

- For each LLM,
  - Compute one-year log growth of total payroll in each year
  - Take LLM-level variance over time
- Correlation with number of establishments in each LLM

# Fact 1: Volatility of Log Total Payment to Labor ► HHI



## Fact 2: Volatility of Establishment-level Employment

- Establishments in larger markets adjust employment more flexibly?
  - Variance of log growth in establishment-level employment
- First residualize estab. yearly employment year FEs

$$\ln \ell_{e,t} = \eta_t + \varepsilon_{e,t}^{\ell}$$

- Second, compute yearly change

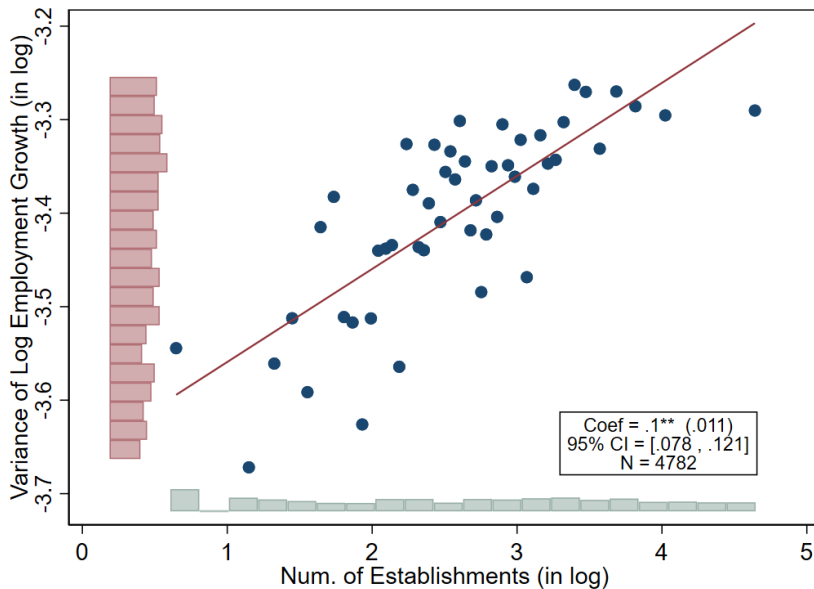
$$\Delta \varepsilon_{e,t,t+1}^{\ell} \equiv \hat{\varepsilon}_{e,t+1}^{\ell} - \hat{\varepsilon}_{e,t}^{\ell}$$

- Then residualize by estab. employment and estab-age FEs

$$\Delta \varepsilon_{e,t,t+1}^{\ell} = \gamma \ln \ell_{e,t} + \eta_{age(e)} + \zeta_{e,t,t+1}$$

- Finally take variance  $Var(\hat{\zeta}_{e,t,t+1})$  across time

## Fact 2: Volatility of Estab-level Employment ► HHI



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# Causal Analysis: Empirical Specification

- Specification

$$\Delta \ln \ell_{e,t,t+1} = \beta_1 \Delta \mu_{e,t,t+1} + \beta_2 (\Delta \mu_{e,t,t+1} \cdot \mathbf{s}_{e,t-1}) + \mathbf{X}'_{e,t} \Gamma + \zeta_e + \zeta_t + \varepsilon_{e,t}, \quad (1)$$

where  $\Delta \ln \ell_{e,t}$  is the change in employment.

- $\Delta \mu_{e,t,t+1}$ : the shift-share demand shock

$$\Delta \mu_{e,t,t+1} = \overline{\text{EXP}_e} \times \left( \sum_c \overline{\omega_{e,c}} \cdot \Delta \text{REX}_{c,t,t+1}^{\text{JPN}} \right) \quad (2)$$

- $\overline{\text{EXP}_e}$ : median export ratio
- $\overline{\omega_{e,c}}$  median exposure of establishment  $e$  to country  $c$  from product mix
- $\Delta \text{REX}_{c,t,t+1}^{\text{JPN}}$  is the change in real exchange rate of the currency

# JPY Appreciation Decreases Establishments' Employment

Regression without the interaction term for the proof of concept of the shock

**Table: Effects of JPY Depreciation on Employment Growth**

	Sales	Employment	Dep. Var.: Log Changes Employment by Types	
			Regular	Non-Regular
AREER Shock				
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓



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**Table: Effects of JPY Depreciation on Employment Growth**

	Sales	Employment	Dep. Var.: Log Changes Employment by Types	
			Regular	Non-Regular
AREER Shock	-3.46 (0.17)			
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

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**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes			
	Sales	Employment	Employment by Types	
			Regular	Non-Regular
AREER Shock	-3.46 (0.17)	-0.25 (0.09)		
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

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**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes			
	Sales	Employment	Employment by Types	
			Regular	Non-Regular
AREER Shock	-3.46 (0.17)	-0.25 (0.09)	-0.29 (0.12)	
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

# JPY Appreciation Decreases Establishments' Employment

Regression without the interaction term for the proof of concept of the shock

**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes			
	Sales	Employment	Employment by Types	
			Regular	Non-Regular
AREER Shock	-3.46 (0.17)	-0.25 (0.09)	-0.29 (0.12)	-2.62 (0.23)
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

# Establishments with Larger Share responds Less

**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62 (0.23)			
AREER Shock $\times$ Payroll Share				
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

# Establishments with Larger Share responds Less

**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62 (0.23)	-2.98 (0.27)		
AREER Shock $\times$ Payroll Share		3.35 (1.26)		
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

# Establishments with Larger Share responds Less

**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62 (0.23)	-2.98 (0.27)	-0.31 (0.44)	
AREER Shock $\times$ Log Payroll			-1.08 (0.14)	
AREER Shock $\times$ Payroll Share		3.35 (1.26)	8.26 (1.41)	
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

# Establishments with Larger Share responds Less

**Table: Effects of JPY Depreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62 (0.23)	-2.98 (0.27)	-0.31 (0.44)	-0.55 (0.44)
AREER Shock $\times$ Log Payroll			-1.08 (0.14)	-1.12 (0.15)
AREER Shock $\times$ Payroll Share		3.35 (1.26)	8.26 (1.41)	
AREER Shock $\times$ (Payroll Share > 3%)				2.43 (0.50)
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓



# Quantitative Model of Granularity

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# The Model Overview

- Small open economy
  - $N$  regions  $n \in \mathcal{N}$
  - continuum of sectors  $j \in \mathcal{J}$
- Timing of the Model:
  1. Continuum of firms  $m_{nj}$  can pay a fixed cost  $\psi$  to *attempt* an entrance in sector  $j$
  2. Random, finite number of firms enter  $E_{nj}$  (Poisson)
  3. Firms get an ex-ante productivity draw  $z_{nje}$  (Pareto)
  4. Workers decide where to live  $n$ , and how much to invest in sector-specific skills  $s_{nj}$
  5. Firm ex-post productivity shocks revealed  $a_{nje}(s)$  (Log-normal)
  6. Workers move labor across establishments and sectors subject to migration frictions

► Worker: Location Choice

► Worker: Skill Choice

► Worker: Ex-Post Labor Choice

► Firm

# Equilibrium

- Firms:
  - earn zero expected profits, conditional on trying to enter;
  - maximize profits taking as given wages, conditional on entering.
- Workers:
  - choose the utility-maximizing location;
  - choose sector-specific skills to maximize expected utility;
  - choose where to work to maximize utility.

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# Intended Calibration

Description	Parameter	Value	Source
Short run labor elasticity across sectors	$\nu$	0.42	Berger et al. (2022)
Short run labor elasticity across firms	$\kappa$	10.85	Berger et al. (2022)
Long run labor elasticity across sectors	$\bar{\nu}$	1	Burstein et al. (2020)
Elasticity of production to labor	$\eta$	0.5	Labor Share (CoM)
Ex-ante firm prod. tail	$\lambda$	2.8	Direct from Regression
Ex-post shock log variance	$\sigma^2$	0.25	Variance of log wages
Migration elasticity	$\theta$	3	Redding (2016)
Congestion externality	$\gamma_u$	-0.25	Redding (2016)
Production externality	$\gamma_z$	0.0025	Combes et al. (2011)

# Size of Externality

1. Agglomeration Externality: Elasticities of wages to the population

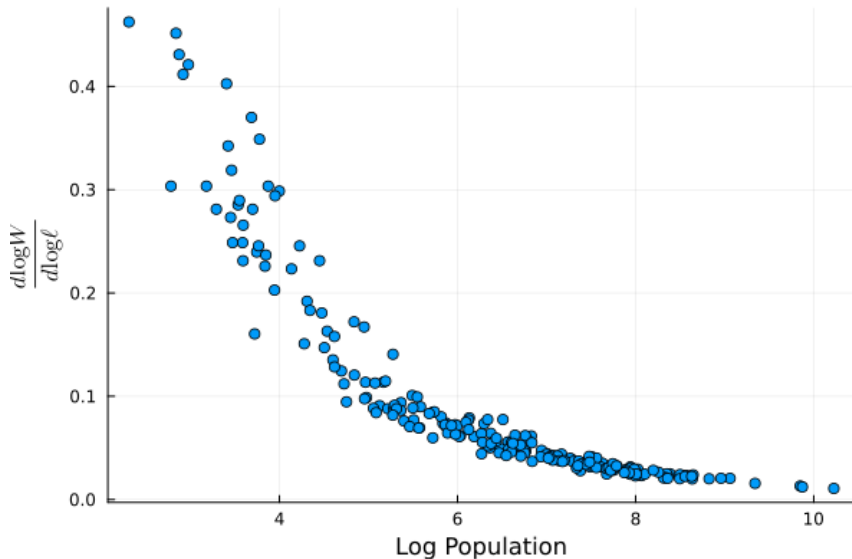
$$\frac{d \log W_n}{d \log \ell_n} := \frac{\gamma_z + \frac{\Psi'(m_n)m_n}{\Psi(m_n)} - (1 - \eta)}{1 - \frac{\Psi'(m_n)m_n}{\Psi(m_n)}},$$

2. Firm Entry Wedge: The percentage difference between expected profits and the expected benefits on production.

$$\frac{\frac{\Pi_n}{m_n}}{\psi_n} - 1 := \frac{1 - \eta}{\frac{\Psi'(m_n)m_n}{\Psi(m_n)}} - 1,$$

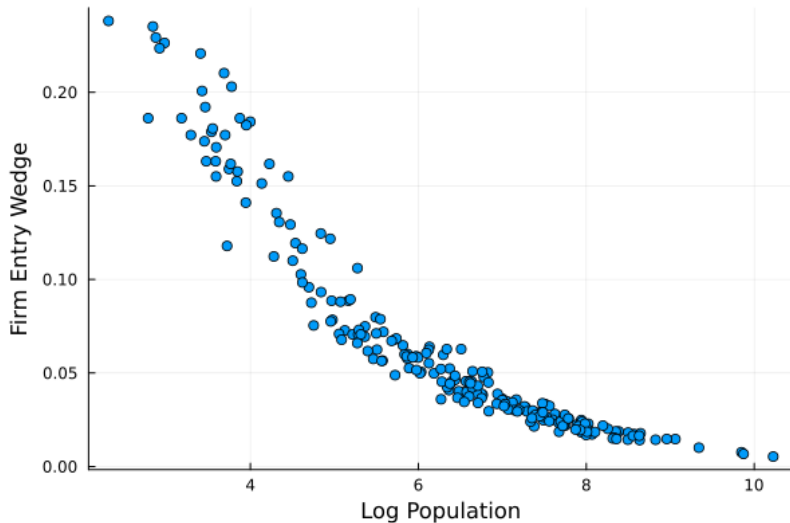
where  $\Pi_n \equiv (1 - \eta) Y_n$  is the expected profit

# Agglomeration Externality is 0.4 in Small Locations





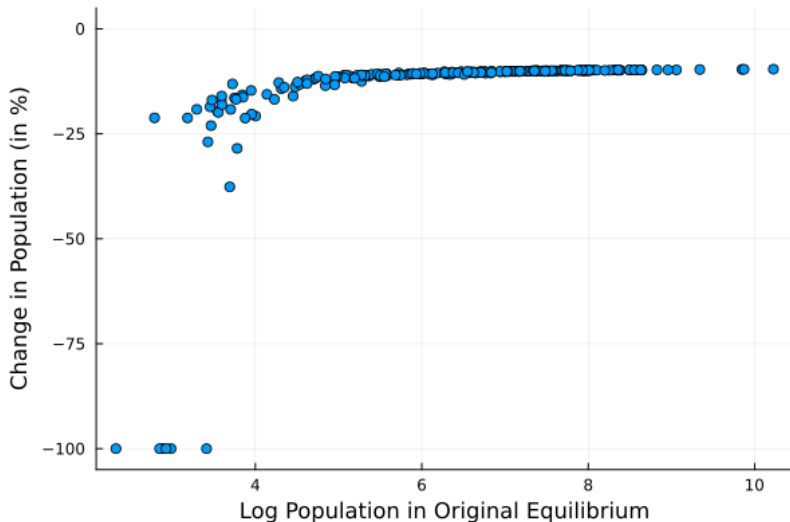
# Firm in Small Locations Capture Less than 80% of Production Benefits



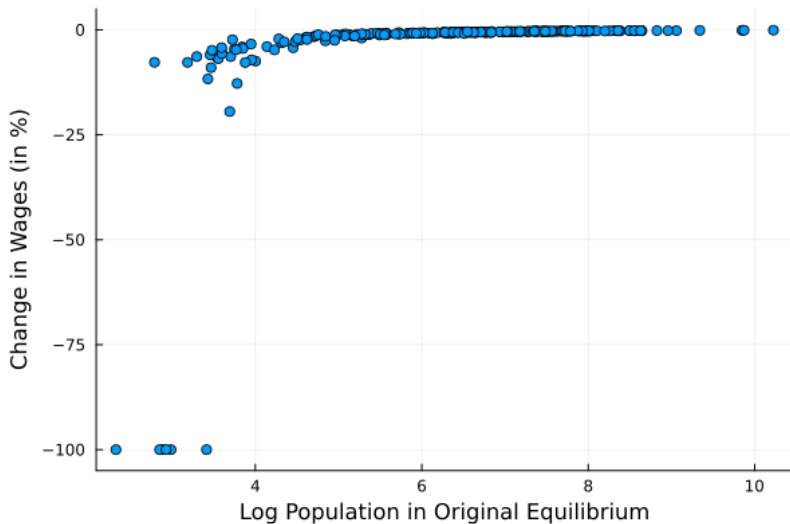
# Counterfactual

- The Japanese working-age population is decreasing
  - NRPSSR: 87 million in 1995, 75 million in 2020, 70 million in 2032
- Simulate 10% drop in population (which is not a crazy scenario)
- See changes in
  1. Population
  2. Wages

# Initially Smaller Locations Become Even Smaller



# Initially Smaller Locations Hit Harder



Tokyo: 9.6% drop in population but 0.1% drop in wages (externality is small)

# Conclusion

- Granularity is an important reason for agglomeration
- Standard economic geography models miss this and give incorrect counterfactual predictions because of it
  - Effects of Demographic Changes on Spatial Distribution
- Lots left to do!
  - How does granularity affect skill acquisition?
  - What is the optimal industrial mix?

# Appendix

# Imperfect Mobility Across Establishments and Labor Markets

- **Key Assumption:** easier to move across establishments within a labor market than moving across labor markets
- We show that this is the case
- We account for this in our quantitative model

► back

# Monopsony Power

- Another force for agglomeration
  - Firms would rather open in small labor markets
  - Workers would rather live in large labor markets
  - Workers “usually” win the tug of war since larger labor markets are more efficient
- Makes our mechanism stronger because distortions are especially bad for good shocks
- Variance of wages understates our mechanism

► back



# Labor Hoarding/Employer Insurance

- If firms have monopsony power, then they should
  1. Hold onto workers during bad years so they can have them when they need them
  2. Provide wage insurance for workers so wages represent “average” contribution
- Both cases strengthen our mechanism
  - In larger labor markets, monopsony power is lower, easier to find workers when you need them, less need for insurance
- Variance of wages understates gains

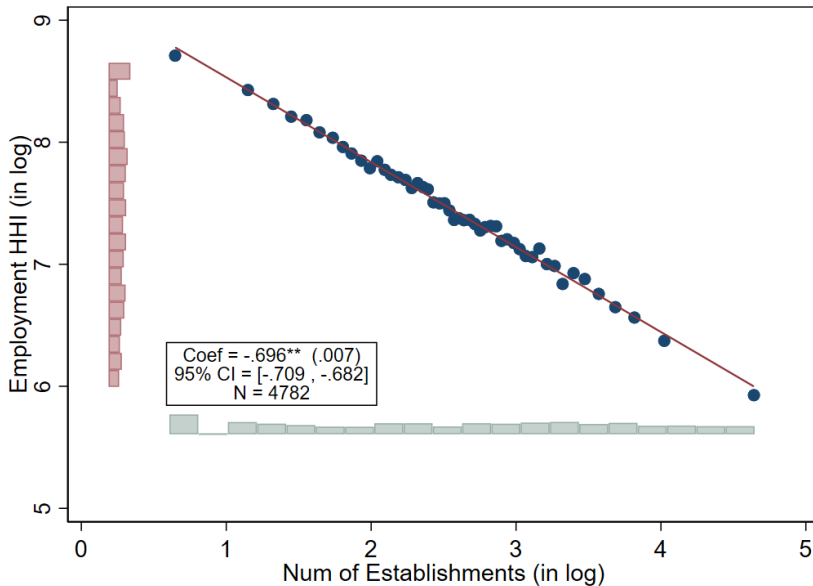
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# Wage Rigidity

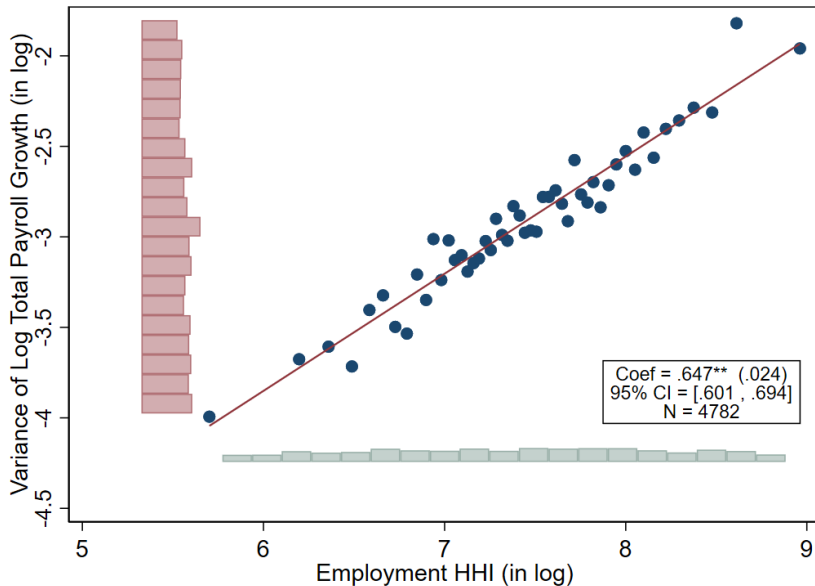
- In large labor markets, variance of marginal product is low
  - Wage rigidity rarely matters
- In small labor markets, will matter a lot!
- Even more inefficient because people become unemployed rather than underemployed
- Wage variance understates the mechanism.

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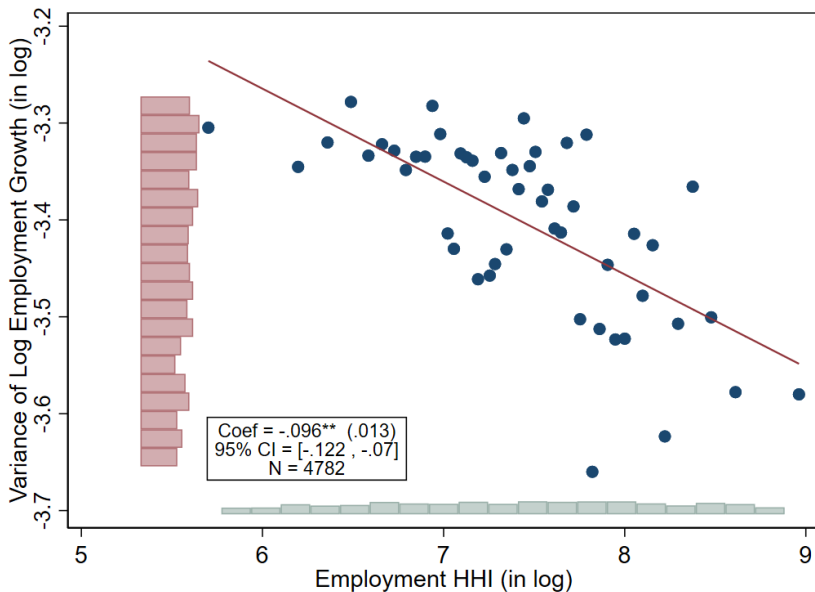
# Number of Establishments and HHI

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# Fact 1: Volatility of LLM-level Payroll

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## Fact 2: Volatility of Establishment-level Employment [► Back](#)



# Today's Plan

Appendix

Quantitative Model

# Workers - Location Choice [▶ Back](#)

- Fundamental utility of location  $n$  is

$$U_n = u_n W_n$$

- Amenities are also subject to spillovers (congestion,  $\gamma_u < 0$ )

$$u_n = \bar{u}_n (\ell_n)^{\gamma_u}.$$

- Workers have Fréchet utility shocks over the different locations

$$\ell_n = \left( \frac{U_n}{U} \right)^{\theta} \ell$$

where

$$U = \left[ \sum_n (U_n)^{\theta} \right]^{\frac{1}{\theta}}$$

# Workers - Ex-ante Skills Choice [▶ Back](#)

- Workers choose skill investments to maximize expected wages

$$\{s_{nj}\}_{j \in \mathcal{J}} \in \underset{s'_j}{\operatorname{argmax}} W_n(\{s'_j\})$$

$$\text{s.t.} \quad 1 = \int_{\mathcal{J}} (s'_j)^{\frac{1+\bar{\nu}}{\bar{\nu}-\nu}} dj$$

- This takes as given number of firms in each sector and ex-ante productivity shocks  $z_{nje}$
- $\nu$  is the short-run elasticity across sectors
- $\bar{\nu} > \nu$  is the long-run elasticity across sectors
- Denote solution by  $W_n$



## Workers - Ex-post Labor Choice [▶ Back](#)

- After the shocks are revealed, workers maximize earnings, taking wages and skills as given

$$\begin{aligned} L_{nje}(s), L_{nj}(s) \in \operatorname{argmax}_{L'_{je}, L'_j} & \int_{\mathcal{J}} \left[ \sum_{e \in \mathcal{E}_{nj}} w_{nje}(s) L'_{je} \right] dj \\ \text{s.t. } L'_j &= \left[ \sum_{e \in \mathcal{E}_{nj}} b_{nje}^{-1/\kappa} (L'_{je})^{\frac{1+\kappa}{\kappa}} \right]^{\frac{\kappa}{1+\kappa}} \\ 1 &= \left[ \int_{\mathcal{J}} s_{nj}^{-1/\nu} (L'_j)^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}} \end{aligned}$$

- Denote solution by  $W_n(\{s_{nj}\})$

Firms maximize profits by taking wages as given

$$\pi_{nje}(s) = \max_{\ell'(s)} z_{nje} a_{nje}(s) \ell'(s)^{\eta_j} - w_{nje}(s) \ell'(s)$$

Free entry is

$$\psi = \frac{1}{m_{nj}} \mathbb{E} \left[ \sum_e^{E_{nj}} \pi_{nje}(s) \middle| m_{nj} \right]$$

- Entry is Poisson

$$\mathbb{P}[E_{nj} = k] = \frac{(m_{nj})^k e^{-m_{nj}}}{k!}$$

- Ex-ante shocks are distributed Pareto

$$z_{nje} \sim \mathcal{P}(z_{nj}, \lambda); \quad z_{nj} = \bar{z}_{nj}(\ell_n)^{\gamma_z}$$

- Ex-post shocks are distributed log-normal

$$a_{nje}(s) \sim \mathcal{LN}(-\sigma^2/2, \sigma^2)$$