Granular Origins of Agglomeration

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- Individual firms play a key role in local labor markets
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  - People can end up unemployed because a single firm had a bad year
  - Firms can have a tough time finding workers to expand

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Research Questions:

- How does granularity affect the geography of economic activity?

- What does granularity imply for optimal policy?

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Therefore, there are **agglomeration** benefits!

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4. Demonstrate quantitative importance of the mechanism
   - An implied wage elasticity of 0.005 in the smallest CZs (total: 0.02-0.05)
   - Optimal subsidies would increase population by $\sim$ 1% in small cities

Related Literature

- Agglomeration: Marshall (1890), Miyauchi (2018), Davis and Dingel (2019), Duranton and Puga (2004) Andersson et al. (2014), Kline and Moretti (2014), Greenstone et al. (2010), Ellison and Glaeser (1997), Rosenthal and Strange (2004)
  This paper: Model and quantify a particular microfoundation
- Labor Market Pooling: Krugman (1992), Overman and Puga (2010), Nakajima and Okazaki (2012), de Almeida and de Moraes Rocha (2018), Moretti and Yi (2024), Conte et al. (2024)
  This paper: New model with new theoretical insights, evidence, and quantification
- Granularity: Gabaix (2011), Bernard et al. (2018), Gaubert and Itskhoki (2021), Gaubert et al. (2021)
  This paper: Implications for economic geography and place-based policy

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- Model Description
- Theoretical Results

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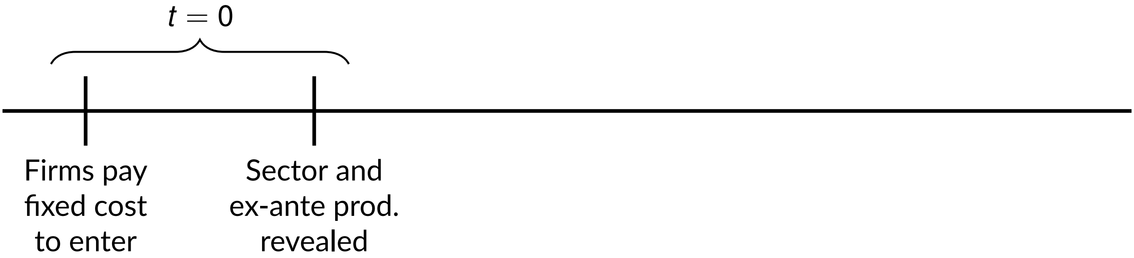

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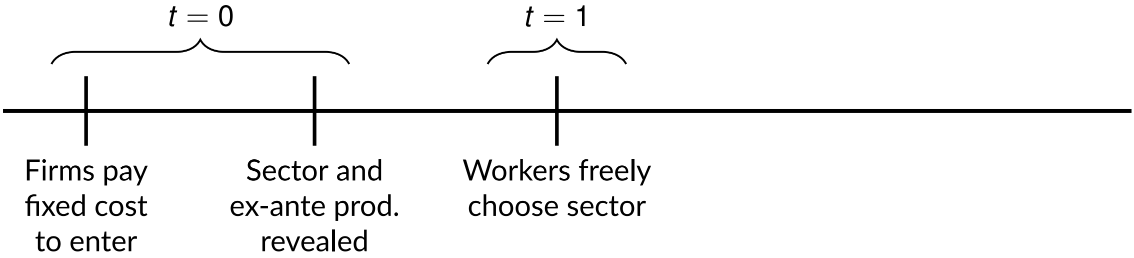

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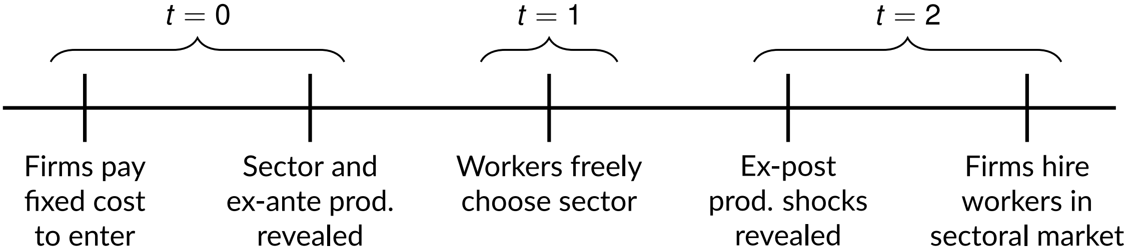

Workers

- There are ℓ_i workers in location i

- Choose sectoral labor allocation L_{is} to maximize expected utility, taking wages as given:

$$\boldsymbol{L_i} \in \underset{\boldsymbol{L_i'} \in \mathcal{L}}{\operatorname{argmax}} \quad \mathbb{E}\left[\int_0^1 w_{is}(\omega) L_{is}' ds\right]$$

  where

$$\mathcal{L} \equiv \left\{ \boldsymbol{L_i'} \middle| \int_0^1 L_{is}' ds \leq 1 \right\}.$$

  Denote the solution by w_i, the average wages in location i.

Firms

- At $t = 2$, choose labor to maximize profits, taking wages as given

$$\ell_{isn}(\omega) \in \underset{\ell'}{\operatorname{argmax}} \quad z_{isn} \cdot a_{isn}(\omega) \cdot (\ell')^{1-\eta} - w_{is}(\omega)\ell'$$

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- Randomly assigned a sector s so each sector has a finite number of firms N_{is} distributed Poisson

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- Randomly assigned a sector s so each sector has a finite number of firms N_{is} distributed Poisson
- Free entry implies expected profits equal fixed cost

$$\psi_i = \frac{\mathbb{E}[\int_{\mathcal{S}} \sum_n \pi_{isn}(\omega) ds]}{m_i}.$$

Equilibrium

A **local equilibrium** consists of wages $w_{is}(\omega)$, labor supply decisions L_{is}, entry decisions m_i, and labor demand decisions $\ell_{isn}(\omega)$ such that

- Workers maximize utility taking wages as given;
- Conditional on entry, firms maximize profits, taking prices and wages as given;
- Firms enter up to the point that expected profits are equal to the fixed cost of entering; and
- Goods and labor markets clear.

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The Regional Production Function

We define a regional production function

$$Y_i(\ell, m) \equiv \max_{L'_s, \ell'_{sn}(\omega)} \int_0^1 \sum_{n \in \mathcal{N}_{is}} z_{isn} \cdot a_{isn}(\omega) \cdot \ell'_{sn}(\omega)^{1-\eta} ds,$$

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2. If Y_i has IRS, then average wages are increasing in population
3. If firms are granular, then there are *labor market pooling effects*

Labor Market Pooling

Relation to Misallocation

Lemma

To second order,

$$Y_i(\ell, m) = \bar{z}_i \cdot \ell^{1-\eta} m^{\eta} \cdot \Phi(m),$$

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Definition

There are labor market pooling effects if the covariance is increasing in *m*.

Equilibrium and Agglomeration Benefits

- Workers are paid their marginal product,

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Proposition

In any stable equilibrium, the average wages are increasing in population if and only if there are labor market pooling effects.

The Granular Origins of Agglomeration

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If firms are subject to idiosyncratic shocks, then average wages are increasing in population, i.e.

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- In large markets, a firm can poach workers from firms in the same sector
- Therefore, labor expands more in response to shocks in large markets

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- $\implies$ Agglomeration benefits

Size of the Mechanism

Proposition

The degree of increasing returns to scale are given by

$$\frac{\partial \log \Phi(m)}{\partial \log m} = -\frac{1}{2} \cdot \frac{1-\eta}{\eta} \cdot \frac{\frac{\partial}{\partial \log m}\left[\int_{\mathcal{S}} \frac{\bar{\ell}_s}{\ell} HHI_s ds\right]}{\Phi(m)} \cdot \sigma_N^2$$

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Corollary

Marginal agglomeration benefits converge to 0 as the market grows, i.e. $\frac{d \log w_i}{d \log \ell_i} \to 0$ *as* $m_i \to \infty$.

Optimal Policy

Proposition

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- Why does the first welfare theorem fail?
  - Entry is not Walrasian
  - Firms ask what their profits would be were they to enter, taking into account their effect on the wage distribution ▸ Details

Taking Stock

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- Turn to quantification with a richer model and test some of the predictions!

A Quantitative, Granular Model of Economic Geography

Extending the Baseline Model

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- **Imperfect Mobility Across Firms and Sectors:** ▸ details
  - Labor perfectly elastic in the long run
  - Short run Elasticity across firms is κ
  - Short run Elasticity across sectors is ν
- **Labor Market Competition:** ▸ details
  - Firms with conduct $f \in \{p, c\}$ commit to a wage schedule in period 0
  - p is Perfect competition, c is Cournot competition

Theoretical Results

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Perfect Competition

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- Quantitative results shift:

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Imperfect Competition

- Mechanism strengthened

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- Qualitative results are the same
- Quantitative results shift:

$$\frac{\partial \log \Phi(m)}{\partial \log m} =$$

$$-\frac{1}{2}\frac{(1-\eta)\left(\frac{1}{\nu}-\frac{1}{\kappa}\right)}{(\eta+\frac{1}{\kappa})(\eta+\frac{1}{\nu})}\frac{\frac{\partial \int_0^1 \frac{\bar{\ell}_{is}}{\ell_i} HHI_{is} ds}{\partial \log m}}{\Phi(m)}\sigma_N^2$$

Imperfect Competition

- Mechanism strengthened
- Workers face markdown

Theoretical Results

Perfect Competition

- Qualitative results are the same
- Quantitative results shift:

$$\frac{\partial \log \Phi(m)}{\partial \log m} = -\frac{1}{2}\frac{(1-\eta)\left(\frac{1}{\nu}-\frac{1}{\kappa}\right)}{(\eta+\frac{1}{\kappa})(\eta+\frac{1}{\nu})}\frac{\frac{\partial \int_0^1 \frac{\bar{\ell}_{is}}{\bar{\ell}_i} HHI_{is} ds}{\partial \log m}}{\Phi(m)}\sigma_N^2$$

Imperfect Competition

- Mechanism strengthened
- Workers face markdown
- Optimal policy requires:
  - Wage subsidy
  - Entry subsidy **OR** tax

Estimation of Granular Driven Agglomeration

Data

- Japanese Census of Manufactures (CoM)
  - Annual survey of all manufacturing establishments with at least 4 employees
  - Employment, payroll, **shipment by product** (2,385 distinct categories)
- Sample Construction: 253,502 unique establishments
  - 2002-2019 + at least 10 employees
  - Manufacturing
- Local Labor Market:
  - JSIC 3-digit manufacturing industry $\times$ commuting zone
  - 148 unique 3-digit manufacturing industries
  - 256 commuting zones

Parameters Estimation Summary

Description	Parameter	Value	Source
A. Labor Demand			
Returns to scale	η		
Ex-ante firm prod. tail	λ		
Variance of sector shocks	σ_S^2		
Variance of idiosyncratic shocks	σ_N^2		
B. Labor Supply			
Short run labor elasticity across firms	κ		
Short run labor elasticity across markets	ν		
C. Economic Geography Parameters			
Migration elasticity	θ		
Average Productivity	z_i		
Amenity	$\overline{u}_i$		
Entry costs	ψ_i		

Parameters Estimation Summary

Description	Parameter	Value	Source
A. Labor Demand			
Returns to scale	η	0.13	Profit Share (Data, FSSC)
Ex-ante firm prod. tail	λ		
Variance of sector shocks	σ_S^2		
Variance of idiosyncratic shocks	σ_N^2		
B. Labor Supply			
Short run labor elasticity across firms	κ		
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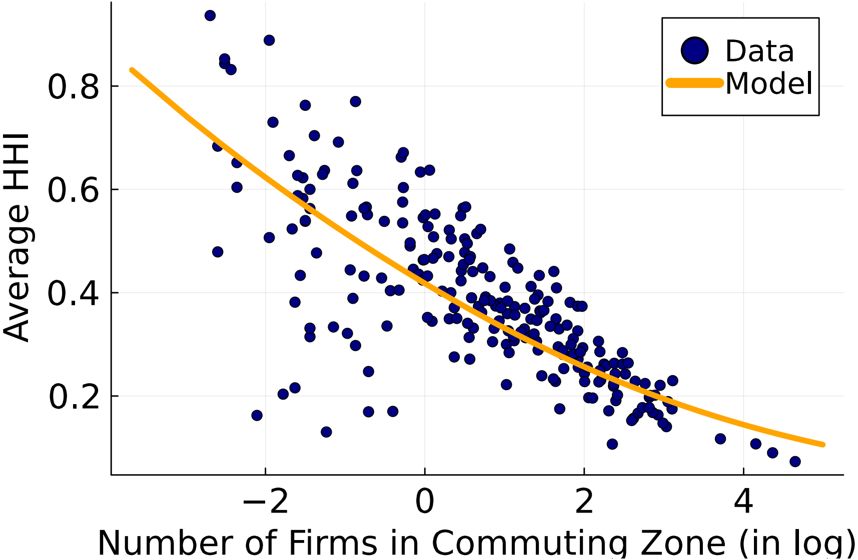
Estimating Firm Producitivity Pareto Tail



Parameters Estimation Summary

Description	Parameter	Value	Source
A. Labor Demand			
Returns to scale	η	0.13	Profit Share (Data, FSSC)
Ex-ante firm prod. tail	λ	10.5	Average HHI (CoM)
Variance of sector shocks	σ_S^2		
Variance of idiosyncratic shocks	σ_N^2		
B. Labor Supply			
Short run labor elasticity across firms	κ		
Short run labor elasticity across markets	ν		
C. Economic Geography Parameters			
Migration elasticity	θ		
Average Productivity	z_i		
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Parameters Estimation Summary

Recall that

$$y_{isn}(\omega) = z_{isn} \cdot a_{isn}(\omega) \cdot \ell_{isn}(\omega)^{1-\eta}$$

Parameters Estimation Summary

Recall that

$$y_{isn}(\omega) = z_{isn} \cdot a_{isn}(\omega) \cdot \ell_{isn}(\omega)^{1-\eta}$$

Estimate productivity changes using first differences

$$\Delta \log \hat{a}_{s,n,t} \equiv \Delta y_{i,s,n,t} - (1 - \hat{\eta}) \Delta \log \ell_{i,s,n,t}$$

Parameters Estimation Summary

Recall that

$$y_{isn}(\omega) = z_{isn} \cdot a_{isn}(\omega) \cdot \ell_{isn}(\omega)^{1-\eta}$$

Estimate productivity changes using first differences

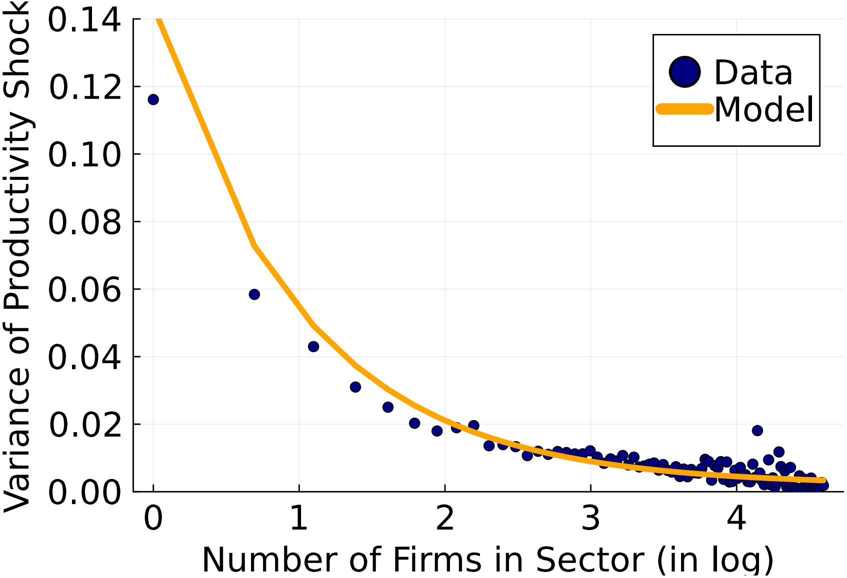
$$\Delta \log \hat{a}_{s,n,t} \equiv \Delta y_{i,s,n,t} - (1 - \hat{\eta})\Delta \log \ell_{i,s,n,t}$$

Estimate σ_S^2 and σ_N^2 using GMM to match the estimated variance of

$$\frac{\sum_{n \in \mathcal{N}_{is}} \Delta \log \hat{a}_{s,n,t}}{N_{is}}$$

for $N = \{1, \ldots, 99\}$.

Fits of the Variances: Data vs Model



Parameters Estimation Summary

Description	Parameter	Value	Source
A. Labor Demand			
Returns to scale	η	0.13	Profit Share (Data, FSSC)
Ex-ante firm prod. tail	λ	10.5	Average HHI (CoM)
Variance of sector shocks	σ_S^2	2.0×10^{-3}	Market shock
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Short run labor elasticity across firms	κ		
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C. Economic Geography Parameters			
Migration elasticity	θ		
Average Productivity	z_i		
Amenity	$\overline{u}_i$		
Entry costs	ψ_i		

Estimating firm labor supply elasticity

- Within-market cross-firm labor supply elasticity: κ

$$\log \ell_{isn}(\omega) = \kappa \log w_{isn}(\omega) - (\kappa - \nu) \log w_{is}(\omega) - \nu \log w_i(\omega) + \tilde{\epsilon}^{w}_{isn}(\omega)$$

Estimating firm labor supply elasticity

- Within-market cross-firm labor supply elasticity: κ

$$\Delta \log \ell_{isnt} = \kappa \Delta \log w_{isnt} + \underbrace{\gamma_{ist}}_{\text{market FE}} + \tilde{\epsilon}^{w}_{isnt}$$

Estimating firm labor supply elasticity

- Within-market cross-firm labor supply elasticity: κ

$$\Delta \log \ell_{isnt} = \kappa \Delta \log w_{isnt} + \underbrace{\gamma_{ist}}_{\text{market FE}} + \tilde{\epsilon}^{w}_{isnt}$$

- Instrument for $\Delta \log w_{isnt}$ using a Bartik IV Δd_{isnt}. 1st stage F-stat 14.26

$$\Delta d_{isnt} = \sum_{p} \overline{s^{p}_{isn}} \cdot d^{\text{national}}_{pt}$$

  - $\overline{s^{p}_{isn}}$: firm n's product mix (median over time, 2002-2019), $\sum_p \overline{s^{p}_{isn}} = 1$
  - $\Delta d^{\text{national}}_{pt}$: national-level log sales growths for product p

Estimating firm labor supply elasticity

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  - $\Delta d^{\text{national}}_{pt}$: national-level log sales growths for product p

- Estimate: $\widehat{\kappa} = 2.48(0.73)$
  - US: 6.52 (Lamadon et al, 2022), 10.85 (Berger et al 2022)
  - Brazil: 1.02 (Felix, 2022)

Robustness

Estimating market labor supply elasticity

- Cross-market labor supply elasticity: ν

$$\log \ell_{is}(\omega) = \nu \log w_{is}(\omega) - \nu \log w_i(\omega) + \varepsilon^{w}_{is}(\omega)$$

where

$$\ell_{is}(\omega) = \left[\sum_{n \in \mathcal{N}_{is}} \left(\frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} \right)^{-\frac{1}{\kappa}} \ell_{isn}(\omega)^{\frac{1+\kappa}{\kappa}} \right]^{\frac{\kappa}{1+\kappa}}, \quad w_{is}(\omega) = \left[\sum_{n \in \mathcal{N}_{is}} \frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} w_{isn}(\omega)^{1+\kappa} \right]^{\frac{1}{1+\kappa}}$$

Estimating market labor supply elasticity

- Cross-market labor supply elasticity: ν

$$\Delta \log \ell_{ist} = \nu \Delta \log w_{ist} + \gamma_{it} + \gamma_{st} + \varepsilon^{w}_{ist}$$

where

$$\ell_{ist} = \left[\sum_{n \in \mathcal{N}_{is}} \left(\frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} \right)^{-\frac{1}{\hat{\kappa}}} \ell_{isnt}^{\frac{1+\hat{\kappa}}{\hat{\kappa}}} \right]^{\frac{\hat{\kappa}}{1+\hat{\kappa}}}, \quad w_{ist} = \left[\sum_{n \in \mathcal{N}_{is}} \frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} w_{isnt}^{1+\hat{\kappa}} \right]^{\frac{1}{1+\hat{\kappa}}}$$

Estimating market labor supply elasticity

- Cross-market labor supply elasticity: ν

$$\Delta \log \ell_{ist} = \nu \Delta \log w_{ist} + \gamma_{it} + \gamma_{st} + \varepsilon^{w}_{ist}$$

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- IV for $\Delta \log w_{ist}$: $\sum_{n'} \frac{\overline{\ell_{isn'}}}{\overline{\ell}_{is}} \cdot \Delta d_{isn't}$. 1st stage F-stat 482

Estimating market labor supply elasticity

- Cross-market labor supply elasticity: ν

$$\Delta \log \ell_{ist} = \nu \Delta \log w_{ist} + \gamma_{it} + \gamma_{st} + \varepsilon^{w}_{ist}$$

where

$$\ell_{ist} = \left[\sum_{n \in \mathcal{N}_{is}} \left(\frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} \right)^{-\frac{1}{\hat{\kappa}}} \ell_{isnt}^{\frac{1+\hat{\kappa}}{\hat{\kappa}}} \right]^{\frac{\hat{\kappa}}{1+\hat{\kappa}}}, \quad w_{ist} = \left[\sum_{n \in \mathcal{N}_{is}} \frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} w_{isnt}^{1+\hat{\kappa}} \right]^{\frac{1}{1+\hat{\kappa}}}$$

- IV for $\Delta \log w_{ist}$: $\sum_{n'} \frac{\overline{\ell_{isn'}}}{\overline{\ell}_{is}} \cdot \Delta d_{isn't}$. 1st stage F-stat 482
- Estimate: $\widehat{\nu} = 1.46(0.07)$
  - US: 4.57 (Lamadon et al, 2022), 0.42 (Berger et al, 2022)
  - Brazil: 0.80 for Brazil (Felix, 2022)

▸ Robustness

Parameters Estimation Summary

Description	Parameter	Value	Source
A. Labor Demand			
Returns to scale	η	0.13	Profit Share (Data, FSSC)
Ex-ante firm prod. tail	λ	10.5	Average HHI (CoM)
Variance of sector shocks	σ_S^2	2.0×10^{-3}	Market shock
Variance of idiosyncratic shocks	σ_N^2	0.14	variance (CoM)
B. Labor Supply			
Short run labor elasticity across firms	κ	2.48	Product-level
Short run labor elasticity across markets	ν	1.46	Bartik shocks (CoM)
C. Economic Geography Parameters			
Migration elasticity	θ		
Average Productivity	z_i		
Amenity	$\overline{u}_i$		
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Parameters Estimation Summary

Description	Parameter	Value	Source
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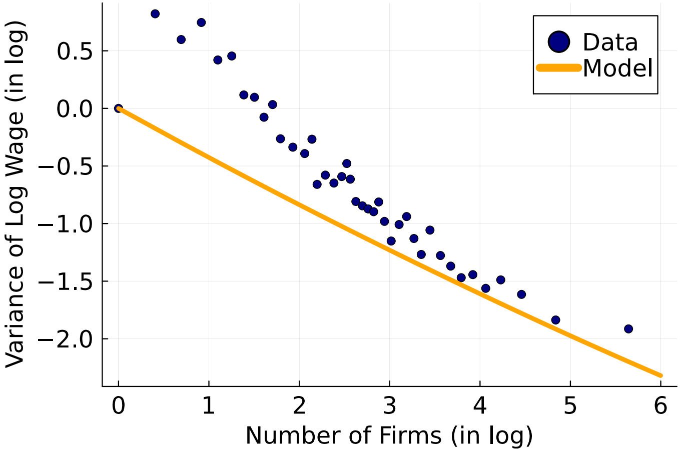
B. Labor Supply			
Short run labor elasticity across firms	κ	2.48	Product-level
Short run labor elasticity across markets	ν	1.46	Bartik shocks (CoM)
C. Economic Geography Parameters			
Migration elasticity	θ	3	Redding (2016)
Average Productivity	z_i		
Amenity	$\bar{u}_i$		Exact hat algebra (2019)
Entry costs	ψ_i		

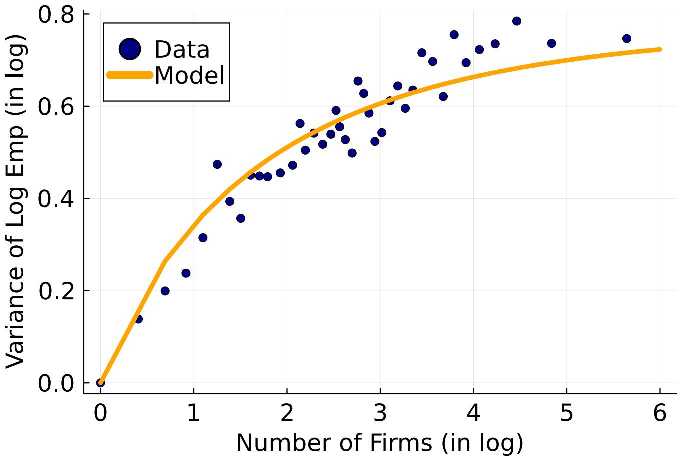
Validation of the Mechanism

Firms are Subject to Idiosyncratic Shocks ▸ math



$$\mathrm{Var}\left(\log\left(\sum_{n\in\mathcal{N}_{is}} w_{isnt}\ell_{isnt}\right)\right)$$

Firms in Larger Markets Expand More - Correlation ▸ math



$$\sum_{n \in \mathcal{N}_{is}} \frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} \operatorname{Var}(\log \ell_{isnt})$$

Firms in Larger Markets Expand More - Reduced Form ▸ math

$$\Delta \log \ell_{isnt} = \frac{1}{\eta + \frac{1}{\kappa}} \left(\Delta \log a_{isnt} - \frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} \frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} \Delta \log a_{isnt} \right)$$

So the implied ratio is

$$-\frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} = -0.35$$

Firms in Larger Markets Expand More - Reduced Form

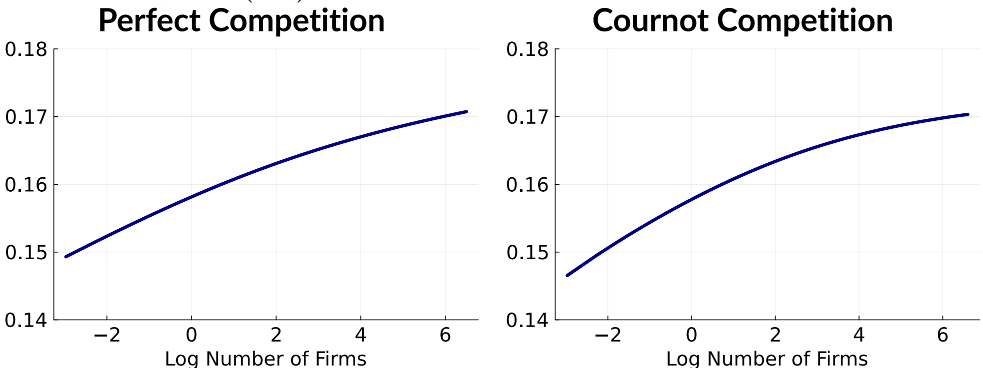
▸ math

	$\Delta \log \ell_{isn,t+1}$	
	(1)	(2)
Shock	0.048	0.048
	(0.002)	(0.002)
Shock $\times \frac{\ell_{isn,t}}{\ell_{is,t}}$	-0.021	-0.023
	(0.005)	(0.005)
Implied Ratio	-0.428	-0.477
95% CI	[-0.615, -0.240]	[-0.651, -0.303]
Observations	1,740,782	1,511,376
Year & Firm FE	✓	✓
Lag. Payroll Share	✓	✓
$\Delta \log \ell_{isn,t}$		✓

$$-\frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} = -0.35$$

Quantification of Granular Driven Agglomeration

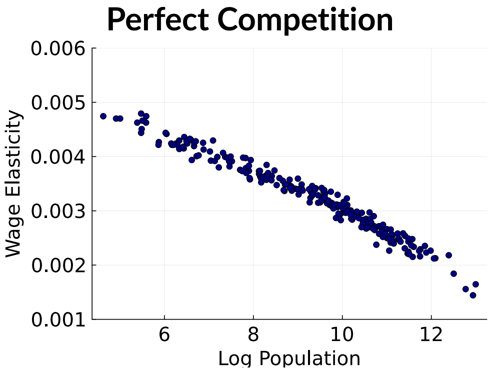
Estimated $\log \Phi^f(m)$ ▸ bertrand version

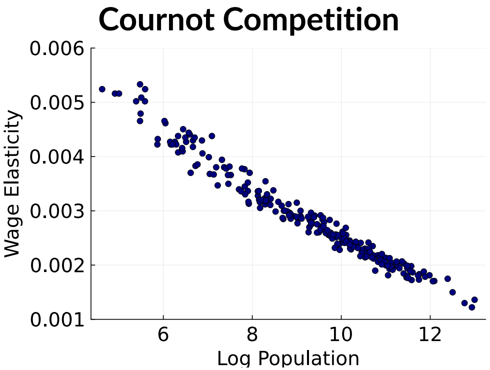


$$Y_i(\ell, m) = z_i \ell^{1-\eta} m^{\eta} \cdot \Phi(m)$$

Implied Wage Elasticity

▸ bertrand version

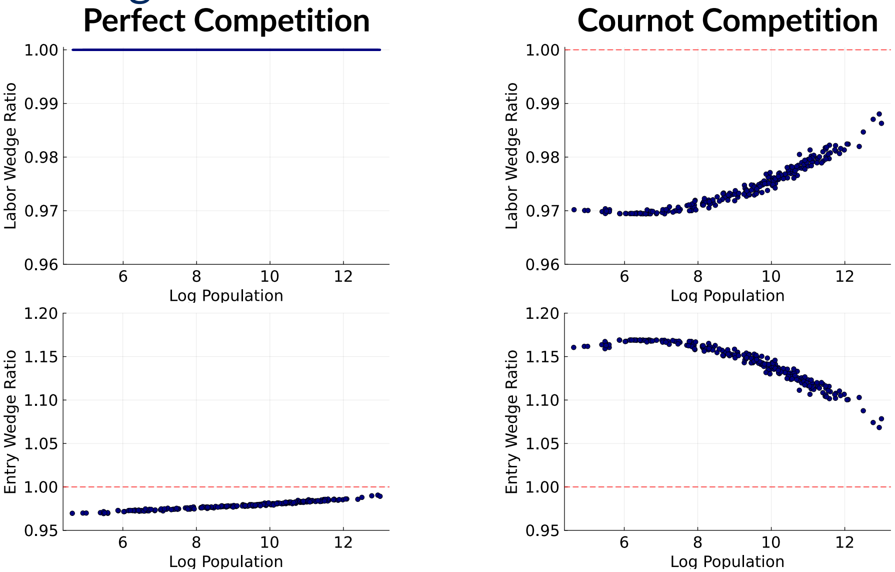




- 0.005 in smallest CZ = 10 – 25% of total agglomeration forces (0.02 – 0.05, Combes et al)

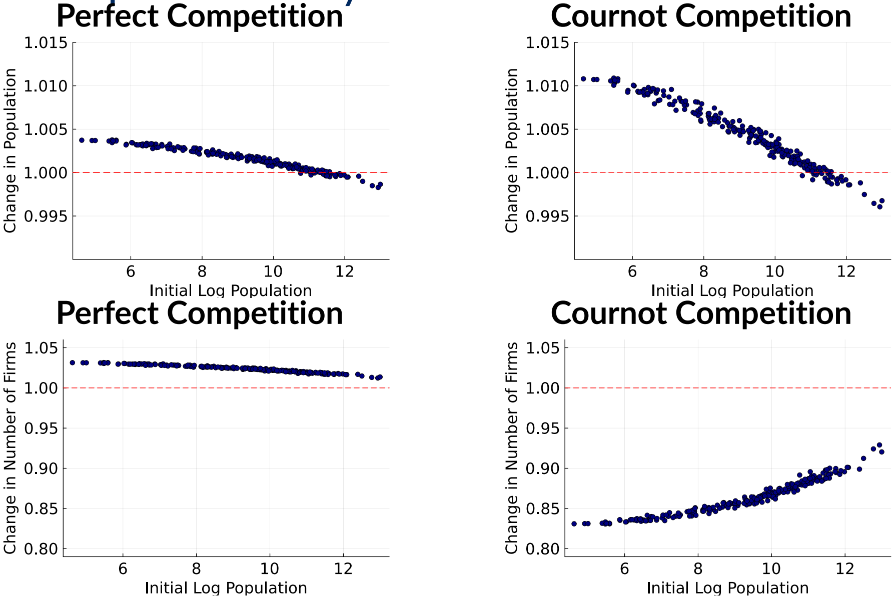
Factor Wedge Ratios

bertrand version



Effects of Optimal Policy

▸ bertrand version



Conclusion

- Labor markets are over-exposed to firm shocks which implies:
  - Larger markets are more productive
  - Too few people live in small locations
- The key object:
  - Covariance between firm productivity and employment
- Quantitatively, granularity means:
  - Tokyo is 2.5% more productive than the smallest cities
  - Small cities should have 1% more people

Appendix

Relation to Misallocation

back

When firms are on their demand curves, one can relate the variance of wages to the covariance,

$$\int_0^1 \frac{\overline{\ell}_{is}}{\overline{\ell}_i} \mathrm{Var}(\log w_s(\omega)) ds = \mathrm{Var}(\log a_{sn}(\omega)) \\ - \eta \int_0^1 \frac{\overline{\ell}_{is}}{\ell_i} \sum_{n \in \mathcal{N}_{is}} \frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} \mathrm{Cov}\left(\log a_{isn}(\omega), \log \ell_{isn}(\omega)\right) ds$$

Relation to Misallocation ‣ back

When firms are on their demand curves, one can relate the variance of wages to the covariance,

$$\int_0^1 \frac{\bar{\ell}_{is}}{\bar{\ell}_i} \mathrm{Var}(\log w_s(\omega)) ds = \mathrm{Var}(\log a_{sn}(\omega)) \\ - \eta \int_0^1 \frac{\bar{\ell}_{is}}{\bar{\ell}_i} \sum_{n \in \mathcal{N}_{is}} \frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} \mathrm{Cov}\left(\log a_{isn}(\omega), \log \ell_{isn}(\omega)\right) ds$$

Then we rewrite the corollary:

Corollary

$Y_i(\ell, m)$ features increasing returns to scale if the average variance of $\log w_s(\omega)$ is decreasing in the number of firms.

Migration Across Regions Details

back

- There is a mass ℓ of workers in the country
- The fundamental utility of living in i is

$$u_i = \bar{u}_i w_i$$

  where $\bar{u}_i$ is the local amenities
- A worker gets utility $u_i \varepsilon_i$ from living in location i where ε_i is the idiosyncratic preference for location i distributed Fréchet with shape parameter θ
- Standard maximization problem implies migration,

$$\ell_i = \left(\frac{u_i}{u}\right)^{\theta} \ell,$$

  where $u = \left(\sum u_i^{\theta}\right)^{1/\theta}$.

Imperfect Mobility Details ▸ back

- Period 1, workers freely allocates labor across firms

$$\mathcal{L} \equiv \left\{ \boldsymbol{L}'_i \middle| \int_0^1 \sum_{n \in \mathcal{N}_{is}} L'_{isn} ds \leq 1 \right\}.$$

Imperfect Mobility Details

back

- Period 1, workers freely allocates labor across firms

$$\mathcal{L} \equiv \left\{ \boldsymbol{L}_i' \middle| \int_0^1 \sum_{n \in \mathcal{N}_{is}} L_{isn}' ds \leq 1 \right\}.$$

- In period 2, worker can then adjust labor subject to the constraint embedded in the set

$$\mathcal{L}_\Omega(\boldsymbol{L}_i) \equiv \Bigg\{ \boldsymbol{L}_i' \Bigg| 1 = \left(\int_0^1 L_{is}^{-\frac{1}{\nu}} L_{is}(\omega)^{\frac{1+\nu}{\nu}} ds \right)^{\frac{\nu}{1+\nu}},$$

$$L_{is}(\omega) = \left(\sum_{n \in \mathcal{N}_{is}} \left(\frac{L_{isn}}{L_{is}} \right)^{-\frac{1}{\kappa}} L_{isn}(\omega)^{\frac{1+\kappa}{\kappa}} \right)^{\frac{\kappa}{1+\kappa}} \Bigg\}.$$

Imperfect Mobility Details ▸ back

- Period 1, workers freely allocates labor across firms

$$\mathcal{L} \equiv \left\{ \boldsymbol{L}_i' \middle| \int_0^1 \sum_{n \in \mathcal{N}_{is}} L_{isn}' ds \leq 1 \right\}.$$

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$$\mathcal{L}_\Omega(\boldsymbol{L}_i) \equiv \left\{ \boldsymbol{L}_i' \middle| 1 = \left(\int_0^1 L_{is}^{-\frac{1}{\nu}} L_{is}(\omega)^{\frac{1+\nu}{\nu}} ds \right)^{\frac{\nu}{1+\nu}}, \right.$$

$$\left. L_{is}(\omega) = \left(\sum_{n \in \mathcal{N}_{is}} \left(\frac{L_{isn}}{L_{is}} \right)^{-\frac{1}{\kappa}} L_{isn}(\omega)^{\frac{1+\kappa}{\kappa}} \right)^{\frac{\kappa}{1+\kappa}} \right\}.$$

- The worker chooses $L_i \in \mathcal{L}$ and $L_i(\omega) \in \mathcal{L}_\Omega(L_i)$ to maximize expected earnings taking wages $w_{isn}(\omega)$ as given

Perfect Competition Details

back

- Compared to the baseline, firms now have individual wages

- Firms choose labor to maximize profits, taking wages as given

$$\ell_{isn}(\omega) \in \underset{\ell'}{\operatorname{argmax}} \quad z_{isn} a_{isn}(\omega)(\ell')^{1-\eta} - w_{isn}(\omega)\ell'.$$

- Market clearing requires that labor clears firm by firm

$$\ell_{isn}(\omega) = L_{isn}(\omega)\ell_i.$$

Cournot Competition Details ▸ back

- Firms commit to a wage profile across all states of the world ω in period 0, $\{w_{isn}(\omega)\}$.
- With Cournot competition, they take as given employment by the other firms in the sector and the wage opportunity in every other sector
- Firms have market power because after a good shock, they expand and have more market share
  - Firms have higher markdowns after good productivity shocks
  - Lower markdowns after bad productivity shocks

Bertrand Competition Details ▸ back

- Firms commit to a wage profile across all states of the world ω in period 0, $\{w_{isn}(\omega)\}$.
- With Bertrand competition, they take as given wages by the other firms in the sector and the wage opportunity in every other sector
- Firms have market power because after a good shock, they expand and have more market share
  - Firms have higher markdowns after good productivity shocks
  - Lower markdowns after bad productivity shocks

Firm Labor Supply Elasticity Robustness

back

	Dep. Var.: Log Employment Growth			
	(1)	(2)	(3)	(4)
Log Wage Growth	2.51	3.21	3.07	3.37
	(0.49)	(0.78)	(0.87)	(1.09)
Observations	1,850,914	1,581,528	1,850,914	1,581,528
1st Stage F-Stat.	31.51	19.14	14.43	10.90
Covariates		✓		✓
Weighted			✓	✓

Market Labor Supply Elasticity Robustness

▸ back

	Dep. Var.: Market-level Log Employment Growth			
	(1)	(2)	(3)	(4)
Market-level Log Wage Growth	1.87	2.13	1.42	1.87
	(0.22)	(0.21)	(0.12)	(0.11)
Observations	215,299	195,330	215,299	195,330
1st Stage F-Stat.	83.56	108.40	158.63	270.78
Covariates		✓		✓
Weighted			✓	✓

Log Wage Bill Variance in the Model ▸ back

- To log first order,

$$\Delta \log \left(\sum_{n \in \mathcal{N}_{is}} w_{isn}(\omega) \ell_{isn}(\omega) \right) \approx \frac{1+\nu}{1+\eta\nu} \sum_{n \in \mathcal{N}_{is}} \frac{\bar{\ell}_{isn}}{\bar{\ell}_{is}} \Delta \log a_{isn}(\omega).$$

- Taking the variance implies

$$\begin{aligned} \text{Var} \log \left(\sum_{n \in \mathcal{N}_{is}} w_{isn}(\omega) \ell_{isn}(\omega) \right) &\approx \frac{1+\nu}{1+\eta\nu} \left(\sigma_S^2 + HHI_{is} \sigma_N^2 \right) \\ &\approx \frac{1+\nu}{1+\eta\nu} \left(\sigma_S^2 + C N_{is}^{-2\left(1 - \frac{1}{\lambda\eta}\right)} \sigma_N^2 \right) \end{aligned}$$

Log Employment Variance in the Model ▸ back

- To log first order

$$\Delta \log \ell_{isn}(\omega) = \frac{1}{\eta + \frac{1}{\kappa}} \left(\Delta \log a_{isn}(\omega) - \frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} \sum_{n' \in \mathcal{N}_{is}} \frac{\overline{\ell}_{isn'}}{\overline{\ell}_{is}} \Delta \log a_{isn'} \right)$$

- Then tedious algebra implies

$$\sum_{n \in \mathcal{N}_{is}} \frac{\ell_{isn}}{\ell_{is}} \mathrm{Var}(\log \ell_{isn}(\omega)) \approx \left(\frac{1}{\eta + \frac{1}{\kappa}} \right)^2 \sigma_S^2 + \left(\frac{1}{\eta + \frac{1}{\kappa}} \right) \sigma_N^2 - \left(\frac{1}{\nu} - \frac{1}{\kappa} \right) \frac{\eta + \frac{1}{\kappa} + \eta + \frac{1}{\nu}}{\left(\eta + \frac{1}{\kappa} \right)^2 \left(\eta + \frac{1}{\nu} \right)^2} \cdot HHI_{is} \cdot \sigma_N^2$$

Reduced Form Labor Response in the Model ▸ back

- To log first order

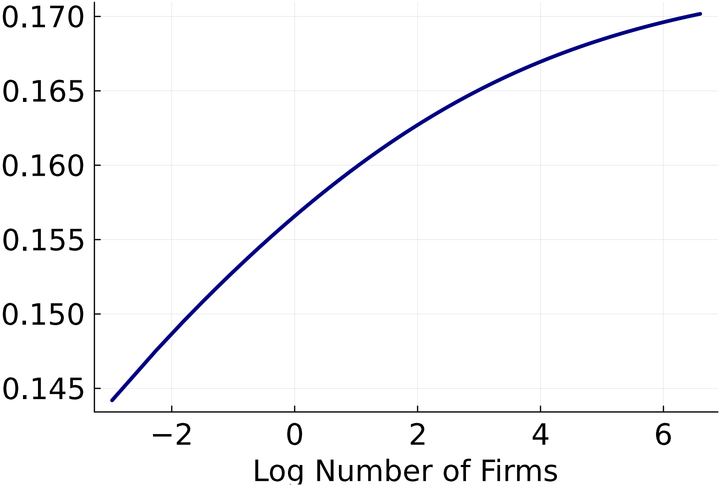
$$\Delta \log \ell_{isn}(\omega) = \frac{1}{\eta + \frac{1}{\kappa}} \left(\Delta \log a_{isn}(\omega) - \frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} \sum_{n' \in \mathcal{N}_{is}} \frac{\overline{\ell}_{isn'}}{\overline{\ell}_{is}} \Delta \log a_{isn'} \right)$$

$$= \frac{1}{\eta + \frac{1}{\kappa}} \left(\Delta \log a_{isn}(\omega) - \frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} \frac{\overline{\ell}_{isn}}{\overline{\ell}_{is}} \Delta \log a_{isn} \right)$$

- Therefore, the implied ratio is

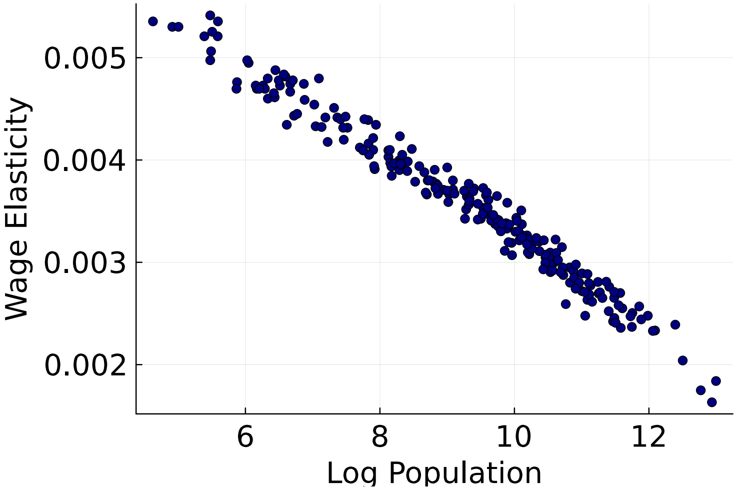
$$-\frac{\frac{1}{\nu} - \frac{1}{\kappa}}{\eta + \frac{1}{\nu}} = -0.359$$

Estimated $\log \Phi^f(m)$ - Bertrand ▸ back



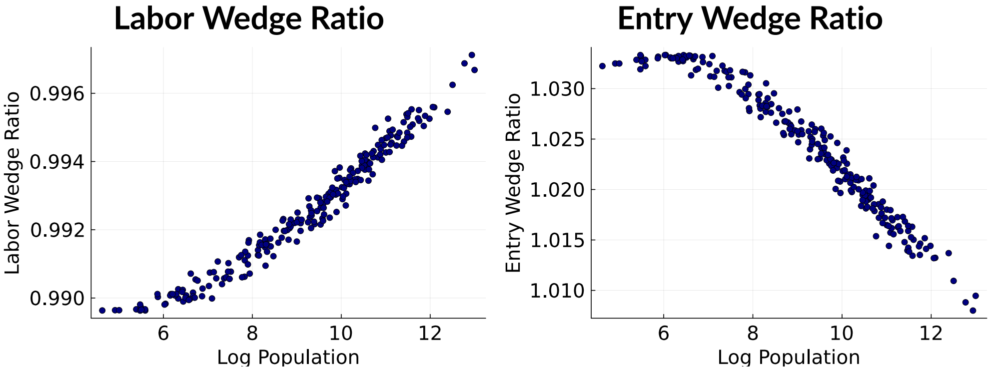
Implied Wage Elasticity - Bertrand

back



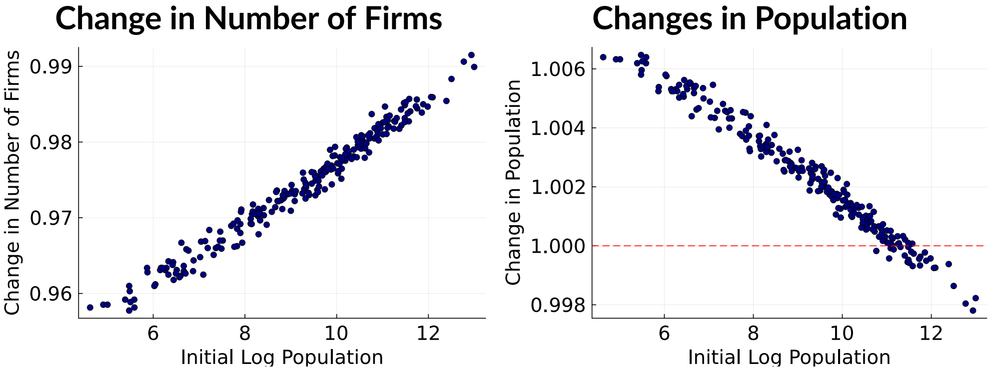
Factor Wedge Ratios - Bertrand

back



Effects of Optimal Policy - Bertrand

back



Entry Details ‣ back

Average profits can be written:

$$
\begin{aligned}
\frac{\mathbb{E}\left[\int_0^1 \sum_{n\in\mathcal{N}_{is}} \pi_{isn}(\omega)\right]}{m_i} &= \frac{\sum_{N=0}^{\infty} \mathbb{E}[\sum_n \pi_{isn}(\omega) | N_{is} = N] p(N, m)}{m_i} \\
&= \sum_{N=0}^{\infty} \pi^e_{i,N} \frac{N}{m_i} p(N, m) \\
&= \sum_{N=0}^{\infty} \pi^e_{i,N+1} \frac{N+1}{m_i} \frac{m_i^{N+1} e^{-m_i}}{(N+1)!} \\
&= \sum_{N=0}^{\infty} \pi^e_{i,N+1} p(N, m_i).
\end{aligned}
$$

Profits of Potential Entrants

back

To second order,

$$\mathbb{E}[\pi_{isn}(\omega)] \approx \overline{\pi}_{isn} \left[\zeta_s - \frac{1-\eta}{\eta^2} \operatorname{Cov}(\log a_{isn}(\omega), \log w_{is}(\omega))\right]$$

Therefore, operating firms (whose productivity shocks are correlated with wages) earn less profits than potentially operating firms.