The Granular Origins of Aggolomeration

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Motivation

- Individual firms play a key role in local labor markets
  - Kodak in Rochester, Toyota in Toyota, Microsoft for engineers in Seattle

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  - Kodak in Rochester, Toyota in Toyota, Microsoft for engineers in Seattle
  - Japanese local labor market (2-digit mfg $\times$ CZ): median of 13 plants
- Firm-specific shocks can have a big impact on the whole labor market
  - People can end up unemployed because a single firm had a bad year
  - Firms can have a tough time finding workers to expand
  - Ideas in Krugman (1991)

What We do

1. A model of a labor market with a finite number of firms s.t. idios. shocks
   - Show that there are increasing returns to scale
   - Derive testable empirical predictions that speak directly to the mechanism
   - Discuss policy implications
2. Tests of the empirical predictions in Japanese microdata
   - The variance of the log wage bill decreases in the size of the labor market
   - The variance of log firm employment increases in the size of the labor market
   - Firms with a larger employment share respond less to demand shocks
3. A quantitative model of economic geography to quantify the mechanism

Related Literature

- Labor Market Pooling: Theory: Marshall (1890), **Krugman (1991)**, Duranton and Puga (2004), Stahl and Walz (2001); Empirics: Overman and Puga (2010), Nakajima and Okazaki (2012), Almeida and Rocha (2018)

  This paper: Stylized model for empirical predictions, direct quantification of the mechanism

- Granularity: Gabaix (2011), Hottman, Redding, Weinstein (2016), Gaubert and Itskhoki (2021)

  This paper: Spatial implications, relevant for medium-sized cities, not just small towns

- Job Search in Large/Thick Markets: Moretti and Yi (WP), Andersson et al. (2014), Gan and Zhang (2006)

  This paper: Similar implications, different mechanism

- Empirics on Agglomeration in Japan: Nakajima et al. (2012), Nakajima and Teshima (2020), Miyauchi (2023), many...

Mickey Mouse Model

Mickey Mouse Model

- Small, open region with E establishments (firms) and a mass ℓ of workers
- **Ex-ante** homogeneous firms (for now)
- In a pre-period, the state of the world $s \in \mathcal{S}$ is revealed $\to$ firm productivity
- Firms then choose labor to maximize profits taking wages and prices as given

$$\ell_e(s) \in \underset{\ell'}{\operatorname{argmax}} \quad a_e(s) f\left(\ell'\right) - w(s)\ell'$$

  - $a_e(s)$ are iid across firms, $f(x) = x^\eta$
- Workers inelastically supply labor

$$\ell = \sum_e \ell_e(s)$$

Robustness | Imperfect Mobility | Monopsony | Labor Hoarding/Insurance | Wage Rigidity

Characterization: Expected Production

- Labor demand characterized by the FOC

$$a_e(s)f'(\ell_e(s)) = w(s)$$

- Wages adjust to clear the labor market in every state of the world s

$$w(s) = \eta \ell^{\eta-1} \left[\sum_{e \in \mathcal{E}} (a_e(s))^{\frac{1}{1-\eta}} \right]^{1-\eta}$$

- Then expected production is

$$Y(\ell, \mathcal{E}) = \mathbb{E} \left[\ell^{\eta} \left[\sum_{e \in \mathcal{E}} (a_e(s))^{\frac{1}{1-\eta}} \right]^{1-\eta} \right]$$

Increasing Returns to Scale

Proposition

If $Var(a_e(s)) > 0$, then expected production has increasing returns to scale.
In math, for any $\ell > 0$, $E \in \mathbb{N}$, and $\alpha > 1$ so that $\alpha E \in \mathbb{N}$,

$$Y(\alpha\ell, \alpha E) > \alpha Y(\ell, E).$$

Comments:

- Larger markets are more productive!
- Without uncertainty, no benefit to being in a larger labor market
- Room for policies ▸ Details

Intuition: In response to shock to $a_e(s)$

- Recall labor demand is

$$a_e(s)f'(\ell_e(s)) = w(s)$$

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$$a_e(s)f'(\ell_e(s)) = w(s)$$

- Sps. that there is one firm $\rightarrow$ must always hire everyone even if unproductive

$$a_e(s)f'(\ell) = w(s)$$

- Sps. that there are many firms $\rightarrow$ can adjust labor as they wish (constant wage)

$$a_e(s)f'(\ell_e(s)) = w$$

Disappearing Agglomeration in Limit

Proposition

As the labor market becomes larger, production converges to constant returns to scale. In math, suppose that $\ell > 0$, $E > 0$ and $\alpha > 1$. Then

$$\frac{Y(\alpha\kappa\ell, \alpha\kappa E)}{\alpha Y(\kappa\ell, \kappa E)} \to 1$$

as $\kappa \to \infty$.

Comments:

- By using models with a continuum of firms, we miss this force.
- Larger market would be largely unaffected

Cross-sectional Implications of the Model

Proposition

To a first-order log-linear approximation around a symmetric equilibrium:

- *The variance of log wage bill is decreasing E:*

$$Var(\log w(s)\ell) \approx \frac{\sigma^2}{E};$$

- *The variance of log employment for an establishment is increasing in E:*

$$Var(\log \ell_e(s)) \approx \frac{\sigma^2}{(1-\eta)^2}\left(1 - \frac{1}{E}\right)$$

where $\sigma^2 = var(\log a_e(s))$.

Comparative Statics Implied by the Model

Proposition

In response to a productivity shock, firms that have a larger share of the labor market expand less.

$$\Delta \log \ell_e(s) \approx \frac{1}{1-\eta} [1 - \mu_e] \Delta \log a_e(s)$$

where $\mu_e = \frac{\ell_e(s)}{\sum_{e'} \ell_{e'}(s)}$ *is the share of labor hired by establishment* e

Comments:

- In larger labor markets, firms are relatively small
- Thus, they can expand without issue

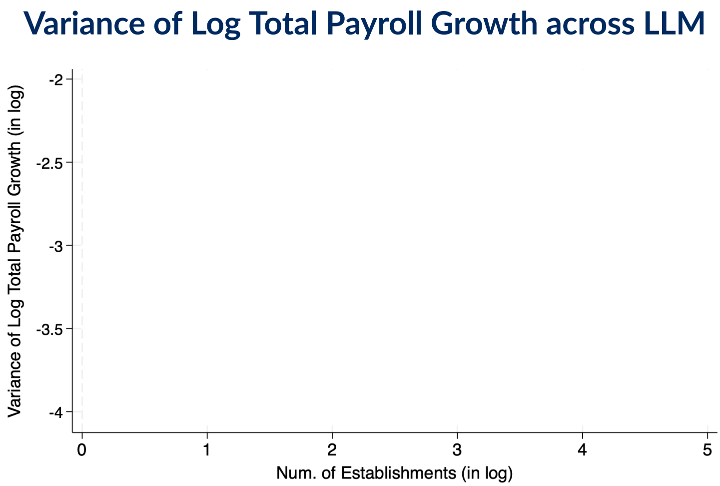
Empirical Evidence

Data

- Japanese Census of Manufactures (CoM)
  - Annual survey of all manufacturing establishments with at least 4 employees
  - For 2011, 2016 (Economic Census)
  - Employment, shipment by product, export share
- Sample Construction: 724,417 unique establishments
  - 1986-2016
  - Manufacturing
  - Must appear for at least 5 years consecutively
- Local Labor Market:
  - JSIC 2 digit manufacturing industry $\times$ commuting zone
  - 25 unique 2-digit manufacturing industries (robust using 3-digit or just MFG)
  - 256 commuting zones

▸ Why Japan? ▸ Workers' Mobility

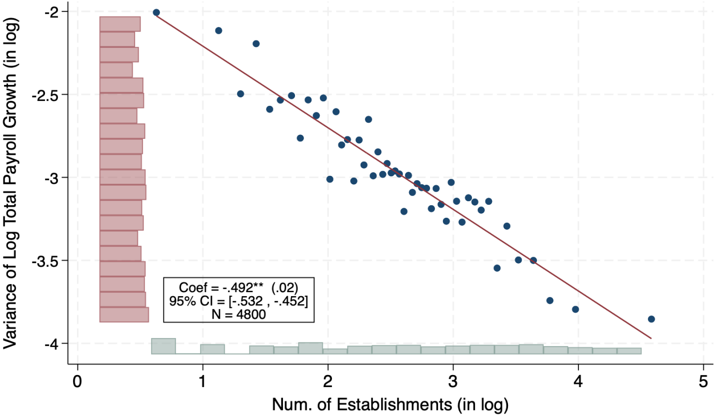
Fact 1: Volatility of Log Total Payment to Labor



▸ Payroll and HHI ▸ Wage and Num of Estabs. ▸ Wage and HHI ▸ German EE Data, Payroll ▸ German EE Data, Wage

Fact 1: Volatility of Log Total Payment to Labor

Variance of Log Total Payroll Growth across LLM



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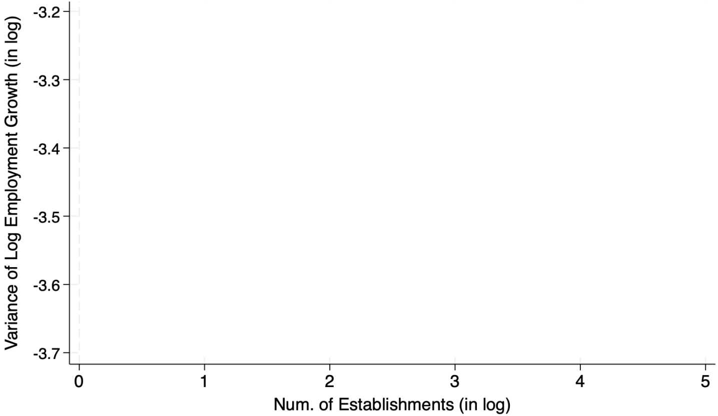
Fact 2: Estabs. Adjust Labor in Larger Market More

Variance of Estab-level Log Emp Growth (residualized), Avg across LLM



▸ Employment and HHI ▸ Non-Reg Emp. and Num of Estabs. ▸ Non-Reg Emp. and HHI ▸ German EE Data ▸ Residualizing Steps

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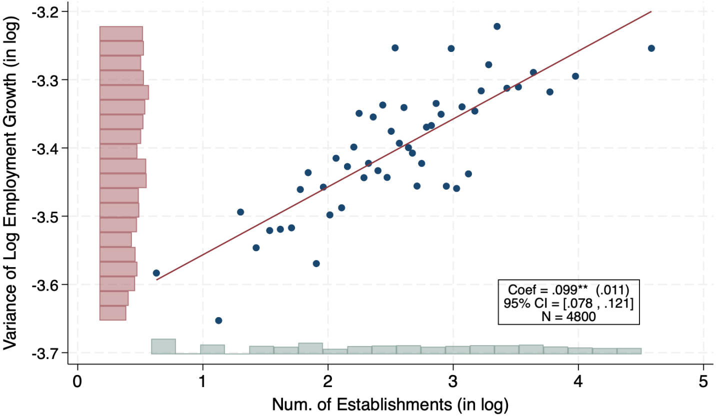

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Causal Analysis: Empirical Specification

- Specification

$$\Delta \ln \ell_{e,t,t+1} = \beta \Delta \mu_{e,t,t+1} + \mathbf{X}'_{e,t}\Gamma + \zeta_e + \zeta_t + \varepsilon_{e,t},$$

  where $\Delta \ln \ell_{e,t}$ is the change in employment
- Shift-share demand shock in a spirit of Yokoyama-Higa-Kawaguchi (2021)

$$\Delta \mu_{e,t,t+1} = \overline{\mathrm{EXP}_e} \times \left(\sum_c \overline{\omega_{e,c}} \cdot \Delta REX^{JPN}_{c,t,t+1} \right)$$

  - $\overline{\mathrm{EXP}_e}$: median export ratio
  - $\overline{\omega_{e,c}}$ median exposure of establishment e to country c from product mix
  - $\Delta REX^{JPN}_{c,t,t+1}$ is the change in real exchange rate of the currency ▸ Shock Time Series
- $\mathbf{X}_{e,t}$: Establishment age squares, log payroll

JPY Appreciation Decreases Establishments' Employment

Regression without the interaction term for the proof of concept of the shock

Table: Effects of JPY Appreciation on Employment Growth

	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock				
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

JPY Appreciation Decreases Establishments' Employment

Regression without the interaction term for the proof of concept of the shock

***Table*: Effects of JPY Appreciation on Employment Growth**

	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock	-3.46			
	(0.17)			
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
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	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock	-3.46	-0.25		
	(0.17)	(0.09)		
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

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	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock	-3.46	-0.25	-0.29	
	(0.17)	(0.09)	(0.12)	
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
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	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock	-3.46	-0.25	-0.29	-2.62
	(0.17)	(0.09)	(0.12)	(0.23)
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

JPY Appreciation Decreases Establishments' Employment

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	Dep. Var.: Log Changes			
			Employment by Types	
	Sales	Employment	Regular	Non-Regular
AREER Shock	-3.46	-0.25	-0.29	-2.62
	(0.17)	(0.09)	(0.12)	(0.23)
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

Next: Interaction with Employment Share within LLM

Establishments with Larger Share responds Less

Table: Effects of JPY Appreciation on Employment Growth

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62			
	(0.23)			
AREER Shock $\times$ Payroll Share				
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

Establishments with Larger Share responds Less

***Table:* Effects of JPY Appreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62	-2.98		
	(0.23)	(0.27)		
AREER Shock $\times$ Payroll Share		3.35		
		(1.26)		
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

Establishments with Larger Share responds Less

Table: Effects of JPY Appreciation on Employment Growth

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62	-2.98	-0.31	
	(0.23)	(0.27)	(0.44)	
AREER Shock $\times$ Payroll Share		3.35	8.26	
		(1.26)	(1.41)	
AREER Shock $\times$ Log Payroll			-1.08	
			(0.14)	
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

Establishments with Larger Share responds Less

***Table*: Effects of JPY Appreciation on Employment Growth**

	Dep. Var.: Log Changes in Non-Regular Emp.			
	(1)	(2)	(3)	(4)
AREER Shock	-2.62	-2.98	-0.31	-0.55
	(0.23)	(0.27)	(0.44)	(0.44)
AREER Shock $\times$ Payroll Share		3.35	8.26	
		(1.26)	(1.41)	
AREER Shock $\times$ Log Payroll			-1.08	-1.12
			(0.14)	(0.15)
AREER Shock $\times$ (Payroll Share $>$ 3%)				2.43
				(0.50)
Observations	1,164,363	1,164,363	1,164,363	1,164,363
Covariates	✓	✓	✓	✓
Year FEs	✓	✓	✓	✓
Establishment FEs	✓	✓	✓	✓

Quantitative Model of Granularity

Qunatitative Model Overview

Summary of the Model

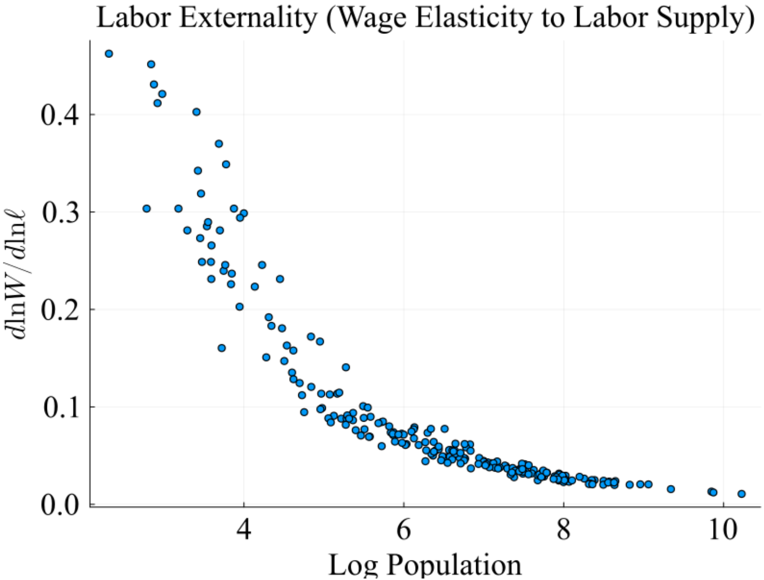
- Small open economy
  - N regions $n \in \mathcal{N}$, Continuum of sectors $j \in \mathcal{J}$
- Endogenous firm entry
- Ex-ante heterogeneous productivity (a lá Gabaix, Pareto)
- Workers can move across labor markets and firms

What We Do

- Quantify the Agglomeration Externality
- Counterfactuals of Population Decline
- Optimal policy (not today)

▸ Model Details ▸ Worker: Location Choice ▸ Worker: Skill Choice ▸ Worker: Ex-Post Labor Choice ▸ Firm ▸ Equilibrium ▸ Calibration

Large Agglomeration Externality in Small Locations



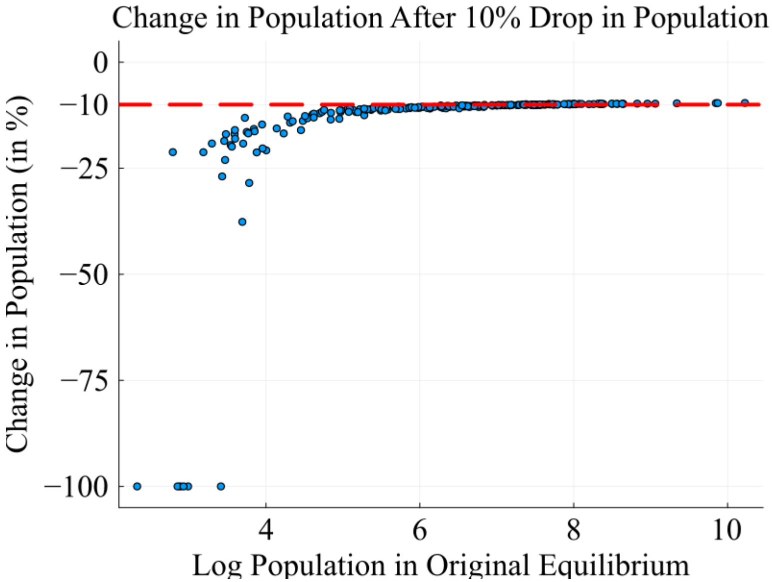
Counterfactual

- The Japanese working-age population is decreasing
  - NRPSSR: 87 million in 1995, 75 million in 2020, 70 million in 2032

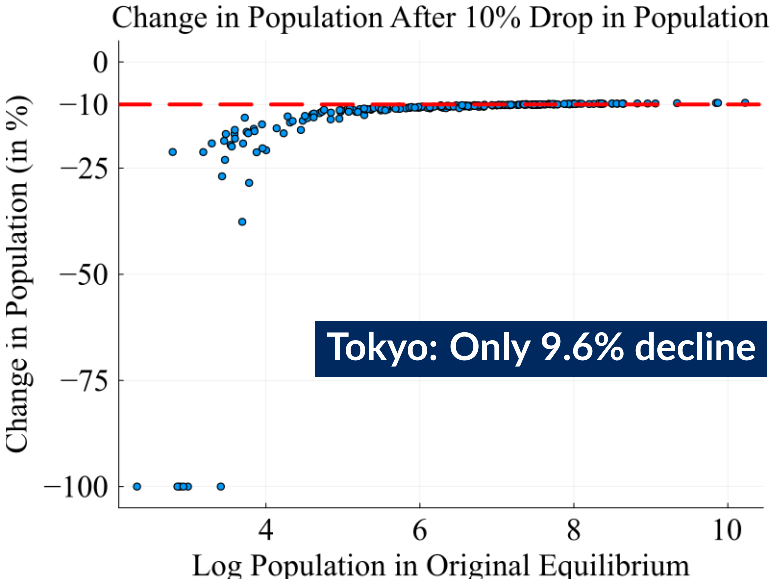
- Simulate **uniform** 10% drop in population
  - Not a crazy scenario in Japan
  - Uniform decline leads to the lower bound of our mechanism
    - Ex-ante smaller locations suffer more lLarger externality in smaller locations)
    - Ex-ante smaller locations will experience a larger drop in population

- See changes in
  1. Population
  2. Wages

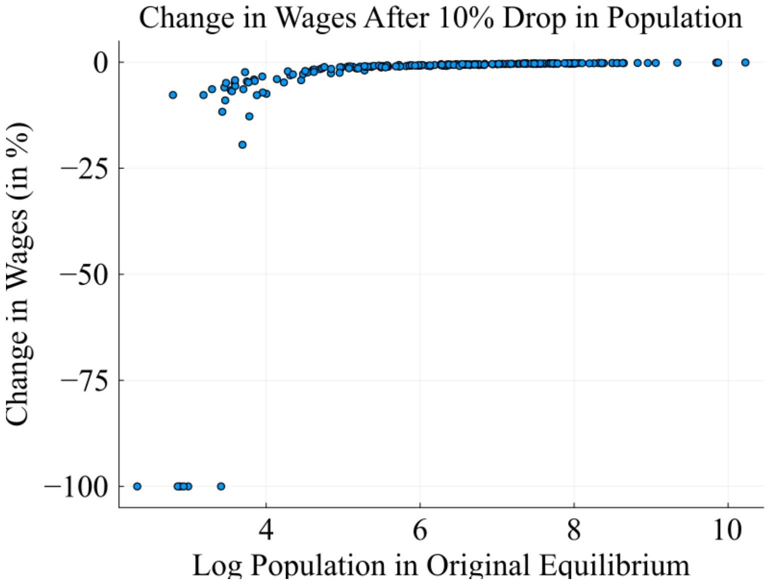
Initially Smaller Locations Become Even Smaller



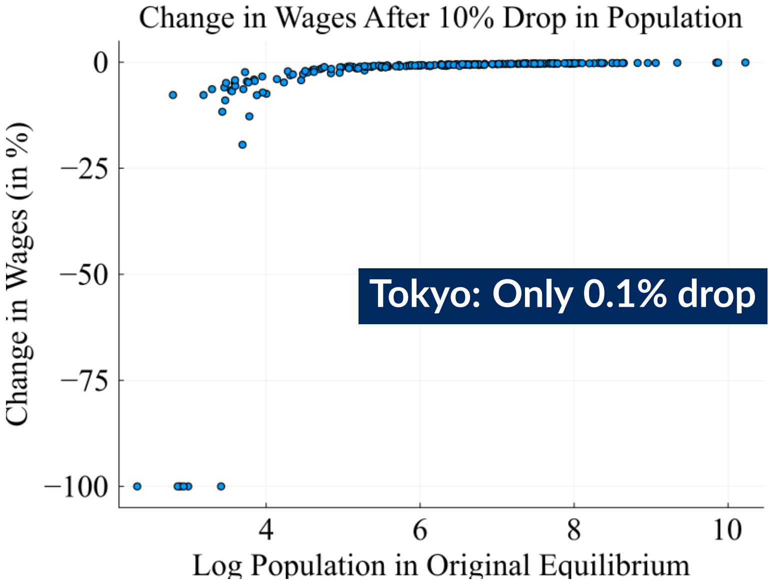
Initially Smaller Locations Become Even Smaller



Initially Smaller Locations Hit Harder



Initially Smaller Locations Hit Harder



Conclusion

- **Granularity** is an important reason for agglomeration
- Standard economic geography models miss this and give incorrect counterfactual predictions because of it
  - Effects of Demographic Changes on Spatial Distribution
- Lots left to do! (We are working on)
  - How does granularity affect skill acquisition?
  - What is the optimal industrial mix? Is sector X *too* large?

Appendix

Model

New Reason for Spatial Policy

Proposition

Adding new firms increases expected production more than the profits those firms would earn.
In math, for $\alpha > 1$,

$$\mathbb{E}[\sum_{e \in \alpha\mathcal{E} \setminus \mathcal{E}} \pi_e(s)] < Y(\ell, \alpha\mathcal{E}) - Y(\ell, \mathcal{E}),$$

where $\pi_e(s) = z_e a_e(s)\ell_e(s)^\eta - w(s)\ell_e(s)$ *are the profits earned when there are* $\alpha\mathcal{E}$ *set of firms operating.*

Comments:

- If the firm entry is somewhat elastic, under-entry
- Violates FWT because it's not Walrasian entry
    - firms internalize the increase in wages when they enter

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Robustness

- Imperfect mobility across establishments and labor markets ▸ Details
- Monopsony power ▸ Details
- Labor hoarding/employer insurance ▸ Details
- Wage rigidity ▸ Details

Imperfect Mobility Across Establishments and Labor Markets

- **Key Assumption:** easier to move across establishments within a labor market than moving across labor markets
- We show that this is the case
- We account for this in our quantitative model

▸ back

Monopsony Power

- Another force for agglomeration
  - Firms would rather open in small labor markets
  - Workers would rather live in large labor markets
  - Workers “usually” win the tug of war since larger labor markets are more efficient
- Makes our mechanism stronger because distortions are especially bad for good shocks
- Variance of wages understates our mechanism

▸ back

Labor Hoarding/Employer Insurance

- If firms have monopsony power, then they should
  1. Hold onto workers during bad years so they can have them when they need them
  2. Provide wage insurance for workers so wages represent "average" contribution
- Both cases strengthen our mechanism
  - In larger labor markets, monopsony power is lower, easier to find workers when you need them, less need for insurance
- Variance of wages understates gains

back

Wage Rigidity

- In large labor markets, variance of marginal product is low
    - Wage rigidity rarely matters
- In small labor markets, will matter a lot!
- Even more inefficient because people become unemployed rather than underemployed
- Wage variance understates the mechanism.

back

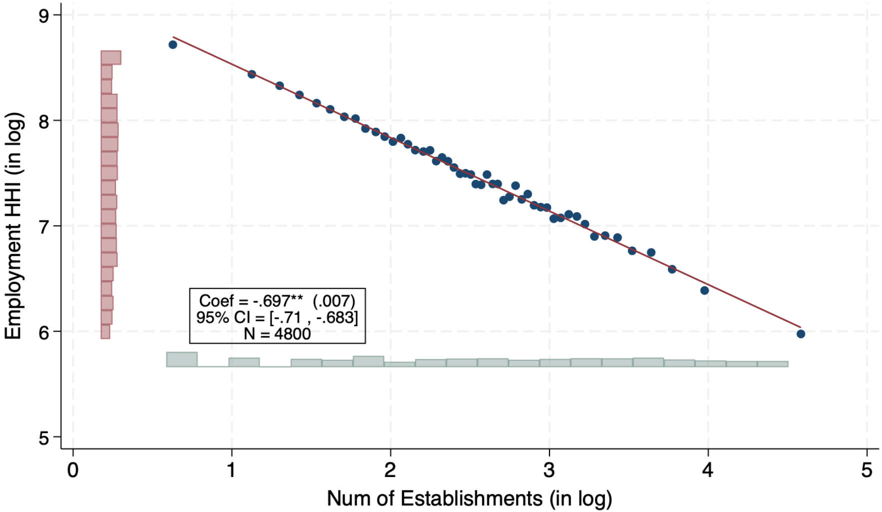
Empirics

Why Japan? Advantages of Japanese CoM

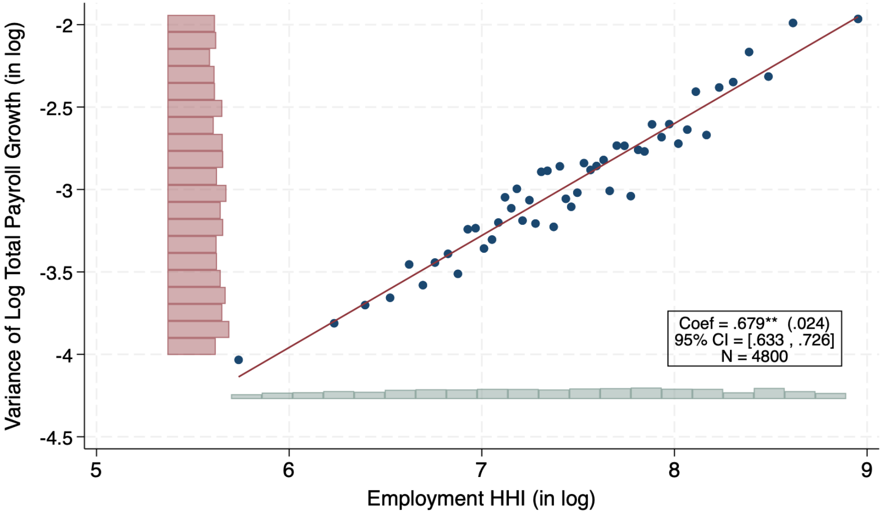
1. Long Panel of Almost Full Samples of Establishments
    - Studying volatility needs panel data
    - Granularity needs almost full samples
    - e.g.1) US CMF: Full-sample every 5 years
    - e.g.2) US LBD: Cannot separate different estab, within states
2. Detailed Product Categories
    - Constructing estab-level shocks needs shipment by estab $\times$ product categories
    - e.g.1) US CMF: Broader categories = 1,000 (US CMF) vs 2,000 (JP CoM)

▸ back

Number of Establishments and HHI ▸ Back

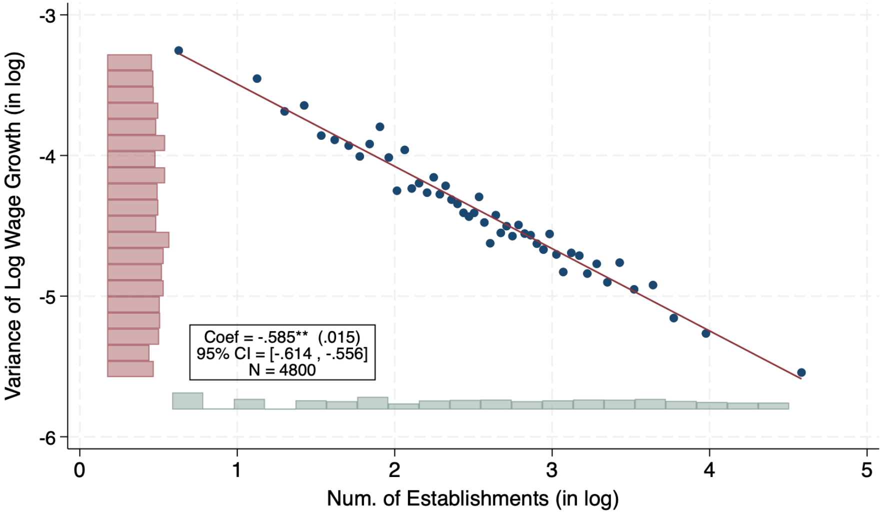


Fact 1: HHI and Volatility of LLM-level Payroll ▸ Back

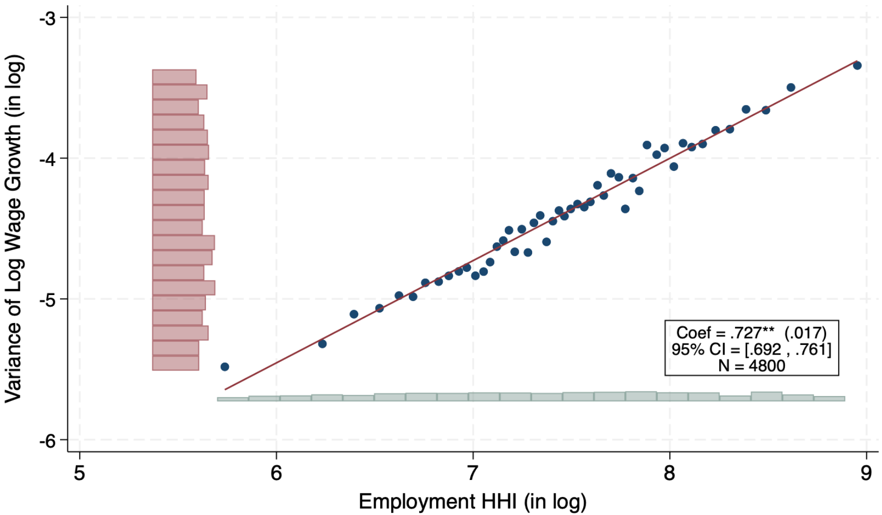


Fact 1: Volatility of LLM-level Wage

Back

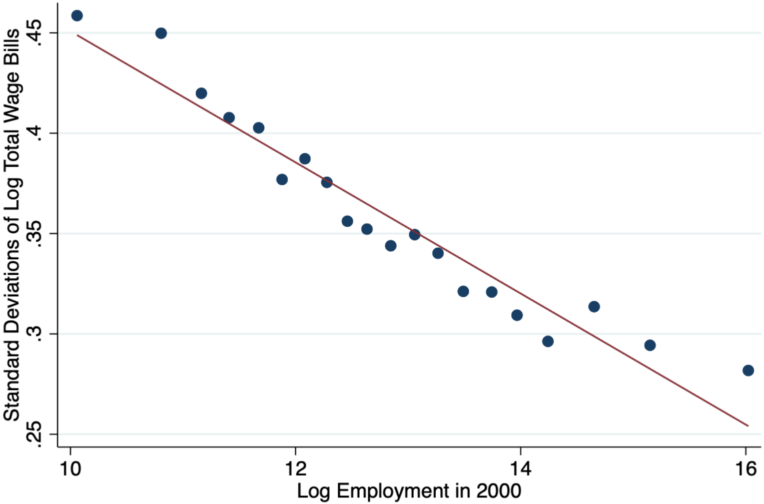


Fact 1: HHI and Volatility of LLM-level Wage ‣ Back



Fact 1: Volatility of LLM-level Payroll: Germany

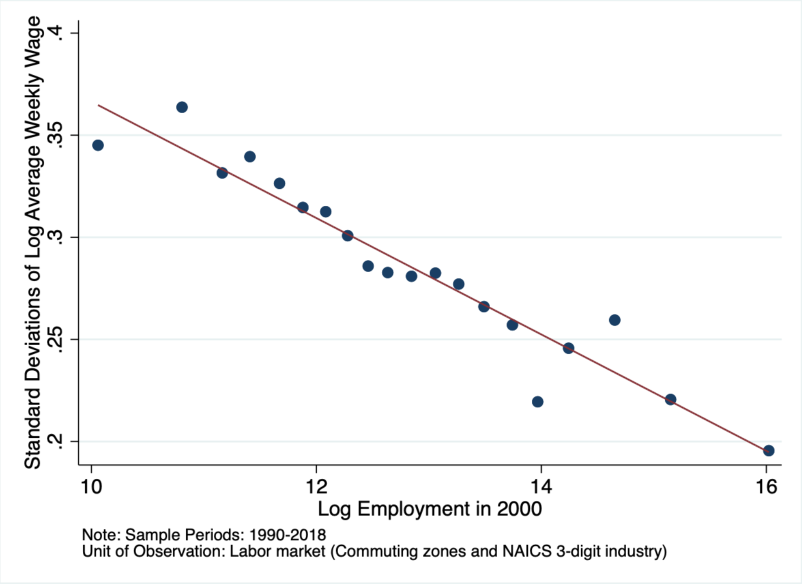
Back



Note: Sample Periods: 1990-2018
Unit of Observation: Labor market (Commuting zones and NAICS 3-digit industry)

Fact 1: Volatility of LLM-level Wage: Germany

▸ Back



Fact 2: Volatility of Establishment-level Employment

- Establishments in larger markets adjust employment more flexibly?
  - Variance of log growth in establishment-level employment
- First residualize estab. yearly employment year FEs

$$\ln \ell_{e,t} = \eta_t + \varepsilon^{\ell}_{e,t}$$

- Second, compute yearly change

$$\Delta\varepsilon^{\ell}_{e,t,t+1} \equiv \hat{\varepsilon}^{\ell}_{e,t+t} - \hat{\varepsilon}^{\ell}_{e,t}$$

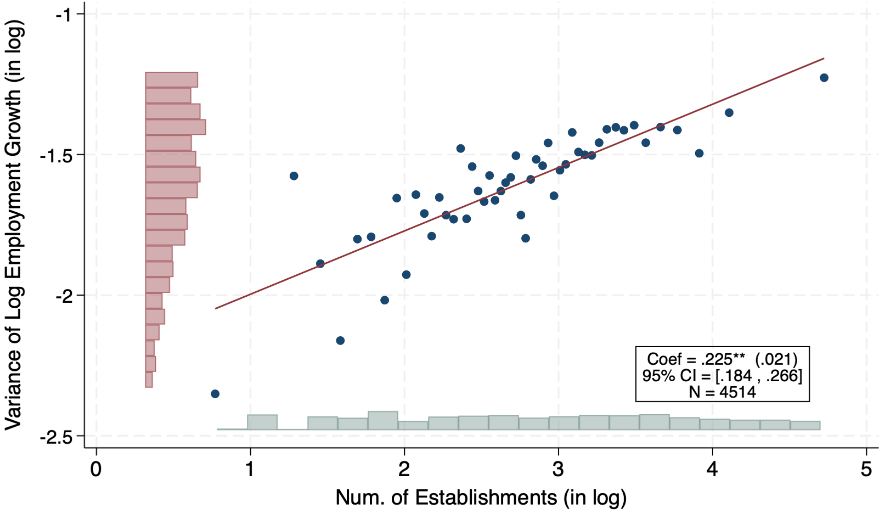
- Then residualize by estab. employment and estab-age FEs

$$\Delta\varepsilon^{\ell}_{e,t,t+1} = \gamma \ln \ell_{e,t} + \eta_{age(e)} + \zeta_{e,t,t+1}$$

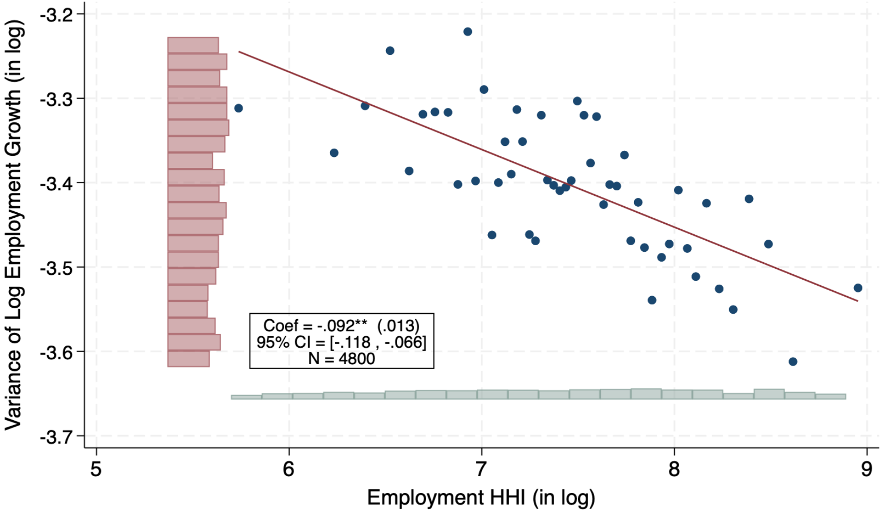
- Finally take variance $Var(\hat{\zeta}_{e,t,t+1})$ across time

Fact 2:Volatility of Establishment-level Non-Regular Employment

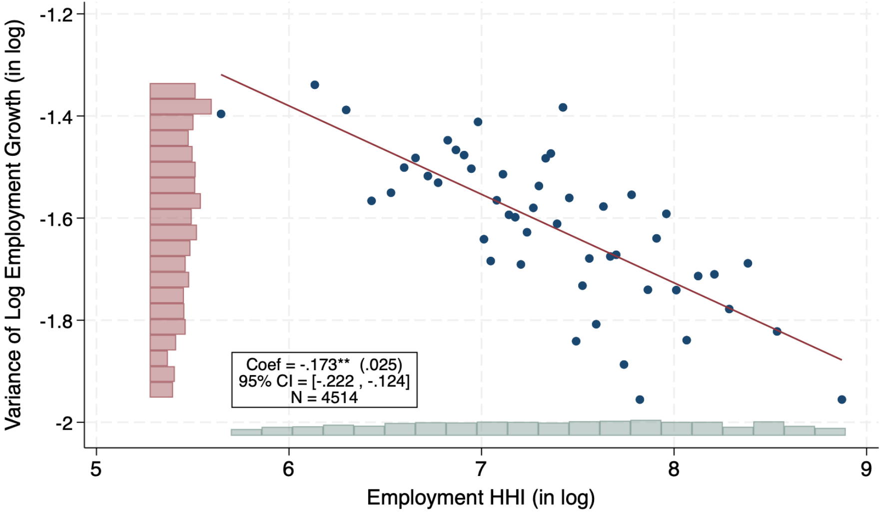
Back



Fact 2: HHI and Volatility of Establishment-level Employment ▸ Back

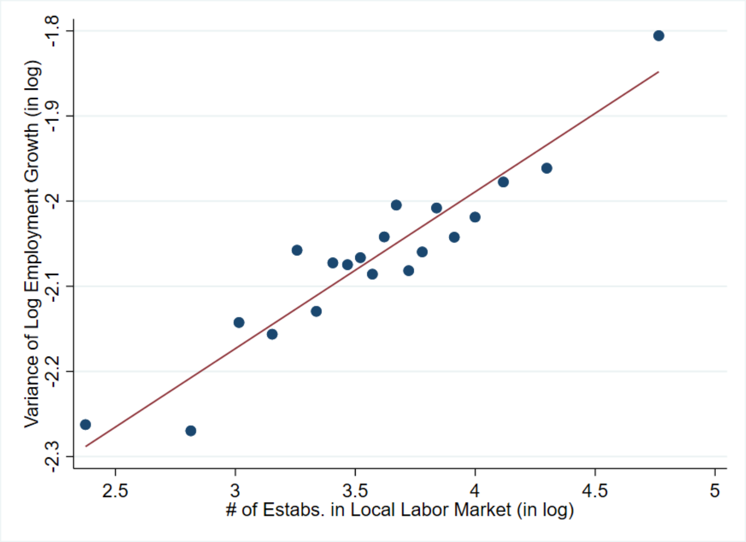


Fact 2: HHI and Volatility of Establishment-level Non-Regular Employment ▸ Back

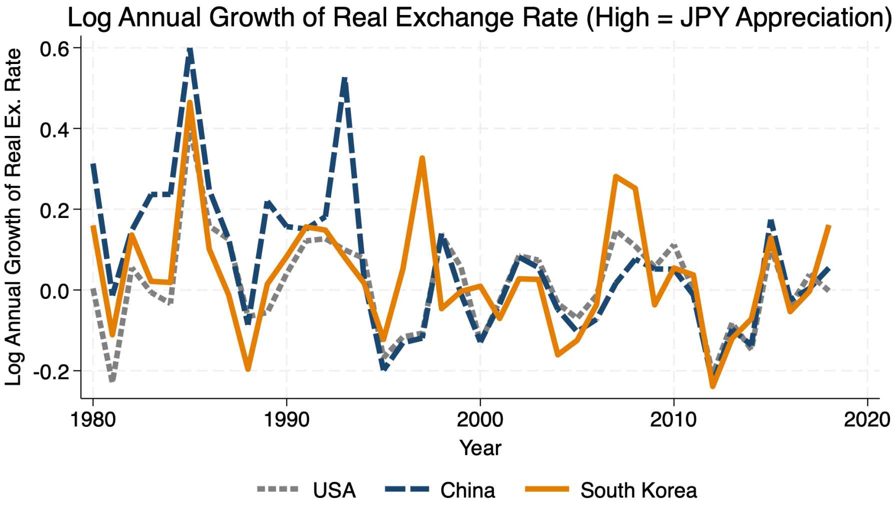


Fact 2: Volatility of Establishment-level Employment: Germany

Back



Real Exchange Rates against Different Currencies ▸ Back



Additional Data

Workers' Mobility in Japan

Back

- Data on worker-level mobilities is rare in Japan
- Japanese Panel Study of Employment Dynamics from Recruit Works Institute
- Some Statistics (aged 25-64, in 2015)
  - Switch jobs within a year: 6.1%
  - Switch jobs across 2-digit sectors: 23% (conditional on switch)
  - Benchmark: Prob. of switch within the same 2-digit sec.: 48.4% (= Sectoral HHI)

Quantitative Model

The Model Overview

- Small open economy
  - N regions $n \in \mathcal{N}$
  - continuum of sectors $j \in \mathcal{J}$
- Timing of the Model:
  1. Continuum of firms can pay a fixed cost to *attempt* an entrance in sector
  2. Random, finite number of firms enter (Poisson)
  3. Firms get an ex-ante productivity draw (Pareto)
  4. Workers decide where to live, and how much to invest in sector-specific skills
  5. Firm ex-post productivity shocks revealed (Log-normal)
  6. Workers move labor across estabs. and sectors subject to migration frictions

▸ Back to Overview

Workers - Location Choice

Back

- Fundamental utility of location n is

$$U_n = u_n W_n$$

- Amenities are also subject to spillovers (congestion, $\gamma_u < 0$)

$$u_n = \overline{u}_n (\ell_n)^{\gamma_u}.$$

- Workers have Fréchet utility shocks over the different locations

$$\ell_n = \left(\frac{U_n}{U} \right)^{\theta} \ell$$

where

$$U = \left[\sum_n (U_n)^{\theta} \right]^{\frac{1}{\theta}}$$

Workers - Ex-ante Skills Choice

Back

- Workers choose skill investments to maximize expected wages

$$\{s_{nj}\}_{j \in \mathcal{J}} \in \underset{s'_j}{\operatorname{argmax}} \; W_n(\{s'_j\})$$

$$s.t. \quad 1 = \int_{\mathcal{J}} (s'_j)^{\frac{1+\bar{\nu}}{\bar{\nu}-\nu}} dj$$

- This takes as given number of firms in each sector and ex-ante productivity shocks z_{nje}
- ν is the short-run elasticity across sectors
- $\bar{\nu} > \nu$ is the long-run elasticity across sectors
- Denote solution by W_n

Workers - Ex-post Labor Choice ▸ Back

- After the shocks are revealed, workers maximize earnings, taking wages and skills as given

$$
\begin{aligned}
L_{nje}(s), L_{nj}(s) \in & \underset{L'_{je}, L'_j}{\operatorname{argmax}} \int_{\mathcal{J}} \left[\sum_{e \in \mathcal{E}_{nj}} w_{nje}(s) L'_{je} \right] dj \\
\text{s.t.} \quad L'_j = & \left[\sum_{e \in \mathcal{E}_{nj}} b_{nje}^{-1/\kappa} \left(L'_{je} \right)^{\frac{1+\kappa}{\kappa}} \right]^{\frac{\kappa}{1+\kappa}} \\
1 = & \left[\int_{\mathcal{J}} s_{nj}^{-1/\nu} \left(L'_j \right)^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}}
\end{aligned}
$$

- Denote solution by $W_n(\{s_{nj}\})$

Firms ‣ Back

Firms maximize profits by taking wages as given

$$\pi_{nje}(s) = \max_{\ell'(s)} \quad z_{nje} a_{nje}(s) \ell'(s)^{\eta_j} - w_{nje}(s)\ell'(s)$$

Free entry is

$$\psi = \frac{1}{m_n}\mathbb{E}\left[\sum_e^{E_{nj}} \pi_{nje}(s) \middle| m_{nj}\right]$$

- Entry is Poisson

$$\mathbb{P}[E_{nj} = k] = \frac{(m_n)^k e^{-m_n}}{k!}$$

- Ex-ante shocks are distributed Pareto

$$z_{nje} \sim \mathcal{P}(z_{nj}, \lambda); \quad z_{nj} = \bar{z}_{nj}(\ell_n)^{\gamma_z}$$

- Ex-post shocks are distributed log-normal

$$a_{nje}(s) \sim \mathcal{LN}\left(-\sigma^2/2, \sigma^2\right)$$

Equilibrium

▸ Back

- Firms:
  - earn zero expected profits, conditional on trying to enter;
  - maximize profits taking as given wages, conditional on entering.
- Workers:
  - choose the utility-maximizing location;
  - choose sector-specific skills to maximize expected utility;
  - choose where to work to maximize utility.

Calibration

Description	Parameter	Value	Source
Short run labor elasticity across sectors	ν	0.42	Berger et al. (2022)
Short run labor elasticity across firms	κ	10.85	Berger et al. (2022)
Long run labor elasticity across sectors	$\overline{\nu}$	1	Burstein et al. (2020)
Elasticity of production to labor	η	0.5	Labor Share (CoM)
Ex-ante firm prod. tail	λ	2.8	Direct from Regression
Ex-post shock log variance	σ^2	0.25	Variance of log wages
Migration elasticity	θ	3	Redding (2016)
Congestion externality	γ_u	-0.25	Redding (2016)
Production externality	γ_z	0.0025	Combes et al. (2011)

Back

Size of Externality

Agglomeration Externality: Elasticities of wages to the population

$$\frac{d\log W_n}{d\log \ell_n} := \frac{\gamma_z + \frac{\Psi'(m_n)m_n}{\Psi(m_n)} - (1-\eta)}{1 - \frac{\Psi'(m_n)m_n}{\Psi(m_n)}},$$

▸ Math