DCIT105
Mathematics for IT Professionals

Session 1 – Number Systems I: Radix

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Session Outline

The topics to be treated in this lecture are:

- Radix
- Signals
- Forms of Number System
    - Decimal
    - Binary
    - Octal
    - Hexadecimal
    - BCD
- Number Conversions
- Application of Number Systems

Topic One

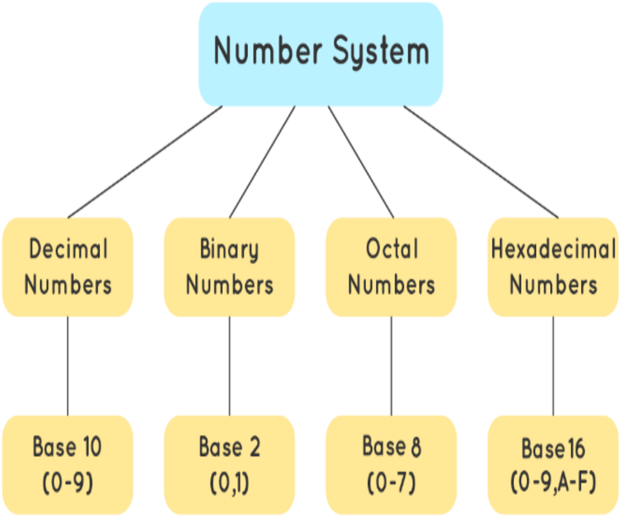
NUMBER SYSTEMS I: RADIX

Introduction

- Decimal or base (radix) ten is used by humans
- Binary or radix two is used by computers
  - Since it only comprehends two states
  - High voltage (+5V) denoted by 1
  - Low voltage (0V) denoted by 0
- Hence, there is the need to convert a number from decimal to binary and vice versa.

Radix in Number Systems

- Number System:
  - naming, representing or expressing numbers in a particular format using digits or symbols.
  - E.g.: Decimal Number System, Binary Number System, etc.



- Radix or Base:
  - number of digits or symbols in a given number system.
  - E.g.: Radix 10, Radix 2, etc.

Why study Number Systems?

1. Understand how the computer represents data before processing
2. Understand the language of the Digital Computer System
3. Convert numbers from one form to another
4. Perform arithmic computation in any form
5. Analyze and design digital electronics

Application of Number Systems (NS)

- Decimal NS:
  - Counting, measuring and labeling items
  - Performing arithmic calculations
  - Used in communication systems such as phone numbers, zip codes, etc
- Binary NS:
  - Used to represent and process data in computers
  - Performing binary arithmic computations
  - Image manipulation and computer graphics
  - Binary operations extract specific information from images

Application of Number Systems (NS)

- Octal NS:
  - Represent file permissions in UNIX OS (Linux and Mac OS)
  - Used for error detection caused by noise or other transmission issues
  - Used to distinguish different aircraft on a radar based on ”squawk” codes

- Hexadecimal NS:
  - Large numbers can be represented using fewer digits
  - MAC addresses are represented using Hex
  - Colours are represented in Hex for graphic design

Application of Number Systems (NS)

- Binary Coded Decimal NS:
  - For encoding product codes using barcodes
  - Used in digital clocks and timekeeping systems
  - Employed in error detection and corrections techniques
    - E.g.: checksums and parity bits
  - For analog-to-digital conversions
  - Used in accounting systems for accurate manipulation of monetary values

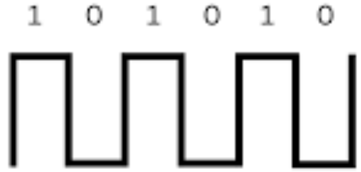
Analog *vs* Digital Signals

- Signal refers to data values converted into electrical pulse readily available for transmission and processing.
  - Signals are used for
    - arithmetic computation
    - communication
    - networking

Analog

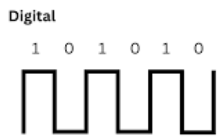
- Forms of numerical value representation
  - Analog signal
  - Digital signal

Digital



Analog *vs* Digital Signals

- Analog signal
  - represents numerical values or figures of a quantity between two expected extreme values.
  - provides an output which is continuous
  - E.g.: Temp from $0^oC – 100^oC$

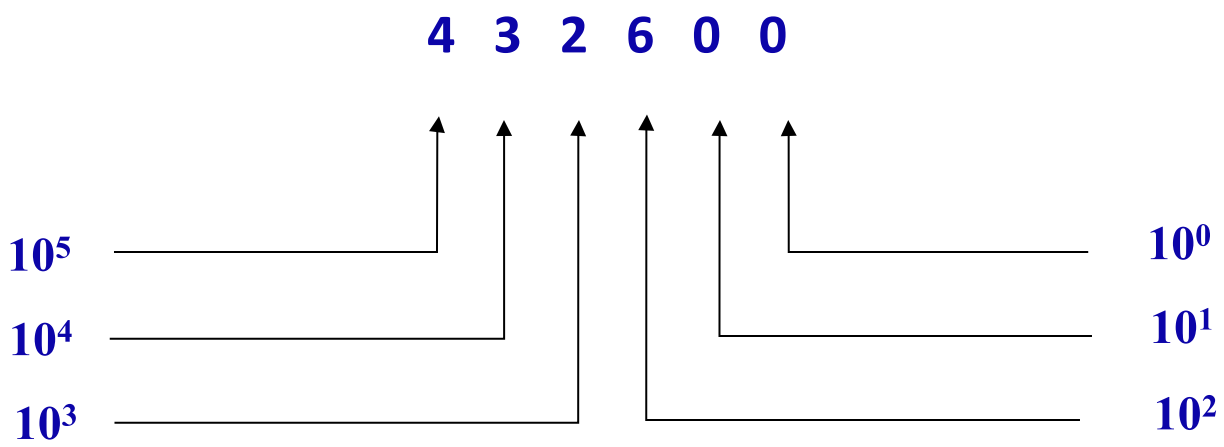


- Digital signal
  - express numerical values or figures of a quantity into steps of values.
  - provides an output which is discrete
  - E.g.: Temp of 9^oC as 00001001

Decimal Number System

- Radix ten or Decimal System
  - {0 1 2 3 4 5 6 7 8 9}

E.g.: 432,600



Powers of ten:

$10^0 = 1$ $10^4 = 10000$

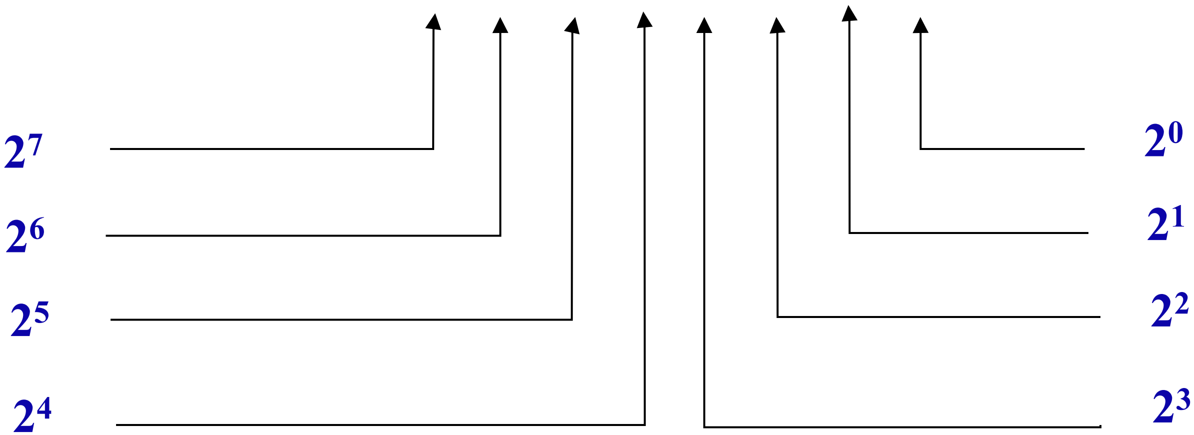
$10^1 = 10$ $10^3 = 1000$ $10^5 = 100000$

Binary Number System (Radix 2) {0, 1}

Binar	Decimal
0	0
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9
1010	10

Example

1 1 0 1 1 0 0 1



Binary to Decimal

Convert 1101 1001 to decimal

$$
\begin{array}{rrcr}
 & 1 \times 2^7 & = & 128 \\
+ & 1 \times 2^6 & = & 64 \\
+ & 1 \times 2^4 & = & 16 \\
+ & 1 \times 2^3 & = & 8 \\
+ & 1 \times 2^0 & = & 1 \\
 & & & 217
\end{array}
$$

Try this!

1. What is 10011010 in decimal?
2. What is 00101001 in decimal?

Solution:

1. 154
2. 41

Decimal to Binary

Steps:

– 1. Determine the greatest power of two that is less than the number
– 2. Deduct the number from that power of two.
– 3. Repeat steps 1 and 2 for the updated outcome until you get to zero.
– 4. Write the binary number

E.g.: Convert 574 to radix 2

- $2^9 = 512$
- $574 - 512 = 62$
- $2^5 = 32$ $\quad$ $62 - 32 = 30$
- $2^4 = 16$ $\quad$ $30 - 16 = 14$
- $2^3 = 8$ $\quad$ $14 - 8 = 6$
- $2^2 = 4$ $\quad$ $6 - 4 = 2$
- $2^1 = 2$ $\quad$ $2 - 2 = 0$
- 1000111110

Decimal to Binary Conversion

- Example: 144
- $2^7 = 128$ $144 - 128 = 16$
- $2^4 = 16$ $16 - 16 = 0$

- Result: 10010000
- 53

  $= 32 + 16 + 4 + 1$

  $= 2^5 + 2^4 + 2^2 + 2^0$

  = 110101 in binary

  = 0011 0101 as a full byte in binary
- Exx. Convert 211 to binary
  - 1101 0011

Fraction(Decimal) to Binary

- **Use Radix to multiply the number**
- **Pick the 0 or 1 integer as a partial result**
- **Pick the fraction part and repeat the process**

Example: $(0.625)_{10}$

		Integer		Fraction	Result
0.625	* 2 =	1	.	25	$x_{-1} = 1$
0.25	* 2 =	0	.	5	$x_{-2} = 0$
0.5	* 2 =	1	.	0	$x_{-3} = 1$

Result: $(0.x_{-1}\ x_{-2}\ x_{-3})_2 = (0.101)_2$

In-Class Exercise

1. For a given binary number system, list the first sixteen digits.
2. Let 0, 1 and X be the independent digits in any arbitrary number system.

(a) What is this number system's radix?

(b) In this number system, list the first ten numbers.

Please try it!

Solution:

1. 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110 and 1111

2(a). The radix of the proposed number system is 3

2(b). The first 10 numbers in this number system would be 0, 1, X, 10, 11, 1X, X0, X1, XX and 100

Octal (Radix 8)

- Has eight different digits because its radix is 8.
- The 8 digits are as follows:

  {0, 1, 2, 3, 4, 5, 6, 7}

- The place values in this number system are
  - $8^0, 8^1, \ldots, 8^7$ (integer)
  - $8^{-1}, 8^{-2}, \ldots, 8^{-7}$ (fraction)

- Exercise:
  - Write the next 10 numbers that follow '7'

Solution: 10, 11, 12, 13, 14, 15, 16, 17, 20 and 21

Decimal to Octal

- Convert 175.3125_{ten} to radix of 8

Converting the integer part: 175_{ten}

	Partial Quotient	Partial Remainder	Result
$175 / 8 =$	21	7	$x_0 = 7$
$21 / 8 =$	2	5	$x_1 = 5$
$2 / 8 =$	0	2	$x_2 = 2$

$175_{ten} = (x_2\, x_1\, x_0)_8 = 257_8$

Converting the fraction part: 0.3125_{ten}

	Partial Integer		Partial Fraction	Result
$0.3125 \times 8 =$	2	.	5	$x_{-1} = 2$
$0.5 \times 8 =$	4	.	0	$x_{-2} = 4$

$0.3125_{ten} = (0.x_{-1}\, x_{-2})_8 = (0.24)_8$

$175.3125_{ten} = 257.24_{eight}$

Binary to Octal

- $8 = 2^3$
- An octal digit is represented by group of 3 bits

Example:

Assume Zeros

$(\ 1\,0\,1\,1\,0\,.\,0\,1\ )_2$

$(\ 2\quad 6\ .\ 2\ )_8$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works both ways (Binary to Octal & Octal to Binary)

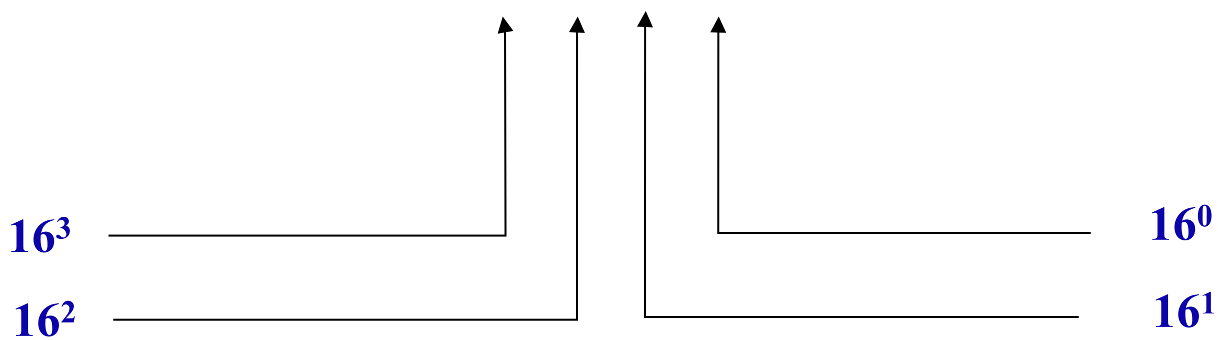
Hexadecimal (Radix 16)

- **A hex digit is represented by group 4 bits**
  {0 1 2 3 4 5 6 7 8 9 A B C D E F}
- **Decimal to Hexadecimal equivalents shown in Table**

Dec	Hex	Dec	Hex
0	0	8	8
1	1	9	9
2	2	10	A
3	3	11	B
4	4	12	C
5	5	13	D
6	6	14	E
7	7	15	F

Example: Hexadecimal to Decimal

3 B 6 E



$3 * 16^3 = 12{,}288$

$11 * 16^2 = 2{,}816$

$6 * 16^1 = 96$

$14 * 16^0 = 14$

$= 15214$

Hexadecimal to Binary

16 = 2^4
Each hexadecimal digit will be represented by 4 bits (nibble)

Binary	Hex	Binary	Hex
0000	0	1001	9
0001	1	1010	A
0010	**2**	1011	B
0011	3	1100	C
0100	**4**	1101	D
0101	5	1110	E
0110	6	1111	F
0111	7		
1000	**8**		

Binary to Hexadecimal

- Put binary numbers in groups of four (nibbles)
  - 1101 1001 0110

- Translate each nibble's representation into hex
- 1101 1001 0110

    D 9 6

Binary to Hexadecimal

- $16 = 2^4$

Example:

Assume Zeros

$$(\,1\;0\;1\;1\;0\;.\;0\;1\,)_2$$

$$(\,1\quad 6\;.\;4\,)_{16}$$

Can be done in both ways

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Octal to Hexadecimal

- **Convert to Binary as a baseline step**

Example:

$$(2\quad 6\ .\ 2)_8$$

Assume Zeros

$$(0\,1\,0\,1\,1\,0\ .\ 0\,1\,0)_2$$

Assume Zeros

$$(1\quad 6\ .\ 4)_{16}$$

Can be done in both ways

Decimal to Hexadecimal

- Eg. 284
  - $16^2 = 256$ $284 - 256 = 28$
  - $16^1 = 16$ $28 - 16 = 12$ (Hex C)
  - Result: 1 1 C
  - **Repeated Division Approach:**

	Quotient	Remainder	Coefficient
284/ 16 =	17	12	$a_0 = C$
17/ 16 =	1	1	$a_1 = 1$
1/ 16 =	0	1	$a_2 = 1$

Answer: $(284)_{10} = (a_2\, a_1\, a_0)_{16} = (11C)_{16}$

  - Exercise: Try 1054
  - Result: 4 1 E

Hexadecimal to Decimal

- Convert 2DB in hex to radix of ten

Place Value	16^2	16^1	16^0
Hexadecimal	2	D	B
	$(16^2 \times 2)$	$(16^1 \times 13)$	$(16^0 \times 11)$
Decimal	512 +	208 +	11 $\boxed{=}$ 731

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Application Example

- ***ASCII*** (American Standard Code for Information Interchange)
  - assigns standard binary codes to letters, numerals, punctuation marks, and other characters used in computers.
  - is a 7-bit character set containing 128 characters
  - the 8th bit is often used for error detection (parity bit)

  - ***E.g.***: Use the ASCII codes to represent ***Digital***

Character	Binary Code	Hexadecimal Code
D	1000100	44
i	1101001	69
g	1100111	67
i	1101001	69
t	1110100	74
a	1100001	61
l	1101100	6C

BCD Number System

- The Binary Coded Decimal (BCD) replaces each decimal digit with a four-bit binary code.

  0 1 2 3 4 5 6 7 8 9

- E.g.: Convert 469_{ten} to BCD
- Answer:

$$(4 \quad 6 \quad 9)_{ten}$$

$$(0100 \quad 0110 \quad 1001)_{BCD}$$

BCD Number System

- Ex.: Find the decimal equivalent of 0110 0100 1011$_{BCD}$

Answer:

$$(\ 0110 \quad 0100 \quad 1011\ )_{BCD}$$

$$(\ 6 \quad 4 \quad \#\ )_{ten}$$

- not possible because 1011 is not a valid BCD

- Assignment:
  - Convert $(9750)_{10}$ to BCD
  - Answer: $(9750)_{10} = (1001011101010000)_{BCD}$

Summary

- Digital electronics use base-two (binary)
  - +5 voltage (1) and 0V voltage (0)
  - High and Low
- Analog electronics operate on continuously varying electrical/physical magnitudes – temperature, pressure, velocity, etc.
- Number conversions
  - Binary, Octal, Hexadecimal, BCD

Reference

Maini, A. K. (2007). "Digital Electronics: Principles, Devices and Applications". *John Wiley & Sons, Ltd*. ISBN: 978-0-470-03214-5 Chapter 1

Thank you

See you next time!