

ECON2103: Financial Economics

Lecture 11

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This week's topics

- CAPM
- Contingent claims and their values
- Capital gains
- The risk-free rate of return
- The expected market rate of return
- The market beta
- Capital market line
- The Sharpe ratio
- Security market line
- Total Risk
- Systematic Risk
- Unsystematic Risk

Story: CAPM

- **Capital Asset Pricing Model (CAPM)** originally depends on the theory which was demonstrated by Markowitz's portfolio selection model (1952).
- In **Markowitz's model**, an investor selects a portfolio at time t that produces a stochastic return at t .
- The model assumes investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return.

Contingent claims and their value

- If management elects to finance the firm's operations with debt, the creditors (lenders) expect the interest and principal—fixed, legal commitments—to be paid back as promised. Failure to pay may result in legal actions by the creditors.
- If the firm finances its operations with equity, the owners expect a return in the form of dividends, an appreciation of the value of their equity interest (i.e., capital gain), or, as is most likely, some combination of both.

Basic idea

- If we know risk, then we can find the appropriate cost of capital.
- If we know cost of capital, then we can find the NPV of a project.
- If we know NPV, then we can find the value of a project.
- If firm choose the best projects according to their returns and risks, there will be benefits from the improvements in financial decision making.

Overview

- CAPM allows us to determine the required rate of return for any risky asset
- Three inputs are needed for CAPM:
 - The risk-free rate of return
 - The expected market rate of return
 - The market beta
- Assets with lower risk have lower expected returns. Assets with higher risk have higher expected returns.

More mathematically

- All investors choose a mixture of the risk-free asset and the market portfolio (m).
- The chosen portfolios are on the straight line and to the right of m are borrowing at the riskless rate, investing both their initial capital and borrowed funds in the tangent portfolio.
- The tangent portfolio of risky assets is widely known as the **market portfolio**.
- Suppose again that there are N risky assets in the market. The market portfolio weight for the i -th asset is defined as

$$w_{mi} = \frac{\text{Asset } i\text{'s Market Value}}{\text{Total Market Value}}$$

The tangent portfolio

- Why does the tangent portfolio turn out to be the simple value-weighted market portfolio?
- Assume that there are only two corporations, Motor and Food, and that both companies have issued stock and no other securities.
- Assume the total numbers of shares issued by Motor and Food are \$100 and \$150, respectively, and that the market price per share of Motor is \$5 and for Food it is \$2.
- The market value of Motor is then \$500 and Food is \$300, so that the total market value is \$800.

The market clearing conditions

- Suppose further that there are only two investors.
- Let I_a and I_b be their total dollar investments in the two stocks.
- Since they hold the same proportions of risky assets, we can assume w is the percentage of the total portfolio invested in Motor, and $1 - w$ is the percentage invested in Food.
- Then, we must have the market clearing conditions:

$$I_a w + I_b w = \$550; \text{ and}$$

$$I_a (1 - w) + I_b (1 - w) = \$300.$$

Capital market line (CML)

- The line representing all the portfolio combinations that can be selected is called the **capital market line** (CML).
- It describes the equilibrium optimal portfolio choice by investors.
- The slope of this line is called the **market price per unit of risk**:

$$Slope = \frac{E(r_m) - r_f}{\sigma(r_m)} \quad (14.2)$$

where $E(r_m)$ is the expected return for the market portfolio, r_f is the risk-free rate of return, and $\sigma(r_m)$ is the standard deviation of the market return.

CAPM using the CML

- The relationship between risk and return of the portfolio chosen by any investor is:

$$E(r_p) = r_f + \left[\frac{E(r_m) - r_f}{\sigma(r_m)} \right] \sigma(r_p) \quad (14.3)$$

where $E(r_p)$ is the expected portfolio return of the individual investor and $\sigma(r_p)$ is the standard deviation of the individual investor's portfolio return.

- If p is a portfolio consisting of only the risk-free asset, then $E(r_p) = r_f$ with zero risk.
- If p is the market portfolio, then $E(r_p) = E(r_m)$ with risk $\sigma(r_p)$.
- The ratio $(E(r_p) - r_f)/\sigma(r_p)$ is called **the portfolio's Sharpe ratio**.

CAPM Formula

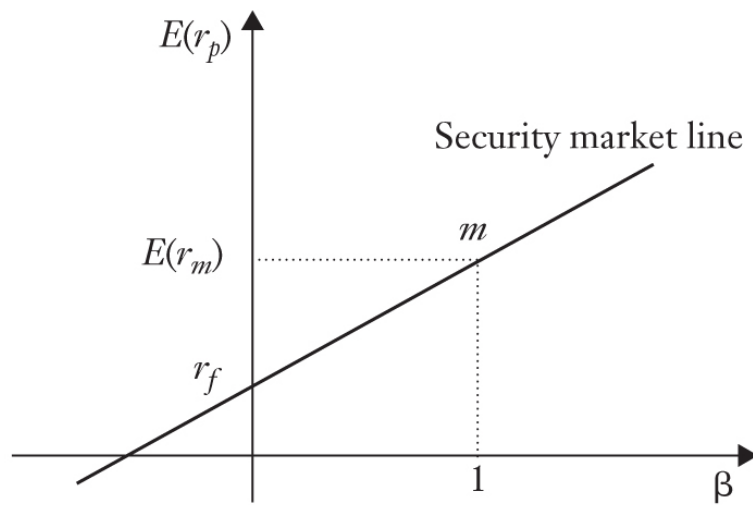
$$E(r_p) = r_f + \beta_p (E(r_m) - r_f)$$

- $E(r_p)$: Expected Return
 - Return required on the investment
- r_f : Risk-free Return
 - Return that can be earned on a risk-free investment
- $E(r_m)$: Average return on all securities
- β_p : The security's beta (systematic) risk factor

Security market line

- First, it is common in finance to refer to β_p as the **security's beta** and to the measure's value as the **security's beta risk**.
- It is a relative measure since it measures risk in relation to the risk of the market portfolio.
- Notice, moreover, that if $\beta_p > 0$, security p moves with the market portfolio, while if $\beta_p < 0$, it moves in the opposite direction.
- Second, the term $E(r_m) - r_f$ is known as the market risk premium.
- It is the return in excess of the risk-free return, and represents the premium required for taking on market risk.

FIGURE 14.3
SECURITY MARKET LINE



Example: Motor and Food

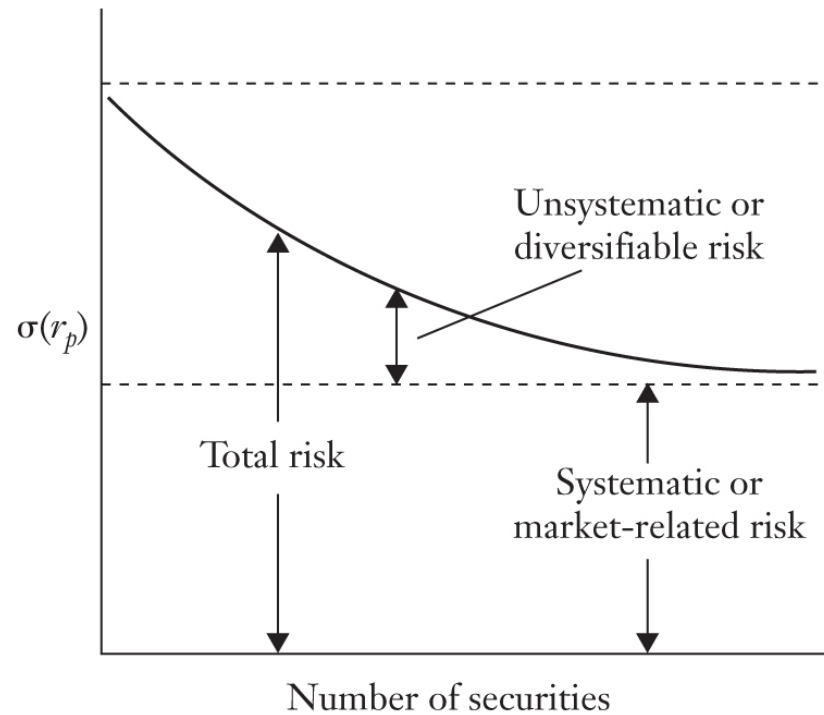
- How much return should an investor expect to get out of an individual security?
- Assume their respective beta risks are 1.5 and 0.14.
- Suppose the risk-free rate is 3% and the expected market return is 12%.
- Then, the market risk premium is 8% ($= 11\% - 3\%$).
- Based on the CAPM, the expected returns will be:
Motor: 15% ($= 3\% + 1.5 \times 8\%$) and Food: 9.4% ($= 3\% + 0.8 \times 8\%$)
- This says that, in comparison with Food, Motor should have a expected return of 15% due to its higher beta risk

Risk

- **Investment risk** is defined as the probability of occurrence of losses relative to the expected return on any particular investment.
- Simply put, the possibility that investors will lose money when they invest in an asset.

Total Risk = Systematic Risk + Unsystematic Risk

FIGURE 14.4
DIMINISHING UNSYSTEMATIC RISK



Key points 1

- The CAPM makes a number of assumptions about investor behavior and about the capital market.
- The key assumptions are that investors have the same information set, and the capital markets are completely competitive and frictionless (i.e., a perfect capital market).
- Under the CAPM assumptions, the optimal portfolios of all investors must be on the security market line.
- Under the CAPM assumptions, the expected security excess return and beta must be on the security market line.

Key points 2

- Under the CAPM, the expected return on any asset is the sum of the risk-free rate return plus the security's risk premium given by the product of the security's beta with the expected market excess return.
- According to the CAPM, the risk of a portfolio can be decomposed into systematic risk and unsystematic risk.
- Systematic risk, also referred to as non-diversifiable risk, is the risk relative to the market and is measured by beta.
- Unsystematic risk is the risk that can be diversified away.
- The CAPM asserts that the only risk for which investors will be compensated is systematic risk, not unsystematic risk.