ECON2103: Financial Economics
Lecture 8

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This week's topics

- St Petersburg paradox
- Probability distribution
- Von Neumann-Morgenstern utility
- Risk averse
- Concave functions
- Stochastic dominance
- Behavioral finance
- The neoclassical paradigm
- Costly information
- Asymmetric information
- Principal-agent relations
- The market for lemons
- Costly transactions
- Scale economies

St Petersburg paradox

- The paradox is illustrated by a lottery stipulating that a fair coin will be tossed until a head appears.
- If the head appears on the first toss, the payoff is $1.
- If it appears on the second toss, then the payoff is $2.
- If the head appears on the third toss, the payoff is $4; on the fourth toss, it is $8.
- If the head appears on the n-th toss, the payoff is 2^{n-1} dollars.

The necessity for utility functions

- The expected payoff is

$$\$1(1/2)+\$2(1/4)+\$4(1/8)+\cdots+\$2^{n-1}/2^{n}+\cdots$$

- Each term equals ½. The payoff is infinite.
- No matter what the price of the lottery is, people should buy the lottery.
- To explain the paradox, Bernoulli suggested that rather than the actual payoff, its utility should be considered:

$$u(1)(1/2)+u(2)(1/4)+u(4)(1/8)+\cdots$$

Probability distribution

- If $u(x) = \log x$, the fair value of the lottery is approximately \$2.
- Technically, a lottery is a probability distribution defined on the set of payoffs.

Von Neumann-Morgenstern expected utility

- Some properties of utility functions reflect preferences likely to be displayed by an entire class of agents.
- For example, all agents who prefer certainty payoffs with higher payoffs are called nonsatiable, and their utility functions are nondecreasing over the range of possible outcomes.
- Thus, if there are two lotteries, one with a certainty payoff of \$100 and another with a certainty payoff of \$200, a nonsatiable agent would never prefer the first opportunity.
- This preference is reflected by writing $u(200) > u(100)$.

Risk averse

- Suppose the agent prefers to receive a certainty outcome that is equal to the expected value of a lottery, rather than the lottery itself.
- In such a case we say the agent is risk averse.
- Assume that the lottery has two possible outcomes, say x_1 with probability p_1 and x_2 with probability $p_2 = 1 - p_1$, $p_1 \in (0, 1)$.
- Then the lottery's expected payoff equals:

$$x_1 p_1 + x_2 (1 - p_1)$$

Concave function

- In terms of the utility function, the risk-aversion property can be expressed as:

$$u[x_1 p_1 + x_2(1-p_1)] \geq u(x_1)p_1 + u(x_2)(1-p_1) \quad \text{for all } x_1, x_2$$
$$\text{and} \qquad (9.1)$$
$$p_1 \in (0,1)$$

- A function that satisfies equation (9.1) is called a concave function, and the utility functions of all risk-averse agents are concave.

Stochastic dominance

- There are some circumstances in which an agent can make decisions without knowing a great deal about the clients' utilities.
- Thus, when a decision maker can make only weak assumptions about individual preferences rather than postulate the exact nature of a utility function, some decisions can still be made correctly.
- For example, some, but not all, probability distributions can be compared for any utility function that, say, merely exhibits risk aversion.

Example

- For example, consider two lotteries *A* and *B* with the following payoffs and associated probabilities:

Lottery A			*Lottery B*	
Payoff	*Probability*		*Payoff*	*Probability*
\$1	0.5		\$0	0.6
\$3	0.5		\$2	0.4

- Lottery *A* would be preferred to lottery *B*, because:

$$E[u(A)] = u(1)\frac{1}{2} + u(3)\frac{1}{2};$$

$$E[u(B)] = u(0)\frac{6}{10} + u(2)\frac{4}{10}.$$

First-degree stochastic dominance

- Then, under the assumption that utility increases in wealth,

$$E[(u(B)] = u(0)\frac{6}{10} + u(2)\frac{4}{10} < u(1)\frac{6}{10} + u(3)\frac{4}{10} < u(1)\frac{1}{2} + u(3)\frac{1}{2} = E[u(A)]$$

- Sometimes the inability to compare distributions whose graphs cross can be resolved by imposing additional assumptions about decision makers' preferences.
- The relationship between A and B is, in this case, expressed by saying A dominates B in the first degree (first-degree stochastic dominance).

Another example

- We suppose all decision makers are both risk averse and nonsatiable, and that we wish to compare lottery *C* and lottery *D* with the following payoffs and associated probabilities:

Lottery C		Lottery D	
Payoff	*Probability*	*Payoff*	*Probability*
\$0	1/3	\$1	1/2
\$2	1/3	\$4	1/2
\$4	1/3		

- To see how it might be shown formally that lottery *D* is actually preferred by any risk-averter, let us ask if $E[u(D)] \geq E[u(C)]$.

Example continued

- If it were, we would have:

$$u(1)\frac{1}{2}+u(4)\frac{1}{2}>u(0)\frac{1}{3}+u(2)\frac{1}{3}+u(4)\frac{1}{3}$$

- Thus:

$$u(1)(\frac{1}{3}+\frac{1}{6})+u(4)(\frac{1}{3}+\frac{1}{6})>u(0)\frac{1}{3}+u(2)(\frac{1}{6}+\frac{1}{6})+u(4)\frac{1}{3}$$

- Then:

$$[u(1)-u(0)](1/3)+[u(4)-u(2)](1/6)>[u(2)-u(1)](1/6)$$

Second degree stochastic dominance

- As *u(1) – u(0) > u(2) – u(1)* by the diminishing marginal utility characteristic of risk-averters, so the original inequality will certainly be satisfied if we can show that:

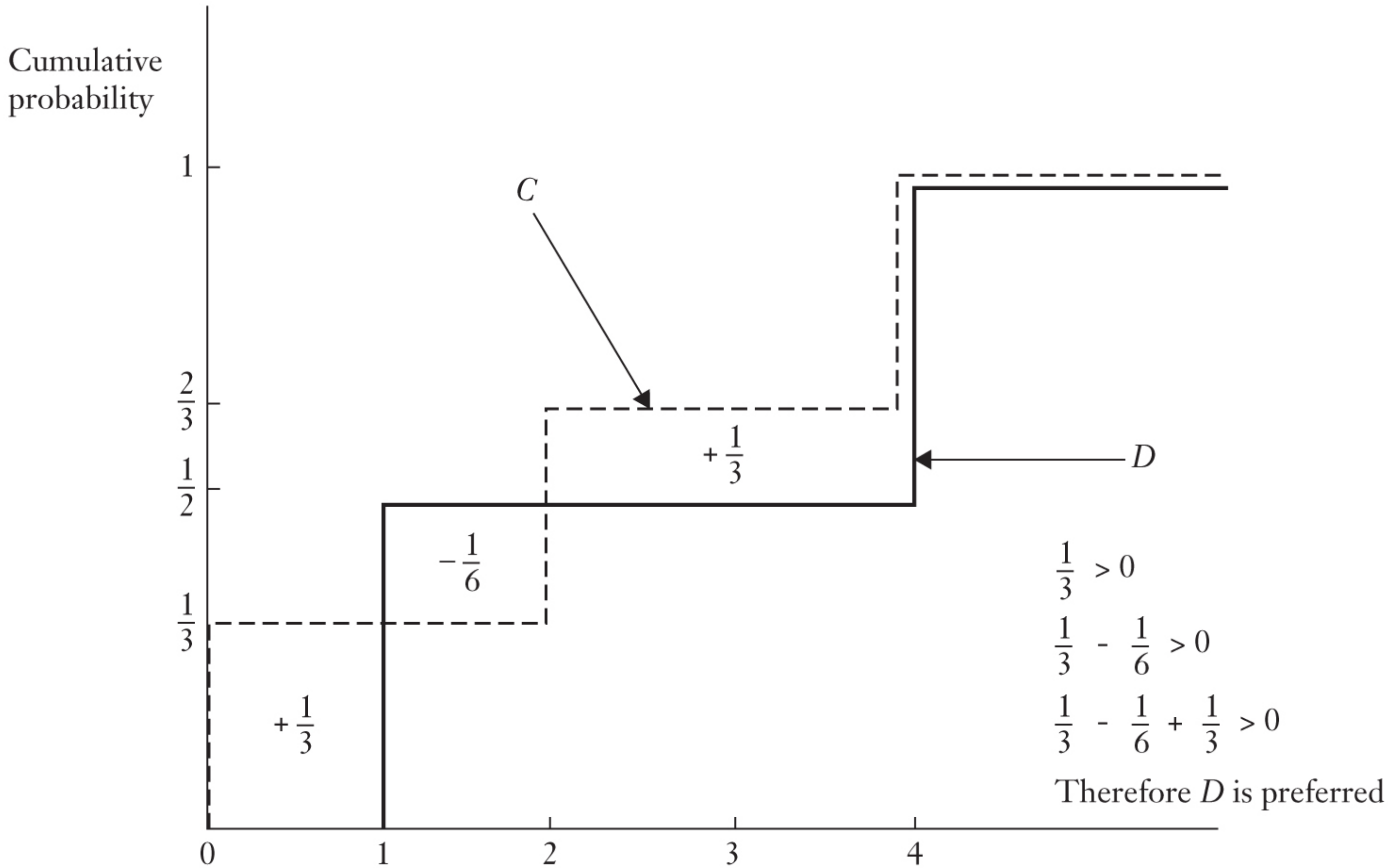
$$[u(1)-u(0)](1/6)+[u(2)-u(1)](1/6)+[u(4)-u(2)](1/6)>[u(2)-u(1)](1/6)$$

- But this is true because

$$[u(1)-u(0)](1/6)+[u(4)-u(2)](1/6)>0$$

- So we obtain *E[u(C)] > E[u(D)].*
- A test for this relationship is known as second-degree stochastic dominance.

FIGURE 9.1
SECOND-DEGREE STOCHASTIC DOMINANCE (START EVALUATION AT LEFT-HAND SIDE OF THE DIAGRAM BECAUSE RISK-AVERSE INVESTORS ATTACH A GREATER NEGATIVE WEIGHT TO DOWNSIDE RISK THAN THEY DO TO UPSIDE POTENTIAL)



Another example

- Not everything can be ranked by second-degree stochastic dominance, for there are pairs of distributions such that the area of the difference between the distributions does change sign as these areas are cumulated from the left.
- For example, consider the following two lotteries, *F* and *G*:

Lottery F		*Lottery G*	
Payoff	*Probability*	*Payoff*	*Probability*
\$0	1/6	\$1	2/3
\$2	1/2	\$4	1/3
\$4	1/3		

Example continued

- These two lotteries have the same payoffs as lotteries *C* as *D*, but with different probability distributions.
- Note that $E(F) = \$2+1/3 > E(G) = \$2$, so that on the basis of the expected value (but not on the basis of lowest outcomes) lottery *F* might prove to be a dominant lottery.
- But we cannot choose between lotteries *F* and *G* without knowing more about the decision maker's preferences.
- A risk-neutral decision maker would prefer lottery *F*, but lottery *G* has a better lowest outcome than lottery *F*, so that a highly risk-averse decision maker would prefer lottery *G*.
- Hence, neither lottery *F* nor lottery *G* satisfies the two conditions necessary for one lottery to dominate the other.

Behavioral finance

- Behavioral finance examines how psychology affects the agents' decisions.
- In the von Neumann-Morgenstern theory, agent expectations are described using objective probabilities (belief).
- Psychology has identified several features of how agents choose probability distributions in practice.
  - Overconfidence
  - Optimism and wishful thinking
  - Representativeness
  - Conservatism
  - Perseverance of belief
  - Anchoring
  - Availability bias

Overconfidence

- Peoples' overconfidence in their own judgment has two principal dimensions.
- First, the confidence limits they assign to judgments are substantially narrower than the confidence limits revealed by statistical studies.
- Second, in practice agents overestimate the probabilities of relatively likely events, and underestimate the probabilities with which relatively unlikely events occur..

Optimism

- Optimism and wishful thinking are also displayed, again in two ways.
- Most people display unrealistically optimistic estimates of personal capabilities such as driving skills, as well as unrealistically optimistic estimates of personal qualities such as a sense of humor.
- They often predict that tasks can be completed much more quickly than usually proves to be the case.

Representativeness

- Representativeness means that people use heuristic methods to conclude that the more closely phenomenon A represents phenomenon B, the more likely people are to believe that the probability of A’s occurring approximates the probability of B’s occurring.
- People put too much weight on apparent similarities and too little weight on the underlying probability of the event.
- People also tend to neglect sample size effects and put too much faith in small samples.

Conservativeness

- Conservativeness is a phenomenon opposite to representativeness.
- Because of past experience, people put too much weight on data that are not representative of models with which they are familiar.

Belief perseverance, anchoring and availability biases

- Belief perseverance is manifest by peoples' holding on to opinions for too long in the face of disconfirming evidence.
- The term anchoring means that people first estimate the probability of an event by using an initial and possibly arbitrary value.
- They then adjust their estimates away from that value in the face of evidence.
- Different anchors will generate widely different concluding estimates in the face of the same evidence.
- Availability biases mean that more recent and more salient events will weigh most heavily in, and therefore distort, estimates.

The neoclassical paradigm

- The neoclassical paradigm refers to formal economic analysis whose results depend on assumptions of individual rationality, homogeneously (symmetrically) distributed information, the absence of transactions costs, and (usually) the assumption of competitive markets.
- Costly information assumes that financial information can be both costly to acquire and costly to use.

Asymmetric information

- Information asymmetry arises when, possibly because of information costs as already recognized, one party to a transaction has payoff-relevant information that the counterparty does not have.
- Information asymmetries lead to problems of both adverse selection and moral hazard.
- In transactions subject to adverse selection, the uninformed party does not have the same information as the informed parties with which he deals, and must perforce deal with the informed parties on the basis of their average characteristics.

Moral hazard

- In transactions involving moral hazard, the uninformed party lacks information about whether an agreed-upon transaction is actually carried out.
- Spence (1973) originally proposed the idea of signalling, arguing that information asymmetries might sometimes be resolved by having informed parties signal uninformed parties, thus credibly transferring information to the counterparty.
- Stiglitz (2002) pioneered the theory of screening, a process by which an under-informed party can induce a counterparty to reveal additional information.

The market for lemons

- A classic paper by Akerlof (1970), aptly titled "The Market for Lemons," considers solutions to asymmetric information problems that involve signaling and screening.
- Some forms of screening involve purchasing conditional probability distributions, while others involve defining a menu of choices so that the informed party's private information is credibly transmitted to the uninformed counterparty.

Principal-agent relations

- Agency theory studies the relationship between a principal and an agent who acts on the principal's behalf.
- Agency theory arises from costly information transmission in that informed agents are viewed as making decisions on behalf of others.
- Thus, agency theory involves both aligning the interests of principal and agent and determining the costs of resolving conflicts between them.
- As one example, the portfolio manager of a mutual fund (a pooled investment vehicle) trades on behalf of the many shareholders in the fund.

Transaction costs

- Transactions costs can explain many different types of organization.
- Demsetz (1968) pointed out that in a world of zero transactions costs, many of the economic arrangements we observe in fact would not be necessary.
- Suppose, for example, that a stock is trading at $48.25, while its fundamental value is $49.
- If the cost per share of transacting exceeds $0.75, it would not pay a decision maker to purchase the undervalued stock even though she knows it to be undervalued.

Scale economies

- Once transaction costs are recognized, the issues of scale economies must also be considered.
- The unit costs paid by an individual to transact may well be larger than the unit costs paid by a financial institution capable of realizing scale economies to transacting.
- For example, the cost per share of transacting might be $0.75 for an individual, but very much less than that for a financial institution.
- As a result, the financial institution could, through large transactions, eliminate price effects that individuals would not find it economic to exploit.

Key points 1

- Expected utility theory is used to prescribe how agents might select among lotteries with different risks and payoffs.
- Different forms of utility functions can be chosen to reflect different agents' underlying preferences.
- Stochastic dominance orderings can rank some risks using only minimal assumptions about agent preferences.
- Behavioral (psychological) approaches to finance attempt to describe how agents actually choose among risky alternatives.
- Behavioral approaches have identified a number of ways in which agents interpret risks and choose among them.

Key points 2

- The neoclassical paradigm that underlies prescriptive approaches to financial economics makes a number of specific assumptions regarding how agents make financial decisions.
- The neoclassical paradigm also assumes information is equally available to all decision makers, but in fact decision makers are not usually equally well informed.
- The neoclassical paradigm further assumes information is freely available, but in practice gathering information is costly.