ECON2103: Financial Economics Lecture 2

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This week's topics

- Opportunity set
- Entrepreneurial firms
- Proprietary firms
- Investment decision made by managers
- Fisher separation theorem
- The market value criterion
- Maximizing market values
- Maximizing profits

- SConsequences for investment and financing decisions
- eparation of operating and financing decisions
- Separation of managers' and owners' decisions
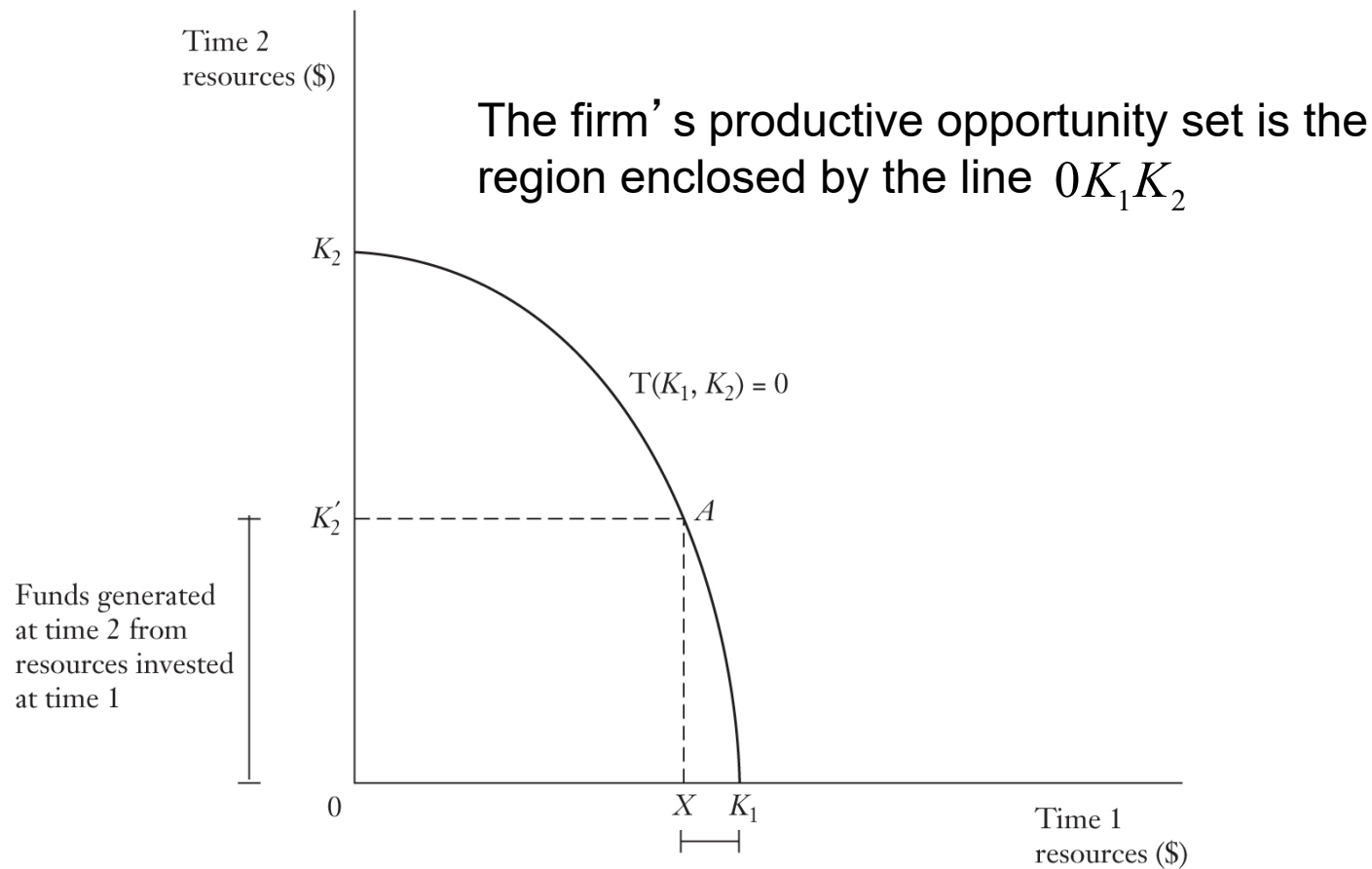
The entrepreneur's opportunity

- The problem we now face is how to represent the entrepreneur's opportunity set, given that some resources are owned and can be used in productive activity.

- We study a transformation curve having the general form

$$T(K_1, K_2) = 0 \,,$$

where K_1 represents the dollar value of funds invested in the firm at time 1 and K_2 the dollar value of funds generated at time 2.

- The slope of a transformation curve may be interpreted as indicating the firm's marginal rate of return.

FIGURE 3.1
PRODUCTIVE OPPORTUNITIES

Time 2
resources ($)

The firm's productive opportunity set is the region enclosed by the line $0K_1K_2$

K_2

$T(K_1, K_2) = 0$

K_2' — — — — — — — — A

Funds generated at time 2 from resources invested at time 1

0   X K_1

Time 1
resources ($)
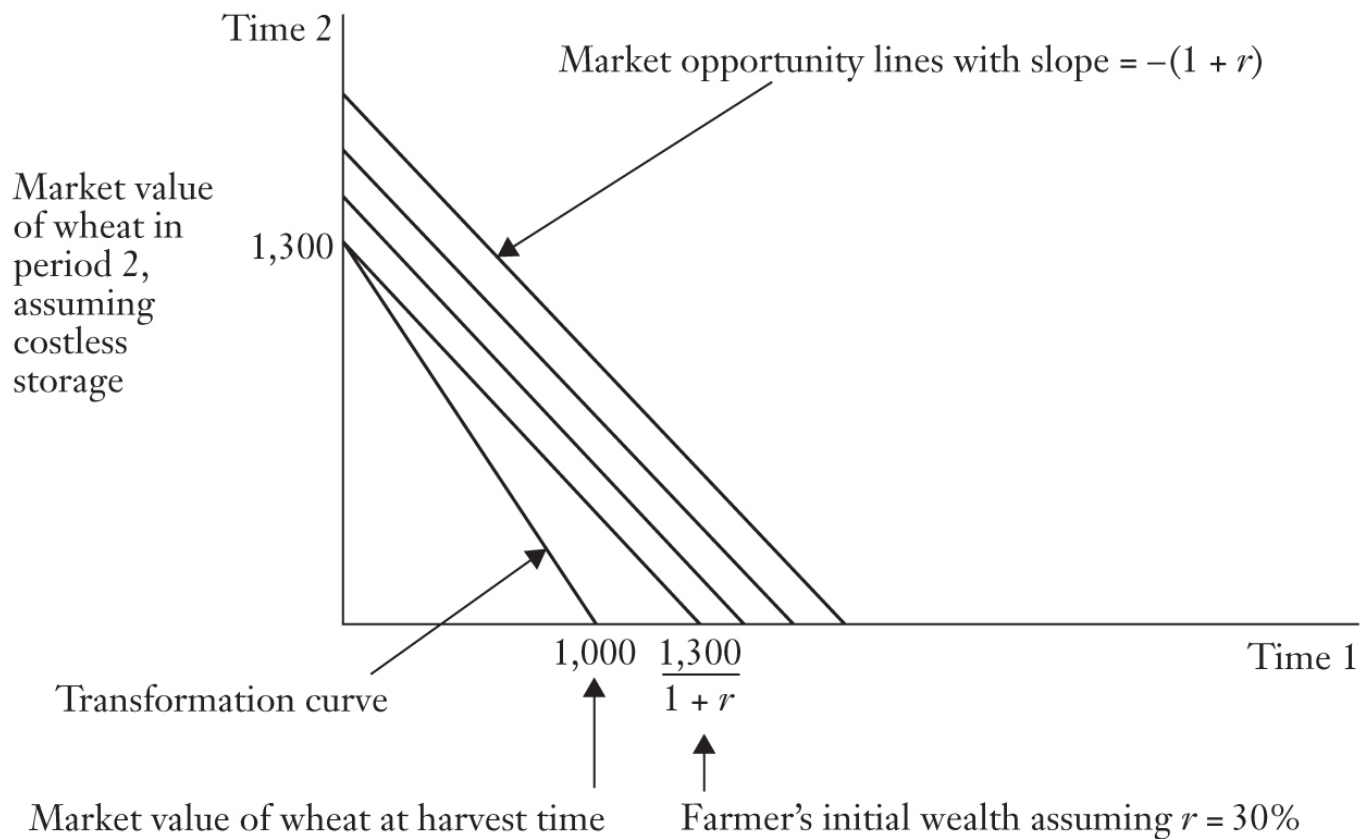
$0K_1$ = market value of available resources
XK_1 = market value of resources used in the firm to produce cash at time 2
$0K$ = market value of resources sold for cash at time 1

Example 1

- Suppose at time 1 a farmer can sell 1,000 units of a commodity (e.g., wheat), which is now in storage.

- The current price of wheat is $1.00 per unit. Alternatively, the farmer can wait until time 2 and sell the 1,000 units at $1.30 per unit.

- Assuming no spoilage and no storage charges, the situation implies a linear transformation curve as shown in Figure 3.2.

FIGURE 3.2
COMMODITY STORAGE EXAMPLE

Figure 3.2 Commodity Storage Example. Market opportunity lines with slope = $-(1 + r)$. Market value of wheat in period 2, assuming costless storage. Time 2 axis value 1,300. Time 1 axis. Transformation curve. Market value of wheat at harvest time: 1,000. Farmer's initial wealth assuming $r = 30\%$: $\frac{1,300}{1 + r}$.

Example 2

- Productive opportunities are represented by a transformation curve of the nonlinear type first introduced.

- In Figure 3.3, if an individual immediately liquidates the initial endowment of resources $0K_1$, an initial cash position with a value of K_1 is obtained.

- This wealth can be converted into time 1 and time 2 cash flows as indicated by the straight line of slope –*(1+r)* passing through K_1 .

- If the individual liquidates only a portion of initial resources and uses the rest $K'_1 K_1$ for investment, we can represent the new set of attainable combinations by a straight line of slope –*(1+r)* passing through the points w_0 and *A*.

FIGURE 3.3
PRODUCTIVE AND CAPITAL MARKET OPPORTUNITIES

Time 2
resources ($)

K_2

K_2^* — — — — — — — B

K_2' — — — — — — — — — — Market opportunity lines with slope $= -(1 + r)$

A

0 K^* K_1' K_1 w_0 w_1 Time 1
resources ($)

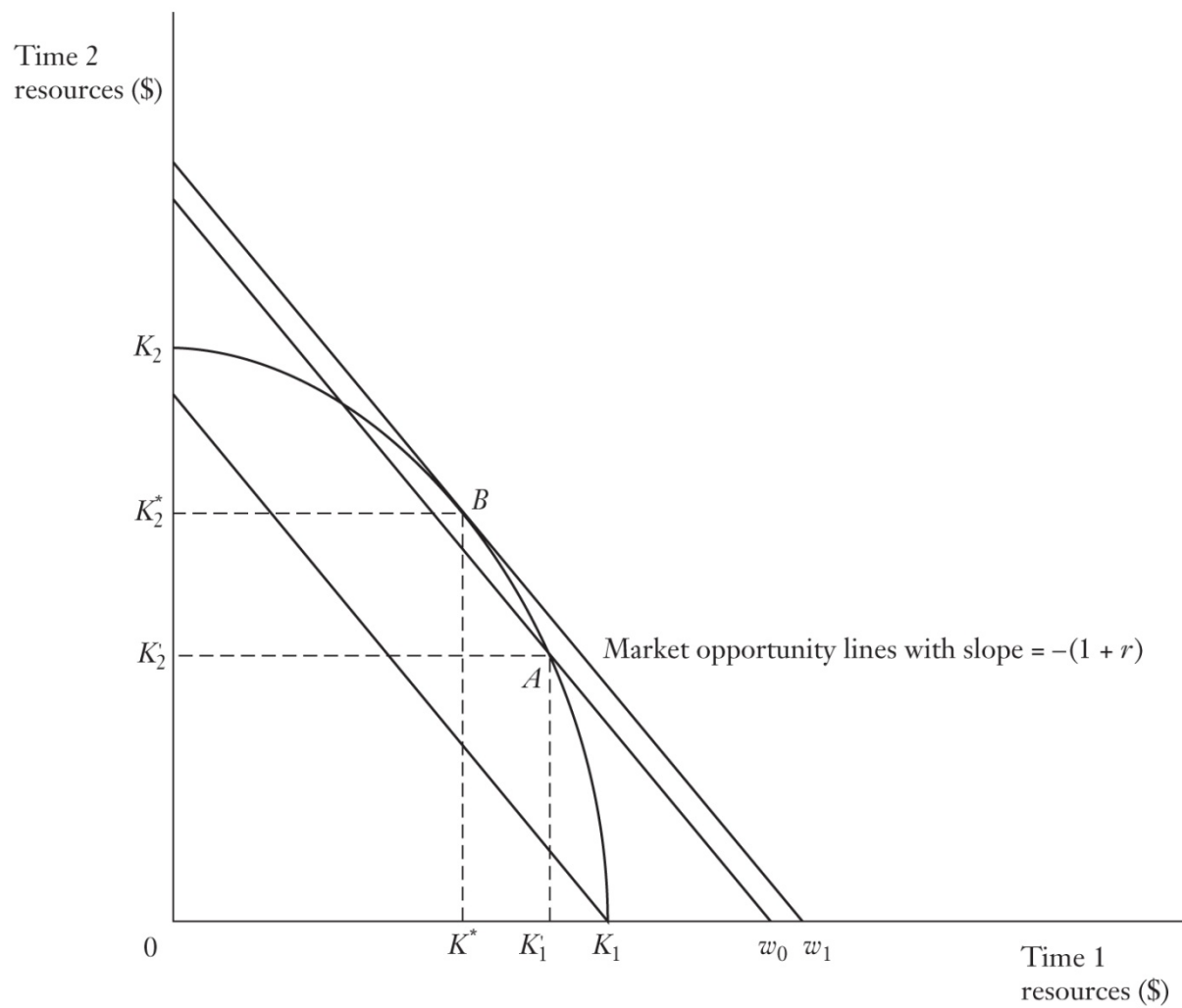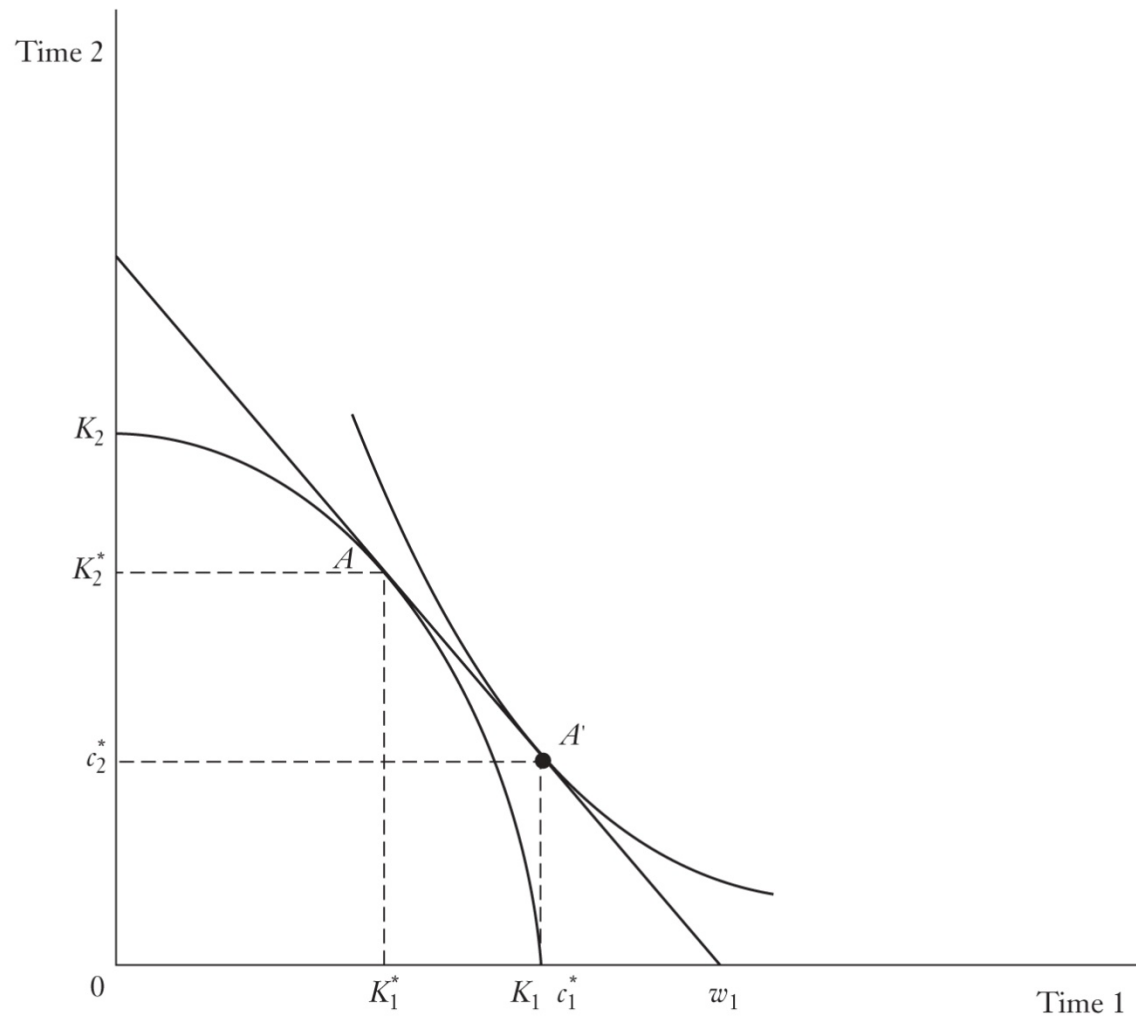FIGURE 3.4
JOINT CHOICE OF PRODUCTION AND CONSUMPTION DECISIONS

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JOINT CHOICE OF PRODUCTION AND CONSUMPTION DECISIONS

Two steps in the decision process

The decision process can be viewed as encompassing two distinct steps:

- Step 1: Choose the optimum production by equating the marginal rate of return on investment with the market required rate of return.

- Step 2: Choose the optimum consumption pattern by borrowing or lending in the capital market until the consumer's subjective rate of time preference equals the market rate of return.
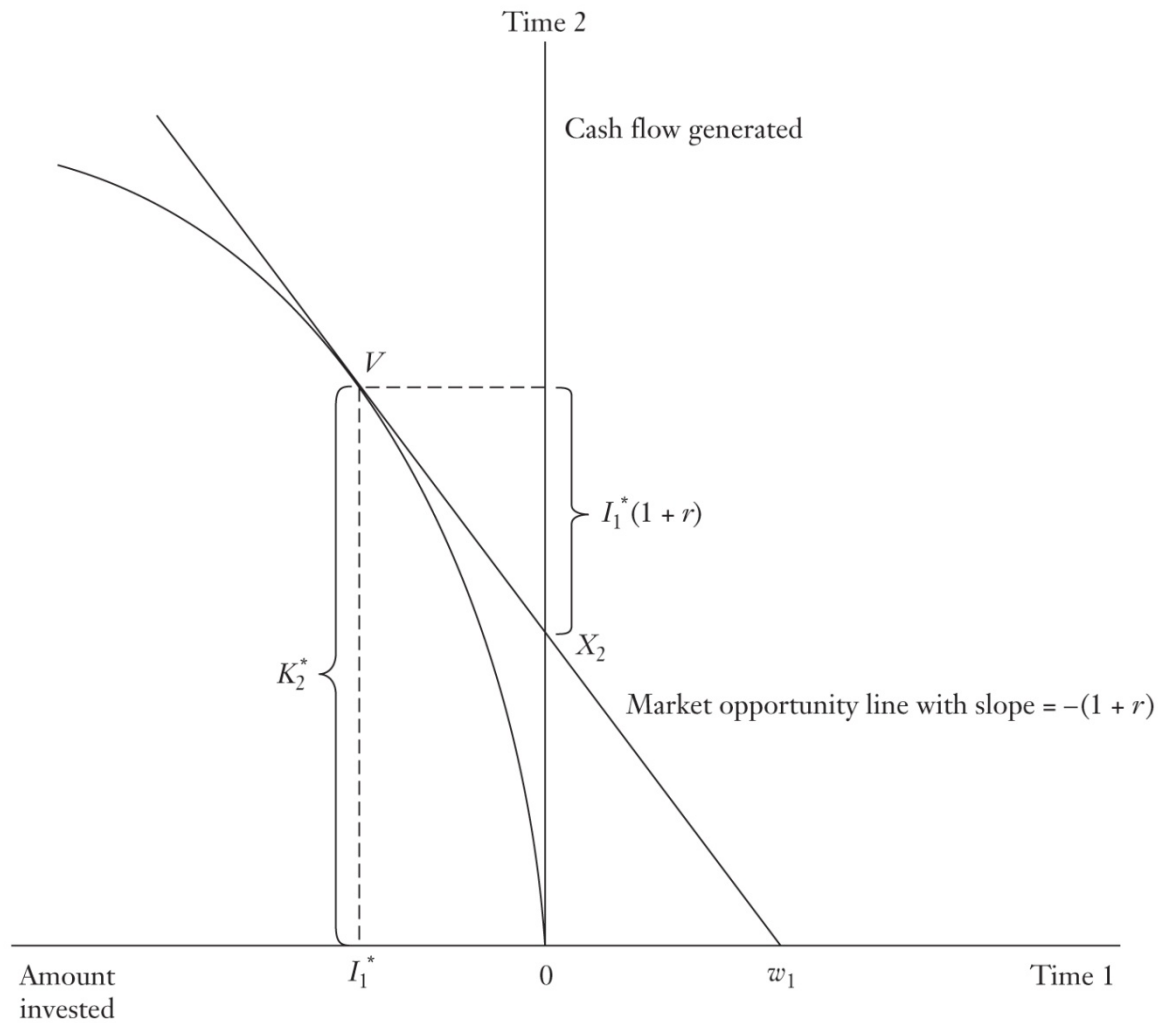
This separation of investment and consumption decisions is sometimes referred to as the *Fisher separation theorem*.

The entrepreneur with no initial resources

- The entrepreneur with no initial resources will be able to go to the capital market at time 1 and borrow all the funds needed to finance production.

- In Figure 3.5, the entrepreneur with no initial resources will be able to go to the capital market at time 1 and borrow all the funds needed to finance production ($0I^*_1$).

- At time 2, the firm repays $I^*_1(1 + r)$, leaving $0X_2$ accruing to the owner.

- In a perfect capital market, ideas can have a marketable value that can be realized by borrowing from others.

FIGURE 3.5
ENTREPRENEUR BORROWS INITIAL CAPITAL

Time 2

Cash flow generated

V

$I_1^*(1 + r)$

X_2

Market opportunity line with slope $= -(1 + r)$

K_2^*

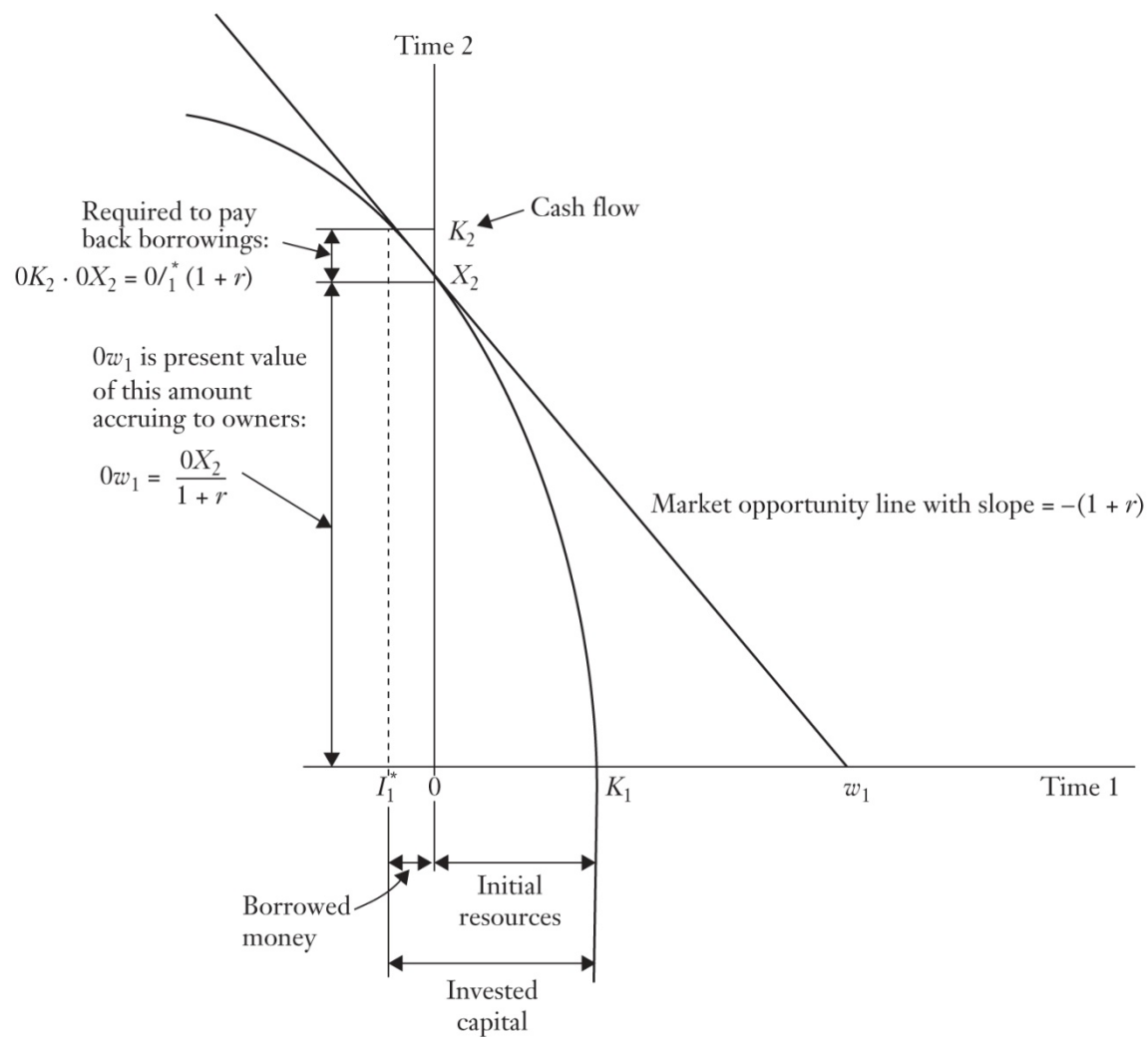Amount invested

I_1^*

0

w_1

Time 1

Partial funds required

- We can also imagine circumstances in which the entrepreneur has some of the funds needed for investment but must borrow the balance.

- We represent this by appropriately positioning the transformation curve.

- In Figure 3.6 the increment to the entrepreneur's wealth $K_1 w_1$ is the same irrespective of whether $0I^*$ is borrowed and $0K_1$ invested or whether $0K_1$ is sold and the entire amount of $I^* K_1$ is borrowed.

- The method of financing a productive investment is irrelevant to determining the value of the opportunity.

FIGURE 3.6
ENTREPRENEUR BORROWS SOME OF THE REQUIRED CAPITAL

Required to pay
back borrowings:

$0K_2 \cdot 0X_2 = 0/_1^* (1 + r)$

$0w_1$ is present value
of this amount
accruing to owners:

$0w_1 = \dfrac{0X_2}{1 + r}$

Time 2

K_2 ← Cash flow

X_2

Market opportunity line with slope $= -(1 + r)$

I_1^* 0 K_1 w_1 Time 1

Borrowed
money

Initial
resources

Invested
capital

Corporations

- Business entities that are responsible for investing most of the funds put to work in an economy are corporations.

This fact leads us to consider two additional issues:

- Issue 1: Corporations have many owners, and there is no reason to assume that the owners' preferences are all the same.

- Issue 2: The owners of a corporation do not make the operating decisions of the firm but rather hire professional managers and delegate decision-making power to these managers.

The market value criterion

- An entrepreneur's optimal consumption, production, and financing decisions were characterized by the simultaneous satisfaction of two tangency conditions between:

1. The transformation curve and the capital market opportunity line and

2. The capital market opportunity line and an indifference curve.

Maximizing market value

- Maximizing market value is equivalent to maximizing profits.
- Let I^*_1 be the total amount invested in a firm at time 1, K_2 be cash flow net of operating expenses at time 2, and K_3 be cash flow net of operating expenses at time 3.
- Assume r is the market rate of interest in both periods.
- Then the firm's market value at time 1 is:

$$MV_1 = \frac{K_3}{(1+r)^2} + \frac{K_2}{1+r} - I^*_1 \qquad (3.1)$$

Maximizing market value continued

- Suppose that the outlays for investment I^*_1 are all borrowed and that a loan repayment schedule is established requiring payment of principal and interest in amounts I_2 and I_3 at times 2 and 3 respectively such that:

$$\frac{I_2}{1+r} + \frac{I_3}{(1+r)^2} = I_1^* \qquad (3.2)$$

- Then we can write

$$MV_1 = \frac{K_2}{1+r} - \frac{I_2}{1+r} + \frac{K_3}{(1+r)^2} - \frac{I_3}{(1+r)^2} \qquad (3.3)$$

To conclude

- if the loan repayments are just equal to economic depreciation, then $K_2 - I_2 = \pi_2$ and $K_3 - I_3 = \pi_3$ so that π_2 and π_3 are economic profits in the two periods:

$$MV_1 \frac{\pi_2}{1+r} + \frac{\pi_3}{(1+r)^2}$$

Example 3: Aging wine

- Consider the problem of a businessperson trying to decide whether to sell a product now or refine it further and sell it later at a higher price.

- For example, assume a vintner who is trying to maximize the market of initially available resources in the form of new wine either by selling the current vintage of wine at time 1 or aging it for sale at time 2.

- Let the market rate of interest be 6%.

Example continued

- The objective function of the vintner will be:

$$\max K_1 + \frac{K_2}{1.06} \qquad (3.4)$$

where K_1 is the value of new wine sold at time 1 and K_2 is the value of new wine sold at time 2.

- Assume the maximization is subject to $K_1^2 + K_2^2 = 212.36$, a transformation curve chosen for its mathematical tractability rather than for its interpretive realism.

- The transformation curve implies that the time 1 cash value of all the raw wine is $(\$212.36)^{1/2}$, or approximately \$14.60.

Rewriting the problem

- We can rewrite the problem (3.4) as

$$\max_{K_2}(212.36 - K_2^2)^{1/2} + \frac{K_2}{1.06} \qquad (3.5)$$

- Taking the derivative with respect to K_2,

$$\frac{\frac{1}{2}(-2K_2)}{(212.36 - K_2^2)^{1/2}} + \frac{1}{1.06} = 0 \qquad (3.6)$$

- This step is equivalent to saying "choose a production plan such that the marginal rate of return equals the market rate of interest."

To conclude

- By the following calculation,

$$\frac{1}{1.06} = \frac{K_2}{(212.36 - K_2^2)^{1/2}}$$

$$K_2 \times 1.06 = (212.36 - K_2^2)^{1/2}$$

$$K_2^2 \times (1.06)^2 = 212.36$$

$$K_2^2 \times [1 + (1.06)^2] = 212.36 - K_2^2$$

$$K_2^2 \times (2.1236) = 212.36$$
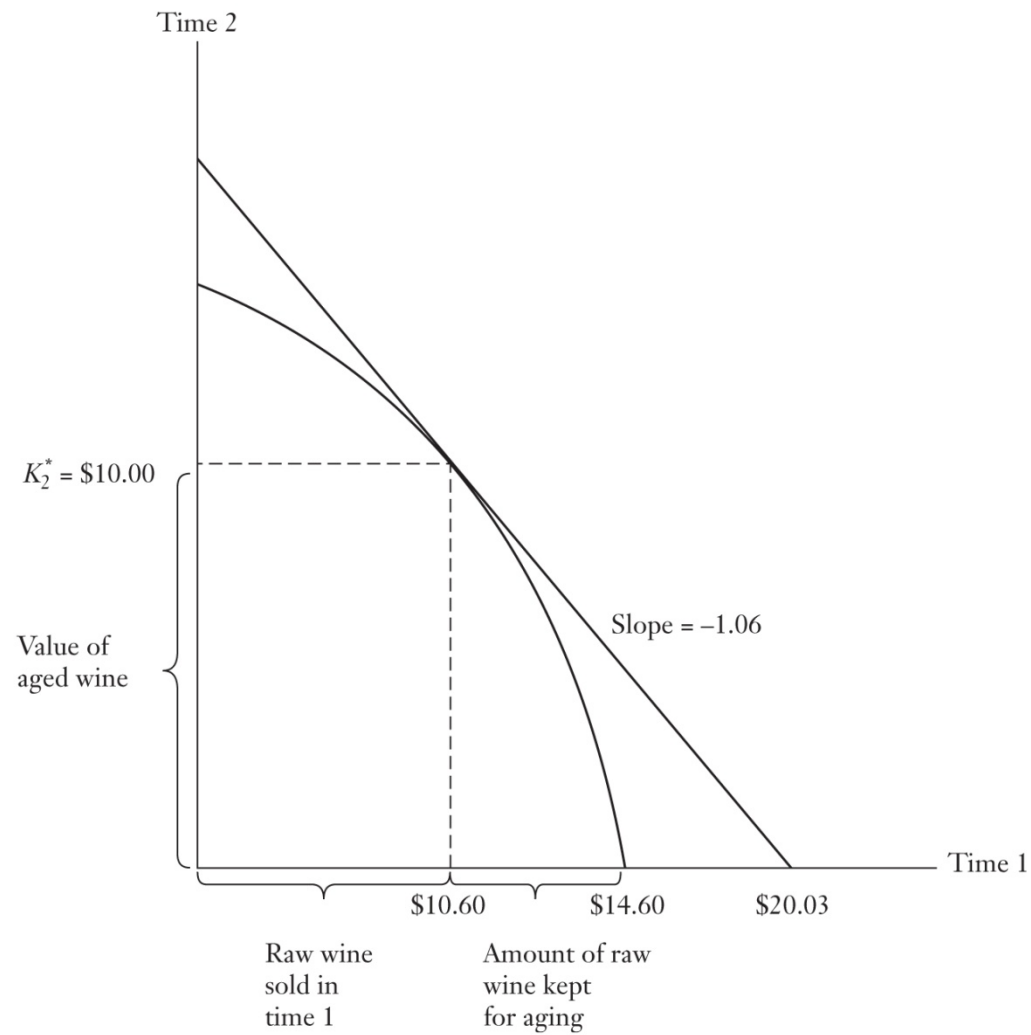
$$K_2^* = \$10.00$$

we obtain

$$K_1^* = (212.36 - 100.00)^{1/2} = \$10.60$$

and

$$MV_1 = 10.60 + \frac{10.00}{1.06} = \$20.03$$

FIGURE 3.7
WINE AGING EXAMPLE

Time 2

$K_2^* = \$10.00$

Value of
aged wine

Slope = −1.06

Time 1

$10.60 $14.60 $20.03

Raw wine
sold in
time 1

Amount of raw
wine kept
for aging

Four major findings

1. The market-value-maximizing firm will invest up to the point where the marginal rate of return equals the market rate of interest. This is also consistent with long-run profit maximization when profits are correctly calculated.

2. Determination of the total investment in a firm depends on two things: (1) the technology and the cash flow it yields and (2) the market rate of interest.

Four major findings continued

3. It does not matter whether the firm is owned by many stockholders or a single owner, since if the market value of the firm is maximized so is any stockholder's proportional interest in that market value.

4. Under assumptions of certainty and a perfect capital market, all sources of financing have the same cost, the market rate of interest.

Separation of operating and financing decisions

- The operational importance of the foregoing separation principle is that the firm's operating plans and the attendant investment decisions need not be considered simultaneously with decisions as to how to finance the investment.

- By an operating decision, we mean the decision to invest X_1K_1, which determines the amount of the firm's output it will produce and sell.

- Properly operated, the firm is worth MV_1.

- If the existing owners do not wish to provide the required investment funds X_1K_1, the funds can be raised in the capital market.

FIGURE 3.8
SEPARATION OF OPERATING AND FINANCING DECISIONS
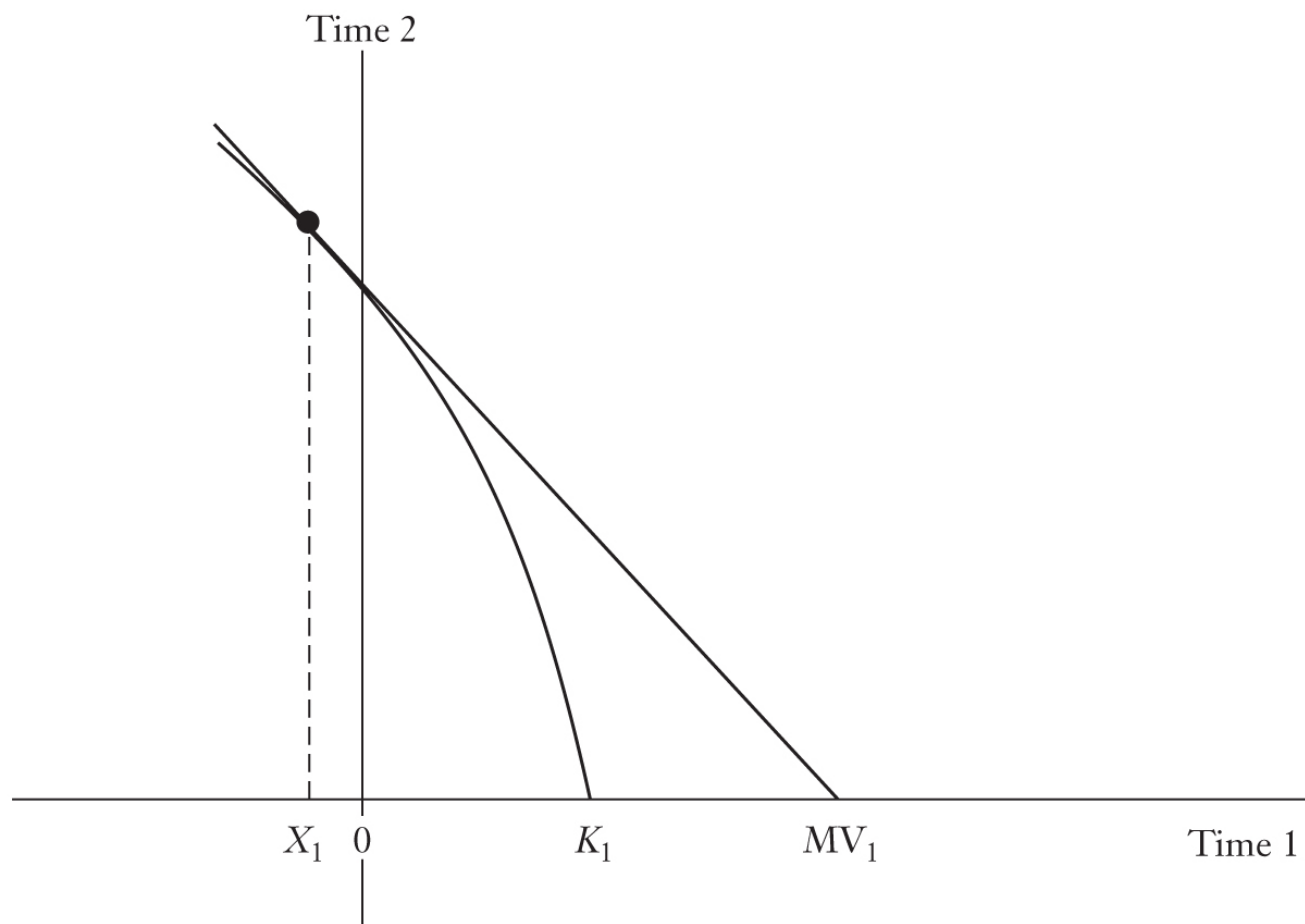
Separation of managers' and owners' decisions

- The separation principle says the firm's managers determine the optimal investment $X_1 K_1$ and hence its market value MV_1.

- If the firm has a single owner, the latter then decides what consumption standards, with a present value of MV_1, are preferred; that is, the owner chooses a position on the market opportunity line.

- If the firm has many owners, a given owner is entitled to only some proportion of market value, say αMV_1 $(0 \leq \alpha \leq 1)$.

- This then defines the market opportunity line along which that particular owner can move.

FIGURE 3.9

SEPARATION OF MANAGERS' AND OWNERS' DECISIONS

Time 2

Time 2 cash
flow for
financing

Time 2 cash
flow to
owners

C_2^*

X_1 K_1 C_1^* MV_1

Time 1
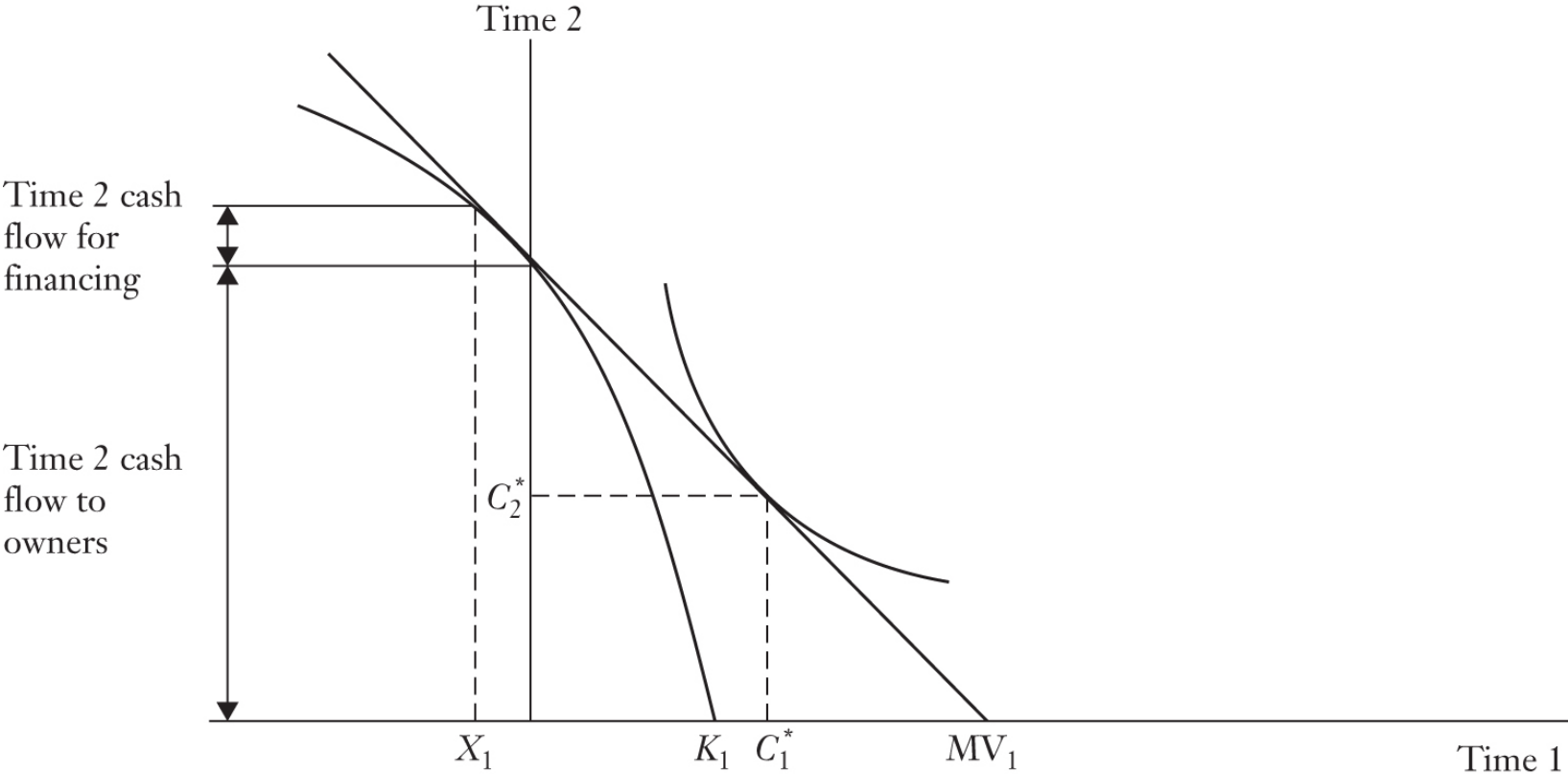
Key points 1

- In a perfect capital market, management can maximize the wealth of a firm's owners by using the market value rule.

- The productive opportunity set of a proprietary firm is graphically depicted by a transformation curve that shows efficient use of funds.

- Efficiency means that the capital investment is put to its best possible use and there are no resources wasted by the rational entrepreneur.

- The slope of the transformation curve may be interpreted as indicating the firm's marginal rate of return.

Key points 2

- The owner's decision process can be viewed as encompassing two distinct steps: (1) selecting the optimum production by equating the marginal rate of return on investment with the market required rate of return and (2) selecting the optimum consumption pattern by borrowing or lending in the capital market until the consumer's subjective rate of time preference equals the market rate of return.

- The separation of investment and consumption decisions is sometimes referred to as the Fisher separation theorem.

- Unlike proprietary firms, managers of corporations are responsible for most of the funds invested in an economy.