ECON2103: Financial Economics Lecture 4

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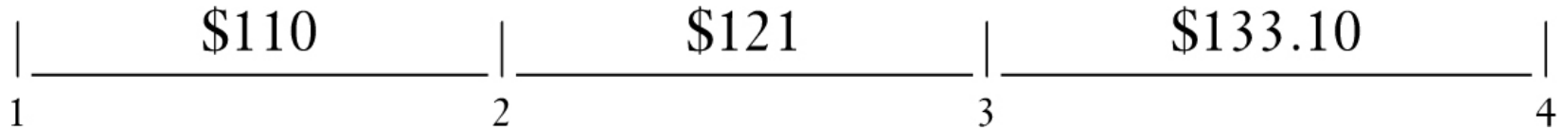
This week's topics

- Market value
- Share price determination
- Valuing the growth firm
- Firm's maximization problem
- Period-by-period maximization problem
- Maximization of present value

Application to share price determination

- Consider



- Let *r = 10%* over all three time periods (in our notation it would be ${}_1r_4$) and suppose that the firm has zero value after time 4.
- Note *s(1)=(1/50)MV(1).*
- Thus,

$$s(1) = \frac{1}{50}\left[\frac{110}{1.1} + \frac{121}{(1+r)^2} + \frac{133.10}{(1.1)^3}\right] = \frac{300}{50} = \$6$$

Calculation

- Therefore

$$s(1) = \frac{1}{50}\left[\frac{D(2)}{1.10} + \frac{MV(2)}{1.10}\right]$$

where *D(2)* is the dividend payment at time 2, distributed to stockholders of record at time 1.

- But,

$$MV(2) = \frac{121}{1.1} + \frac{133.10}{(1.1)^2} = \$220$$

- Finally

$$s(1) = \frac{1}{50}\left[100 + \frac{1}{1.1}(220)\right]$$

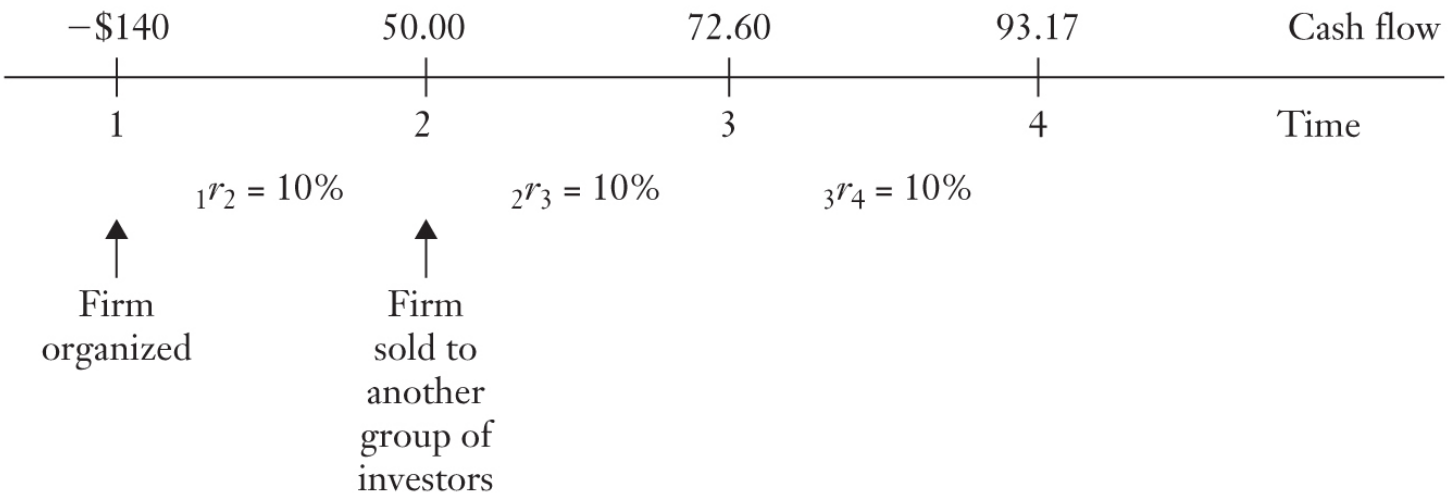
$$= \frac{d(2)}{1.1} + \frac{s(2)}{1.1}$$

where *d(2)* is the time 2 dividend to a single share and *s(2)* is the price of the share.

Applications to valuing the growing firm

- Consider the following situation

FIGURE 4.2
VALUING THE GROWING FIRM



Value of the firm

- Then

TABLE 4.1
VALUES OF THE FIRM AT TIMES 1 THROUGH 4

Time	*Market value of incomes not yet received*[a]
1	\$50.00 + \$60.00 + \$70.00 = \$180.00
2	\$55.00 + \$66.00 + \$77.00 = \$198.00
3	\$72.60 + \$84.70 = \$157.30
4	\$93.17 = \$ 93.17

[a]Note that the data are obtained by appropriate discounting of the cash flows. For example, \$93.17/1.10 = \$84.70, and so on.

Additional example

- Since we have already, and will again, show that several time points can be incorporated in our analyses, the present example uses only two time points so as to focus more sharply on other matters.
- Suppose that a firm announces an investment of $2,200 at time 1, that the cash flow realized at time 2 on this investment will be $3,520, and that the investment will be financed by a new share issue.
- How many shares must be issued, and what will the market price of all shares be after the new issue?

Example continued

- Let us denote with a superscript 0 the values for the variables before the announcement of the new investment.
- Let's assume the following in our example:

$V^0(2)=\$11,000; r=0.10; N^0(1)=1,000 \text{ shares}; s^0(1)=\$10 \text{ per share}$

- After the announcement, we will then have:

$$V(2)=\$11,000+\$3,520=\$14,520$$

$$I(1)=\$2,200$$

where $I(1)$ is the amount of the new investment.

The number of new shares

- Denoting the number of new shares as *M(1)*, we write the time 1 value of each share outstanding after the new issue as:

$$\frac{V(2)}{1.10}\left[\frac{1}{N^0(1)+M(1)}\right]$$

- Therefore,

$$I(1)=\frac{V(2)}{1.10}\left[\frac{M(1)}{N^0(1)+M(1)}\right]$$

or

$$\$2,200=\frac{\$14,520}{1.10}\left[\frac{M(1)}{N^0(1)+M(1)}\right]$$

so that

$$M(1)=200$$

What is the issue price?

- All shares must earn the market rate of interest from period 1 to period 2.
- Therefore,

V(1)=V(2)/1.1=$13,200 and s(1)=$13,200/$1,200=11

Mathematical example

- Suppose, as in the previous example, new investors can buy the stock only at time 2, 3, and 4; time 1 is an organizing time before the founders sell any shares in the financial markets:

- Also let K_1 denote the capital stock purchased at time 1, π_t denote earnings at time t+1, t = 1, 2, 3.

Example continued

- The firm's objective is to maximize the present value of the original owners' investment, a value determined by earnings it can generate through setting up the firm and producing and selling a single product.
- Assume that production is given by

$$q_t = K_1^{1/2} \text{ in each of the three time periods,}$$

where K_1 is the capital stock purchased at time 1, a level that cannot subsequently be altered, and q_t, $t = 2, 3, 4$, the outputs produced from the capital stock at those times.

The maximization problem

- The maximization problem is

$$\max_{K_1} \frac{p_2 K_1^{1/2}}{1+_1 r_2} + \frac{1}{1+_1 r_2}\left(\frac{p_3 K_1^{1/2}}{1+_2 r_3}\right) + \frac{1}{1+_1 r_2}\left(\frac{1}{1+_2 r_3}\right)\left(\frac{p_4 K_1^{1/2}}{1+_3 r_4}\right) - c_1 K_1 \qquad (4.15)$$

where p_t is the price of output at time *t, t = 2, 3, 4*, and c_1 is the unit cot of capital stock acquired at time 1.

Solving the maximization problem

- When we assume

$$p_2 = p_3 = p_4 = 3 \; and \; c_1 = 1$$

and $$_1r_2 =_2 r_3 =_3 r_4 = \frac{1}{2} \; (i.e., 50\%)$$

then (4.15) can be rewritten as

$$\max_{K_1} \frac{3K_1^{1/2}}{\frac{3}{2}} + \frac{3K_1^{1/2}}{\left(\frac{3}{2}\right)^2} + \frac{3K_1^{1/2}}{\left(\frac{3}{2}\right)^3} - K_1$$

$$= \max_{K_1} \frac{38}{9} K_1^{1/2} - K_1 \qquad (4.16)$$

Some computation

- Then we obtain:

$$\frac{1}{2}\left(\frac{38}{9}\right)K_1^{-1/2}-1=0$$

$$\frac{19}{9}K_1^{-1/2}=1$$

$$\frac{19}{9}=K_1^{1/2}$$

$$K_1^{*}=\left(\frac{19}{9}\right)^2$$

The market value

- To obtain the market value of the firm at time 1,

$$MV(1) = \left(\frac{38}{9}\right)\left(\frac{19}{9}\right) - \left(\frac{19}{9}\right)^2 = \left(\frac{19}{9}\right)^2$$

- Assuming that the firm distributes earnings to stockholders as quickly as the earnings are generated, the market value of the firm at time 2 is the present value, at that time, of all future earnings the firm can generate:

$$\left[\frac{3}{\frac{3}{2}} + \frac{3}{\left(\frac{3}{2}\right)^2}\right]\left(\frac{19}{9}\right) = \left(\frac{30}{9}\right)\left(\frac{19}{9}\right)$$

- We can also compute MV(3) as $\frac{3}{\frac{3}{2}}\left(\frac{19}{9}\right) = 2\left(\frac{19}{9}\right)$

Calculation of return

- Suppose that the firm was sold by the founders at time 2.
- Then the one-period return on the original owner's investment is

$$\text{return} = \frac{\text{revenues in period 1} + \text{sales price of firm} - \text{original investment}}{\text{original investment}}$$

$$= \frac{p_1 (K_1^*)^{1/2} + MV(2) - K_1^*}{K_1^*}$$

$$= \frac{3(\frac{19}{9}) + \frac{30}{9}(\frac{19}{9}) + (\frac{19}{9})^2}{(\frac{19}{9})^2} = 100\%$$

- If they continue to hold shares in the firm after period 1, their subsequent returns, when calculated on the basis of the firm's market values, just equal market rates of interest.

The rate of return between times 3 and 4

- For example, the rate of return between times 3 and 4 is

$$\text{return} = \frac{p_1 (K_1^*)^{1/2} - MV(3)}{MV(3)}$$

$$= \frac{3(\frac{19}{9}) - \frac{18}{9}(\frac{19}{9})}{\frac{18}{9}(\frac{19}{9})} = 50\%$$

Interpretation

1. The value of the firm at any point in time can be viewed as the sum of two terms: the value of current period earnings on the one hand and the discounted future value of the firm on the other.
2. The discounted future value used for planning over more than a single period is the market value of the firm at the end of that single planning period.
3. The original owners of the firm get any excess returns because, as and when they sell the firm, they demand and can obtain a market price such that the new owners receive only the market rate of interest on their investment

Relation to profit maximization

- The example of the last section can be used to indicate how period-by-period profit maximization can, if correctly carried out, lead to the same result as maximization of present value.
- A time 1 decision yields time 2 earnings composed of both current earnings and future capital gains.
- In the example, period 1 earnings are thus, when evaluated in time 1 magnitudes, equal to the time 1 values of earnings received at time 2, plus asset value at time 1, less asset acquisition costs on the market value of the firm at time 1:

$$\frac{3\left(\frac{19}{9}\right)}{\frac{3}{2}} + \frac{\frac{10}{3}\left(\frac{19}{9}\right)}{\frac{3}{2}} - \left(\frac{19}{9}\right)^2 = \left(\frac{19}{9}\right)^2$$

Zero-earning calculations

1. The rate of return on invested assets valued at market prices is the market rate of return.
2. No new value is created subsequent to the initial asset acquisition decision.
3. Moreover, the initial value creation stemmed from acquiring assets at a cost less than the present value of the earnings those assets could generate.

Review: Key points 1

- When management follows the market value rule, the value of the firm is equal to the present value of cash flows to be generated by the firm's existing assets plus the present value of cash flows to be generated from future growth opportunities.
- The firm's value depends only on its future earnings stream and not on the source of funds needed to finance creation of the earnings stream.
- A growth firm is one in which the expanding assets generate returns in excess of the market rate of interest.

Review: Key points 2

- The price-earnings ratio is the ratio of the current market price of the stock to the firm's earnings per share.
- In a perfect capital market, financial instruments outstanding over several periods must have market prices such that they yield the market rates of interest prevailing in each time period.