ECON2103: Financial Economics
Lecture 9

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This week's topics

- States of the world
- A contingent claim
- Primary security
- Arrow-Debreu security
- An investor's utility maximization problem
- Short sales
- The MM theorem revisited
- Complete market
- Incomplete market
- Contingent strategies
- Noncongingent strategies
- Formulating contingent strategies

States of the world

- The idea of states of the world is useful for thinking about convenient ways to model risky payoffs.
- In a two-time-point model, states of the world are defined as those future events that matter to the decision problem being considered.
- These states of the world are defined by the decision maker to be mutually exclusive and collectively exhaustive.

Example

- Suppose the investor defines (1) “states” to represent economic conditions and (2) “future prices” to be the following list of possible stock prices that may be realized at the time a given state is actually realized:

State	*Future Prices*
1	\$10
2	\$ 8
3	\$ 6

A contingent claim

- A unit contingent claim is a security that will pay an amount of \$1 if a certain state of the world is actually realized, but nothing otherwise.
- A claim that pays \$1 if state *i* is realized is frequently called a unit claim on state *i*.
- A unit contingent claim is also referred to as a primary security or Arrow-Debreu security.
- A contingent claim can be: Ten unit claims on state 1; Eight unit claims on state 2; Six unit claims on state 3.

Example

- Suppose that we can describe the world using two states and that two stocks are available, stock A and stock B.
- We assume the stocks' future prices have the following distributions:

State	*Future Prices Stock A*	*Future Prices Stock B*
1	\$10	\$7
2	\$ 8	\$9

- Let C_1 and C_2 represent the time 1 prices of unit claims on states 1 and 2.
- Then $10C_1 + 8C_2 = \$6$ and $7C_1 + 9C_2 = \$5$ implying $C_1 = \$\ 7/17$ and $C_2 = \$4/17$.

Risk-free rate

- Since a risk-free instrument is one that offers the same payoff irrespective of which state of the world obtains, we wish to find a combination of the two stocks that gives the same time 2 payoff, here denoted k, in either state of the world.
- That is, the following equation must be solved for α:

$$10\alpha + 7(1 - \alpha) = 8\alpha + 9(1 - \alpha)$$

Implying $2\alpha = 2(1 - \alpha)$ and thus $\alpha = ½$.

The riskless payoff

- The riskless payoff is

$$\tfrac{1}{2}(10) + \tfrac{1}{2}(7) = \$8.5 \text{ and } \tfrac{1}{2}(6) + \tfrac{1}{2}(5) = \$5.5.$$

- The risk-free rate of return is

$$(\$8.5-\$5.5)/\$5.5 = 6/11 = 54.55\%.$$

Investors' utility maximization

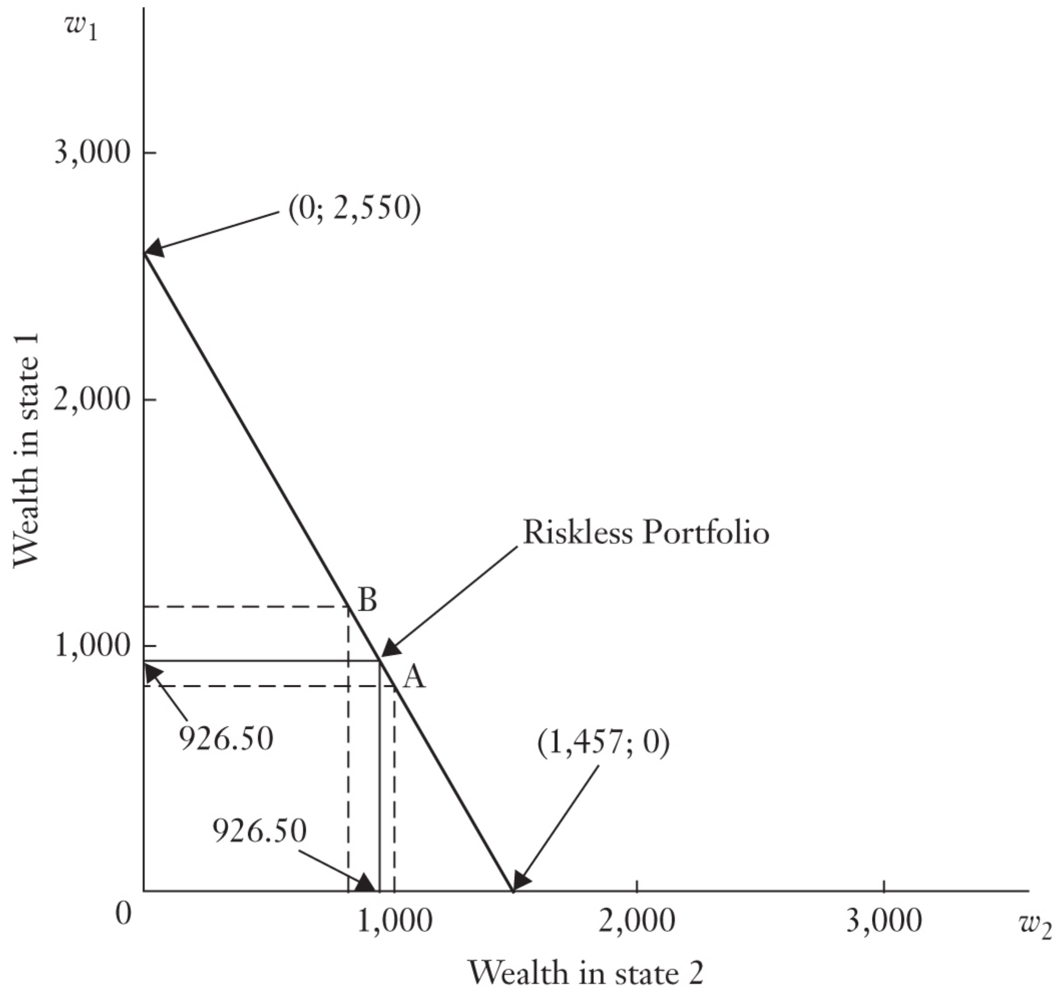
- We continue with Stocks A and B.
- Assume that the investor's initial wealth is $600.
- Consider the following scenario:

Table 10.1
Summary of Terminal Wealth in Two States

	No. of shares Purchased	*Terminal wealth*	
		State 1	*State 2*
Purchase A only	100	$1,000	$ 800
Purchase B only	120	$ 840	$1,080

- Each security generates a riskless terminal wealth position of $w_1 = w_2 = \$926.5$.

Figure 10.1
Market Opportunity Line Showing Implied Prices of Unit Claims



Computation

- The straight line in Figure 10.1 is $w_2 = a - bw_1$.
- For the time 1 price of stock A

$$\$800 = a - \$1000b$$

- For the time 1 price of stock B

$$\$1080 = a - \$840b$$

- Therefore $A = \$2,550$ and $b = 1.75$.
- Also when $w_1 = 0$, $w_2 = \$2,550$ and when $w_2 = 0$, $w_1 = \$1,457$.
- If $w_2 = 0$, we have the case of a claim (primary security) on state 1. The price is $\$600/\$1,457 = 0.41 = 7/17$.
- Similarly, the price of primary security 2 is $\$600/\$2,550 = 0.24 = 4/17$.

Short sales

- Note that in Figure 10.1 the investor's time 1 position is some point on the line from *A* to *B*.
- How could the investor obtain a terminal wealth position lying beyond these points?
- The investor could engage in short sales, that is, selling shares not currently owned, for delivery when the unknown future state of the world is revealed.

Illustration

- To illustrate, consider point $w_1 = \$1,457$, $w_2 = 0$.
- Let n_A be the number of shares of stock A and n_B the number of shares of stock B purchased.
- If state 1 occurs, the terminal wealth will be:

$$10\, n_A + 7\, n_B = \$1,457.$$

- If state 2 occurs, we must have:

$$8\, n_A + 9\, n_B = 0.$$

- Solving these equations simultaneously, we find $n_B = -\,343$.

Incomplete markets for contingent claims

- A market is said to be a complete market when economic agents can structure any set of future state payoffs by investing in a portfolio of unit continent claims (i.e., primary securities).
- A financial market is said to be incomplete if the number of (linearly) independent securities traded in it is smaller than the number of distinct states of the world.
- Since the number of states of the world used to describe a typical financial market is likely to be large, the possibility that real-world financial markets will be incomplete is a very real one.

Example

- Consider the following:

TABLE 10.2
MARKET VALUES OF TWO SECURITIES AT TIME 1

	States of the World		
Security	*1*	*2*	*3*
1	1	0	0
2	0	1	0

Modigliani-Miller Revisited

- Let *p(s)* be the price at time 1 of receiving \$1 at time 2 if state *s* is realized.
- Consider the following:

State	*Operating earnings*	*Bondholders*	*Stockholders*
1	\$1,700	\$500	\$1,200
2	\$ 800	\$500	\$ 300

- The value of the bond at time 1 is $B(1) = \$500p(1)+\$500p(2)$.
- The time 1 value of equity is $S(1) = \$1{,}200p(1)+\$300p(2)$.
- The value of both securities is $V(1) = \$1{,}700p(1)+\$800p(2)$.

A second case

- Consider the following:

State	*Operating earnings*	*Bondholders*	*Stockholders*
1	\$1,700	\$1,000	\$700
2	\$ 800	\$ 800	\$ 0

- The value of the bond at time 1 is *B(1) = \$1,000p(1)+\$800p(2).*
- The time 1 value of equity is *S(1) = \$700p(1)+\$0p(2).*
- The value of both securities is *V(1) = \$1,700p(1)+\$800p(2).*

Contingent strategies

- Just as investor satisfaction can increase if more kinds of contingent claims become available, a firm can improve its earnings distribution by using a contingent strategy.
- To recognize the possibility of taking contingencies into account in decision making, we say a decision maker uses contingent planning when instead of merely saying "I will do *X*," the person announces "I will do X_1 if state 1 is realized, X_2 if state 2 is realized," and so on.
- The details of contingency planning are referred to as formulating a contingent strategy.

Example

- Esoteric Electronics is a manufacturer of components used in both industrial applications and in space exploration.
- At the present time, the company is planning its production for the next two quarterly periods.
- It has to decide whether to produce either a or b components in each quarter, since it cannot produce both components simultaneously.
- Steady production of either one component or the other for both quarters eliminates setup charges.

Example continued

- Revenues from continued production of *b* will be affected by the success or failure of a space exploration mission, the results of which will become known before the end of the first quarter but after the time for making the first quarter's production decision has passed.
- The revenue from a, a non-space-industry-utilized component, is independent of the mission's outcome.

Decision sequences and their payoffs

Table 10.3
Decision Sequences and Their Payoffs

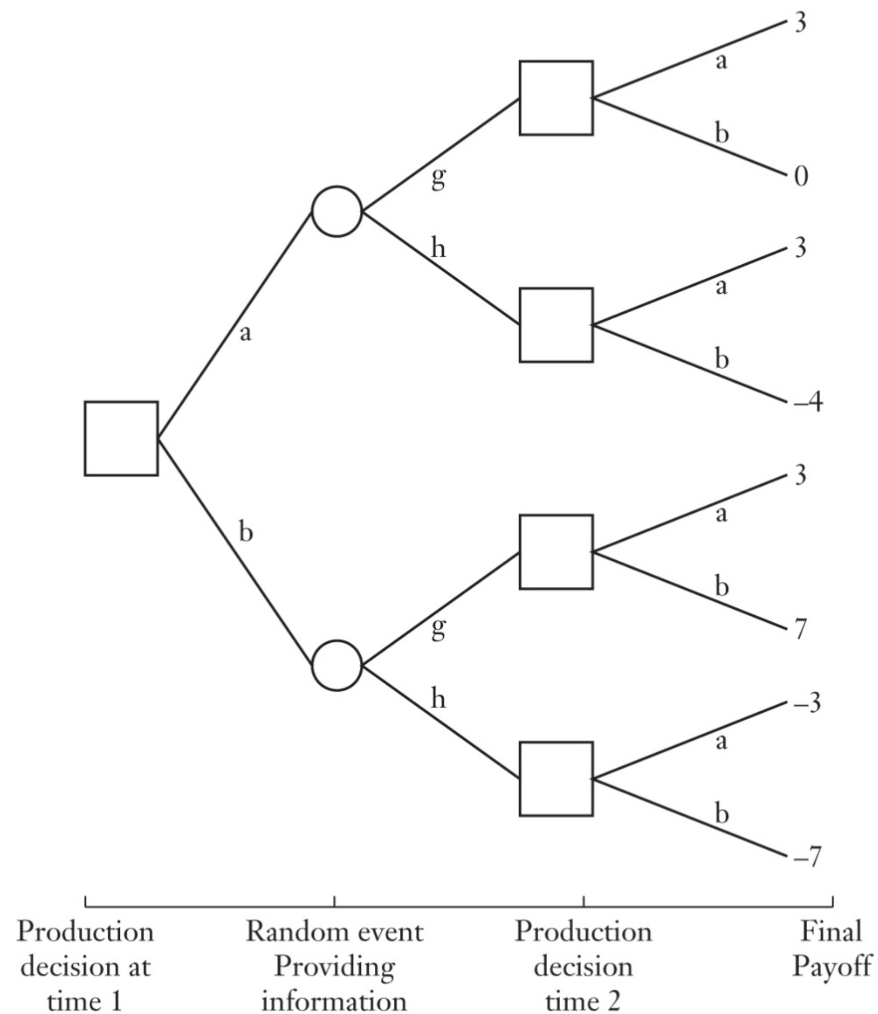
Decision sequence	*Payoff in state g*	*Payoff in state h*
Aa	\$3	\$3
Ab	\$0	−\$4
Ba	\$3	−\$3
Bb	\$7	−\$7

Example continued

- The foregoing considerations are captured in Table 10.3, where production plan payoffs are shown to depend on the state of the world (i.e., the mission outcome).
- A successful mission outcome is denoted by g and an unsuccessful outcome by h.

Figure 10.2

Illustration of a Contingent Strategy



Noncontingent strategies

- First consider

Table 10.4
Noncontingent Strategies and Expected Payoffs

Noncontingent Strategy	*Expected Payoff*
$a(ga, ha)$	\$3(0.65) + \$3(0.35) = \$3.00
$a(gb, hb)$	\$0(0.65) + \$4(0.35) = \$1.40
$b(ga, ha)$	\$3(0.65) + \$3(0.35) = \$0.90
$b(gb, hb)$	\$7(0.65) + \$7(0.35) = \$2.10

Note: The notation $c(gd, hd)$ means that production of c is planned for the first quarter, followed by production of d in the second quarter regardless of whether the mission succeeds or fails.

Contingent strategies

Table 10.5
Contingent Strategies and Expected Payoffs

Contingent Strategy	*Expected Payoff*
$a(ga, hb)$	\$3(0.65) + \$4(0.35) = \$0.55
$a(gb, ha)$	\$0(0.65) + \$3(0.35) = \$1.05
$b(ga, hb)$	\$3(0.65) + \$7(0.35) = \$0.50
$b(gb, ha)$	\$7(0.65) + \$3(0.35) = \$3.50

Illustration

- We see that the optimal strategy is *b(gb, ha).*
- In other words, management begins by producing *b* and continues with *b* if the mission is successful but switches to a if the mission is unsuccessful.
- Note this strategy has a higher expected value than the noncontingent strategy that management initially considered.
- Incorporating flexibility into the firm's decision making encompasses a wider range of possibilities, and the extra flexibility gained never does any harm (except for the costs of making extra computations).

Key points 1

- Contingent claims analysis and contingent strategies are tools for dealing with risk in financial decision making.
- Contingent claims analysis uses the notion of states of the world in assessing future risky payoffs.
- A unit contingent claim (also known as a primary security or Arrow-Debreu security) is a security that has a payoff of $1 if a certain state of the world is actually realized, but nothing in all other states.
- A contingent claim that pays off $1 if state *i* is realized is also referred to as a unit claim on state *i*.

Key points 2

- If the number of (linearly) independent securities traded is smaller than the number of distinct states of the world, the financial market is said to be incomplete.
- Because the number of states of the world necessary to describe a well-functioning financial market is likely to be large, the possibility that real-world financial markets will be incomplete is a very real one.
- Contingent strategies can be used by a firm’s management to recognize the possibility of taking contingencies into account in financial decision making.
- Contingent strategies can improve the payoffs obtained from financial decision making.