ECON2103: Financial Economics
Lecture 10

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This week's topics

- Uncertainty
- Risk
- Wealth-based criteria
- Decision-based criteria
- Target-based criteria
- Risk transfer
- Hedging
- Risk management
- Insuring
- Deversifying
- Expected utility
- Expected value
- Variance
- Standard deviation
- Covariance
- Risk averse
- Certainty equivalent value

Uncertainty and risk

- Uncertainty itself can be divided into two classifications.
- The first of which refers to situations where qualitative descriptions are so difficult to generate that it is not practical even to define possible future states of the world.
- The second kind of uncertainty refers to situations in which it is possible to define future states of the world, but not to attach probability distributions to their possible realizations.
- Risk refers to situations in which it is possible both to define future states of the world and the probabilities with which they might occur.

Decision criteria

- Under risk, decision makers' criteria include expected wealth maximization, minimizing maximum regret, and the customary expected utility maximization.
- Different choices of criteria will help manage certain aspects of a risk-management problem, but the solutions will not necessarily be consistent with those obtained using other criteria.

Wealth-based and decision-based criteria

- Wealth-based criteria emphasize the distribution of a decision maker's final wealth, usually focusing on such measures of central tendency as expected value.
- Dispersion-based criteria emphasize the spread of a random variable.
- They are also both symmetric measures, and as a result consider variation above the mean as serious as variation below the mean.
- In contrast, many decision makers prefer to use criteria that weight losses more heavily than gains.

Target-based criteria

- In some instances, management may exhibit risk aversion through attempts to avoid downside risk (unfavorable outcomes).
- We shall refer to techniques of this type as target-based approaches or chance-constrained approaches.

Example

- Suppose the management of a firm is trying to allocate liquid assets to two accounts, one of which (e.g., a bank checking account) is riskless but pays no interest, while the other offers a risky return (money market investments, for example, if we assume they will not necessarily be held to maturity).
- Assume the rate of return r on the second account is uniformly distributed over the range *[-0.5, 0.7].*
- If R is the amount currently available for allocation to the two accounts, the value of invested resources next period will be: $(R-S)+S(1+r)$ where S is invested in the risky asset and the remainder R - S is kept in the non- interest-bearing riskless account.

Example continued

- Suppose management would like to make the next period investment value as large as possible but subject to the condition that $R + Sr$ not fall below *95%* (an arbitrarily chosen percentage) of the original value of R too often.
- In this example, suppose that if the investment falls below *95%* of its original value, it should not do so more than *25%* of the time (another arbitrary choice).

Formal statement of the problem

- Management wishes to

$$\max_{S} E\left[R\left(1+\frac{S}{R}r\right)\right] \qquad (11.3)$$

subject to

$$\Pr[R+Sr \geq 0.95R] \geq 0.75 \qquad \text{and} \qquad 0 \leq S \leq R$$

where Pr means cumulative probability.

A simpler form

- Equation (11.3) can be rewritten as

$$\max_{S} E\left(1+\alpha r\right)$$

subject to

$$\Pr[(1+\alpha r) \geq 0.95] \geq 0.75 \qquad \text{and} \qquad 0 \leq \alpha \leq 1 \quad (11.4)$$

where *α=S/R.*

The solution to this problem is α=0.25*.

A different constraint

- The constraint requiring that downside risk be controlled according to equation (11.3) imposes an opportunity cost on the firm, since if more risk were taken, expected return would be higher.
- One way of assessing this opportunity cost is to allow the probability of losses to increase and recalculate the solution to the original problem.
- Suppose instead of (11.4) that we impose:

$$\Pr[(1+\alpha r) \geq 0.95] \geq 0.67 \qquad \text{and} \qquad 0 \leq \alpha \leq 1$$

- The solution to this problem is $\alpha^{*}=0.50$.

Methods of risk transfer

- Risk management often involves transferring risks to the agents best equipped to bear them.
- Risk transfer can make it possible for agents to undertake new risks that they would otherwise avoid, and hence improve an economy's resource allocation.
- Many attempts to manage risks involve selecting among forms of risk transfer according to a pre-selected criterion.
- Using the notion of a decision tree to represent the management problem, one can think of risk transfers as actions intended to affect the impacts of certain outcomes.

Hedging

- Hedging means eliminating the possibility of realizing either a gain or a loss.
- A hedge can be arranged either by selling the risky prospect to another party, or by buying an offsetting risky prospect.
- For example, suppose a decision maker has a long position in a random variable X that promises to pay \$4 with probability ½ and - \$2 with probability ½.
- If the decision maker now sells short the same random variable, that position becomes $X - X$, offering a certainty outcome of zero irrespective of the outcome of X.
- In such a situation the decision maker is said to be fully hedged against the risk.

Risk management

- The decision maker has, of course, eliminated the potential for either loss or gain in this example.
- If the offsetting short position is to be arranged through a market transaction, the hedger must be able to find a suitable counterparty—say, a speculator who might assume a long position in *X*.
- Hedging is a form of risk management that involves risk sharing.

Insuring

- Insuring means reducing the probability of one or more downside outcomes by buying insurance protection.
- The price paid for the protection is referred to as an insurance premium.
- Upside outcomes are not usually affected by the purchase of insurance.
- To see the difference between hedging and insuring, consider a variant on the above example.
- Suppose it is now possible to purchase, for price p, an insurance contract P that allows the insured to sell to the insurance company the variable X at a price of $1.5.

Example

- The decision maker's payoffs, exclusive of the insurance premium *p*, are then:

Instrument	X	P	$X + P$
First outcome	\$4.0	\$0.0	\$4.0
Second outcome	−\$2.0	\$1.5	−\$0.5

- The decision maker's loss exposure has now been reduced from \$2 to \$0.5, by paying a price of *p*, so that if things turn out badly, the decision maker's total loss is \$0.5 + *p*.
- On the other hand, the decision maker's gross gain remains at \$4, and the net gain including the insurance premium is \$4 - *p*.

Diversifying

- Diversifying means combining different prospects in ways designed to reduce down-side risks.
- To see how diversification can lower risk in relation to return, consider investing in just two financial instruments, *X* and *Y*.

Joint probabilities and returns

- Denoting realized returns on the two financial instruments by r_X and r_Y, Table 11.1 shows the joint probabilities:

Table 11.1
Joint Probabilities of Returns

	r_Y		
r_X	*4%*	*7%*	*10%*
1%	1/9	1/9	1/9
3%	1/9	1/9	1/9
5%	1/9	1/9	1/9

The expected return

- The expected return on either financial instrument in Table 11.1 is given by the sum of the outcomes multiplied by the probability of realizing each possible outcome.
- The probabilities of the outcomes of r_X are given by the row sums of the joint probabilities, while the probabilities of the outcomes of r_Y are given by the column sums of the joint probabilities.
- Thus,

$$E(r_X) = (1/3)(0.01) + (1/3)(0.03) + (1/3)(0.05) = 0.03$$

and

$$E(r_Y) = 0.07.$$

The variance of returns

- The variance of returns are both measures of how dispersed returns can be—the greater the dispersion, the greater the variance.
- The variance is defined as the expected value of the square of the differences between outcomes and their mean:

$$var(r_X) = \sigma^2(r_X) = E[r_X - E(r_X)]^2.$$

- For example, letting $\sigma^2(r_X)$ denote the variance of return on financial instrument X,

$$\sigma^2(r_X) = E[r_X - E(r_X)]^2$$

$$= (1/3)(0.01 - 0.03)^2 + (1/3)(0.03 - 0.03)^2 + (1/3)(0.05 - 0.03)^2$$

$$= 0.000267.$$

The standard deviation of return

- The square root of variance is the standard deviation.
- Note that the standard deviation of return on financial instrument *X* is

$$\sigma(r_X) = (0.000267)^{1/2}.$$

- Similarly

$$\sigma(r_Y) = (0.0006)^{1/2}.$$

Covariance

- The covariance between r_X and r_Y, denoted by $cov(r_X , r_Y)$, is a measure of the statistical association between the returns on the two financial instrument.
- Covariance is defined as:

$$cov(r_X, r_Y) = E(r_X r_Y) – E(r_X)E(r_Y).$$

- In the present example this covariance is equal to zero because the two returns for the financial instruments are distributed independently.
- You can see the returns are statistically independent by noting that regardless of which outcome you consider for r_X, the probabilities of the three outcomes for r_Y are all equal.

Expected utility

- An intuitive way to characterize a risk averter's behavior is to say that downside risk is regarded more seriously by a risk averter than an equal upside potential is valued, meaning that the risk-averse investor requires to be compensated if she is to accept risk.
- Consider the lottery X: payoff of \$10 and - \$10 with equal probability.
- This lottery's mean is zero.

Certainty equivalent value

- One can calculate the certainty equivalent value for this lottery, which is defined as an amount of cash, paid or received with certainty, that the decision maker regards as just equal to the value of the lottery.
- Hence, a risk-averse investor's certainty equivalent value for lottery *X* will be negative.
- In the present example (continuing with the practice of not including dollar signs in the utility function),

$$U(c) \equiv E[u(X)] = (1/2)u(10) + (1/2)u(-10) \leq u(0)$$

where *c* is the certainty equivalent value of the lottery and "≡" means "is defined to be."

Key points 1

- The environments in which financial decisions are taken can be viewed as either uncertain or as risky.
- Uncertainty refers to an environment that is difficult to specify, even qualitatively.
- In contrast to decision making under uncertainty, decision making under risk allows taking advantage of quantitative problem formulations.
- Nearly all of the models used in financial economics assume that decisions are taken under risk.
- A decision maker facing risk may strive to maximize the expected utility of final possible outcomes.

Key points 2

- Many financial decision problems focus on transferring risk.
- There are three main types of risk transfer—hedging, insurance, and diversification.
- Hedging involves eliminating the possibility of making either gains or losses.
- Insurance involves paying premiums to eliminate certain downside outcomes.
- Diversification involves combining different prospects in ways designed to reduce downside risks without selling them off.