ECON2103: Financial Economics
Lecture 3

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This week's topics

- Dividend
- The firm's value
- A valuation model
- A growth firm
- Constant dividend growth
- The value of a share
- The Gordon growth model
- Price-earnings ratio
- Capital gain
- A sustained dividend growth model
- The geometric mean rate of interest
- The instrument's average yield to maturity
- Market value
- Share price determination
- Valuing the growth firm

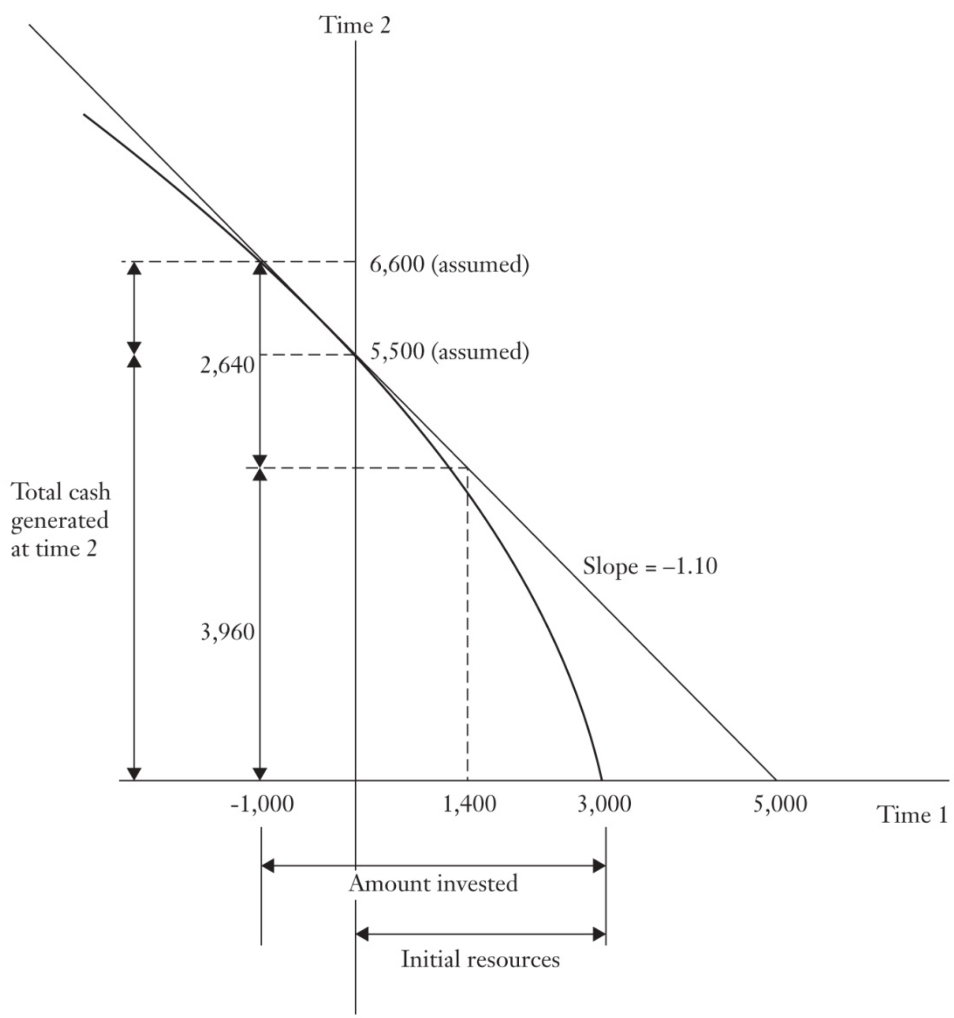
Dividend

- At any point in time, the value of a firm to its stockholders is based on the firm's capacity to generate investor returns.
- This value may be determined by capitalizing either the dividends or the future earnings to which the original stockholders are entitled.
- Dividends are cash payments made by a corporation to its owners and the determination of their amount, if any, is solely at the discretion of the corporation's board of directors.

The size of dividend and the firm's value

- Consider the situation, shown in Figure 4.1, of a firm with initial resources equal to $3,000.
- That amount is assumed to be generated from previous operations. Optimal operation of the firm, according to management's estimates, requires an investment of $4,000, so the firm needs $1,000 in additional capital.
- The firm investment creates $6,600 in time 2 cash flow that can be distributed to the various parties financing the operation (which we assume are only new stockholders).
- We assume the market rate of return required by investors is 10%.

Figure 4.1
First Step in Establishing that Earnings and Dividend Approaches to Valuing the Firm are Equivalent in a Perfect Capital Market



Example: The firm's value

- The value of the firm is equal to \$5,000 because

Initial resources	\$3,000
Optimal time 1 investment required	\$4,000
Cash generated at time 2 by investment	\$6,600
Market rate of return required	10%

And thus we compute:

$$3,000 + \frac{6,600}{1.10} - 4,000 = \$5,000$$

A different approach 1

- With no dividend being paid,

All initial resources invested	\$3,000
Funds raised in the market at a cost of 10%	\$1,000
Cost of funds obtained in the market (0.1 X \$1,000)	\$100
Available to original stockholders at time 2 (\$6,600 - \$1,000 - \$100)	\$5,500

And thus we compute \$5,500/1.1 = \$5,000.

A different approach 2

- When \$1,400 dividend is paid at time 1, initial resources invested by original stockholders is \$3,000 - \$1,400 = \$1,600.
- Suppose that funds raised in the market is \$2,400.
- Cost of funds obtained in the market is \$2,400 X 10% = \$240.
- The available amount to original stock holders from time 2 flows is \$6,600 - \$2,640 = \$3,960.
- In summary, for the original stockholders, the case flow in this case is \$1,400 at time 1, which is the assumed dividend payment, and \$3,960 at time 2.
- Discounting the time 1 cash flow of \$3,960 at 10% and adding that to the \$1,400 time 1 cash flow gives a present value of \$5,000.

A valuation model based on earnings

- In a perfect capital market, the equilibrium price of a share at time 1 obeys the following condition:

$$s(1) = \frac{d(2) + s(2)}{1 + r} \qquad (4.1)$$

where *s(t)* is the ex-dividend price of a share, *d(t)* is the dividend accruing to stockholders of record at time *t-1 (t = 2,3),* and *r* is the market interest rate from investing between times 1 and 2.

More computation

- Let *N(1)* be the number of shares outstanding at *t = 1*.
- Then the market value of the firm at time 1, denoted by *V(1)*, assuming all funds are financed by the sale of common stock, is:

$$V(1) = N(1)s(1) = \frac{N(1)d(2) + N(1)s(2)}{1+r} \qquad (4.2)$$

- At time 2 the value of the firm, denoted by *V(2)*, is:

$$V(2) = N(2)s(2) = [N(1) + M(2)]s(2)$$

where *M(2)* refers to the number of new shares (if any) issued at time 2.

Computation continued

- Therefore, by substitution we can write:

$$V(1) = \frac{N(1)d(2) + V(2) - M(2)s(2)}{1+r} \qquad (4.3)$$

- But *N(1)d(2) = D(2)* is the entire time 2 dividend.
- Moreover, the firm either invests funds or pays them out as dividends.
- Thus $$M(2)s(2) + X(2) = D(2) + I(2) \qquad (4.4)$$
- Finally

$$V(1) = \frac{D(2) + V(2)X(2) - D(2) - I(2)}{1+r}$$

$$= \frac{X(2) - I(2) + V(2)}{1+r} \qquad (4.6)$$

Equation (4.7)

- Using equation (4.6) for all *t*, assuming each period's market interest rate r is the same, and proceeding to eliminate *V(2), V(3)*, . . . by successive substitution, we obtain:

$$V(1) = \sum_{t=1}^{\infty} \frac{X(t+1) - I(t+1)}{(1+r)^t} \qquad (4.7)$$

where *X(t)* is net cash earnings at time *t* and *I(t)* is cost of new investments at time *t*.

- Cash earnings *X(t)* account for all earnings, both existing and those resulting from new investments, while the *I(t)* reflect the costs of adopting the new investments.

The meaning of a growth firm

- A growth firm is one in which the expanding assets generate returns in excess of the market rate of interest.
- To see this more precisely, consider the time 1 value of the original stock- holders' investment in a firm which will exist for just one more period, so that it is valued as follows:

$$V^0(1) = \frac{X^0(2)}{1+r} \qquad (4.9)$$

where $V^0(1)$ is the market value of existing operations at time 1, $X^0(2)$ is the earnings from existing operations, including disposal value, if any, of assets at time 2, and r is the market interest rate between times 1 and 2.

A growth firm continued

- Equation (4.9) can be rewritten as follows:

$$V(1) = \frac{X(2)}{1+r} - I(1) = \frac{X^0(2) + I(1)(1+r^*)}{1+r} - I(1)$$

or

$$V(1) = \frac{X^0(2) + I(1)(r^* - r)}{1+r} \qquad (4.10)$$

Constant dividend growth

- We now examine the case of a firm whose assets grow continuously, earning rates of return in excess of the market, with the result that management increases dividends at the same rate.
- For analytic simplicity, we consider a firm with earnings and dividends that grow at a constant rate for all future time periods.

The value of a share

- The value of a share is given in this case by the following:

$$s(1) = \sum_{t=2}^{\infty} \frac{d(2)(1+g)^{t-2}}{(1+r)^{t-1}} \qquad (4.11)$$

where *s(1)* is the ex-dividend share price at time 1, *d(2)* is the dividend paid at time 2 to a stockholder of record at time 1, *g* is the rate of growth of dividend, and *r* is the market rate of interest.

The Gordon growth model

- Equation (4.11) can be rewritten as:

$$s(1) = \frac{d(2)}{1+g} \sum_{t=2}^{\infty} \left(\frac{1+g}{1+r} \right)^{t-1}$$

- Summing the infinite series, we obtain:

$$s(1) = \frac{d(2)}{1+g} \left(\frac{1}{1-[(1+g)/(1+r)]} - 1 \right)$$

- Then we obtain

$$s(1) = \frac{d(2)}{r-g} \qquad (4.12)$$

- The model given by (4.12) is called the Gordon growth model.

Price-earnings ratio

- The price-earning ratio, denoted by *P/E*, is the ratio of the current market price of the stock to the firm's earnings per share of common stock.
- Price-earnings ratios for firms with constantly growing dividends may conveniently be expressed as

$$s(1) = \frac{ke(2)}{r-g}$$

where *k* refers to the proportion of earnings paid as dividends (payout ratio) and *e(2)* to time 2 earnings per share.

- Then we obtain $$\frac{P}{E} = \frac{s(1)}{e(2)} = \frac{ke(2)}{e(2)(r-g)} = \frac{k}{r-g} \qquad (4.14)$$

Example: (4.14)

- Suppose that: $r = 20\%$, $k = 30\%$, $e(2) = \$10$, *and* $s(1) = \$100$.
- What rate must the earnings be growing for the price-earnings ratio of 10 to be obtained?
- By (4.14), $g = 0.17 = 17\%$ because

$$P/E = \$100/\$10 = 10 = 0.3/(0.2-g).$$

- Observe that with an earnings growth rate of 17%, e(3) will be \$11.70, and hence, s(2) will be \$117.
- The dividend d(2) will be 30% of \$10 or \$3.
- Hence, for \$100 invested at time 1, one obtains \$117 + \$3 = \$120 at time 2, for a return of 20%.

Example continued

- In the perfect capital market now being considered, the equilibrium return on investment will be the same for all firms, even though price-earnings ratios for two stocks differ substantially.
- Suppose that: $r = 20\%$, $k = 30\%$, $e(2) = \$10$, and $s(1) = \$200$.
- By (4.14), $g = 0.185 = 18.5\%$ because

$$P/E = \$200/\$10 = 20 = 0.3/(0.2-g).$$

Are low P/E stocks underpriced?

- Low *P/E* stocks are not necessarily underpriced.
- In a perfect capital market they cannot be cheap.
- Suppose that: $r = 20\%$, $k = 30\%$, $e(2) = \$10$, *and* $s(1) = \$20$.
- By (4.14), $g = 0.05 = 5\%$ because

$$P/E = \$20/\$10 = 2 = 0.3/(0.2-g).$$

Capital gains and dividends

- Let's now consider relations between capital gains and income earned on the shares of the kinds of firms we have been examining.
- In the first instance, we show that even a firm that does not grow can issue shares yielding capital gains.
- Consider a firm that will pay just one dividend and that dividend is paid at time 3.
- The values of a share at times 1 and 2, assuming that the market rate of interest is constant, are

$$s(1) = \frac{d(3)}{(1+r)^2} \ (at\, time\, 1); \qquad s(2) = \frac{d(3)}{(1+r)} (at\, time\, 2).$$

A sustained dividend growth model

- We next consider a sustained dividend growth model.
- In the infinite stream model with a constant payout ratio [equation (4.13)], we have:

$$k = \frac{d(2)}{e(2)} = \frac{d(3)}{e(3)} = \frac{d(2)(1+g)}{e(2)(1+x)}$$

where *g* is the dividend growth rate and *x* is the earnings growth rate.

- Assuming *x* = *g*, we obtain

$$s(1) = \frac{ke(2)}{r-g} \quad and \quad s(2) = \frac{ke(3)}{r-g} = \frac{ke(2)(1+g)}{r-g}.$$

A sustained dividend growth model continued

- Because

$$s(1) = \frac{d(2) + e(2)}{1 + r} \; and \; s(2) = s(1)(1 + g),$$

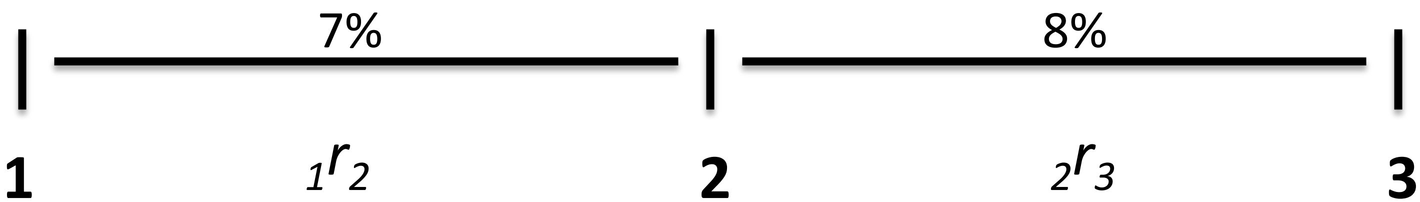
we obtain

$$\frac{d(2)}{gs(1)} = \frac{r - g}{g}.$$

- If dividends and capital gains are taxed at differential rates, the price-earnings ratios and equations for stock values of this chapter must be amended to take into account the different forms in which stockholders’ returns are received.

Returns on financial instruments outstanding for several periods

- Consider a simple numerical example with interest rates as shown in the following diagram:



- Suppose \$100 is invested on the terms indicated by the diagram for two periods (i.e., from time 1 to time 3). The value of the investment at the end of two periods will be:

$$\$100(1+{}_1r_2)(1+{}_2r_3)$$

which can be modified to

$$\$100(1+{}_1r_3)^2 \quad where \quad 1+{}_1r_3=[(1+{}_1r_2)(1+{}_2r_3)]^{1/2}$$

The geometric mean rate of interest

- Thus, ${}_1r_3$ is defined as the geometric mean rate of interest (or geometric average rate of interest) earned in each of the two periods.
- Then $1 + {}_1r_3 = [1.07(1.080)]^{1/2} = 1.07499.$
- The average interest rate earned over each of the two periods is 7.499%.
- This rate is frequently termed the instrument's average yield to maturity.

Market values and time horizons

- The foregoing notions can also be used to determine the market value of a firm at different points in time, and this allows us to consider decisions just in terms of their present effects plus those on market values one period hence.
- To see this, observe that a firm's market value can be determined in two ways.
- The first is by the stream of earnings accruing to the original stockholders,

$$MV(1) = \frac{\pi(1)}{1 +_1 r_2} + \frac{\pi(2)}{(1 +_1 r_2)(1 +_2 r_3)} + \frac{\pi(3)}{(1 +_1 r_2)(1 +_2 r_3)(1 +_3 r_4)} + \cdots$$

where *MV(1)* is the market value at the beginning of period 1 and *π(t)* is the net earnings for period.

The second way

- The second is by the discounted current earnings plus the period 1 equivalent of the firm's market value one period later,

$$MV(1) = \frac{\pi(1)}{1+{}_1r_2} + \frac{MV(2)}{1+{}_1r_2}$$

where *MV(2)* is the market value at the end of time period 2.

- The market value at time 2 is the time 2 value of all earnings accruing to the original stockholders from time 2 onwards.
- We can look at market value either in terms of an entire earnings stream or in terms of current earnings plus a future market value.

Application to share price determination

- Consider

	\$110		\$121		\$133.10	
1		2		3		4

- Let *r = 10%* over all three time periods (in our notation it would be ${}_1r_4$) and suppose that the firm has zero value after time 4.
- Note *s(1)=(1/50)MV(1).*
- Thus,

$$s(1)=\frac{1}{50}\left[\frac{110}{1.1}+\frac{121}{(1+r)^2}+\frac{133.10}{(1.1)^3}\right]=\frac{300}{50}=\$6$$

Calculation

- Therefore

$$s(1) = \frac{1}{50}\left[\frac{D(2)}{1.10} + \frac{MV(2)}{1.10}\right]$$

where *D(2)* is the dividend payment at time 2, distributed to stockholders of record at time 1.

- But,

$$MV(2) = \frac{121}{1.1} + \frac{133.10}{(1.1)^2} = \$220$$

- Finally

$$s(1) = \frac{1}{50}\left[100 + \frac{1}{1.1}(220)\right]$$

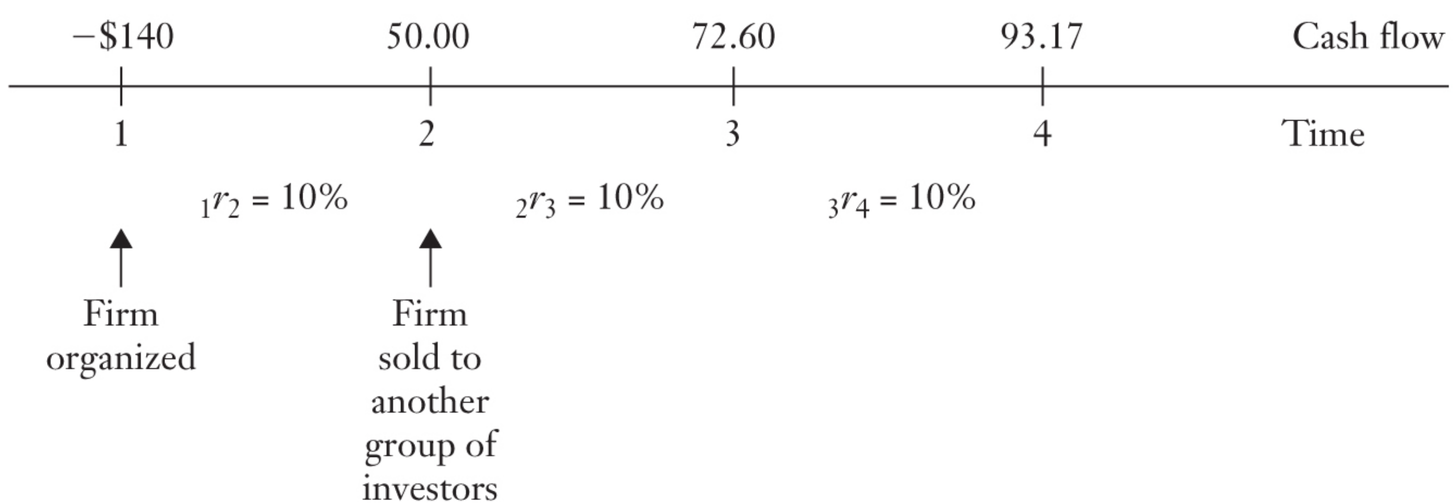
$$= \frac{d(2)}{1.1} + \frac{s(2)}{1.1}$$

where *d(2)* is the time 2 dividend to a single share and *s(2)* is the price of the share.

Applications to valuing the growing firm

- Consider the following situation

FIGURE 4.2
VALUING THE GROWING FIRM



Value of the firm

- Then

TABLE 4.1
VALUES OF THE FIRM AT TIMES 1 THROUGH 4

Time	*Market value of incomes not yet received*[a]
1	\$50.00 + \$60.00 + \$70.00 = \$180.00
2	\$55.00 + \$66.00 + \$77.00 = \$198.00
3	\$72.60 + \$84.70 = \$157.30
4	\$93.17 = \$ 93.17

[a]Note that the data are obtained by appropriate discounting of the cash flows. For example, \$93.17/1.10 = \$84.70, and so on.

Additional example

- Since we have already, and will again, show that several time points can be incorporated in our analyses, the present example uses only two time points so as to focus more sharply on other matters.
- Suppose that a firm announces an investment of $2,200 at time 1, that the cash flow realized at time 2 on this investment will be $3,520, and that the investment will be financed by a new share issue.
- How many shares must be issued, and what will the market price of all shares be after the new issue?

Example continued

- Let us denote with a superscript 0 the values for the variables before the announcement of the new investment.
- Let's assume the following in our example:

$V^0(2)=\$11{,}000;\ r=0.10;\ N^0(1)=1{,}000$ *shares*; $s^0(1)=\$10$ *per share*

- After the announcement, we will then have:

$$V(2)=\$11{,}000+\$3{,}520=\$14{,}520$$

$$I(1)=\$2{,}200$$

where *I(1)* is the amount of the new investment.

The number of new shares

- Denoting the number of new shares as *M(1),* we write the time 1 value of each share outstanding after the new issue as:

$$\frac{V(2)}{1.10}\left[\frac{1}{N^0(1)+M(1)}\right]$$

- Therefore,

$$I(1)=\frac{V(2)}{1.10}\left[\frac{M(1)}{N^0(1)+M(1)}\right]$$

or

$$\$2,200=\frac{\$14,520}{1.10}\left[\frac{M(1)}{N^0(1)+M(1)}\right]$$

so that

$$M(1)=200$$

What is the issue price?

- All shares must earn the market rate of interest from period 1 to period 2.
- Therefore,

$$V(1)=V(2)/1.1=\$13,200 \text{ and } s(1)=\$13,200/\$1,200=11$$

Key points 1

- When management follows the market value rule, the value of the firm is equal to the present value of cash flows to be generated by the firm's existing assets plus the present value of cash flows to be generated from future growth opportunities.
- The firm's value depends only on its future earnings stream and not on the source of funds needed to finance creation of the earnings stream.
- A growth firm is one in which the expanding assets generate returns in excess of the market rate of interest.

Key points 2

- The price-earnings ratio is the ratio of the current market price of the stock to the firm’s earnings per share.
- In a perfect capital market, financial instruments outstanding over several periods must have market prices such that they yield the market rates of interest prevailing in each time period.