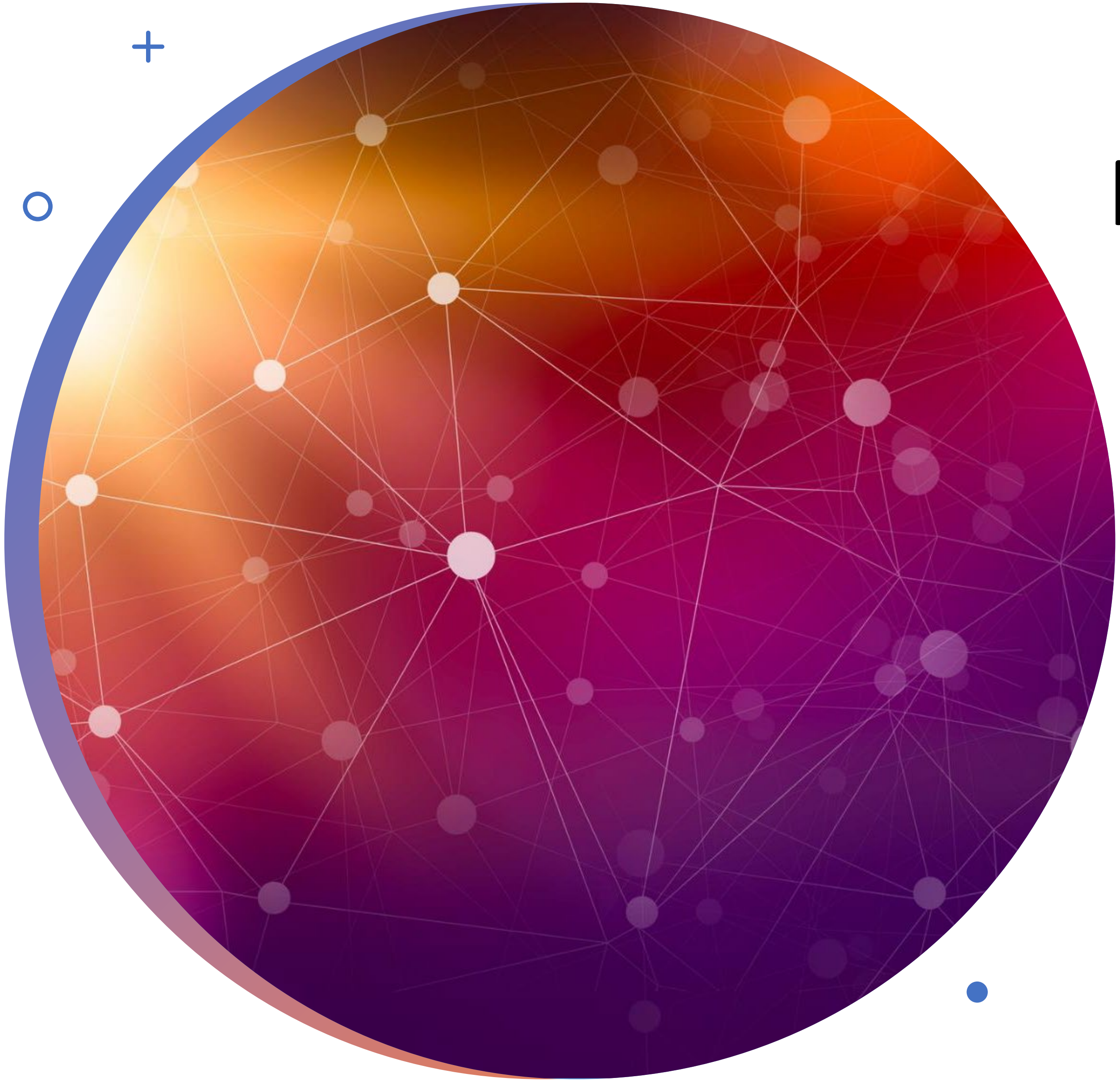

Financial Economics ECON2103

Lecture 6:

CAPM

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Introduction

- CAPM and risk management are important tools in the financial world, providing a structured approach to valuing assets and safeguarding investments against potential risks. Today we will learn these essential tools and explore how they are theoretically structured.

Agenda

Introduction

Topic one: Insurance, Hedging and Diversifying

Topic two: Decision Criteria & Target-based Approach

Topic three: CAPM

Topic four: Introduction to Efficient Market Hypothesis

Summary

Learning Objectives

- Explore risk management strategies.
- Examine the strategies of insurance, diversifying and hedging.
- Introduce a concept of decision criteria.
- Understand what CAPM is.
- Introduce Efficient Market Hypothesis

Topic one

Insurance, Hedging and Diversifying

Risk management (Review)

Risk management often involves transferring risks to the agents best equipped to bear them.

Risk transfer can make it possible for agents to undertake new risks that they would otherwise avoid, and hence improve an economy’s resource allocation.

Insurance (Review)

- Suppose that there are two possible outcomes for a project, of which each arises with probability 0.5.
- Suppose it is now possible to purchase, for price $\$c$, an insurance contract P such that the insured will receive payment $\$P$ from the insurance company contingent on an outcome.
- The price paid for the protection is referred to as an insurance premium.
- The decision maker's payoffs, exclusive of the insurance premium $\$c$, are given by:

Instrument	Gains (X)	Payment (P)	$X + P$
First outcome	\$4.0	\$0.0	\$4.0
Second outcome	-\$2.0	\$1.5	-\$0.5

Buy or not?

- Because both outcomes are equally likely, his expected payoff from buying the insurance is:

$$\left(\frac{1}{2}\right) \times (4 - c) - \left(\frac{1}{2}\right)(0.5 + c) = 1.75 - c.$$

- His expected payoff from not buying it is:

$$\left(\frac{1}{2}\right) \times 4 - \left(\frac{1}{2}\right) \times 2 = 1.$$

- If $1.75 - c > 1$, he should buy the insurance.

Expected Utility

- Suppose $u(w) = 2w - \frac{1}{2}w^2$ and $c = 1$.
- When buying insurance,
  - Payoff in the first outcome: \$4.0 – \$1.0 = \$3.0
  - Payoff in the second outcome: - \$0.5 - \$1.0 = - \$1.5
- Buying insurance:
  - $Eu(w, buy) = \frac{1}{2}\left(2 \cdot 3 - \frac{1}{2} \cdot 9\right) + \frac{1}{2}\left(2 \cdot (-1.5) - \frac{1}{2} \cdot (-1.5)^2\right) = -\frac{5.25}{4}.$
- Not buying insurance
  - $Eu(w, not\ buy) = \frac{1}{2}\left(2 \cdot 4 - \frac{1}{2} \cdot 16\right) + \frac{1}{2}\left(2 \cdot (-2) - \frac{1}{2} \cdot (-2)^2\right) = -3.$
- You would buy the insurance.

Insurance: Variation

- Suppose the initial wealth is \$20.
- Write wealth level in each outcome $i = 1,2$ as w_i.
- Suppose $u(w) = \log w$ and *c=1*.
- The option is to purchase *h* units of insurance.

Instrument	Gains (X)	Payment (P)	$X + P$	Wealth (w)
First outcome	\$4.0	\$0.0	\$4.0	$w_1 = 20 + 4 - h$
Second outcome	-\$20.0	\$1.5h	-\$20+1.5h	$w_2 = 20 - 20 + 1.5h - h$

- The expected payoff is

$$0.5 \cdot \log(20 + 4 - h) + 0.5 \cdot \log(20 - 20 + 1.5 \cdot h - h)$$

- Note the derivative of $a \cdot \log(b + k \cdot h)$ w.r.t. h is $\frac{a \cdot k}{b + k \cdot h}$.

Review from ECON1050 (recommended pre-requisite):

$$g(x) = \log x \implies g'(x) = \frac{1}{x}$$

$$F(x) = g(f(x)) \implies F'(x) = g'\big(f(x)\big) f'(x)$$

- The expected payoff is

$$0.5 \cdot \log(24 - h) + 0.5 \cdot \log(0.5 \cdot h)$$

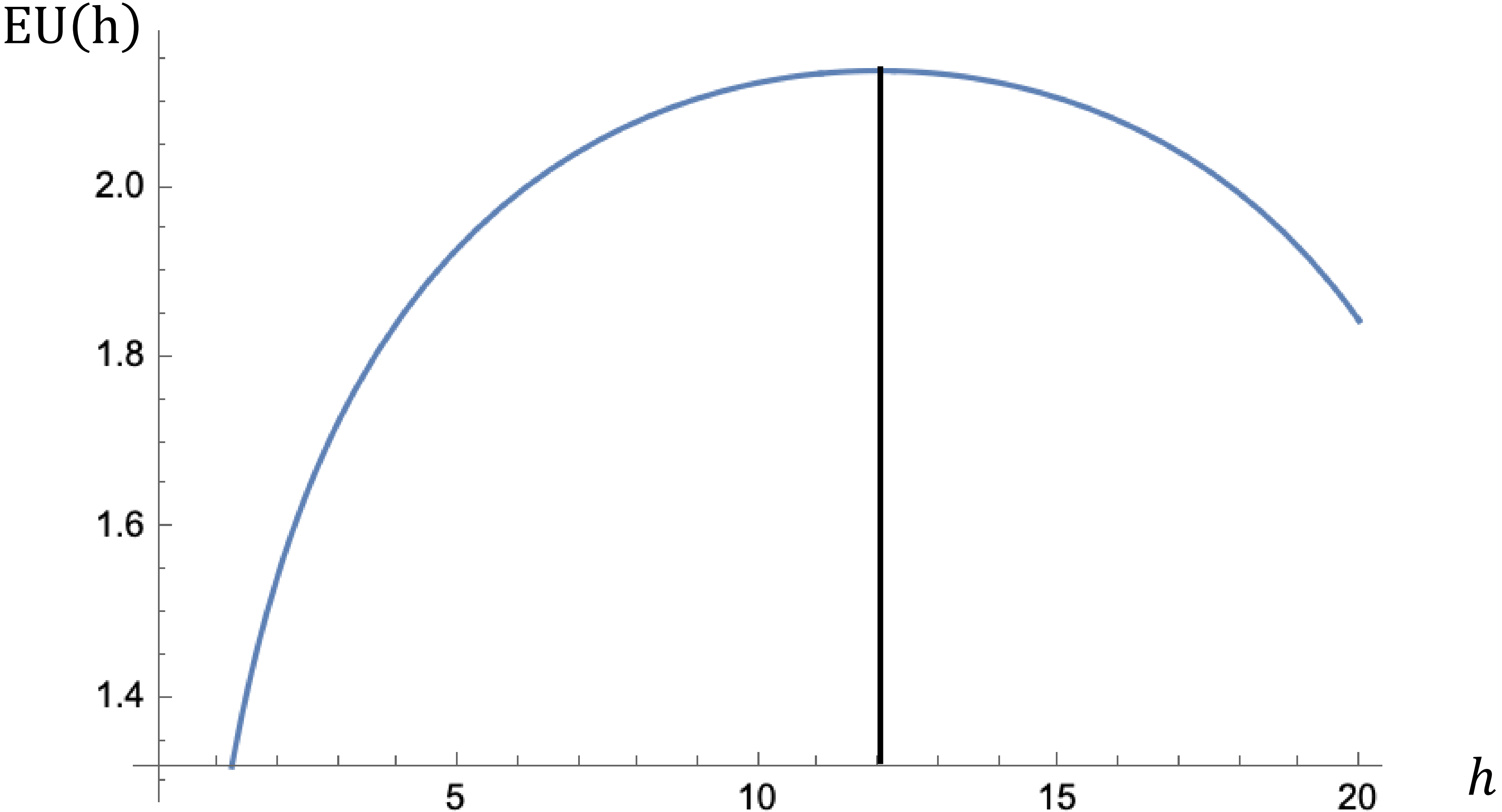
- The maximization w.r.t. h gives

$$\frac{0.5 \cdot (-1)}{24 - h} + \frac{0.5 \cdot (0.5)}{0.5h} = 0$$

- Thus,

$$h = 12$$

$$\mathrm{EU(h)} = 0.5 \cdot \log(24 - h) + 0.5 \cdot \log(0.5 \cdot h)$$



Strategies: Risk Management

- Insurance involves paying premiums to eliminate certain downside outcomes.
- Hedging involves eliminating the possibility of making either gains or losses.
- Diversification involves combining different prospects in ways designed to reduce downside risks without selling them off.

Hedging

- Hedging means eliminating the possibility of realizing either a gain or a loss.
- A hedge can be arranged either by selling the risky prospect to another party, or by buying an offsetting risky prospect.
- For example, suppose a decision maker has a long position in a security for *X* shares that promises to pay *$4* with probability ½ and - *$2* with probability ½.
- If the decision maker now sells short the same security, that position becomes *X* - *X*, offering a certainty outcome of *zero* irrespective of the outcome.
- In such a situation the decision maker is said to be fully hedged against the risk.

Diversifying

- Diversifying means combining different prospects in ways designed to reduce down-side risks.
- To see how diversification can lower risk in relation to return, consider investing in just two financial instruments, *X* and *Y*.
- Suppose the following payoff schedule of a share price:

	State 1 (0.5 chance)	State 2 (0.5 chance)
Stock X	\$10	\$2
Stock Y	\$2	\$10

- Suppose that both are traded at \$5.
- In the worst case, both stocks just give \$2.
- By purchasing 0.5 share for each, the payoff is \$6 in either state.

Q1: Investment

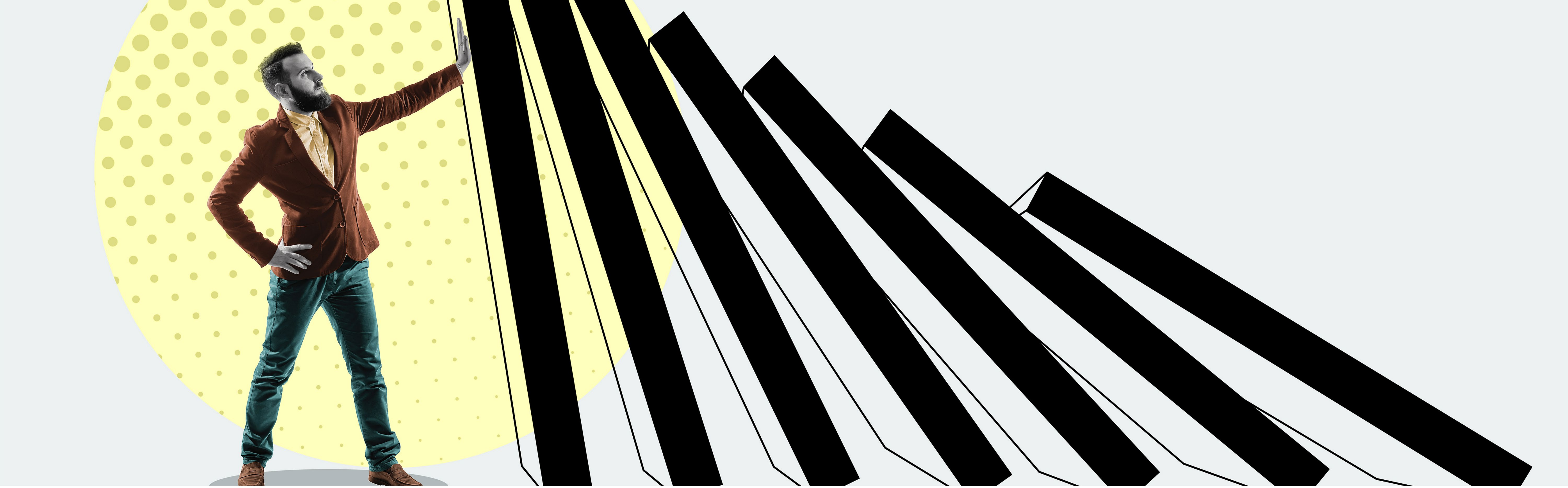
- Suppose you are a business that regularly imports goods from another country, and you need to pay your supplier in their local currency.
- The current exchange rate is 1 USD = 1.5 AUD.
- You need to pay 100,000 USD to your supplier.
- If the exchange rate worsens to 1 USD = 1.6 AUD, you would now need to pay 100,000 USD*1.6 = 160,000 AUD, resulting in higher costs for your business.

Q1: Forward Contract

- To mitigate the risk, you decide to use a financial instrument called a forward contract.
- You enter into a forward contract to buy 100,000 USD in three months at the current exchange rate of 1 USD = 1.5 AUD.
- Regardless of the future exchange rate, you are obligated to buy the agreed-upon amount of USD at the agreed-upon rate.

Q1: Is this hedging or diversifying?

1. Hedging
2. Diversifying



Q1: Hedging

- If the exchange rate worsens to 1 USD = 1.6 AUD, you still get to buy the USD at the agreed-upon rate of 1.5, saving money compared to the spot rate.
- You pay 100,000 USD * 1.5 = 150,000 USD, effectively hedging against the adverse movement in the exchange rate.

Q2: Investment Portfolio

- Suppose you have $50,000 that you want to invest.
- Scheme 1
    - You decide to invest the entire $50,000 in a single stock (Stock A).
    - Initial Investment in Stock A: $50,000
- Scheme 2
    - You decide to invest into two different assets: stock and bond.
    - Allocate $30,000 to Stock A.
    - Allocate $20,000 to a bond.

Q2: Is this hedging or diversifying?

1. Hedging
2. Diversifying

Q2: Diversifying

- Scheme 1

  If the stock performs well, you make a profit; if it performs poorly, you incur losses.

- Scheme 2

  If Stock A performs poorly, the impact on your overall portfolio is mitigated by the potential positive performance of the bond.

Topic two

Investment Decision & Target-based Criteria

Target-based approach

- Decision criteria helps manage certain aspects of a risk-management problem.
  - Wealth-based criteria emphasize the distribution of a decision maker’s final wealth.
  - Dispersion-based criteria emphasize the spread of a random variable.
- Target-based approaches emphasizes risk aversion through attempts to avoid downside risk (unfavourable outcomes).
- Suppose the management of a firm is trying to allocate liquid assets with amount R to two accounts, one of which is riskless but pays no interest, while the other offers a risky return r.
- Let S denote the amount invested in the risky account.
- The value of invested resources next period will be: $(R - S) + S(1 + r)$.

Formal statement of the problem

- Suppose management would like to make the next period investment value as large as possible but subject to the condition that $R + Sr$ not fall below *95% of the original value of* R more than *25% of the time*.
- Management wishes to

$$\max_S E[R + Sr] \quad = \max_S E\left[R\left(1 + \frac{S}{R}r\right)\right] \quad (11.3)$$

subject to $(\Pr[R + Sr \le 0.95R] \le 0.25)$

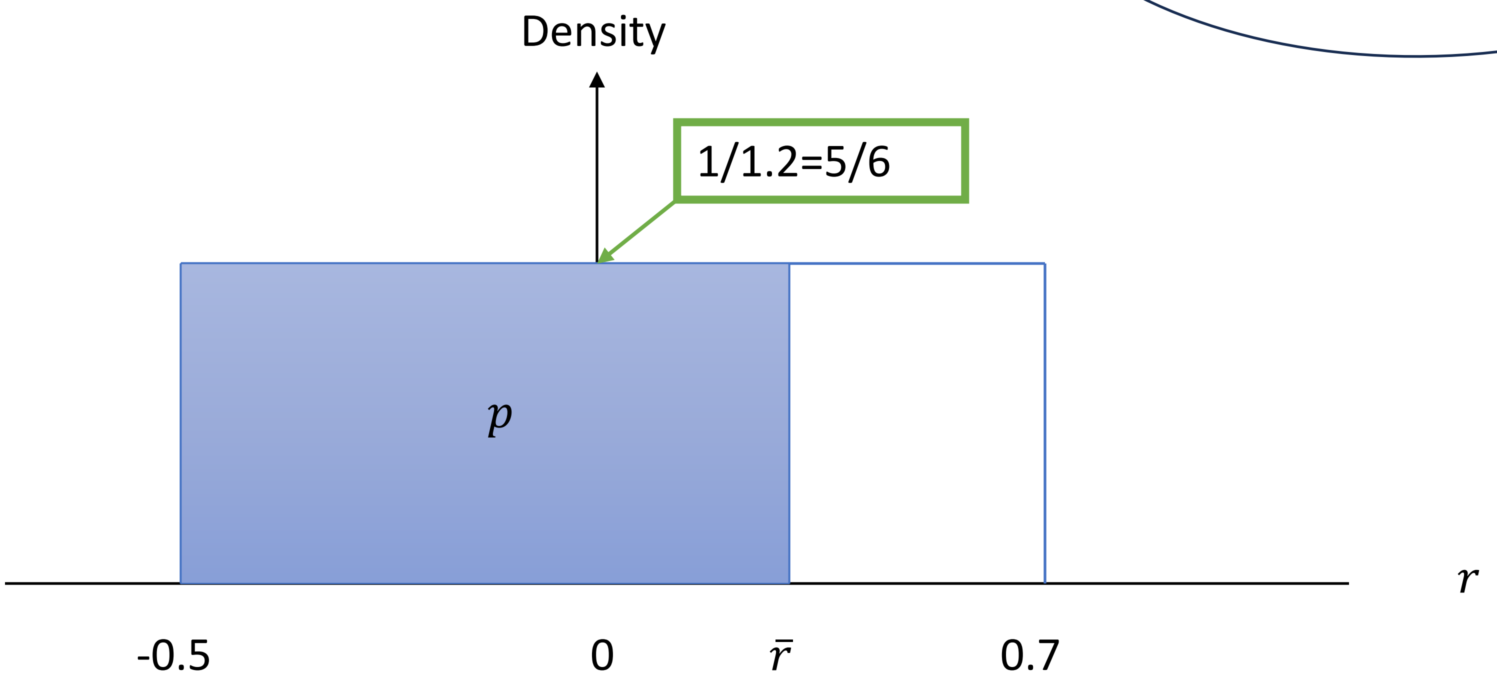
$$\Pr[R + Sr \ge 0.95R] \ge 0.75 \text{ and } 0 \le S \le R$$

where $\Pr$ means cumulative probability.

Assume the rate of return r on the second account is uniformly distributed over the range [-0.5, 0.7].

- $\Pr[r \leq \bar{r}] = p$: Cumulative probability

The probability that r is at most $\bar{r}$ is given by the colored area p.



A simpler form

- Equation (11.3) can be rewritten as

$$\max_{\alpha} E(1+\alpha r)$$

subject to

$$\Pr[(1+\alpha r) \geq 0.95] \geq 0.75 \; and \; 0 \leq \alpha \leq 1 \quad (11.4)$$

where $\alpha = S/R$.

- $\Pr[(1+\alpha r) \geq 0.95] \geq 0.75$ implies $\Pr[\alpha r \geq -0.05] \geq 0.75$.

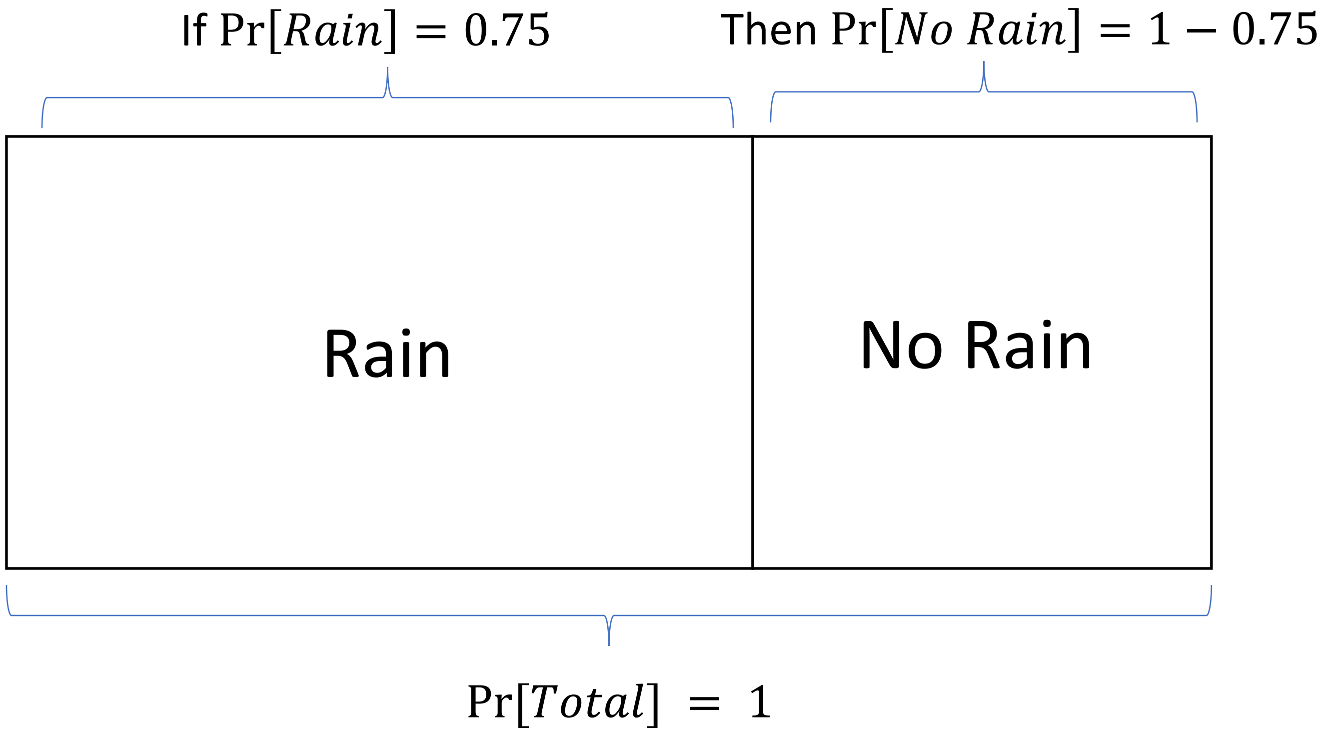
Q3: Probability

- Suppose that the probability of raining tomorrow is at least 75%.
- What is the probability of not-raining tomorrow at most?

1. 25%
2. 50%
3. 75%
4. 100%

Q3: 1. 25%

- $\Pr[Rain] \geq 0.75$ implies $\Pr[No\ Rain] \leq 1 - 0.75 = 0.25$



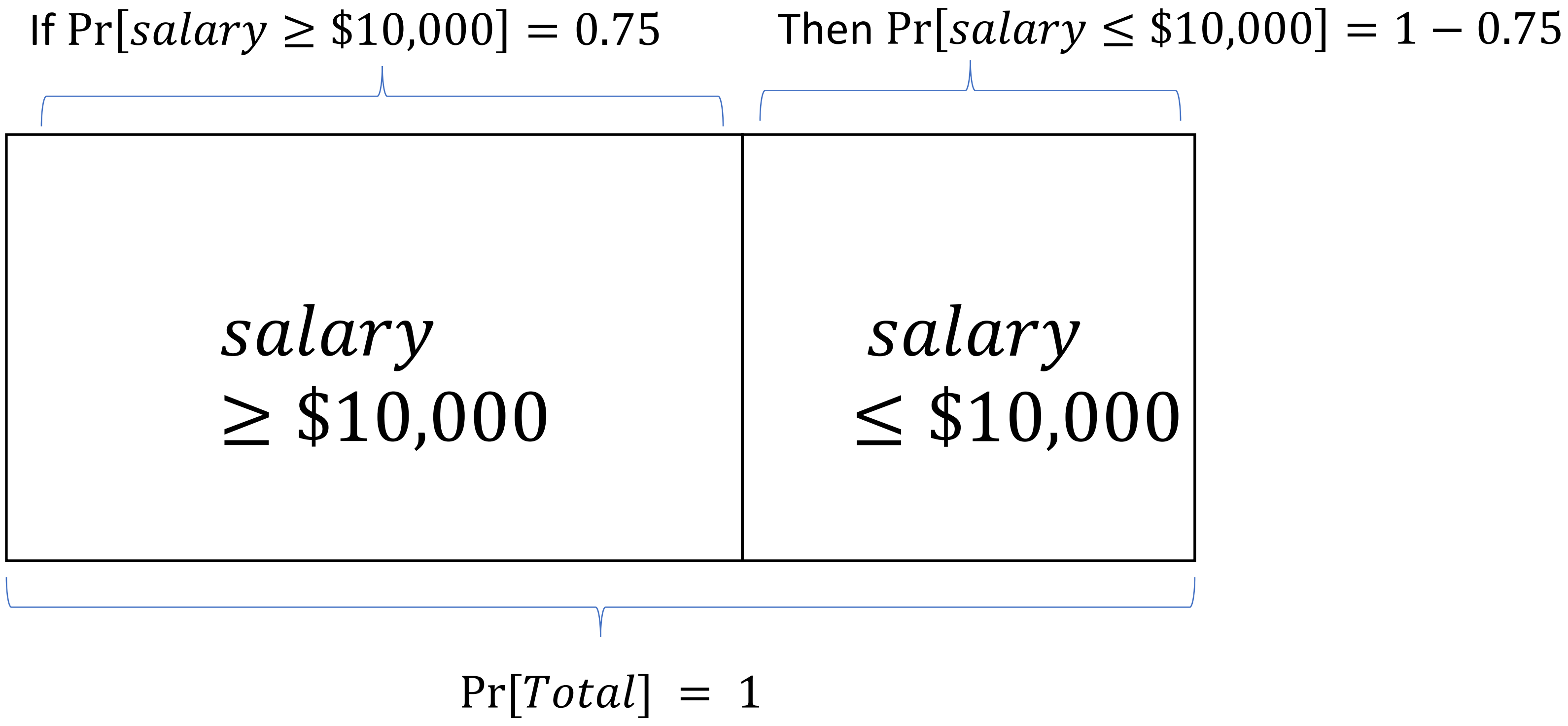
Q4: Continuous Case--Probability

- Suppose that the probability that your salary is higher than \$10,000 is at least 75%.
- What is the probability that your salary is lower than \$10,000 at most?

1. 25%
2. 50%
3. 75%
4. 100%

Q4: 1. 25%

- $\Pr[salary \geq \$10{,}000] \geq 0.75$ implies $\Pr[salary \leq \$10{,}000] \leq 1 - 0.75 = 0.25$



Mathematical Description

- We have started with

$$\Pr[(1 + \alpha r) \leq 0.95] \leq 0.25 \text{ and } 0 \leq \alpha \leq 1 \quad (11.4)$$

- Then, $\Pr[\alpha r \geq -0.05] \geq 0.75$ which implies

$$\Pr[\alpha r \leq -0.05] \leq 1 - 0.75 = 0.25.$$

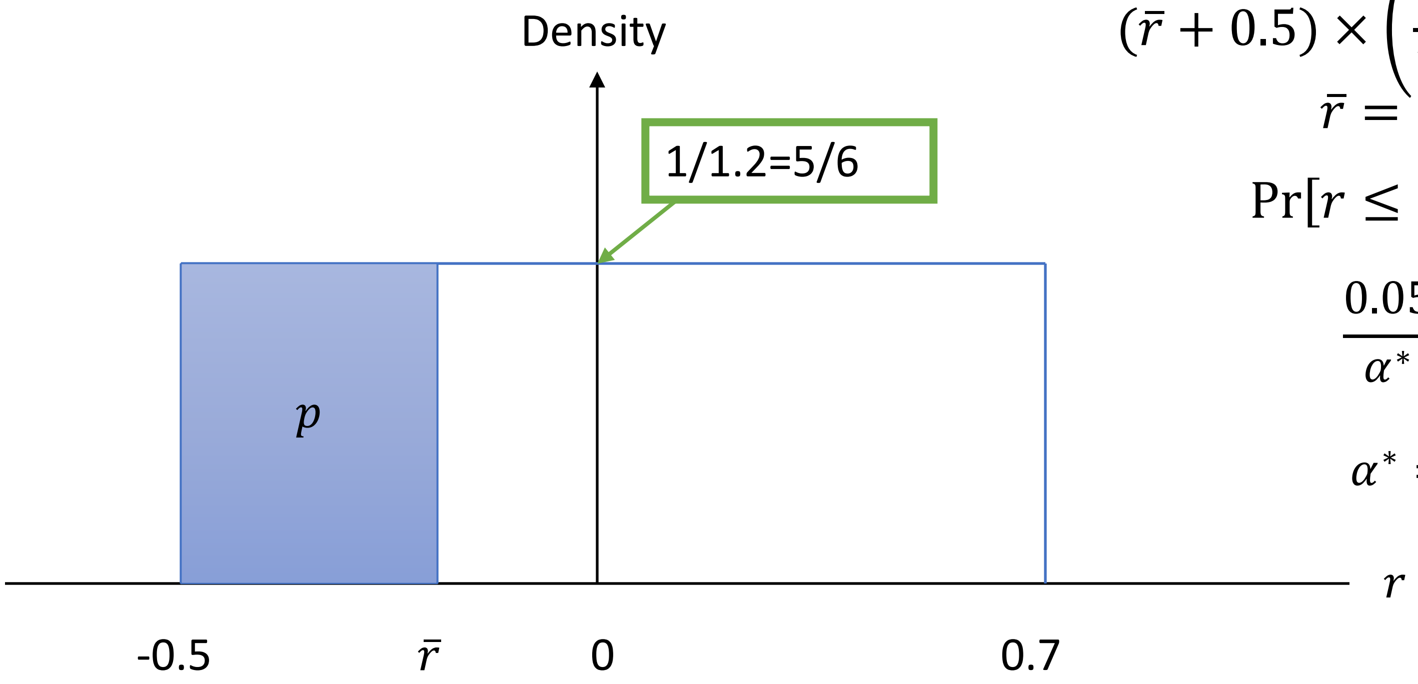
- Then,

$$\Pr\left[r \leq -\frac{0.05}{\alpha}\right] \leq 0.25$$

$$\Pr\left[r \le -\frac{0.05}{\alpha}\right] \le 0.25$$

When $\Pr\left[r \le -\frac{0.05}{\alpha}\right] = 0.25$, what is α?

When $\Pr[r \le \bar{r}] = 0.25$, what is $\bar{r}$?



$$(\bar{r} + 0.5) \times \left(\frac{1}{1.2}\right) = 0.25$$

$$\bar{r} = -0.2$$

$$\Pr[r \le -0.2] = 0.25$$

$$\frac{0.05}{\alpha^*} = 0.2$$

$$\alpha^* = 0.25$$

$\Pr\left[r \le -\frac{0.05}{\alpha} \le -\frac{0.05}{\alpha^*} = -0.2\right] \le 0.25$ for all $\alpha \le \alpha^*$ (As α decreases, r decreases.)

Math Practice Questions (will be reviewed in Tutorial 6)

A “random variable” Y is distributed as $\Pr[Y \leq x] = f(x)$ where f is uniform in $[a, b]$.

1. When $[a, b] = [-1,1]$, what is the probability that Y is smaller than ½?

In the following, let $[a, b] = [0,2]$.

2. What is the probability that Y is greater than 1.5?
3. If the probability that Y is smaller than x is 0.8, what is x?
4. If the probability that Y is smaller than x is 0.8, what is x at most?
5. If the probability that Y is greater than x is 0.5, what is x at least?
6. When $\Pr[Y \leq x + 1] \geq 0.2$, what is x at least?
7. When $\Pr[Y \leq x + 1] \leq 0.3$, what is x at most?

Answers:

1. $3/4$;
2. $1/4$;
3. 1.6;
4. 1.6;
5. 1;
6. -0.6;
7. -0.4.

Topic three

CAPM

Capital Asset Pricing Model (CAPM)

- CAPM allows us to determine the required rate of return for any risky asset.
- If we know risk, then we can find the appropriate cost of capital.
- If we know cost of capital, then we can find the NPV of a project.
- If we know NPV, then we can find the value of a project.
- All investors choose a mixture of the risk-free asset and the market portfolio (*m*).
  - A market portfolio is a theoretical bundle of investments that includes every type of asset available in the investment universe, with each asset weighted in proportion to its total presence in the market.

CAPM Formula: $E(r_p) = r_f + \beta_p (E(r_m) - r_f)$

$E(r_p)$: Expected Return of security p

r_f : Risk-free return rate

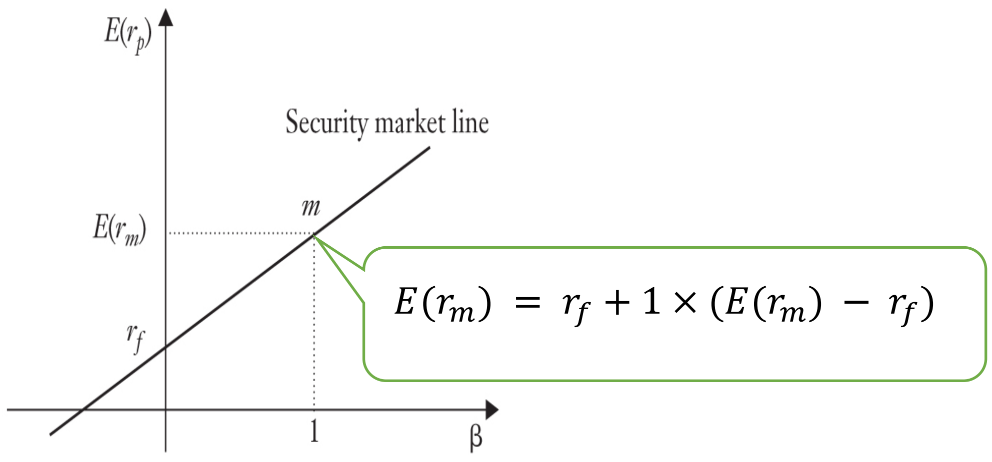
$E(r_m)$: Average return on all securities

β_p: The security's beta risk factor

Intuition

- Assets with lower risk have lower expected returns.
- Assets with higher risk have higher expected returns.

FIGURE 14.3
SECURITY MARKET LINE



Example: Motor and Food

- Assume that there are only two corporations, Motor and Food, and that both companies have issued stock and no other securities.
- Assume the total numbers of shares issued by Motor and Food are 100 and 150, respectively, and that the market price per share of Motor is $5 and for Food it is $2.
- The market value of Motor is then $500 and Food is $300, so that the total market value is $800.


Example: Motor and Food

- How much return should an investor expect to get out of an individual security?
- Assume their respective beta risks (β_p) are 1.5 and 0.8.
- Suppose the risk-free rate (r_f) is 3% and the expected market return ($E(r_m)$) is 11%.
- Then, the market risk premium ($E(r_m) - r_f$) is 8% (= 11% - 3%).
- Based on the CAPM, the expected returns will be:

  Motor: 15% (= 3% + 1.5X8%) and Food: 9.4% (= 3% + 0.8X8%)

- This says that, in comparison with Food, Motor should have an expected return of 15% due to its higher beta risk

Security market line

1. It is common in finance to refer to β_p as the security's beta and to the measure's value as the security's beta risk.
   - It is a relative measure since it measures risk in relation to the risk of the market portfolio.
   - Notice that if $\beta_p > 0$, security p moves with the market portfolio, while if $\beta_p < 0$, it moves in the opposite direction.
2. The term $E(r_m) - r_f$ is known as the market risk premium.
   - It is the return in excess of the risk-free return, and represents the premium required for taking on market risk.

Risk

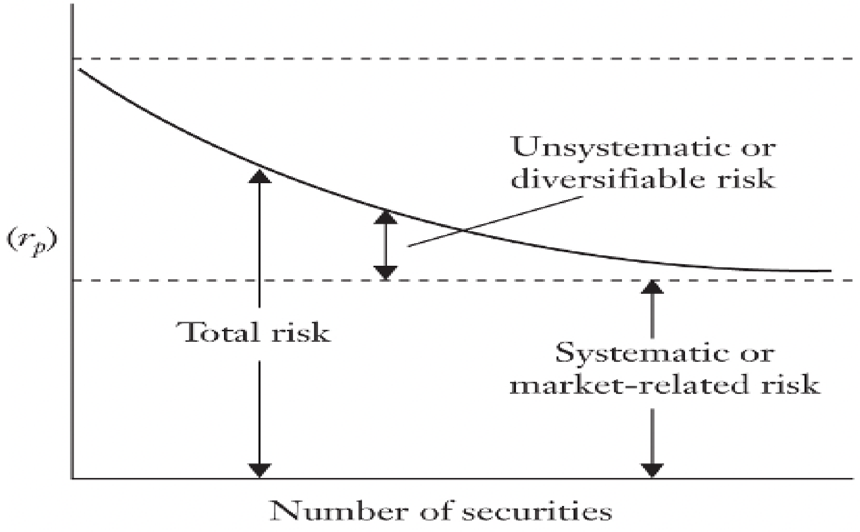
- Investment risk is defined as the probability of occurrence of losses relative to the expected return on any particular investment.
- Simply put, the possibility that investors will lose money when they invest in an asset.

Total Risk = Systematic Risk + Unsystematic Risk

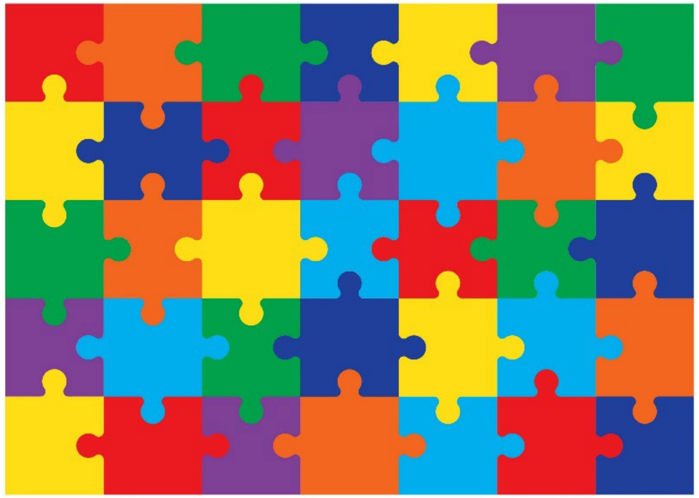
- The variance of returns are both measures of how dispersed returns can be—the greater the dispersion, the greater the variance:

$$\mathrm{var}(r_p) = \sigma^2(r_p) = E[r_p - E(r_p)]^2.$$

- $\sigma(r_p)$ is the standard deviation of the individual investor's portfolio return.
- Ex: When r_p is 10% with ½ chance and -10% with ½ chance, $E(r_p) = 0$, $\mathrm{var}(r_p) = 0.01$ and $\sigma(r_p) = 0.1$; When r_p is 20% with ½ chance and -20% with ½ chance, $E(r_p) = 0$, $\mathrm{var}(r_p) = 0.04$ and and $\sigma(r_p) = 0.2$



- Systematic risk, also referred to as non-diversifiable risk, is the risk related to the market and is measured by beta.
- Unsystematic risk is the risk that can be diversified away.



- CAPM has been criticized for its assumptions, including the assumption that all investors have the same information.
- Nevertheless, it remains a popular and widely used tool for evaluating investment opportunities and assessing portfolio risk.

Criticism for CAPM

Topic four

Introduction to Efficient Market Hypothesis

Efficient Market Hypothesis

- Adaptive Expectation: Expectations are formed from past experience only.
- Rational Expectation: Expectations will be identical to optimal forecasts using all available information.
- The application of the theory of rational expectations to financial markets is called the **efficient market hypothesis (EMH)**.

Eugene Fama, the originator of EMH, and Robert Shiller, one of the founders of behavioural finance, won the Nobel prize in 2013.
Source: *The Guardian*

Summary

In conclusion, understanding CAPM and risk management equips us with the knowledge needed to navigate the complexities of financial markets and make informed decisions that align with our financial objectives. Today we have learnt those important concepts, and critiques against CAPM as well.

Feedback for Topic 6

- https://padletuq.padlet.org/Shino/econ2103-shared-thoughts-topic-6-go7p766agjqrf64v

Midterm Course Survey

- Please help us improve the course for the later half.
- URL: https://uniofqueensland.syd1.qualtrics.com/jfe/form/SV_8vHutqkG4xxWBUi

Thank You

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