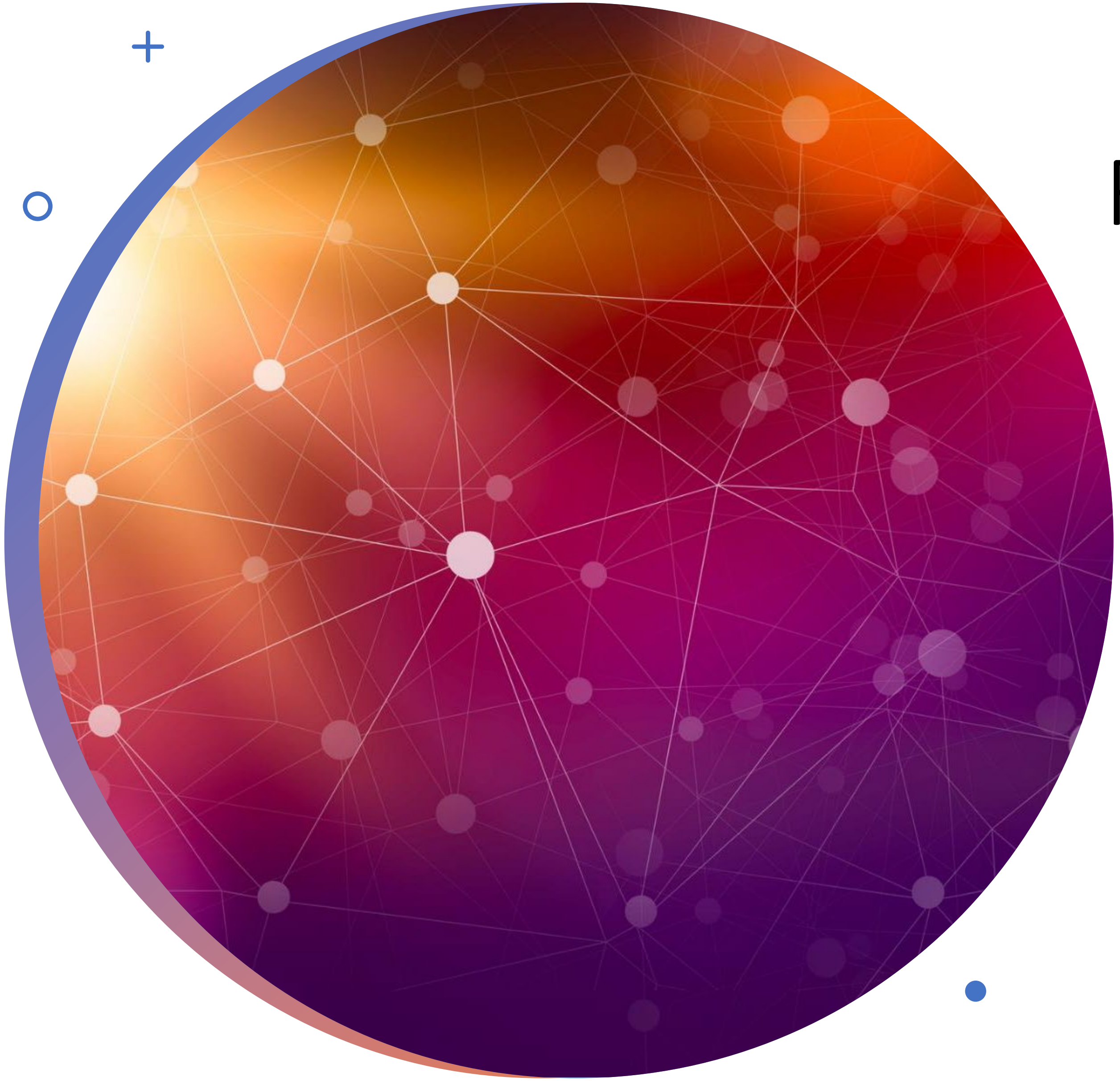

Financial Economics ECON2103

Lecture 3:

Investment Decision and Asset Pricing

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Introduction

- We will learn which project to be chosen, how the price of new stocks is determined, and risk-free rates, which are important for investment decision. Then we study about short sales.

Agenda

Introduction

Topic one: Investment Decision

Topic two: Determination of Stock Prices

Topic three: States of the World

Topic four: Risk-Free Rates

Topic five: Short Sales

Summary

Learning Objectives

- Explore how an investment decision is made.
- Study how economic theory determines stock prices.
- Investigate how economic theory defines uncertainty in the future.
- Explore how a risk-free rate is determined.
- Understand the concept of short sales.

Topic one

Investment Decision

Types of projects

1. An independent project is one whose cash flows are not related to the cash flows of any other project.
   - The net cash flows of an opportunity are not affected by the level of operation of any other opportunities (economic independence).
   - The fact that economically independent projects' values add to the market value of the firm is sometimes referred to as the value additivity principle.
2. Contingent projects are dependent on the acceptance of another project. Ex. a new character, Pippy, and a line of Pippy cards.
3. Complementary projects are such that the investment in one enhances the cash flows of one or more other projects. Ex. a manufacturer of personal computer equipment and software.

Capital budgeting

- Suppose a firm invests in a new project, Project *X*.
- Consider cash flows Project *X* generates and the amount needed to compensate investors for the risk they bear on this project.
- If the expected change in the value of the company from an investment is:
  - Positive: the project returns > the cost of capital.
  - Negative: the project returns < the cost of capital.
  - Zero: the project returns = the cost of capital.
- Capital budgeting is the process of identifying and selecting investments in long-lived assets, that is, assets expected to produce benefits over more than one year.

A general formulation

The market value criterion deals with the totality of the firm's investments.

- CF_t: net cash flow for period t
- I: initial investment outlay
- N: the number of periods
- r: one-period required rate of return for the project.
- Then,

$$Net\ Present\ Value\ (NPV) = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} - I$$

A project's incremental cash flow

- When a firm invests in new assets, it expects the future cash flows to be greater than without this new investment.
- The difference between them is called the project's incremental cash flows.
- The change in a firm's value is the difference between project benefits and costs.
- Another way of evaluating the change in the firm's value is to break down the project's cash flows into:

1. The cash flows from the project's operating activities (revenues and operating expenses), referred to as the project's operating cash flows;
2. The investment cash flows, that is, the expenditures needed to acquire the project's assets and any cash flows from disposing the project's asset.

As a manager

Project X

Project Y

Both projects require an initial investment outlay of $10 million, and a required rate of return of return of 10%.

Example: Project X

Year	Net Cash Flow (CF_t)	Present Value of Net Cash Flow at 10% ($CF_t(1+0.1)^{-t}$)
1	$0	$0
2	$2,000,000	$1,652,893
3	$3,000,000	$2,253,944
4	$9,000,000	$6,147,121

Because the initial investment $I = \$10{,}000{,}000$, we have

$$Net\ Present\ Value\ (NPV) = \sum_{t=1}^{4} \frac{CF_t}{(1.1)^t} - I$$

$$= \$10{,}053{,}958 - \$10{,}000{,}000 = \$53{,}958$$

Example: Project Y

Year	Net Cash Flow (CF_t)	Present Value of Net Cash Flow at 10% ($CF_t(1+0.1)^{-t}$)
1	$3,250,000	$2,954,545
2	$3,250,000	$2,685,950
3	$3,250,000	$2,441,773
4	$3,250,000	$2,219,794

Because the initial investment $I = \$10{,}000{,}000$, we have

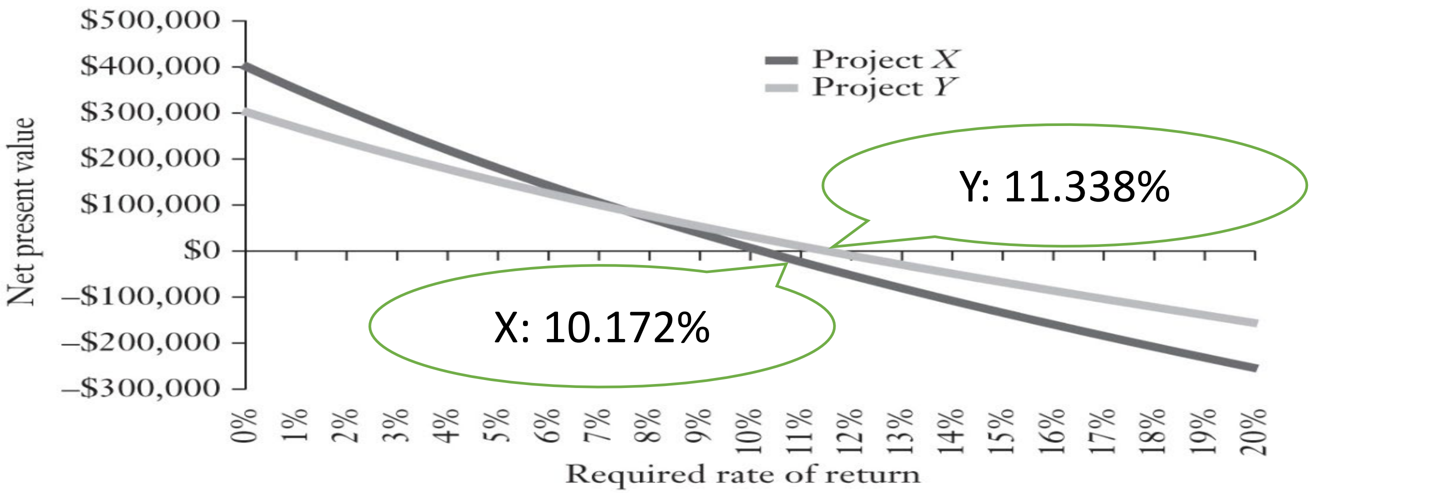
$$Net\ Present\ Value\ (NPV) = \sum_{t=1}^{4} \frac{CF_t}{(1.1)^t} - I$$

$$= \$10{,}302{,}062 - \$10{,}000{,}000 = \$302{,}062$$

Investment Profiles and Internal Rate of Return

- The investment profile is a graphical depiction of the relation between the NPV of a project and the discount rate.
- The internal rate of return (IRR) is the discount rate that makes the sum of the present value of the net cash flows equal to zero.

FIGURE 6.1
INVESTMENT PROFILES OF INVESTMENTS X AND Y



Topic two

Determination of Stock Prices

- A firm needs to finance their new project by issuing new stocks.
- How many shares does the firm need to issue?
- How much should a share of the new stock be?

Example: New Issue

- There are two periods $t = 1, 2$.
- Suppose
  - that a firm announces an investment of *$4,500* at time *1*;
  - that the cash flow realized at time *2* on this investment will be *$5,500*;
  - that the investment will be financed by a new share issue.
- How many shares must be issued, and what will the market price of all shares be after the new issue?

Existing Share's Price

- Let us denote with a superscript *0* the values for the variables before the announcement of the new investment.
- Denote the market value of the existing operation in time 2 by $V^0(2)$.
- Denote the market interest rate, the number of the existing shares in time 1 by r, and $N^0(1)$.
- Let's assume the following in our example:

$$V^0(2) = \$44{,}000; r = 0.1; N^0(1) = 1{,}800.$$

Q1. What is the existing share price in time 1, $s^0(1)$?

1. $s^0(1) = \frac{V^0(2)}{N^0(1)} = \frac{\$44{,}000}{1{,}800} = \$\frac{220}{9}$; or
2. $s^0(1) = \frac{V^0(2)}{(1+r)\cdot N^0(1)} = \frac{\$44{,}000}{1.1\times 1{,}800} = \$\frac{200}{9}$

Explanation: Q1

- It is 2:

$$s^0(1) = \frac{V^0(2)}{(1+r)\cdot N^0(1)}$$

$$= \frac{\$44{,}000}{1.1 \times 1{,}800} = \$\,\frac{200}{9}$$

- $V^0(2)$ is the value in time 2.
- It must be revalued at time 1

After the Announcement

- After the announcement, the market value in time 2 is

$$V(2) = \$44{,}000 + \$5{,}500 = \$49{,}500.$$

- Denoting the number of new shares as $M(1)$, we write the time 1 value of each share outstanding after the new issue as:

$$\frac{V(2)}{1.1}\left(\frac{1}{N^0(1) + M(1)}\right).$$

- This value multiplied by the number of new shares must equal the required investment.

The Number of New Shares & the Issue Price

- Therefore, $I(1) = \frac{V(2)}{1.1}\left(\frac{M(1)}{N^0(1)+M(1)}\right)$.
- Hence, $\$4{,}500 = \frac{\$49{,}500}{1.1}\left(\frac{M(1)}{1{,}800+M(1)}\right)$.
- Finally, we obtain $M(1) = 200$.
- All shares must earn the market rate of interest from period *1* to period *2*.
- Therefore,

$$s(1) = \frac{\$45{,}000}{2{,}000} = \$22.5$$

Q2. What is the market value of the operation in time 1, $V(1)$?

1. $V(1) = \frac{V(2)}{1.1} = \frac{\$49{,}500}{1.1} = \$45{,}000;$

   or

2. $V(1) = \frac{V^0(2)}{1.1} = \frac{\$44{,}000}{1.1} = \$40{,}000$

Explanation: Q2

- It is 1:

$$V(1) = \frac{V(2)}{1.1} = \frac{\$49{,}500}{1.1} = \$45{,}000$$

- The market value includes both the pre-existing operation and the new investment.
- So $V(2)$ must be used to calculate $V(1)$.

Topic three

States of the World

States of the world

- The idea of states of the world is useful for thinking about convenient ways to model risky payoffs.
- In a two-time-point model, states of the world are defined as those future events that matter to the decision problem being considered.
- These states of the world are defined by the decision maker to be mutually exclusive and collectively exhaustive.
- Risk refers to situations in which it is possible both to define future states of the world and the probabilities with which they might occur.

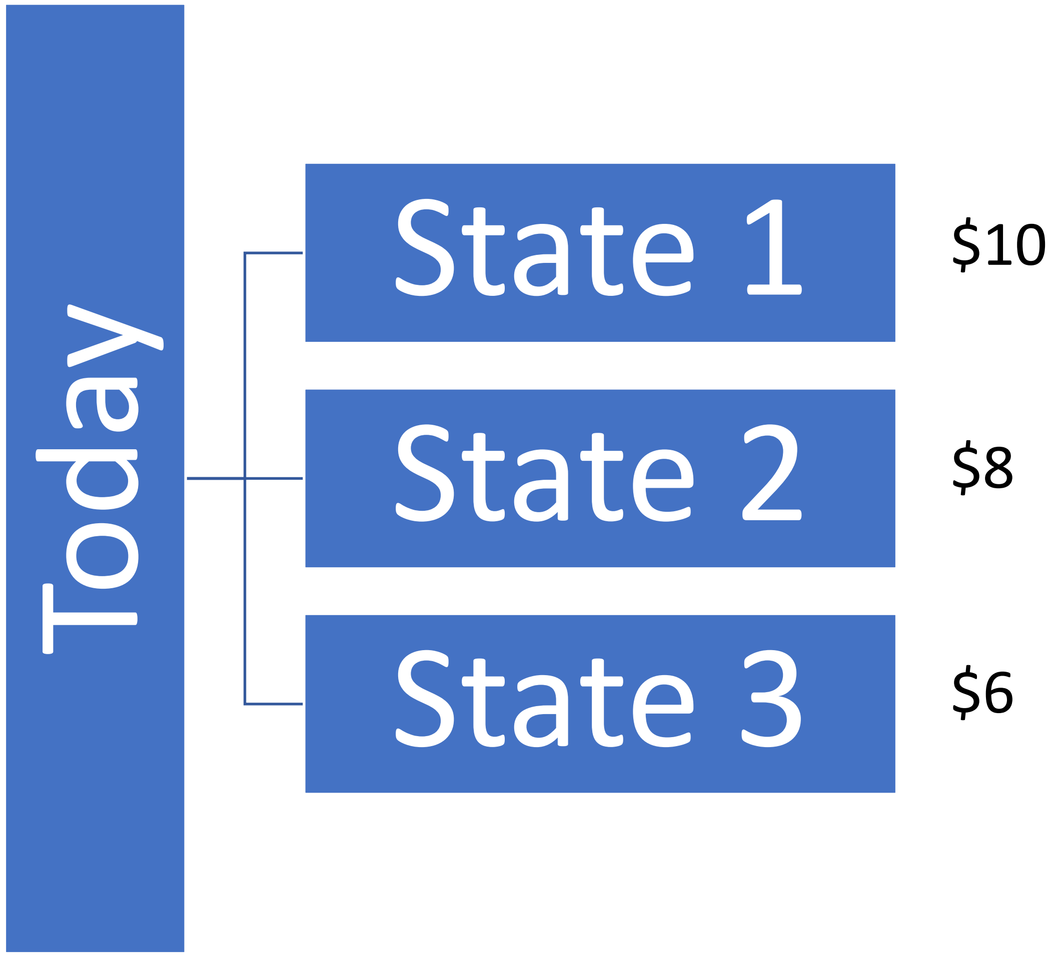
Example

- Suppose the investor defines

(1) "states" to represent economic conditions and

(2) "future prices" to be a list of possible stock prices that may be realized at the time a given state is actually realized.

Event tree



Expected payoff

- Assume that the lottery has two possible outcomes, say x_1 with probability p_1 and x_2 with probability
$$p_2 = 1 - p_1, \qquad p_1 \epsilon (0, 1).$$
- Then the lottery's expected payoff equals:
$$x_1 p_1 + x_2 (1 - p_1)$$

Q3. A company issued a contingent claim contract to an investor, which will pay \$500 if it rains tomorrow. If it does not rain, the contract will be worthless. It rains with 1/2 chance. What is the expected value of this contract?

- 250 + 0 = 250; or
- 500

Explanation: Q3

- If it rains tomorrow, the contract pays \$500 with a probability of 1/2 (= 0.5).
- If it does not rain, the contract pays \$0 with a probability of 1/2 (= 0.5).
- Now, we can calculate the expected value:

$$\$500 \times 0.5 + \$0 \times 0.5 = \$250$$

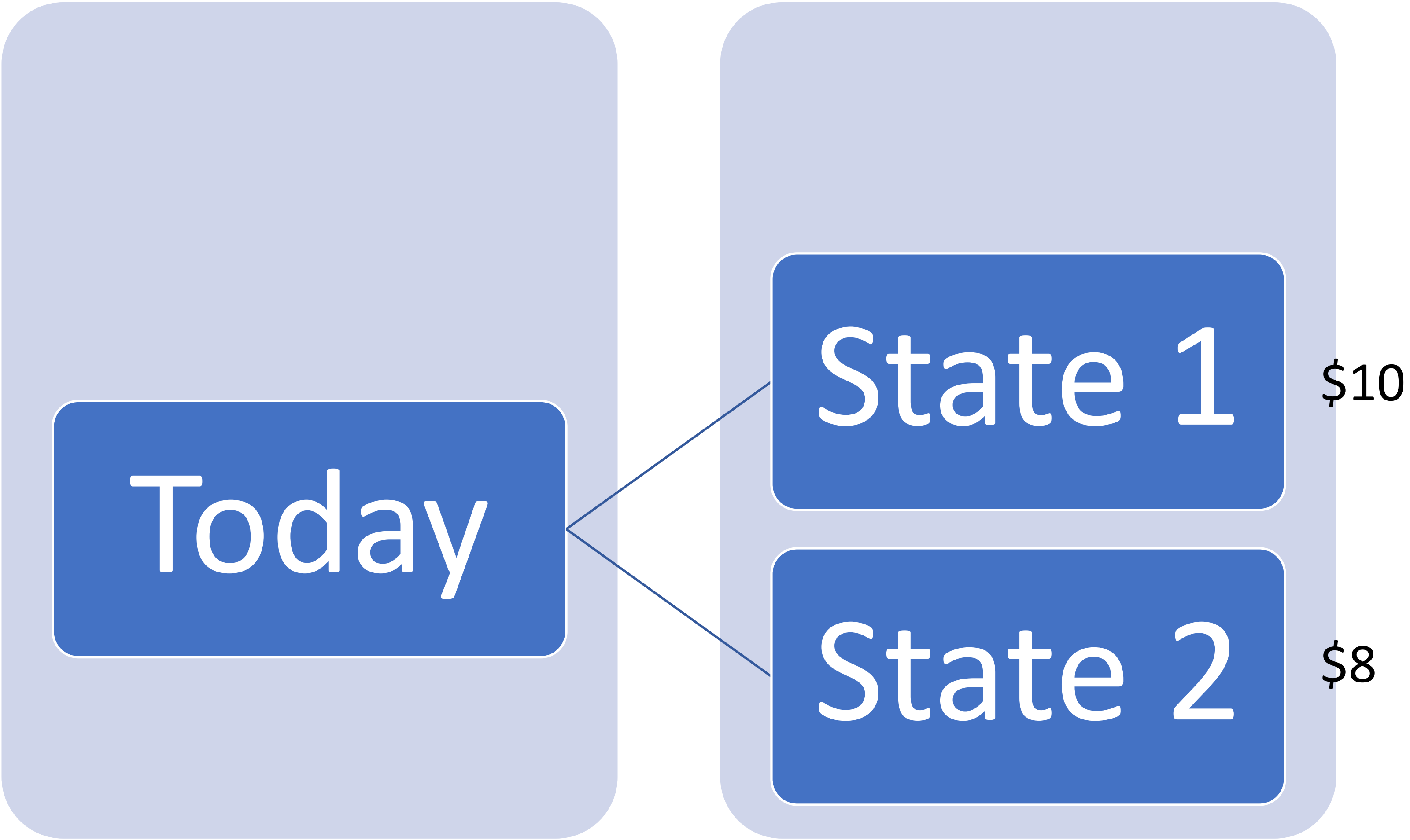
- So, the expected value of the contingent claim contract is \$250.

Example

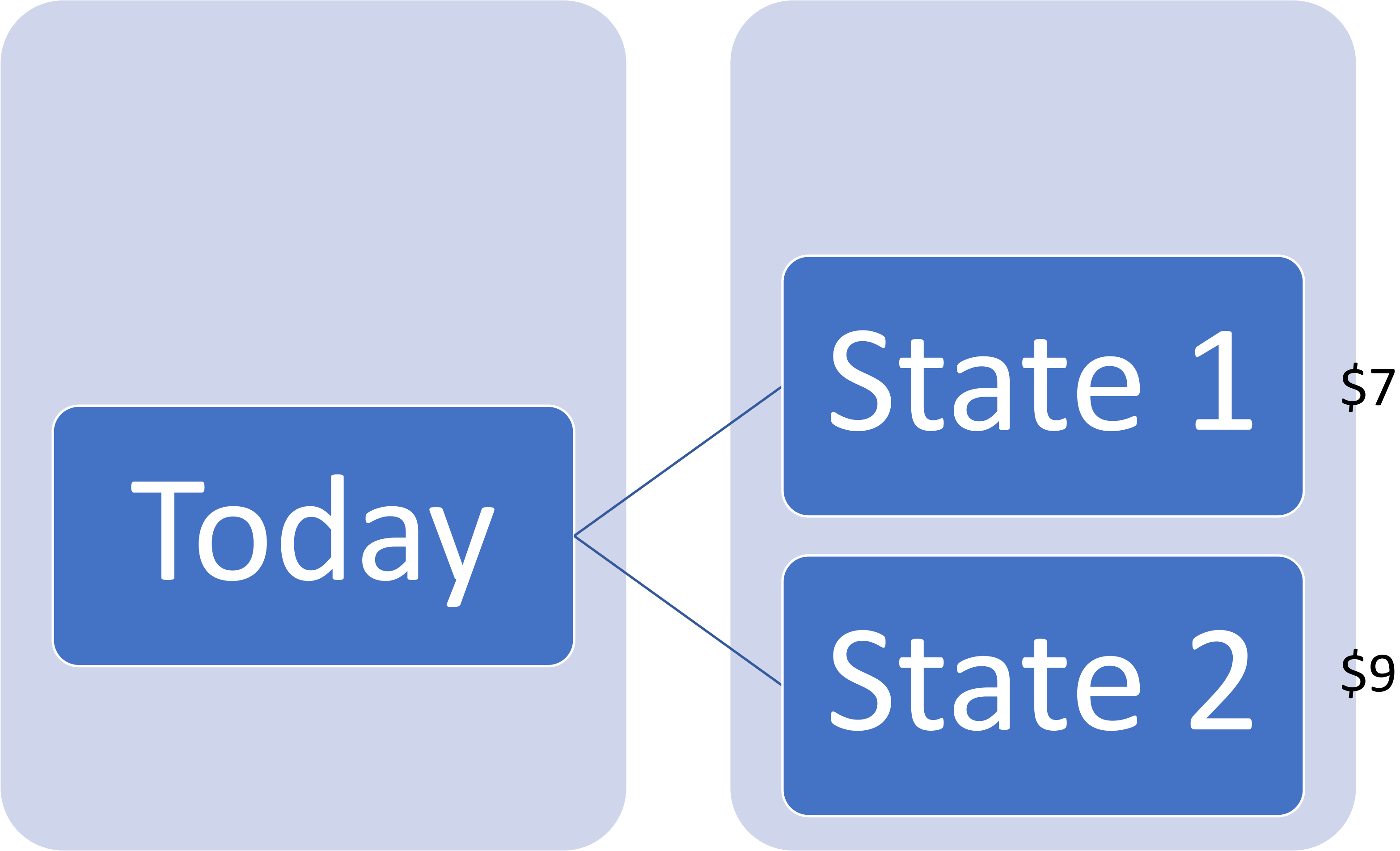
- Suppose that we can describe the world using two states and that two stocks are available, stock A and stock B.
- We assume the stocks' future prices have the following distributions:

State	Future Prices Stock A	Future Prices Stock B
1	\$10	\$7
2	\$8	\$9

Stock A



Stock B



A Unit Contingent Claim

- A unit contingent claim is a security that will pay an amount of \$1 if a certain state of the world is actually realized, but nothing otherwise.
- A claim that pays \$1 if state *i* is realized is frequently called a unit claim on state *i*.
- A unit contingent claim is also referred to as a primary security (or Arrow—Debreu Security).
- A contingent claim can be: Ten-unit claims on state 1; Eight-unit claims on state 2; Six-unit claims on state 3.


Example Continued

- Let *A(1)* = \$6 denote the time 1 price of stock A, and *B(1)* = \$5 the time 1 price of stock B.
- Purchasing stock A for \$6 is equivalent to buying a package of 10 unit claims on state 1 and 8 unit claims on state 2.
- Purchasing stock B for \$5 is equivalent to buying a package of 7 unit claims on state 1 and 9 unit claims on state 2.

Example: A Unit Contingent Claim

- Let C_1 and C_2 represent the time 1 prices of unit claims on states 1 and 2.
- Buying a unit claim on state *i* at each price gives $1 when state *i* is realized.
- For example, buying ten-unit of a unit claim on state *1* is 10 C_1 .
- Purchasing stock A for $6 is equivalent to $10\ C_1 \ + \ 8\ C_2$.
- Purchasing stock B for $5 is equivalent to $7\ C_1 \ + \ 9\ C_2$.

Example Continued

- When the unit claims comprising the two stocks are perfect substitutes, they must sell for the same prices in a perfect market.
- Hence, we can write:

$$10C_1 + 8C_2 = \$6 \text{ and } 7C_1 + 9C_2 = \$5$$

$$(70C_1 + 56C_2 = \$42 \text{ and } 70C_1 + 90C_2 = \$50 \text{ -> } 34C_2 = \$8)$$

$$\text{implying } C_1 = \$\,7/17 \text{ and } C_2 = \$4/17.$$

Topic four

Risk Free Rate

Risk-free Rate

- Let α denote the portion of stock A's share to the total purchasing amount M.
- You purchase αM shares of stock A and $(1-\alpha)M$ shares of stock B.
- We wish to find a combination of the two stocks that gives the same time 2 payoff in either state of the world.
- Then, the following equation must be solved for α:

$$10\alpha M + 7(1 - \alpha)M = 8\alpha M + 9(1 - \alpha)M$$

implying $2\alpha = 2(1 - \alpha)$ and thus $\alpha = ½$.

Risk-free Rate and Rate of Return

- Since a risk-free instrument is one that offers the same payoff irrespective of which state of the world obtains, we wish to find a combination of the two stocks that gives the same time 2 payoff in either state of the world.

The Risk-free (Riskless) Payoff & the Risk-free Rate of Return

- As calculated, we have $\alpha = \frac{1}{2}$.
- The risk-free (riskless) payoff is:

$$\frac{1}{2}(10) + \frac{1}{2}(7) = \$8.5.$$

- The price for this combination is:

$$\frac{1}{2}(6) + \frac{1}{2}(5) = \$5.5.$$

- The risk-free rate of return is:

$$\frac{\$8.5 - \$5.5}{\$5.5} = \frac{6}{11} = 54.55\%.$$

Notation

- w_1: wealth if state 1 occurs.
- w_2: wealth if state 2 occurs.
- We want to calculate the value of the claims' combinations that could arise tomorrow.
- We have computed the risk-free rate is 50% (for each stock holding).

Maximization

- $A(1) = \$6$: the time 1 price of stock A
- $B(1) = \$5$: the time 1 price of stock B
- Q: the number of shares for stock A under a risk-free rate (50%)
- Each security generates a riskless terminal wealth position of

$$w_1 = w_2 = \$926.5$$

because

$$5 \times Q + 6 \times Q = 600 ☞ Q = 600/11 = 54.5$$

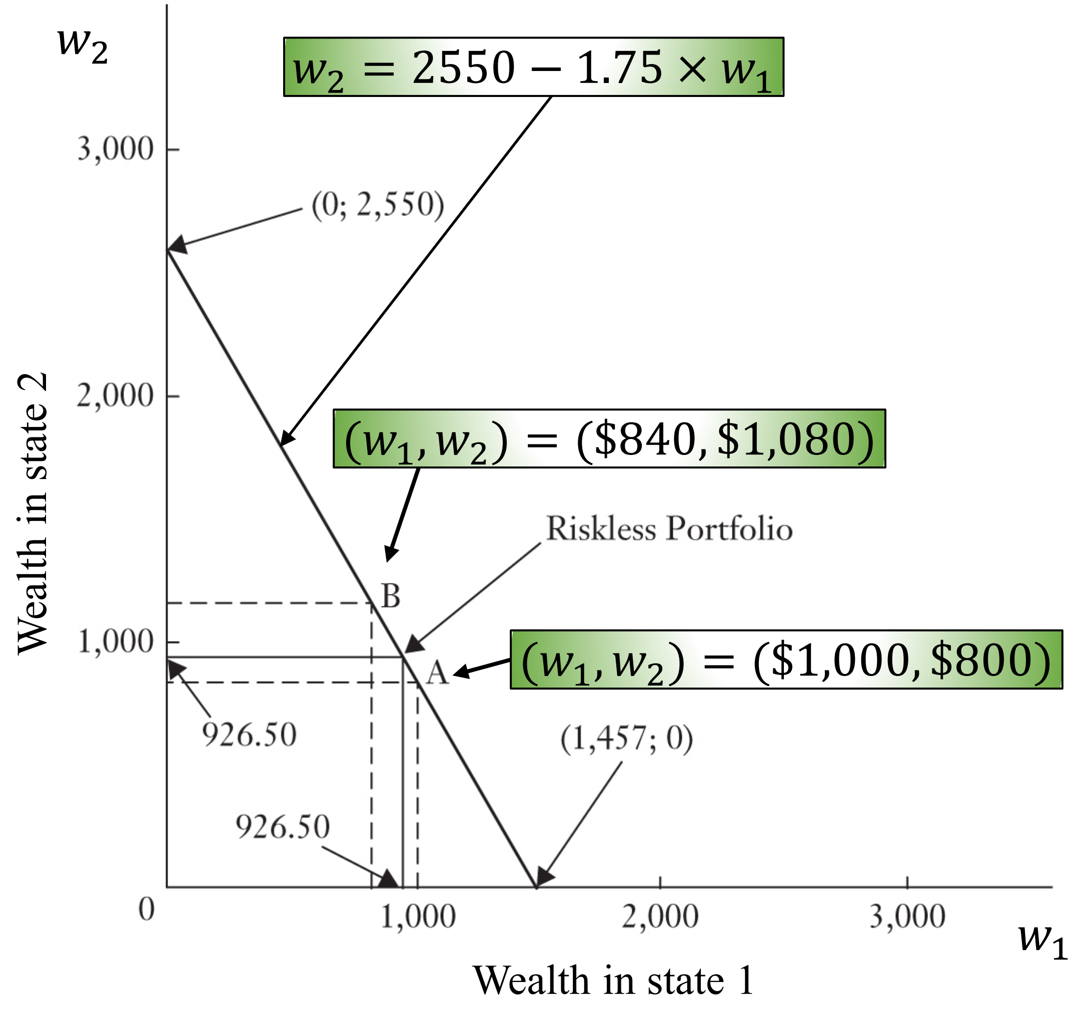
$$\begin{aligned} & 10 \times 54.5 + 7 \times 54.5 \\ &= 8 \times 54.5 + 9 \times 54.5 \\ &= 926.5 \end{aligned}$$

Claim for Each State

- We want to know possible combinations of claims for each state, namely (w_1, w_2) for purchasing stocks A and B by $600.
- We write the relationship as $w_2 = a - bw_1$.
- We substitute numbers from Table 10.1.
- For exclusively purchasing stock A, purchasing *100* shares, and $(w_1, w_2) = (\$1{,}000, \$800)$ implying
$$\$800 = a - \$1000b$$
- For exclusively purchasing stock B, purchasing *120* shares, and $(w_1, w_2) = (\$8{,}40, \$1080)$ implying
$$\$1080 = a - \$840b$$
- Therefore $a = \$2{,}550$ and $b = 1.75$.

FIGURE 10.1

MARKET OPPORTUNITY LINE SHOWING IMPLIED PRICES OF UNIT CLAIMS



Topic five

Short Sales

Short sales

- Note that in Figure 10.1 the investor's time 1 position is some point on the line from *A* to *B*.
- Points A and B represent the situations where exclusively Stocks A and B are purchased, respectively.
- How could the investor obtain a terminal wealth position lying beyond these points?
- The investor could engage in short sales, that is, selling shares not currently owned, for delivery when the unknown future state of the world is revealed.

Illustration for short sales

- To illustrate, consider point $w_1 = \$1{,}457,\ w_2 = 0.$
- Let n_A be the number of shares of stock *A* and n_B the number of shares of stock *B* purchased.
- If state 1 occurs, the terminal wealth will be:

$$10\, n_A + 7\, n_B = \$1{,}457.$$

- If state 2 occurs, we must have:

$$8\, n_A + 9\, n_B = 0.$$

- Solving these equations simultaneously, we find $n_B = -343.$

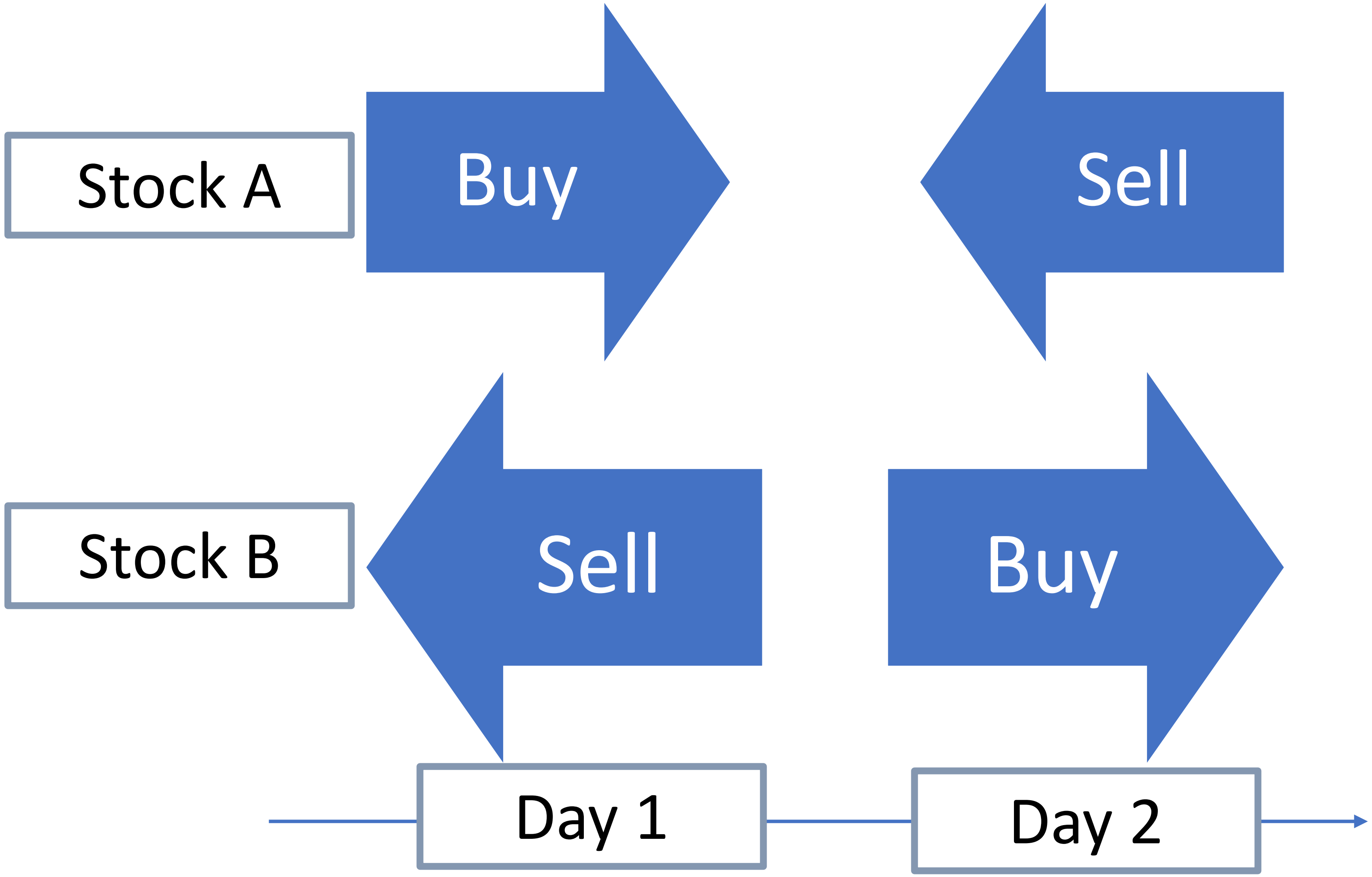
Explanation: Short Sales

- If the investor sells short 343 shares of stock B at the current price of $\$5$, he will receive $\$1{,}715$.
- Combining this with the initial wealth of $\$600$ gives $\$2{,}315$, so this investor may buy $\$2{,}315/\$6 = 386 = n_A$.
- So, $(n_A, n_B) = (386, -343)$.
- In state 1, the investor receives $\$3{,}860$ ($10 times 386 shares of Stock A) but now must pay $\$2{,}401$ ($7 times 343 shares of Stock B) shares to cover the short position.
- The net terminal wealth in state 1 is $\$3{,}860 - \$2{,}401 = \$1{,}459$.

State 2?

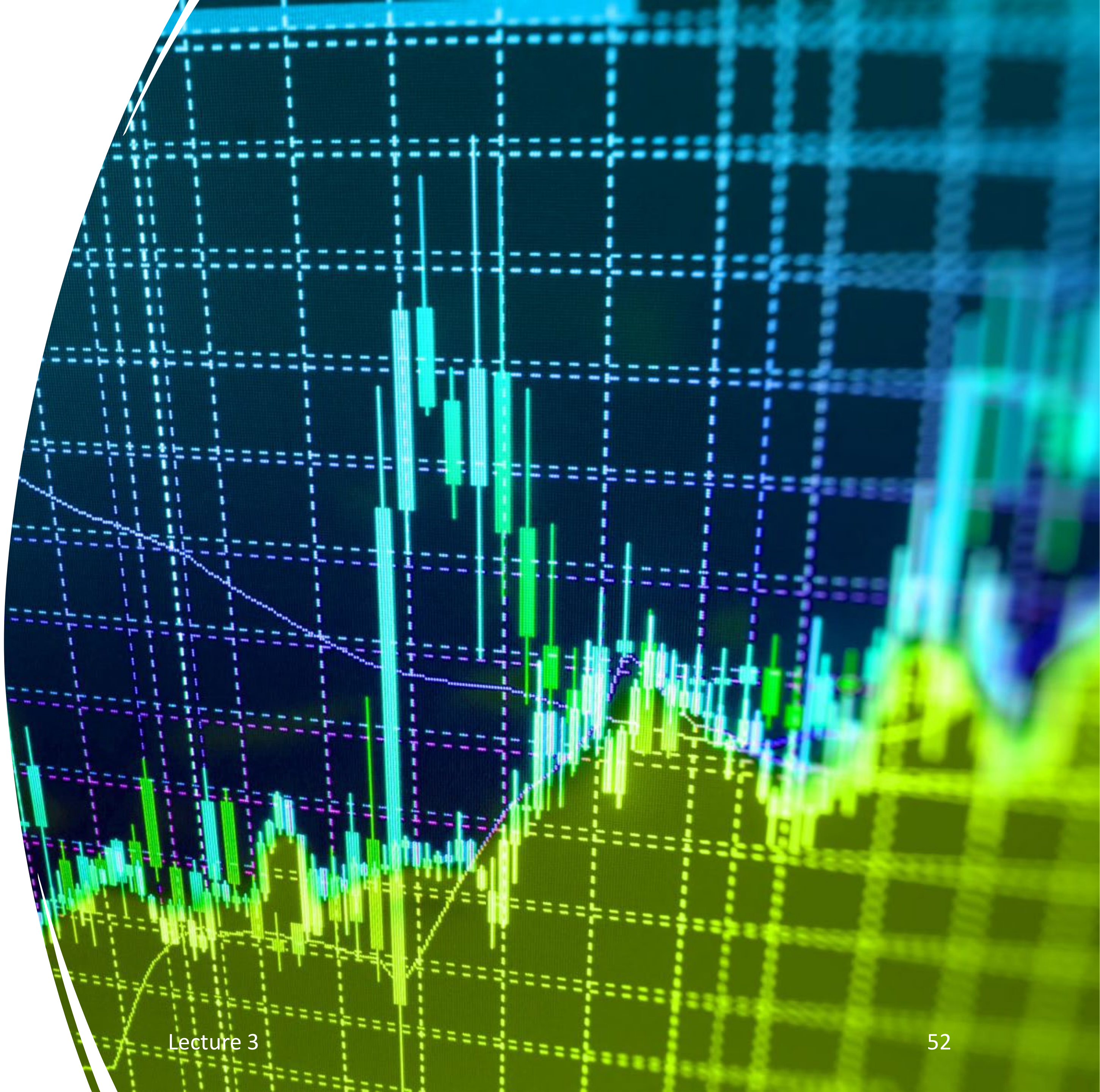
- In state 2, the terminal wealth is $\$1$.
- Stock A: $\$3{,}088$ (386 shares times $\$8$ per share of Stock A) reduced by
- Stock B: $\$3087$ (343 shares times $\$9$ per share of Stock B) gives
- $1 ($386 \times 8 - 343 \times 9 = 3088 - 3087$).

Short Sales: Illustration
For Day 1 and Day 2



Q4. What is a short sale in the context of the stock market?

1. Buying a stock with the expectation that its price will increase; or
2. Borrowing and selling a stock with the expectation that its price will decrease.



Explanation: Q4

- Answer: 2 -- Borrowing and selling a stock with the expectation that its price will decrease.
- The investor aims to buy back the same number of shares at a lower price to return them to the broker, pocketing the difference as profit.
- This strategy is used to profit from a falling stock price.

Summary

In this lecture, we have learnt how the price of new stocks is determined, and risk-free rates, which are important for investment decision. Then we also studied about short sales and how short sales could expand investment opportunities.

Feedback for Topic 3

- https://padletuq.padlet.org/Shino/econ2103-shared-thoughts-topic-3-1epe65eo04jfm5a9

Thank You

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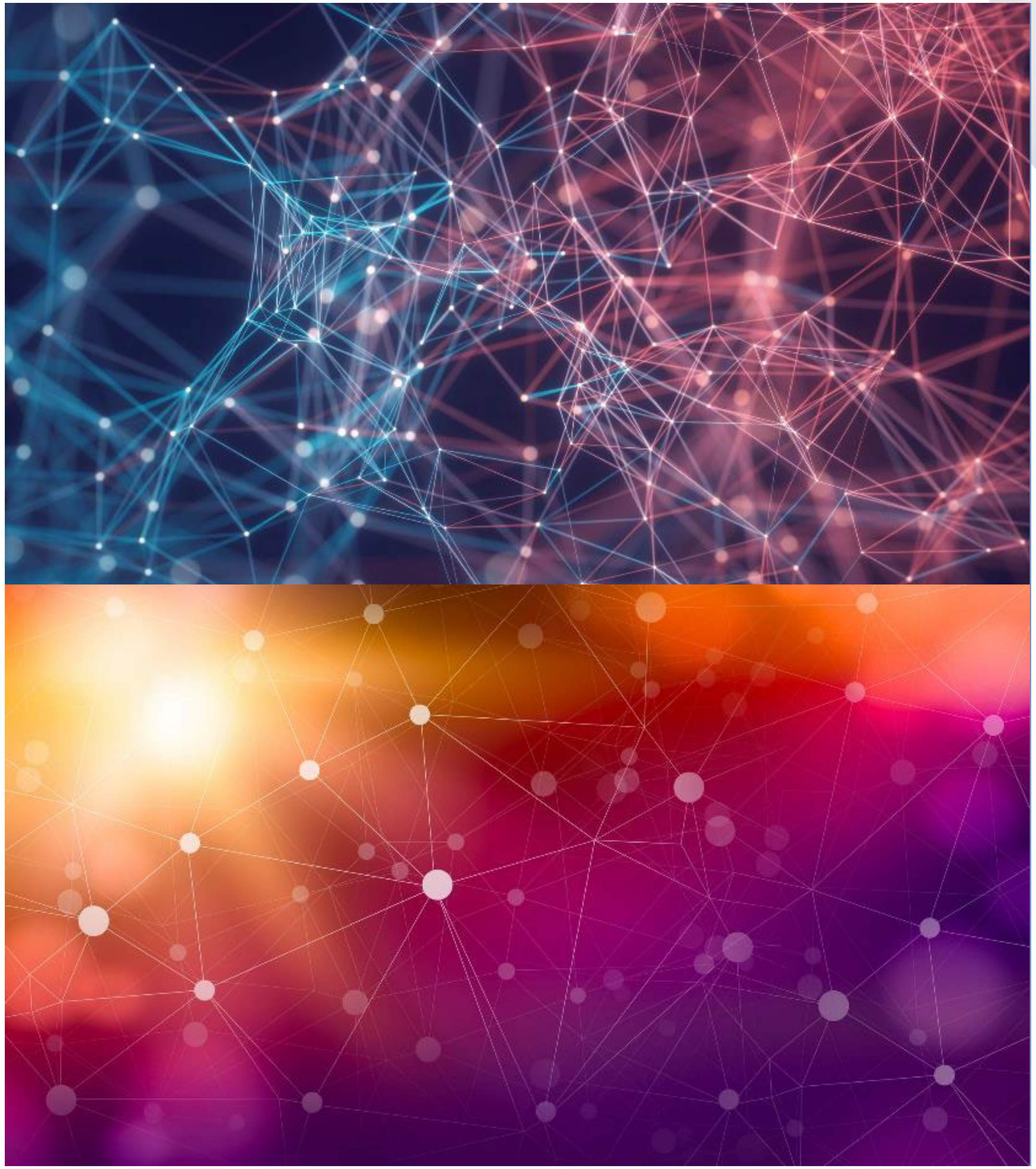

Source

- Chapter 4, “How Investors Value Firms,” in “*Financial Economics*,” by F. J. Fabozzi, E.H. Neave, and G. Zhou, Wiley.
- Chapter 9, “The Microeconomics Foundation of Financial Economics,” in “*Financial Economics*,” by F. J. Fabozzi, E.H. Neave, and G. Zhou, Wiley.
- Chapter 10, “Contingent Claims and Contingent Strategies,” in “*Financial Economics*,” by F. J. Fabozzi, E.H. Neave, and G. Zhou, Wiley.
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