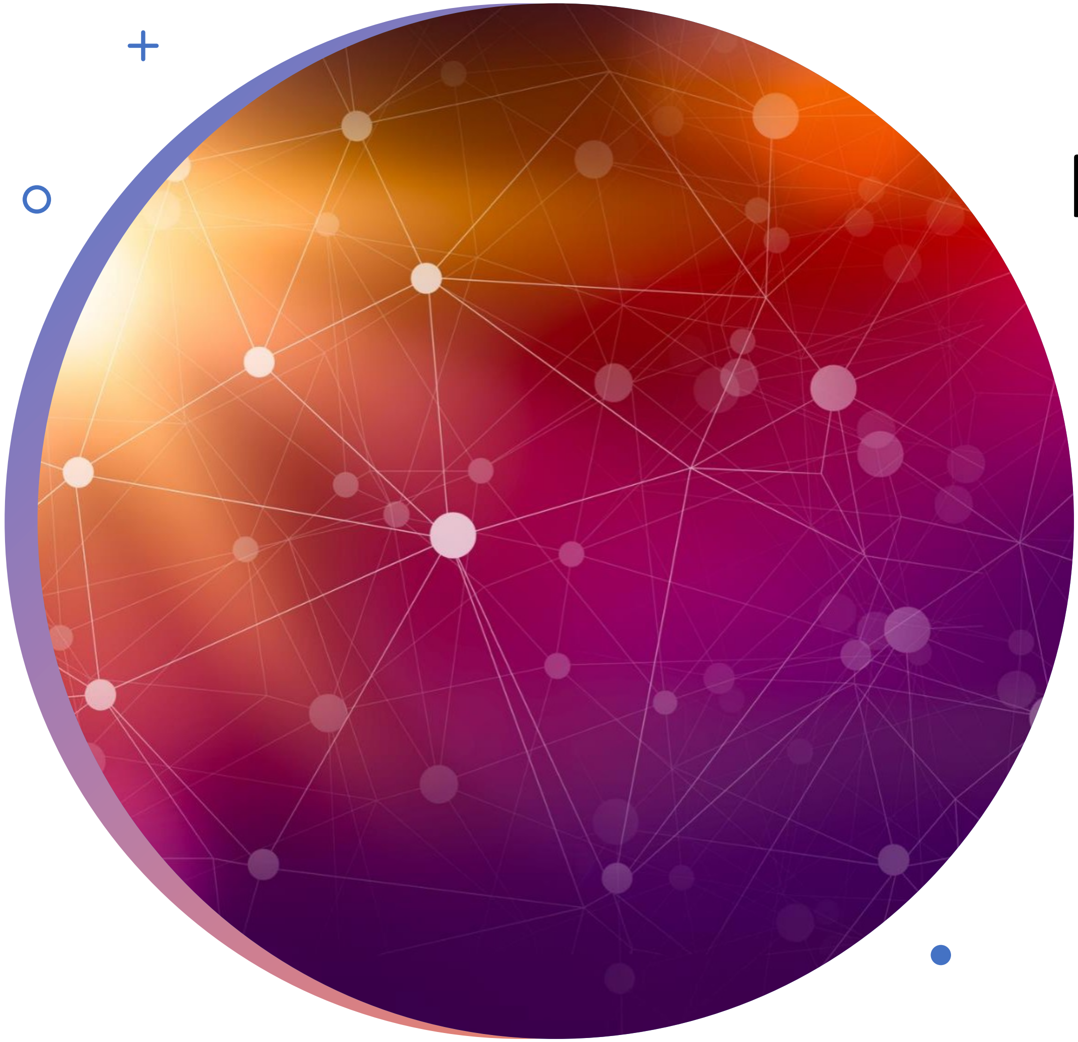


Financial Economics ECON2103

Lecture 5

Risk and Insurance

Dr Shino Takayama

University of Queensland

2024

Introduction

- In the dynamic landscape of life and business, we face various uncertainties. Whether it's the unpredictability of health or accidents, risks we face can have significant financial consequences. Thus, insurance becomes crucial. By the end of this lecture, you'll have a better understanding of the vital link between insurance and risk management.

Agenda

Introduction

Topic one: Utility Function

Topic two: Risk Management and Insurance

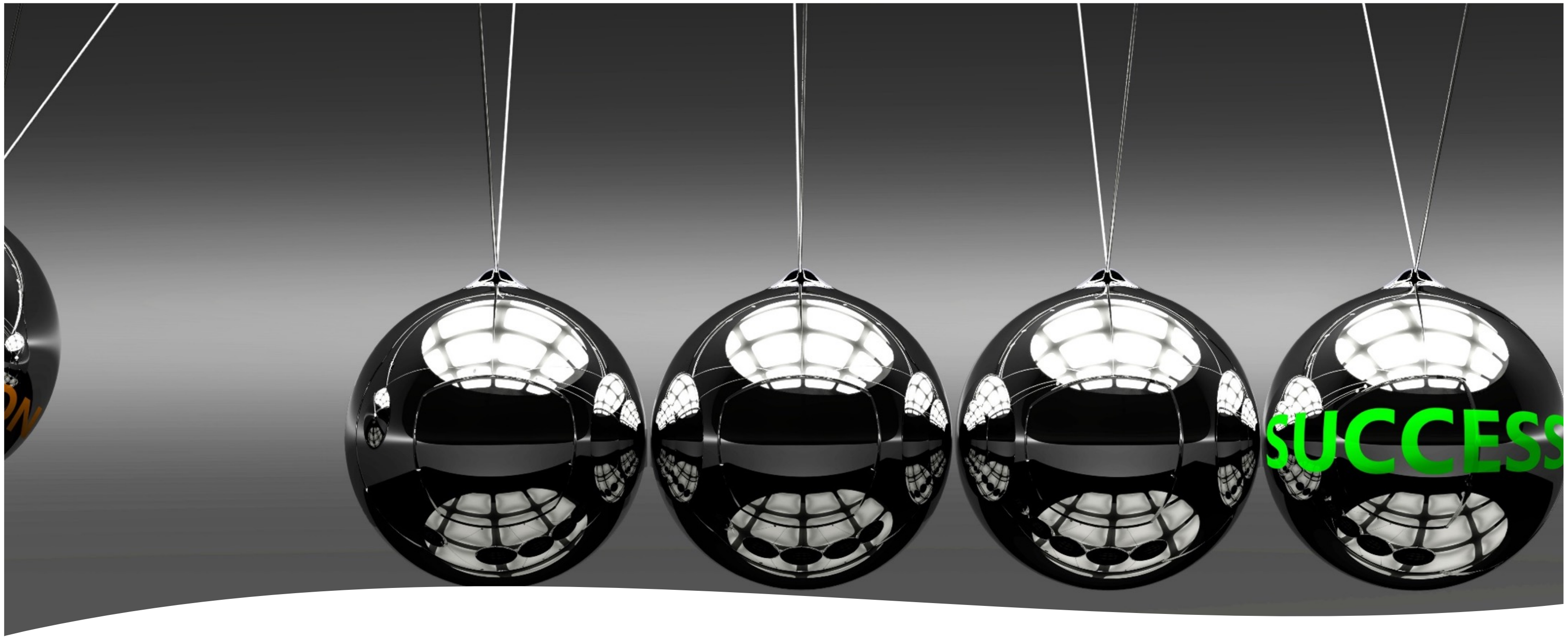
Summary

Learning Objectives

- Describe why we need to define expected utility.
- Define a probability distribution.
- Explore how we can define risk attitudes in a utility function.
- Introduce the concept of risk management.
- Explore why we need insurance.

Topic one

Utility Function



Von Neumann-Morgenstern expected utility

- We can use the concept of an expected utility to describe the sum of a utility in each state.
- Risk averse is a property of preference such that an individual prefers a less risk situation.

St Petersburg paradox

- The paradox is illustrated by a lottery stipulating that a fair coin will be tossed until a head appears.
- If the head appears on the first toss, the payoff is \$1.
- If it appears on the second toss, then the payoff is \$2.
- If the head appears on the third toss, the payoff is \$4; on the fourth toss, it is \$8.
- If the head appears on the n-th toss, the payoff is 2^{n-1} dollars.

The necessity for utility functions

- The expected payoff is

$$\$1 \cdot \left(\frac{1}{2}\right) + \$2 \cdot \left(\frac{1}{4}\right) + \$4 \cdot \left(\frac{1}{8}\right) + \cdots + \$2^{n-1} \cdot \left(\frac{1}{2^n}\right) + \cdots$$

- Each term equals ½. The payoff is infinite.
- No matter what the price of the lottery is, people should buy the lottery.
- To explain the paradox, Bernoulli suggested that rather than the actual payoff, its utility should be considered:

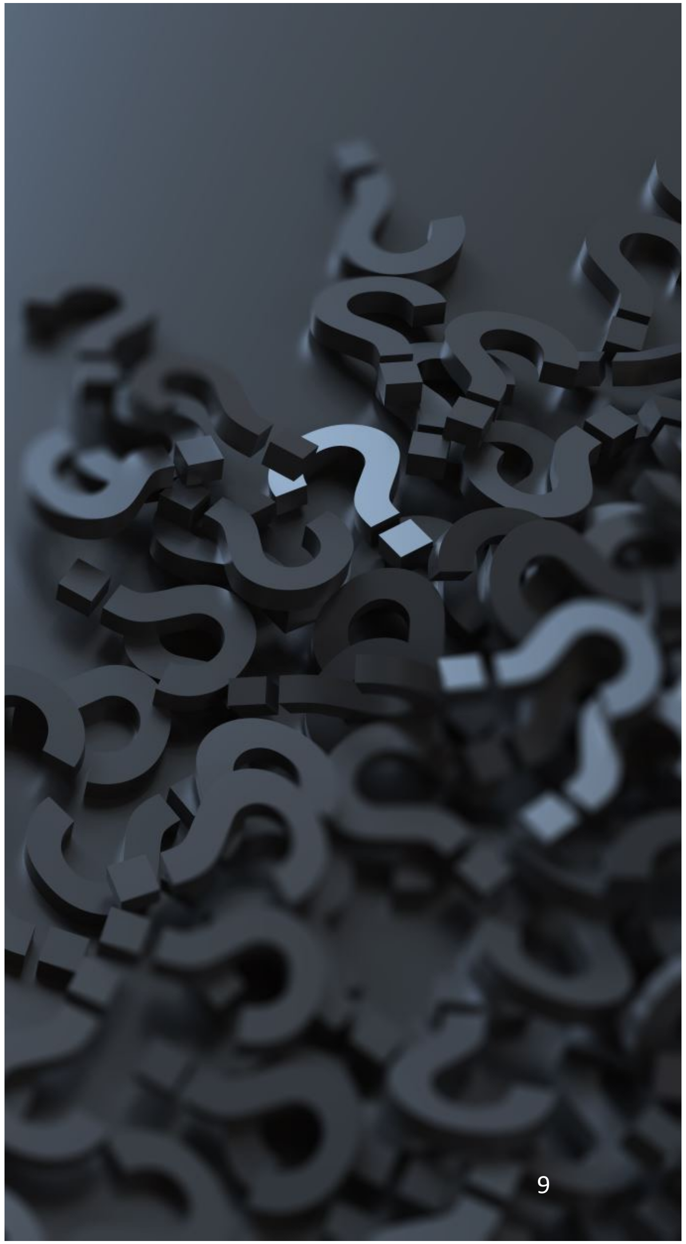
$$u(\$1) \cdot \left(\frac{1}{2}\right) + u(\$2) \cdot \left(\frac{1}{4}\right) + u(\$4) \cdot \left(\frac{1}{8}\right) + \cdots + u(\$2^{n-1}) \cdot \left(\frac{1}{2^n}\right) + \cdots$$

- If $u(x) = \log x$, the fair value of the lottery is approximately \$2.

Probability distribution

- A lottery is a probability distribution defined on the set of payoffs.
- Some properties of utility functions reflect preferences likely to be displayed by an entire class of agents.
- For example, all agents who prefer certainty payoffs with higher payoffs are called nonsatiable.
- If there are two lotteries, one with a certainty payoff of \$100 and another with a certainty payoff of \$200, a nonsatiable agent would never prefer the first opportunity.
- This preference is reflected by writing

$$u(200) > u(100).$$



Risk averse

- Suppose the agent prefers to receive a certainty outcome that is equal to the expected value of a lottery, rather than the lottery itself.
- In such a case we say the agent is risk averse.
- Assume that the lottery has two possible outcomes, say x_1 with probability p_1 and x_2 with probability $p_2 = 1 - p_1$, $p_1 \epsilon (0, 1)$.
- Then the lottery's expected payoff equals:

$$x_1 p_1 + x_2 (1 - p_1)$$

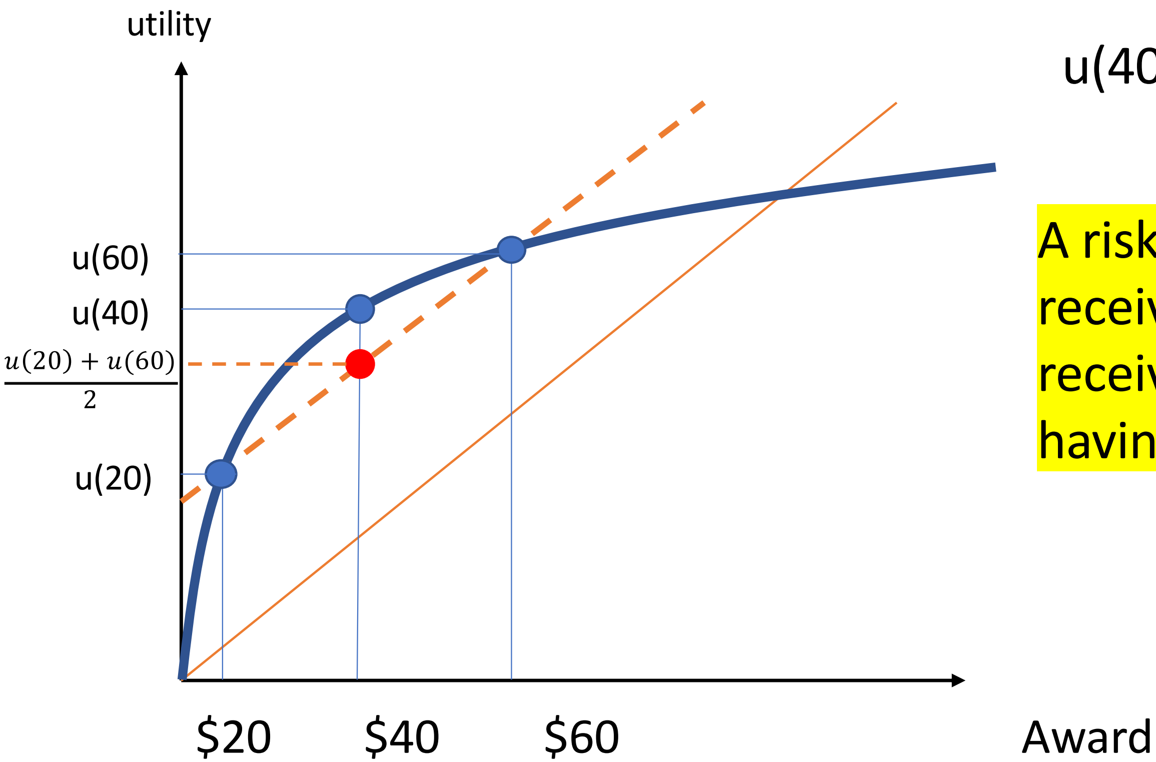
Concave function

- In terms of the utility function, the risk-aversion property can be expressed as:

$$u\big(x_1p_1 + x_2(1-p_1)\big) \geq u(x_1)p_1 + u(x_2)(1-p_1)$$
$$\text{for all } x_1, x_2 \text{ and } p_1 \in (0,1) \qquad (9.1)$$

- A function that satisfies equation (9.1) is called a concave function, and the utility functions of all risk-averse agents are concave.

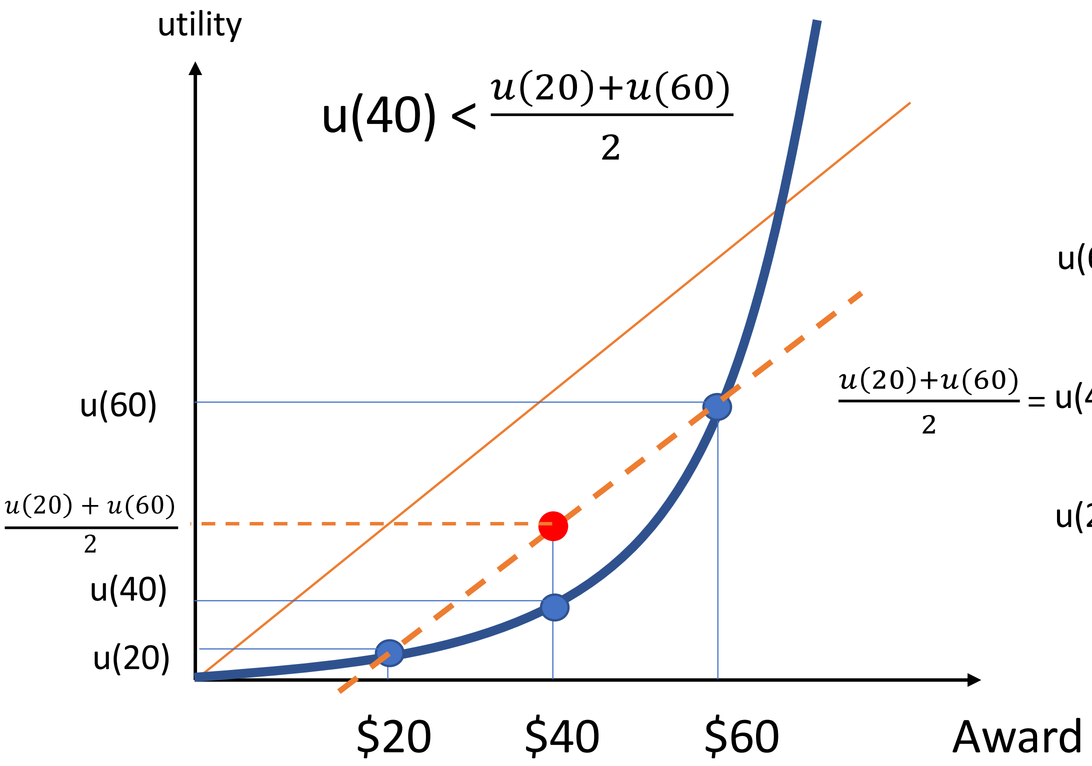
Risk Averse: Concave



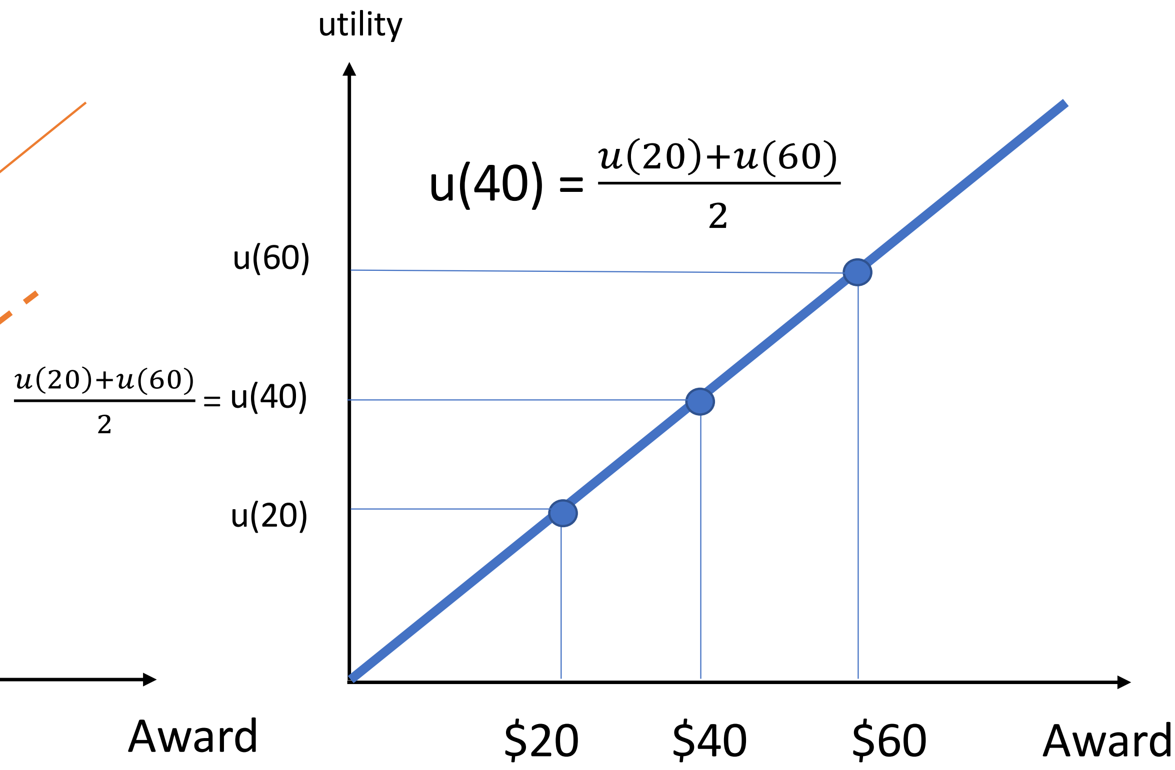
u(40) > $\frac{u(20)+u(60)}{2}$

A risk averse individual prefers receiving $40 with certainty to receiving $20 or $60 with each having an equal chance.

Risk Lover: Convex



Risk Neutral: Linear

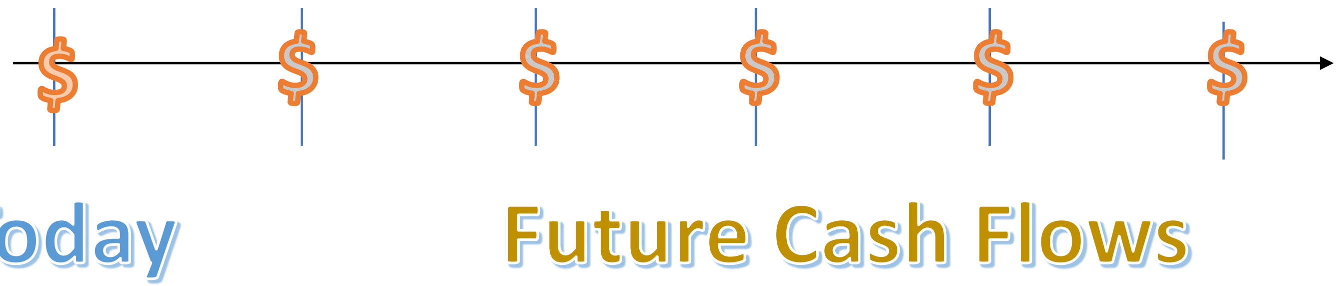


Topic three

Risk Management and Insurance

Cash flow risk

- We need to evaluate the risk of these future cash flows, referred to as *cash flow risk*, in order to understand the risk of any investment opportunity on the value of the company.
- Two basic risks
  - Sales risk: The risk related to the firm's output that will be sold and the price of the goods or services; and
  - Operating risk: The risk concerning operating cash flows that arises from the particular mix of fixed and variable operating costs.



Risk management

- Risk management often involves transferring risks to the agents best equipped to bear them.
- Risk transfer can make it possible for agents to undertake new risks that they would otherwise avoid, and hence improve an economy's resource allocation.

Insuring

- Insuring means reducing the probability of one or more downside outcomes by buying insurance protection.
- The price paid for the protection is referred to as an insurance premium.
- Upside outcomes are not usually affected by the purchase of insurance.
- Suppose it is now possible to purchase, for price \$c, an insurance contract P such that the insured will receive payment \$$P$ from the insurance company contingent on an outcome.

Insurance

- Suppose that there are two possible outcomes for a project, of which each arises with probability 0.5.
- Suppose it is now possible to purchase, for price $\$c$, an insurance contract P such that the insured will receive payment $\$P$ from the insurance company contingent on an outcome.
- The price paid for the protection is referred to as an insurance premium.
- The decision maker's payoffs, exclusive of the insurance premium $\$c$, are given by:

Instrument	Gains (X)	Payment (P)	$X+P$
First outcome	\$4.0	\$0.0	\$4.0
Second outcome	-\$2.0	\$1.5	-\$0.5

Buy or not?

- Because both outcomes are equally likely, his expected payoff from buying the insurance is:

$$\left(\frac{1}{2}\right)\times(4-c) - \left(\frac{1}{2}\right)(0.5+c) = 1.75 - c.$$

- His expected payoff from not buying it is:

$$\left(\frac{1}{2}\right)\times 4 - \left(\frac{1}{2}\right)\times 2 = 1.$$

- If $1.75 - c > 1$, he should buy the insurance.

Expected Utility

- Suppose $u(w) = 2w - \frac{1}{2}w^2$ and $c = 1$.
- When buying insurance,
  - Payoff in the first outcome: \$4.0 – \$1.0 = \$3.0
  - Payoff in the second outcome: - \$0.5 - \$1.0 = - \$1.5
- Buying insurance:
  - $Eu(w) = \frac{1}{2}\left(2 \cdot 3 - \frac{1}{2} \cdot 9\right) + \frac{1}{2}\left(2 \cdot (-1.5) - \frac{1}{2} \cdot (-1.5)^2\right) = -\frac{5.25}{4}$.
- Not buying insurance
  - $Eu(w) = \frac{1}{2}\left(2 \cdot 4 - \frac{1}{2} \cdot 16\right) + \frac{1}{2}\left(2 \cdot (-2) - \frac{1}{2} \cdot (-2)^2\right) = -3$.
- You would buy the insurance.

Math Practice Questions

(will be reviewed in Tutorial 5)

Compute $f'(x)$ for:

1. $f(x) = \log(2x)$;
2. $f(x) = \frac{1}{2} \times \log(2x)$
3. $f(x) = \log(1 - x)$;
4. $f(x) = \frac{1}{2} \times \log(1 - x)$;
5. $f(x) = \log(4 - x^2)$;
6. $f(x) = \frac{1}{2} \times \log(4 - x^2)$

Answers:

1. $f'(x) = \frac{1}{x}$;
2. $f(x) = \frac{1}{2x}$;
3. $f(x) = -\frac{1}{1-x}$;
4. $f(x) = -\frac{1}{2(1-x)}$;
5. $f(x) = \frac{-2x}{4-x^2}$;
6. $f(x) = \frac{-x}{4-x^2}$.

Summary

In conclusion, insurance serves as a financial safety net. We have studied why we need insurance. For that, we also explored various risk attitudes and how economic theory can model them in utility function.

Feedback for Topic 5

- https://padletuq.padlet.org/Shino/econ2103-shared-thoughts-topic-5-bx7b1rnqwtzhhxk0

Midterm Course Survey

- Please help us improve the course for the later half.
- URL: https://uniofqueensland.syd1.qualtrics.com/jfe/form/SV_8vHutqkG4xxWBUi

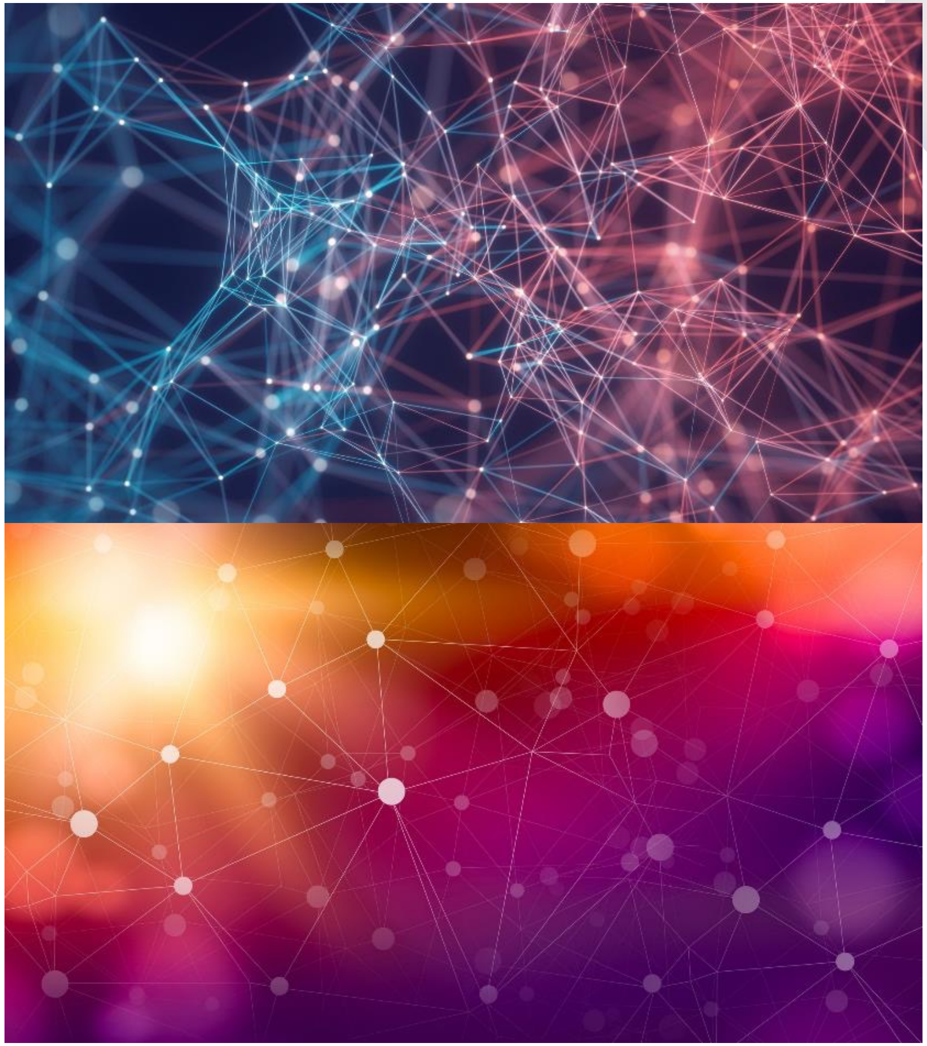
Thank You

Shino Takayama

s.takayama1@uq.edu.au

shino.mclennan@gmail.com

www.shinotakayama.com



Source

- Chapter 10 -- 11, "Contingent Claims and Contingent Strategies," in "*Financial Economics*," by F. J. Fabozzi, E.H. Neave, and G. Zhou, Wiley.
- Image sources: iStock, Commons Wikimedia and Adobe Stock.