Economics of Financial Markets – Lecture 2

Shino Takayama

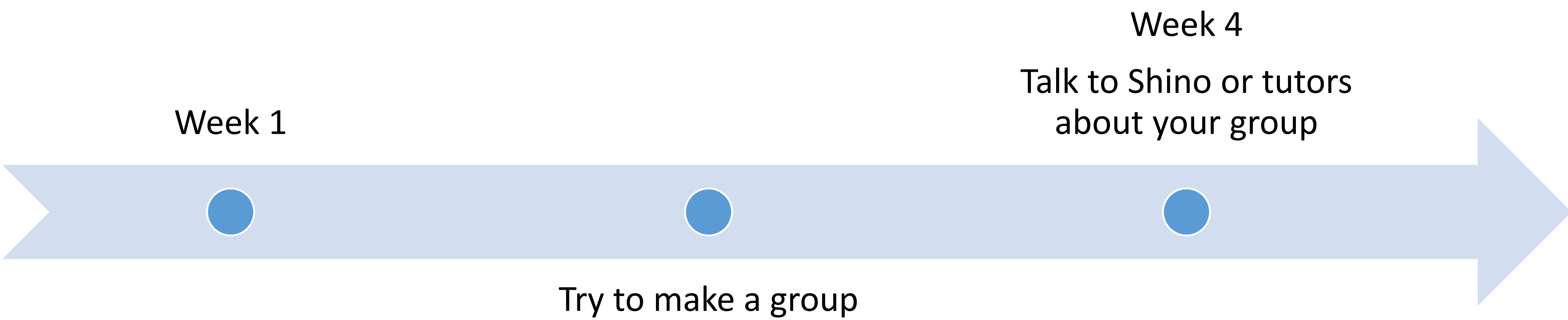
School of Economics

Just Making Sure:

- You can submit your answers for tutorial preparations at the assessment/Tutorial attendance and participation.
- Only submission via Blackboard will be accepted.
- Due date: **10am each Monday**, from teaching week 2
- The mark for the preparation cannot be awarded if the student does not attend the tutorial.
- You have to attempt all the questions but do not need to be correct.
- If you have questions, please let me know.

About Research Group Project

- Preferably make a group with people in the same tutorial session with you by Week 3 or 4.
- Please let us know if you make a group, or cannot make a group.



Preview

- Before we can go on with the study of money, banking, and financial markets, we must understand exactly what the phrase “interest rates” means. Today, we see that a concept known as the **yield to maturity** is the most accurate **measure of interest rate**.

Learning Objectives

- Calculate the **present value of future cash flows** and the **yield to maturity** on the four types of credit market instruments.
- Recognize the distinctions among **yield to maturity**, **current yield**, **rate of return**, and **rate of capital gain**.
- Interpret the distinction between **real** and **nominal interest rates**.

Measuring Interest Rates

- **Present value**: a dollar paid to you one year from now is less valuable than a dollar paid to you today.
  - Why: a dollar deposited today can earn interest and become \$1 x (1+i) one year from today.

A Simple Loan

- The lender provides the borrower with an amount of funds (called the *principal*) that must be repaid to the lender at the *maturity date*, along with an additional payment for the interest.
- Suppose that you made your friend, Jane, a simple loan of $100 for one year, you would require her to repay the principal of $100 in one year's time along with an additional payment for interest; say, $10.

Simple Interest Rate

- In the case of a simple loan, the interest payment divided by the amount of the loan is a natural and sensible way to measure the *interest rate*.
- Therefore,

$$i = \frac{\$10}{\$100} = 0.1.$$

Present Value

Let $i = .10$

In one year: $100 X (1+ 0.10) = $110

In two years: $110 X (1 + 0.10) = $121

or $\$100 \text{ X } (1 + 0.10)^2$

In three years: $121 X (1 + 0.10) = $133

or $\$100 \text{ X } (1 + 0.10)^3$

In n years

$\$100 \text{ X } (1 + i)^n$

Simple Present Value

$$\text{PV} = \text{today's (present) value}$$

$$\text{CF} = \text{future cash flow (payment)}$$

$$i = \text{the interest rate}$$

Equation 1: $$\text{PV} = \frac{\text{CF}}{(1 + i)^{\text{n}}}$$

Simple Present Value

•Cannot directly compare payments scheduled in different points in the time line

	\$100	\$100	\$100	\$100
Year	0	1	2	n
PV	100	$100/(1+i)$	$100/(1+i)^2$	$100/(1+i)^n$

Intuition of Equation 1

- Intuitively, what Equation 1 tells us is that if you are promised $1 for certain ten years from now, this dollar would not be as valuable to you as $1 is today,
- because if you had the $1 today, you could invest it and end up with more than $1 in ten years.

Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond

A Fixed Payment Loan

- The lender provides the borrower with an amount of funds, which must be repaid by making the same payment every period (such as a month), consisting of part of the principal and interest for a set number of years.
- For example, if you borrowed $1,000, a fixed-payment loan might require you to pay $126 every year for 25 years.
- Instalment loans (such as auto loans) and mortgages are frequently of the fixed-payment type.

Coupon Bond

- A **coupon bond** pays the owner of the bond a fixed interest payment (coupon payment) every year until the maturity date, when a specified final amount (**face value** or **par value**) is repaid.
- A coupon bond is identified by three pieces of information
  - First is the corporation or government agency that issues the bond.
  - Second is the maturity date of the bond.
  - Third is the bond's **coupon rate**, the dollar amount of the yearly coupon payment expressed as a percentage of the face value of the bond.
- Capital market instruments such as U.S. Treasury bonds and notes and corporate bonds are examples of coupon bonds.

Example: Coupon Bond

- A coupon bond with \$1,000 face value might pay you a coupon payment of \$100 per year for ten years, and at the maturity date repay you the face value amount of \$1,000.
- The **coupon rate** is then \$100/\$1,000, which is 0.10, or 10%.

Discount Bond

- A **discount bond** (also called a **zero-coupon bond**) is bought at a price below its face value (at a discount), and the face value is repaid at the maturity date.
- Unlike a coupon bond, a discount bond does not make any interest payments; it just pays off the face value.
- For example, a discount bond with a face value of $1,000 might be bought for $900; in a year's time the owner would be repaid the face value of $1,000.
- U.S. Treasury bills, U.S. savings bonds, and long-term zero-coupon bonds are examples of discount bonds.

Yield to Maturity

- **Yield to maturity**: the interest rate that equates the **present value of cash flow payments** received from a debt instrument with **its value today**

Yield to Maturity on a Simple Loan

$$PV = \text{amount borrowed} = \$100$$

$$CF = \text{cash flow in one year} = \$110$$

$$n = \text{number of years} = 1$$

$$\$100 = \frac{\$110}{(1+i)^1}$$

$$(1+i)\,\$100 = \$110$$

$$(1+i) = \frac{\$110}{\$100}$$

$$i = 0.10 = 10\%$$

For simple loans, the simple interest rate equals the yield to maturity

Fixed-Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

Equation 2: $$\text{LV} = \frac{\text{FP}}{1+i} + \frac{\text{FP}}{(1+i)^2} + \frac{\text{FP}}{(1+i)^3} + \ . \ . \ . + \frac{\text{FP}}{(1+i)^n}$$

Pocket Calculator will Tell you

- For example, in the case of the 25-year loan with yearly payments of \$126, the yield to maturity that solves Equation 2 is 12%.

$$\$1000 = \frac{\$126}{1+i} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \cdots + \frac{\$126}{(1+i)^{25}}.$$

Coupon Bond

Using the same strategy used for the fixed-payment loan:

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

Equation 3: $$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Coupon Bond

- The price of a coupon bond and the yield to maturity are <u>negatively</u> related.
- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
- The yield to maturity is greater than the coupon rate when the bond price is below its face value.

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}.$$

Table 1: From Equation 3

TABLE 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Coupon Bond

- **Consol** or **perpetuity**: a bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$P = C / i_c$$

$$P_c = \text{price of the consol}$$

$$C = \text{yearly interest payment}$$

$$i_c = \text{yield to maturity of the consol}$$

$$\text{can rewrite above equation as this} : i_c = C / P_c$$

For coupon bonds, this equation gives the **current yield**, easy to calculate approximation to the yield to maturity

- Let x=1/(1+i) and then

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots$$

$$= C\ (x + x^2 + x^3 + \cdots\ ) = C \times \frac{x}{1-x} = C \times \frac{1-(1-x)}{1-x}$$

$$= C \times (\frac{1}{1-x} - 1\ ) = C \times (\frac{1}{1-1/(1+i)} - 1\ )$$

$$= C \times (\frac{1+i}{1+i-\ (1+i)/(1+i)} - 1\ ) = C \times (\frac{1+i}{i} - \frac{i}{i}\ ) = \frac{C}{i}$$

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \quad for \quad x < 1$$

$$S = 1 + x + x^2 + x^3 + \cdots$$

$$xS = \quad x + x^2 + x^4 + \cdots$$

$$(1-x)S = 1$$

$$S = \frac{1}{1-x} \quad for \quad x < 1$$

Discount Bond

For any one year discount bond

$$i = \frac{F - P}{P}$$

F = Face value of the discount bond

P = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.

$$\text{Present Value}\,(PV) = \frac{\text{Cash Flow}\,(CF)}{(1+i)^n}$$

When $n = 1$,

$$PV = \frac{CF}{(1+i)}$$

$$PV(1+i) = CF$$

$$PV + PV \times i = CF$$

$$i = \frac{CF - PV}{PV} = \frac{F - P}{P}$$

Why?

- The yield to maturity equals the increase in prices over the year divided by the initial price.
- The yield to maturity is <u>negatively</u> related to the current bond price.

$$i = \frac{F - P}{P} = \frac{F}{P} - 1 .$$

Example: Discount Bond

- Let us consider a discount bond such as a one-year Treasury bill, which pays off a face value of \$1,000 in one year's time.
- If the current purchase price of this bill is \$900, then equating this price to the present value of the \$1,000 received in one year, using Equation 1, gives:

$$\$900 = \frac{\$1000}{1+i}.$$

Solving for *i* gives:

$$(1 + i) \times \$900 = \$1{,}000$$

$$\$900 + \$900i = \$1{,}000$$

$$\$900i = \$1{,}000 - \$900$$

$$i = \frac{\$1{,}000 - \$900}{\$900} = 0.111 = 11.1\%$$

We have obtained $i = \dfrac{F - P}{P}$.

The Distinction Between Interest Rates and Returns

- Rate of Return (RET):

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

RET = return from holding the bond from time t to time $t + 1$

P_t = price of bond at time t

P_{t+1} = price of the bond at time $t + 1$

C = coupon payment

$\frac{C}{P_t}$ = current yield = i_c

$\frac{P_{t+1} - P_t}{P_t}$ = rate of capital gain = g

One More Time

- Rate of return is given by the following relationship:

$$\text{Rate of Return} = R = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}.$$

- Thus, Rate of Return = the Current Yield + the Rate of Capital Gain.
- Because

$$\text{Current Yield} = \frac{C}{P_t}.$$

$$\text{Rate of Capital Gain} = \frac{P_{t+1} - P_t}{P_t}.$$

One More Time: Equation 3

- Consider the situation where interest rates rise from 10% to 20%.
- We study One-Year Returns on Different-Maturity 10% Coupon-Rate and Bonds.

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}.$$

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

The Distinction Between Interest Rates and Returns

- The return equals the yield to maturity only if the holding period equals the time to maturity.
- A *rise* in interest rates is associated with a fall in *bond prices*, resulting in a *capital loss* if time to maturity is longer than the holding period.
- The *more distant* a bond's maturity, the *greater* the size of the percentage price change associated with an interest-rate change.

The Distinction Between Interest Rates and Returns

- The *more* distant a bond's maturity, the *lower* the rate of return that occurs as a result of an increase in the interest rate.
- Even if a bond has a substantial initial interest rate, its return can be *negative* if interest rates *rise*.

Maturity and the Volatility of Bond Returns: Interest-Rate Risk

- Prices and returns for *long-term bonds* are *more volatile* than those for shorter-term bonds.
- There is *no interest-rate risk* for any bond whose time to maturity matches the holding period.
- The risk level associated with an asset's return that results from interest-rate change is called **interest-rate risk**.

The Distinction Between Real and Nominal Interest Rates

- **Nominal interest rate** makes no allowance for inflation.
- **Real interest rate** is adjusted for changes in price level so it more accurately reflects the cost of borrowing.
  - *Ex ante real interest rate* is adjusted for expected changes in the price level
  - *Ex post real interest rate* is adjusted for actual changes in the price level

Fisher Equation

$$i = i_r + \pi^e$$

i = nominal interest rate

i_r = real interest rate

π^e = expected inflation rate

When the real interest rate is low,

there are greater incentives to borrow and fewer incentives to lend.

The real interest rate is a better indicator of the incentives to borrow and lend.

Example: Fisher Equation

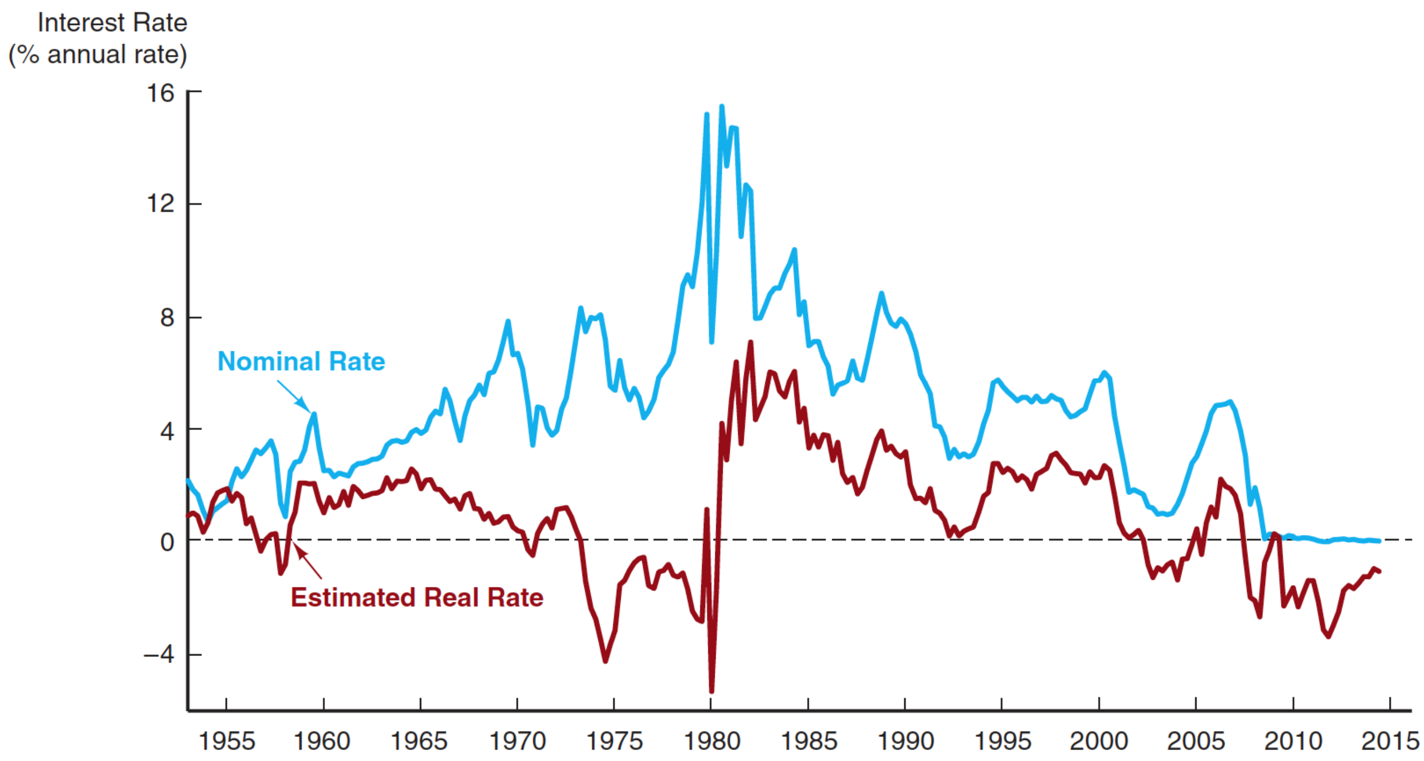
- Suppose that you have made a one-year simple loan with a 5% interest rate (*i* =5%) and you expect the price level to rise by 3% over the course of the year (π^e= 3%).
- In this case, the interest rate you have earned in terms of real goods and services is 2%.
- Now what if the interest rate rises to 8%, but you expect the inflation rate to be 10% over the course of the year?
- Although you will have 8% more dollars at the end of the year, you will be paying 10% more for goods.
- This is also exactly what the Fisher definition tells us, because:

$$i^r = 8\% - 10\% = -2\%.$$

Incentives to borrow and lend

- As a lender, you are clearly less eager to make a loan in this case, because in terms of real goods and services you have actually earned a negative interest rate of 2%.
- As the borrower, you fare quite well because at the end of the year, the amounts you will have to pay back will be worth 2% less in terms of goods and services—you as the borrower will be ahead by 2% in real terms.
- ***When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.***

Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2014



Sources: Nominal rates from Federal Reserve Bank of St. Louis FRED database: http://research.stlouisfed.org/fred2/. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," Carnegie-Rochester Conference Series on Public Policy 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends, and then subtracting the expected inflation measure from the nominal interest rate.