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- Module Ledger -
 1 [
    High level specification of DLT Ledger, expressed as a single state machine without MVCC
    validation
 6 EXTENDS Sequences, Integers, TLAPS, Datatype
    State variables of this module
    VARIABLES state,
                                current state of the ledger state machine.
                   chain,
                                blockchain, a list of received transactions.
12
                   index
                                unprocessed TX index at the blockchain.
13
    vars \triangleq \langle state, chain, index \rangle
    Type invariant
    ChainEntry \stackrel{\Delta}{=} [tx: TX, is\_valid: BOOLEAN \cup \{NULL\}]
19
    Chain \triangleq Seq(ChainEntry)
20
    TypeInv \triangleq
         \land \ state \ \in State
22
         \land index \in Nat
23
         \wedge index > 0
24
          Each TX in the blockchain has a flag if it's valid or not. Before the TX
25
          is processed, its value is NULL.
26
         \land chain \in Chain
27
28
    Initial condition
   Init \triangleq
32
         \land state = InitState
                                      state is at the initial state, and
33
         \wedge index = 1
34
35
         \wedge chain = \langle \rangle
                                      empty transaction queue.
37 F
    Actions
    SubmitTX: Client appends a transaction to the transaction queue.
    SubmitTX(tx) \triangleq
45
         \land chain' = Append(chain, [tx \mapsto tx, is\_valid \mapsto NULL])
46
         \land UNCHANGED \langle state, index \rangle
47
    ProcessTx: Ledger processes the oldest unprocessed TX and updates its state by applying f
    ProcessTX\_OK \triangleq
54
        LET
55
             f \triangleq chain[index].tx.f
56
        IN
57
               \land Len(chain) \ge index
58
              \land index \in domain chain
59
              \wedge chain' = [chain \ EXCEPT \ ![index].is\_valid = TRUE] update validity flag
60
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\wedge index' = index + 1
                                                  increment the index.
 61
                 \land state' \in f[state]
                                                  perform non-deterministic state transition by f.
 62
      ProcessTX\_ERR \triangleq
 64
           LET
 65
                f \triangleq chain[index].tx.f
 66
 67
                  \land Len(chain) \ge index
 68
 69
                 \land index \in domain chain
                 \land chain' = [chain \ EXCEPT \ ![index].is\_valid = FALSE] see above.
 70
                 \wedge index' = index + 1
                                                see above.
                 \land UNCHANGED state
                                                state does not change due to invalid TX.
 72
      Next \triangleq (\exists tx \in TX : SubmitTX(tx)) \lor ProcessTX\_OK \lor ProcessTX\_ERR
      Specification
     Spec \stackrel{\triangle}{=} Init \wedge \Box [Next]_{vars}
 79
      Invariants
     Finality \stackrel{\Delta}{=} TRUE TODO
      Safety \stackrel{\triangle}{=} Finality
 86
       Invariant (safety) on the blockchain
 88
      ChainInv \triangleq
 89
             chain = (processed part) + (unprocessed part)
 90
            \land \forall i \in 1 ... index - 1 : chain[i].is\_valid \in BOOLEAN
 91
            \land \forall i \in \{i \in Nat : index \leq i\} \cap DOMAIN \ chain : chain[i].is\_valid = NULL
      Inv \stackrel{\Delta}{=} TypeInv \wedge ChainInv
 94
       Invariant (safety) on the high-level Ledger
 96
     THEOREM LedgerInv \stackrel{\Delta}{=} Spec \Rightarrow \Box Inv
 97
 98
      PROOF
           \langle 1 \rangle 1 \; Init \Rightarrow Inv
 99
                By InitStateAxiom def Init, Inv, TypeInv, ChainInv, Chain
100
           \langle 1 \rangle 2 \; Inv \wedge [Next]_{vars} \Rightarrow Inv'
101
                (2)1 SUFFICES ASSUME TypeInv, ChainInv, [Next]<sub>vars</sub>PROVE Inv'BY DEF Inv
102
                \langle 2 \rangle2Case Next
103
                      \langle 3 \rangle use def Inv, Next
104
                      (3) USE DEF TypeInv, ChainInv, Chain, ChainEntry
105
                      \langle 3 \rangle1CASE (\exists tx \in TX : SubmitTX(tx))
106
                           \langle 4 \rangle USE DEF SubmitTX
107
                           \langle 4 \ranglea \forall i \in \text{DOMAIN } chain : chain[i] = chain'[i]BY \langle 3 \rangle 1
108
                           \langle 4 \rangle 1 \ TypeInv'BY \langle 2 \rangle 1, \langle 3 \rangle 1
109
                           \langle 4 \rangle 2 \ ChainInv'
110
                                \langle 5 \rangle 1 ChainInv!1'OBVIOUS
111
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\langle 5 \rangle 2 ChainInv!2'
112
                                                   \langle 6 \ranglea domain chain' = domain <math>chain \cup \{Len(chain) + 1\} By TypeInv, \langle 3 \rangle 1
113
                                                   \langle 6 \rangle 1 pick tx \in TX : SubmitTX(tx)by \langle 3 \rangle 1
114
                                                   \langle 6 \rangle 2 Take i \in (\{i \in Nat : index \leq i\} \cap DOMAIN \ chain)'
115
                                                    \langle 6 \rangle3CASE i \in (\{j \in Nat : index \leq j\} \cap \{Len(chain) + 1\})BY \langle 2 \rangle 1, \langle 4 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 3
116
                                                    \langle 6 \rangle4CASE i \in (\{j \in Nat : index \leq j\} \cap DOMAIN \ chain)BY \langle 2 \rangle 1, \langle 4 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 4
117
                                                    \langle 6 \rangle QED BY \langle 2 \rangle 1, \langle 6 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 2, \langle 6 \rangle 3, \langle 6 \rangle 4
118
                                             \langle 5 \rangle QED BY \langle 5 \rangle 1, \langle 5 \rangle 2
119
                                     \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
120
                              \langle 3 \rangle2CASE ProcessTX\_OK
121
                                     \langle 4 \rangle USE DEF ProcessTX\_OK
122
                                     \langle 4 \rangle 1 TypeInv'by \langle 2 \rangle 1, \langle 3 \rangle 2 DEF TX, Operation, TotalFunc
123
                                     \langle 4 \rangle 2 ChainInv'
124
                                            ⟨5⟩ ChainInv!1′OBVIOUS
125
                                            \langle 5 \rangle ChainInv!2'BY \langle 2 \rangle 1, \langle 3 \rangle 2
126
                                            \langle 5 \rangle QED OBVIOUS
127
                                     \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
128
                              \langle 3 \rangle3CASE ProcessTX\_ERR
129
                                     \langle 4 \rangle USE DEF ProcessTX\_ERR
130
                                     \langle 4 \rangle 1 TypeInv'BY \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 3 \rangle 3 DEF TX, Operation, TotalFunc
131
                                     \langle 4 \rangle 2 \ ChainInv'
132
                                            \langle 5 \rangle ChainInv!1'OBVIOUS
133
                                            \langle 5 \rangle ChainInv!2'BY \langle 2 \rangle 1, \langle 3 \rangle 3
134
                                            (5) QED OBVIOUS
135
136
                                     \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
                              \langle 3 \rangle QED
137
                                     BY \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 3 \rangle 1, \langle 3 \rangle 2, \langle 3 \rangle 3
138
                       \langle 2 \rangle3CASE UNCHANGED vars
139
                             BY \langle 2 \rangle 1, \langle 2 \rangle 3 DEF Inv, TypeInv, ChainInv, vars
140
                       \langle 2 \rangle QED BY \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 2 \rangle 3
141
                \langle 1 \rangle QED BY PTL, \langle 1 \rangle 1, \langle 1 \rangle 2 DEF Spec
142
         \* Modification History
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