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— Module Ledger —
1 [
    High level specification of DLT Ledger, expressed as a single state machine without MVCC
    validation
 6 EXTENDS Sequences, Integers, TLAPS
    Constants
   CONSTANTS State, InitState a set of states, and
    ASSUME InitStateAxiom \stackrel{\triangle}{=} \overline{InitState \in State} the designated initial state.
    NULL \triangleq \text{CHOOSE } x : x \notin \text{BOOLEAN}
    State variables of this module
    VARIABLES state,
                                  current state of the ledger state machine.
18
19
                    chain.
                                  blockchain, a list of received transactions.
20
                    index
                                  index of the blockchain.
    vars \stackrel{\Delta}{=} \langle state, chain, index \rangle
    Datatype definition
    TotalFunc(S1, S2) \stackrel{\Delta}{=} [S1 \rightarrow S2 \setminus \{\{\}\}] a set of total function from S1 to S2
    Operation is a function from a state to a state, can be non-deterministic, but required to be total.
    Operation \triangleq TotalFunc(State, SUBSET State)
      Operation \stackrel{\Delta}{=} [State \rightarrow (SUBSET State) \setminus \{\{\}\}]
     TX \stackrel{\Delta}{=} [f: Operation] a transaction. note that "f" is just a label
    Type invariant
    ChainEntry \triangleq [tx : TX, is\_valid : BOOLEAN \cup \{NULL\}]
     Chain \triangleq Seq(ChainEntry)
     TypeInv \triangleq
          \land \ state \ \in State
44
          \land index \in Nat
45
          \wedge index > 0
46
           Each TX in the blockchain has a flag if it's valid or not. Before the TX is processed, its value is NULL.
47
          \land chain \in Chain
48
49
    Initial condition
    Init \stackrel{\triangle}{=}
53
          \land state = InitState
                                        state is at the initial state, and
54
          \wedge index = 1
55
          \wedge chain = \langle \rangle
                                         empty transaction queue.
56
    Actions
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SubmitTX: A client appends a transaction to the transaction queue.

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SubmitTX(tx) \triangleq
           \land chain' = Append(chain, [tx \mapsto tx, is\_valid \mapsto NULL])
 67
           \land UNCHANGED \langle state, index \rangle
 68
     CommitTx: Ledger processes the oldest unprocessed TX and
     ProcessTX\_OK \triangleq
 73
          LET
 74
               f \triangleq chain[index].tx.f
 75
 76
          IN
                 \land Len(chain) \ge index
 77
                \land index \in domain chain
 78
                \land chain' = [chain \ EXCEPT \ ![index].is\_valid = TRUE] update validity flag
 79
                \wedge index' = index + 1
                                               increment the index.
 80
                \land state' \in f[state]
                                               perform non-deterministic state transition by f.
 81
      ProcessTX\_ERR \triangleq
 83
          LET
 84
               f \triangleq chain[index].tx.f
 85
          IN
 86
 87
                 \land Len(chain) \ge index
                \wedge index \in DOMAIN \ chain
 88
                \land chain' = [chain \ EXCEPT \ ! [index].is\_valid = FALSE] see above.
 89
                \wedge index' = index + 1 see above.
 90
                \land UNCHANGED state
                                             state does not change due to invalid TX.
 91
     Next \triangleq (\exists tx \in TX : SubmitTX(tx)) \lor ProcessTX\_OK \lor ProcessTX\_ERR
 93
     Specification
     Spec \stackrel{\Delta}{=} Init \wedge \Box [Next]_{vars}
100 |
     Invariants
104 Finality \stackrel{\triangle}{=} TRUE TODO
105 Safety \stackrel{\triangle}{=} Finality
       Invariant (safety) on the blockchain
107
      ChainInv \triangleq
108
            chain = (processed part) + (unprocessed part)
109
           \land \forall i \in 1 ... index - 1 : chain[i].is\_valid \in BOOLEAN
110
           \land \forall i \in \{i \in Nat : index \leq i\} \cap \text{domain} : chain[i].is\_valid = NULL
111
     Inv \stackrel{\Delta}{=} TypeInv \wedge ChainInv
     THEOREM LedgerInv \triangleq Spec \Rightarrow \Box Inv
     PROOF
116
117
           \langle 1 \rangle 1 \; Init \Rightarrow Inv
               BY InitStateAxiom DEF Init, Inv, TypeInv, ChainInv, Chain
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\langle 1 \rangle 2 \; Inv \wedge [Next]_{vars} \Rightarrow Inv'
119
                      (2)1 Suffices assume TypeInv, ChainInv, [Next]<sub>vars</sub>Prove Inv'by def Inv
120
                      \langle 2 \rangle2Case Next
121
                             \langle 3 \rangle USE DEF Inv, Next
122
                             (3) USE DEF TypeInv, ChainInv, Chain, ChainEntry
123
                             \langle 3 \rangle1CASE (\exists tx \in TX : SubmitTX(tx))
124
                                   \langle 4 \rangle USE DEF SubmitTX
125
                                   \langle 4 \ranglea \forall i \in DOMAIN \ chain : chain[i] = chain'[i]BY \langle 3 \rangle 1
126
                                   \langle 4 \rangle 1 \ TypeInv'BY \langle 2 \rangle 1, \langle 3 \rangle 1
127
                                   \langle 4 \rangle 2 \ ChainInv'
128
                                          \langle 5 \rangle 1 ChainInv!1'OBVIOUS
129
                                          \langle 5 \rangle 2 \ ChainInv!2'
130
                                                 \langle 6 \ranglea domain chain' = domain \ chain \cup \{Len(chain) + 1\} By TypeInv, \langle 3 \rangle 1
131
                                                 \langle 6 \rangle 1 pick tx \in TX : SubmitTX(tx)by \langle 3 \rangle 1
132
                                                 \langle 6 \rangle 2 take i \in (\{i \in Nat : index \leq i\} \cap DOMAIN \ chain)'
133
                                                 \langle 6 \rangle3CASE i \in (\{j \in Nat : index \leq j\} \cap \{Len(chain) + 1\})BY \langle 2 \rangle 1, \langle 4 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 3
134
                                                 \langle 6 \rangle4CASE i \in (\{j \in Nat : index \leq j\} \cap DOMAIN \ chain)BY \langle 2 \rangle 1, \langle 4 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 4
135
                                                 \langle 6 \rangle QED BY \langle 2 \rangle 1, \langle 6 \rangle a, \langle 6 \rangle 1, \langle 6 \rangle 2, \langle 6 \rangle 3, \langle 6 \rangle 4
136
                                          \langle 5 \rangle QED BY \langle 5 \rangle 1, \langle 5 \rangle 2
137
                                   \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
138
                             \langle 3 \rangle2CASE ProcessTX\_OK
139
                                   \langle 4 \rangle USE DEF ProcessTX\_OK
140
                                   \langle 4 \rangle 1 TypeInv'by \langle 2 \rangle 1, \langle 3 \rangle 2 Def TX, Operation, TotalFunc
141
                                   \langle 4 \rangle 2 \ ChainInv'
142
                                          ⟨5⟩ ChainInv!1′OBVIOUS
143
                                          \langle 5 \rangle ChainInv!2'BY \langle 2 \rangle 1, \langle 3 \rangle 2
144
                                          (5) QED OBVIOUS
145
                                   \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
146
                             \langle 3 \rangle3CASE ProcessTX\_ERR
147
                                   \langle 4 \rangle USE DEF ProcessTX\_ERR
148
149
                                   \langle 4 \rangle 1 TypeInv'by \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 3 \rangle 3 DEF TX, Operation, TotalFunc
                                   \langle 4 \rangle 2 \ ChainInv'
150
                                          \langle 5 \rangle ChainInv!1'OBVIOUS
151
                                          \langle 5 \rangle ChainInv!2'BY \langle 2 \rangle 1, \langle 3 \rangle 3
152
                                          (5) QED OBVIOUS
153
                                   \langle 4 \rangle QED BY \langle 4 \rangle 1, \langle 4 \rangle 2
154
                             \langle 3 \rangle QED
155
                                   By \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 3 \rangle 1, \langle 3 \rangle 2, \langle 3 \rangle 3
156
                      \langle 2 \rangle3CASE UNCHANGED vars
157
                            BY \langle 2 \rangle 1, \langle 2 \rangle 3 DEF Inv, TypeInv, ChainInv, vars
158
                      \langle 2 \rangle QED BY \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 2 \rangle 3
159
               \langle 1 \rangle QED BY PTL, \langle 1 \rangle 1, \langle 1 \rangle 2 DEF Spec
160
162
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