Properties: Property 1.1 — Domain Restrictions

The domain of an expression is the set of all real numbers for which the expression is defined. The following restrictions must be satisfied:

1. **Division by zero.** A denominator may not be zero.
2. **Even radicals.** An even–indexed radical requires a nonnegative radicand.
3. **Logarithms.** The argument of $\log_b(x)$ (in particular $\ln x$) must satisfy $x > 0$.

Properties: Property 1.2 — Relationship Between e^x and $\ln x$

The exponential function e^x and the natural logarithm $\ln x$ are inverse functions. In particular,

1. $e^{\ln x} = x, \quad x > 0;$
2. $\ln(e^x) = x, \quad$ for all real x.

Note. $\ln(x)$ is defined as the logarithm base e, Euler's number; that is, $\ln(x) = \log_e(x)$.

Properties: Property 1.3 — Laws of Exponents

Let a and b be positive real numbers, and let $x, y \in \mathbb{R}$. Then

1. $b^x\, b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Properties: Property 1.4 — Laws of Logarithms

For $a > 0,\ A > 0,\ B > 0$, and $x \in \mathbb{R}$,

1. $\ln(AB) = \ln A + \ln B$
2. $\ln\left(\dfrac{A}{B}\right) = \ln A - \ln B$
3. $\ln(e^x) = x$
4. $\ln 1 = 0$
5. $\ln(a^x) = x \ln a$
6. $a^x = e^{x \ln a}$

Note. Although most examples use natural logarithms (ln), these identities hold for all logarithmic bases.

Definition: Definition 1.5 — Composition of Functions

A composition function is a function where the output of one function is used as the input of another, creating a new combined function.

$$(f \circ g)(x) = f(g(x)).$$

Difficulty: Easy ($\star$)

1. Find the domain and write in interval notations of $f(x) = \sqrt{x-5}$

2. Find the domain and write in interval notations of $f(x) = \dfrac{1}{\sqrt{x-5}}$

3. Find the domain and write in interval notations of $f(x) = e^{-x}$

4. Find the domain and write in interval notations of $f(x) = \ln(x + 2)$

5. Find the domain and write in interval notations of $f(x) = \ln(x^2 - 4)$

6. Find the domain and write in interval notations of $f(x) = \dfrac{1}{\ln(x^2 - 4)}$

7. Solve for x the following $4e^{x-1} = 4$

8. Solve for x the following $2e^{5x} - 1 = 0$

9. Solve $3 + 2\ln\left(\frac{x}{7} + 3\right) = -4$

10. Compute the following terms;

    (a) $a^5 \times a^7$

    (b) $\frac{a^3}{a^2}$

    (c) $(3a^2)^2$

    (d) $2^5 \times 4^{-2}$

11. Simplify the following terms;

(a) $\ln(8) + \ln(2)$

(b) $\ln(54) - 2\ln(2) + \ln(2)$

Difficulty: Intermediate ($\star\star$)

1. Find the domain and write in interval notations of $f(x) = \ln(\ln(x^2 - 4)))$

2. Find the domain and write in interval notations of $f(x) = \sqrt{154 - x^2}$

3. Solve for t the following $\dfrac{A}{1 + Be^{t/2}} = C$ for $B, C \neq 0$.

4. Solve $2\ln(\sqrt{x}) - \ln(1 - x) = 2$

5. Find the domain of the following function.
   $h(x) = (f \circ g)(x)$, where

$$f(x) = \sqrt{x}$$

$$g(x) = \sin(x) \ x \in [0, 2\pi].$$

6. Solve the following equation:

$$(\sqrt[3]{2})^x = 8^{10-x}.$$

Difficulty: Challenging ($\star\star\star$)

1. Let the functions $f(x), g(x)$, and $h(x)$ be defined as follows:

$$f(x) = \frac{1}{x-2},$$
$$g(x) = \sqrt{3x-1},$$
$$h(x) = \ln(x+1).$$

   Find the composition function $(f \circ g \circ h)(x)$ and determine its domain.

2. A classmate from your Math 154 course attempted this problem. Identify if they've solved it correctly. If you find any errors, point out the step where the mistake occurred and provide the correct solution.
Questions: Solve the following inequality for x:

$$\ln(x) + \ln(x + 4) = 1$$

Student's solution:
Step 1:

$$\ln\left(x(x + 4)\right) = 1$$

Step 2:

$$\mathrm{e}^{\ln(x(x+4))} = \mathrm{e}^1$$

Step 3:

$$x(x + 4) = \mathrm{e}$$

step 4:

$$x^2 + 4x - e = 0$$

step 5: By quadratic formula,

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-\mathrm{e})}}{2(1)}$$

Therefore, $x = \dfrac{-4 \pm \sqrt{16 + 4\mathrm{e}}}{2}$