

私見: キャビティ問題

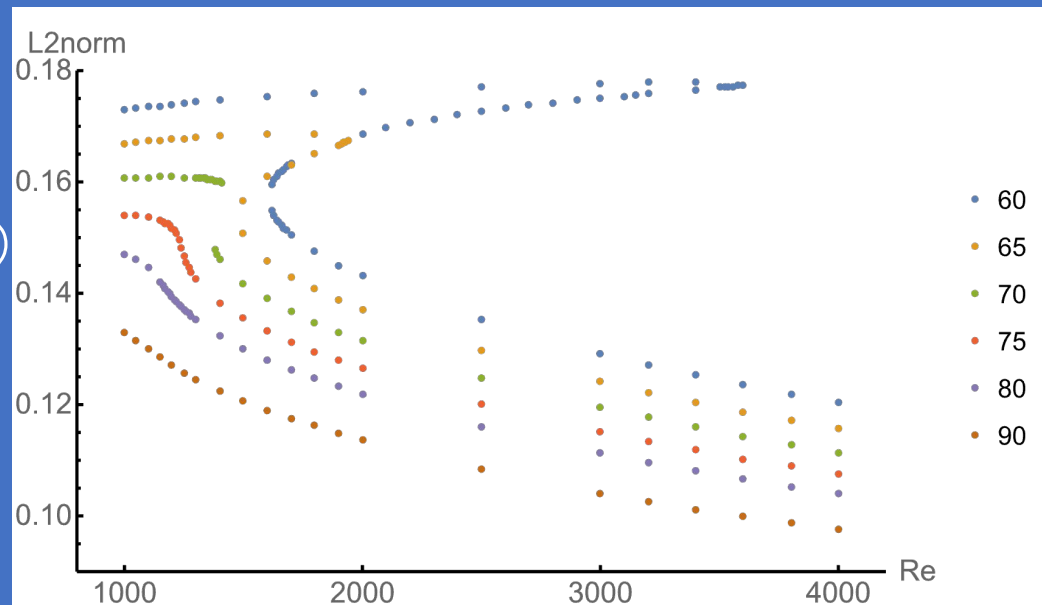
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(北海道大学 電子科学研究所)

TFDA 会合

2025年12月28日

新高岡駅観光交流センター



1. はじめに
 1. なぜキャビティ問題？
2. 四角形キャビティ問題の定常解の分岐図
3. 三角形キャビティ問題の分岐図と周期解, カオティックな振る舞い
4. 頂角に着目した三角形キャビティ流れの定常解の分岐図 (U.)

2次元キャビティ流

定常問題

Find $(\mathbf{u}, p) : \Omega_\theta \rightarrow \mathbf{R}^2 \times \mathbf{R}$ such that

$$(\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{0} \quad \text{in } \Omega_\theta$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_\theta$$

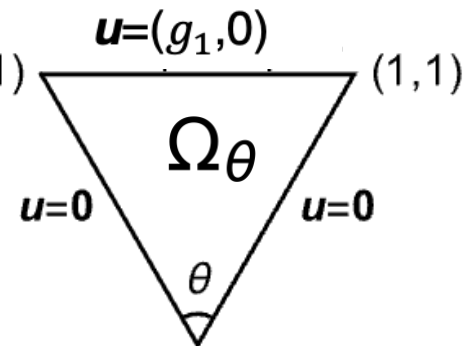
非定常問題

Find $(\mathbf{u}, p) : \Omega \times (0, T) \rightarrow \mathbf{R}^2 \times \mathbf{R}$ such that

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \Delta \mathbf{u} + \nabla p = \mathbf{0} \quad \text{in } \Omega_\theta \times (0, T)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_\theta \times (0, T)$$

$\mathbf{u}=(1,0)$ がよく使われている



二等辺三角形. θ [°] は“頂角”

Reの定義も文献により異なることがある.
(代表長を掛けてない等)

$\nu > 0$ は粘性係数を表す定数. $Re = \nu^{-1}$ とする.

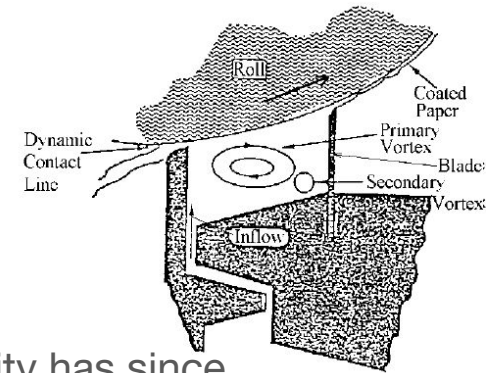
なぜキャビティ問題？

- **Shankar and Deshpande 2000** (<https://doi.org/10.1146/annurev.fluid.32.1.93>) の記述

- driven cavity flows offer an ideal framework in which meaningful and detailed comparisons can be made between results obtained from experiment, theory, and computation

- **Shankar and Deshpande 2000** (*Annu. Rev. Fluid Mech.*) の例

- short-dwell coater (Aidun et al (1991), 右の (a))
- drag-reducing riblets



- **An et al. 2019** (<https://doi.org/10.1017/jfm.2019.512>) の記述

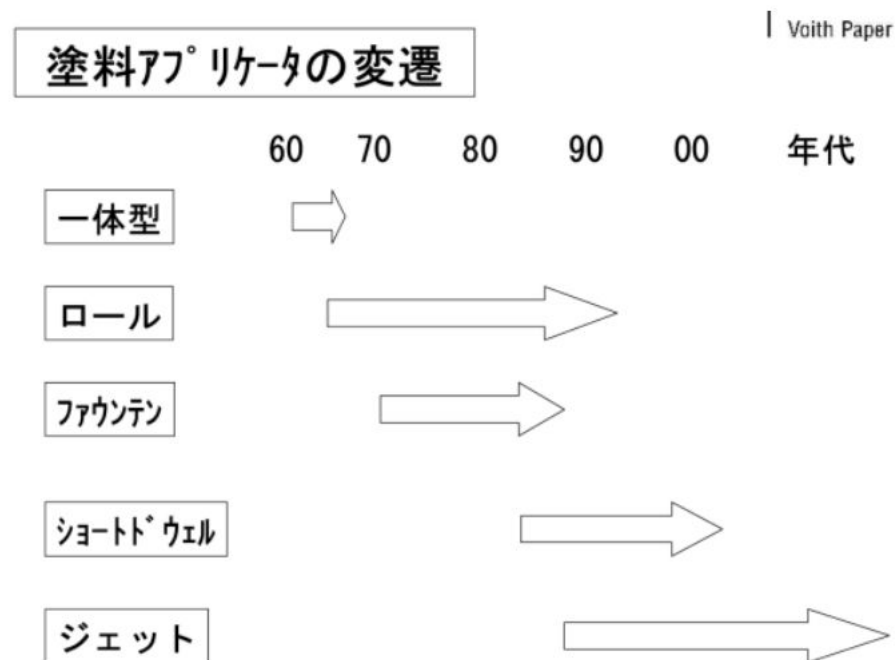
- The incompressible shear-driven flow inside a wall-bounded cavity has since become a classical problem in fluid mechanics (Shankar & Deshpande 2000; Kuhlmann & Romano (a) 2019). Its geometrical simplicity yet rich flow phenomenology make it a perfect set-up for studying a wealth of complex phenomena such as **corner eddies**, **vortex dynamics**, **flow instabilities** (centrifugal and shear) and turbulent transition.
- lid-driven square cavity (LDSC) flow, has become a **model problem** against which **all sorts of numerical codes**

- **An et al. 2019** の目的

- to unfold the **transition process from the laminar base state to chaotic motion**
- (この論文は三角形問題に着目)
- The motivation behind our choice of a triangular geometry stems from the way in which the vortex arrangement becomes increasingly intricate with Reynolds number escalation, even before the flow becomes non-stationary.

ブレードコータの変遷

(株)アイ・エイ・アイ フォイト パーパーステクノロジー エンジニアリング本部 森田 博文



正方形キャビティ問題の文献

- Ghia et al. 1982 [https://doi.org/10.1016/0021-9991\(82\)90058-4](https://doi.org/10.1016/0021-9991(82)90058-4)
 - 正方形, 定常問題でのベンチマーク解
- Shen 1991 [https://doi.org/10.1016/0021-9991\(91\)90261-I](https://doi.org/10.1016/0021-9991(91)90261-I)
 - 正方形, Hopf 分岐
- Nuriev et al. 2016 <https://iopscience.iop.org/article/10.1088/0169-5983/48/6/061405>
 - 正方形, 定常解の計算. 分岐
- 鈴木厚 <https://www.ljll.fr/~suzukia/FreeFempp-tutorial-JSIAM2016/>
 - 正方形, FreeFEM によるあるレイノルズ数帯の定常解の計算

四角形キャビティ問題の分岐 Nuriev et al. 2016

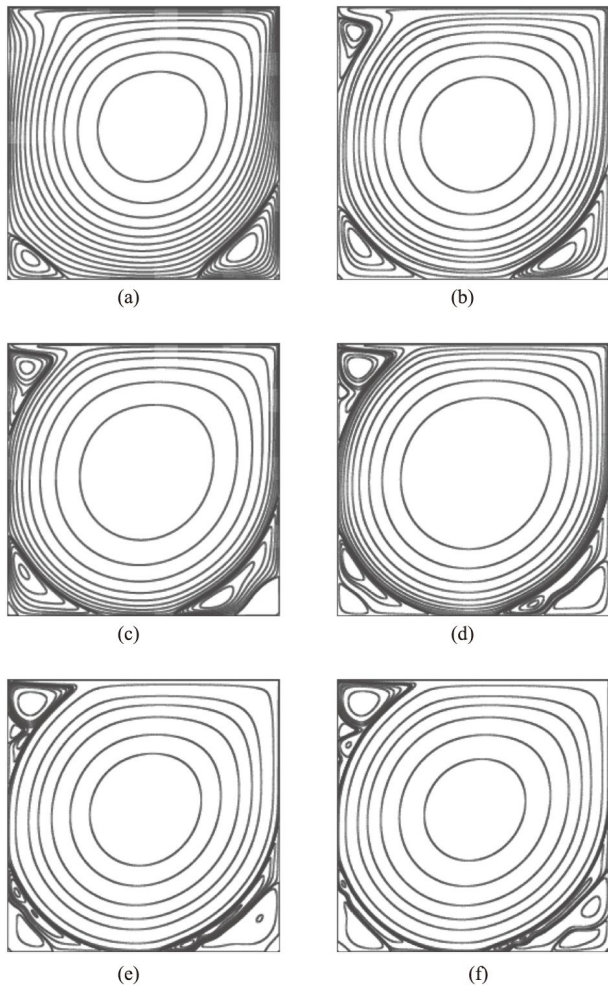


Figure 2. The evolution of the flow pattern in the cavity with the increasing Reynolds number: (a)— $Re = 1000$, (b)— $Re = 5000$, (c)— $Re = 10000$, (d)— $Re = 20000$, (e)— $Re = 40000$, (f)— $Re = 80000$.

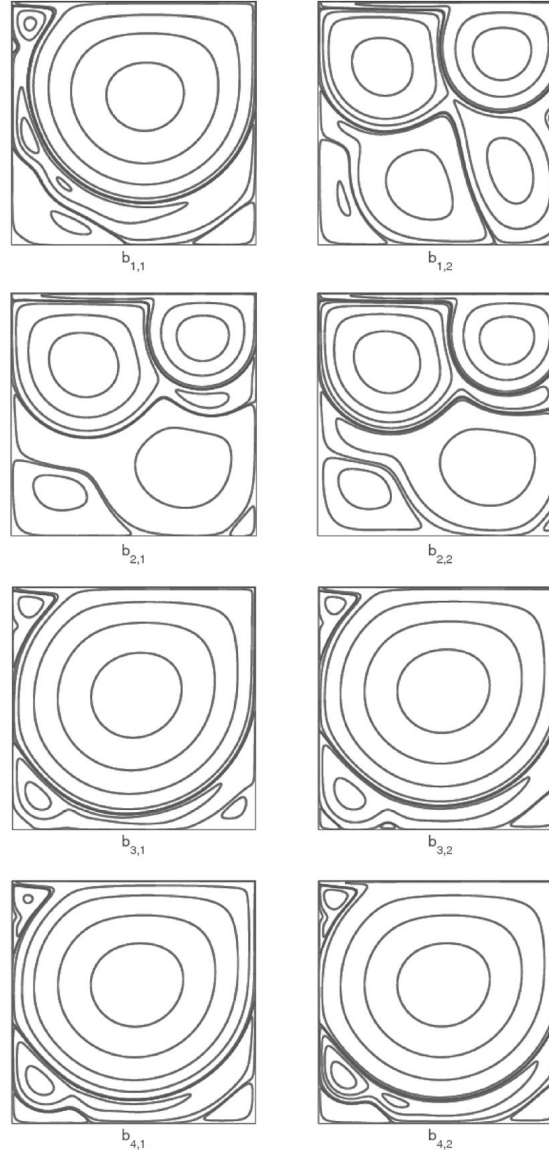


Figure 6. Flow patterns for additional solutions at $Re = 16000$.

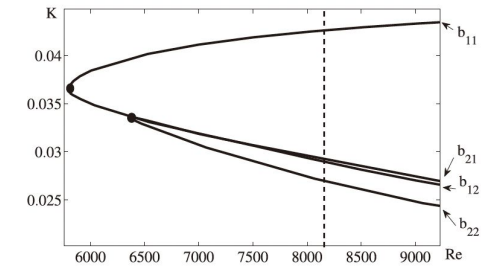


Figure 4. Additional solution branches computed after the first localization. Markers indicate fold bifurcation points.

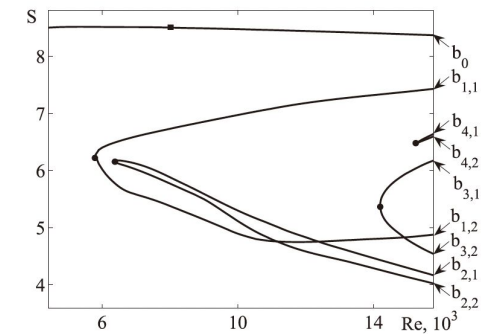
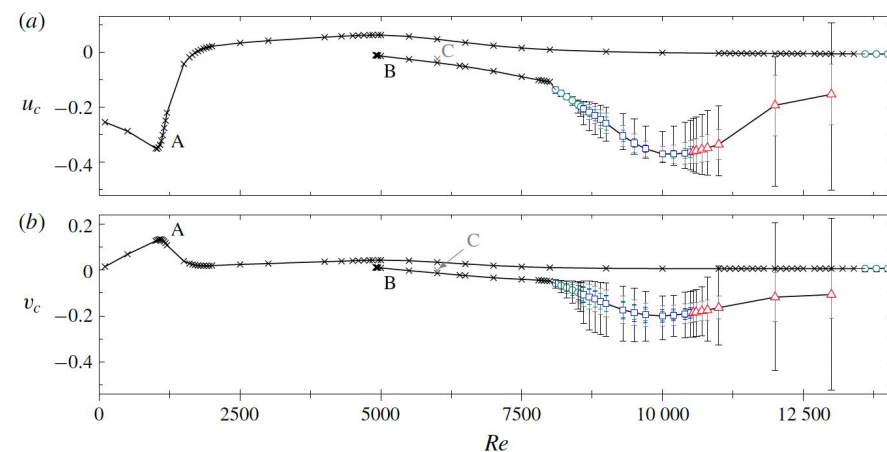
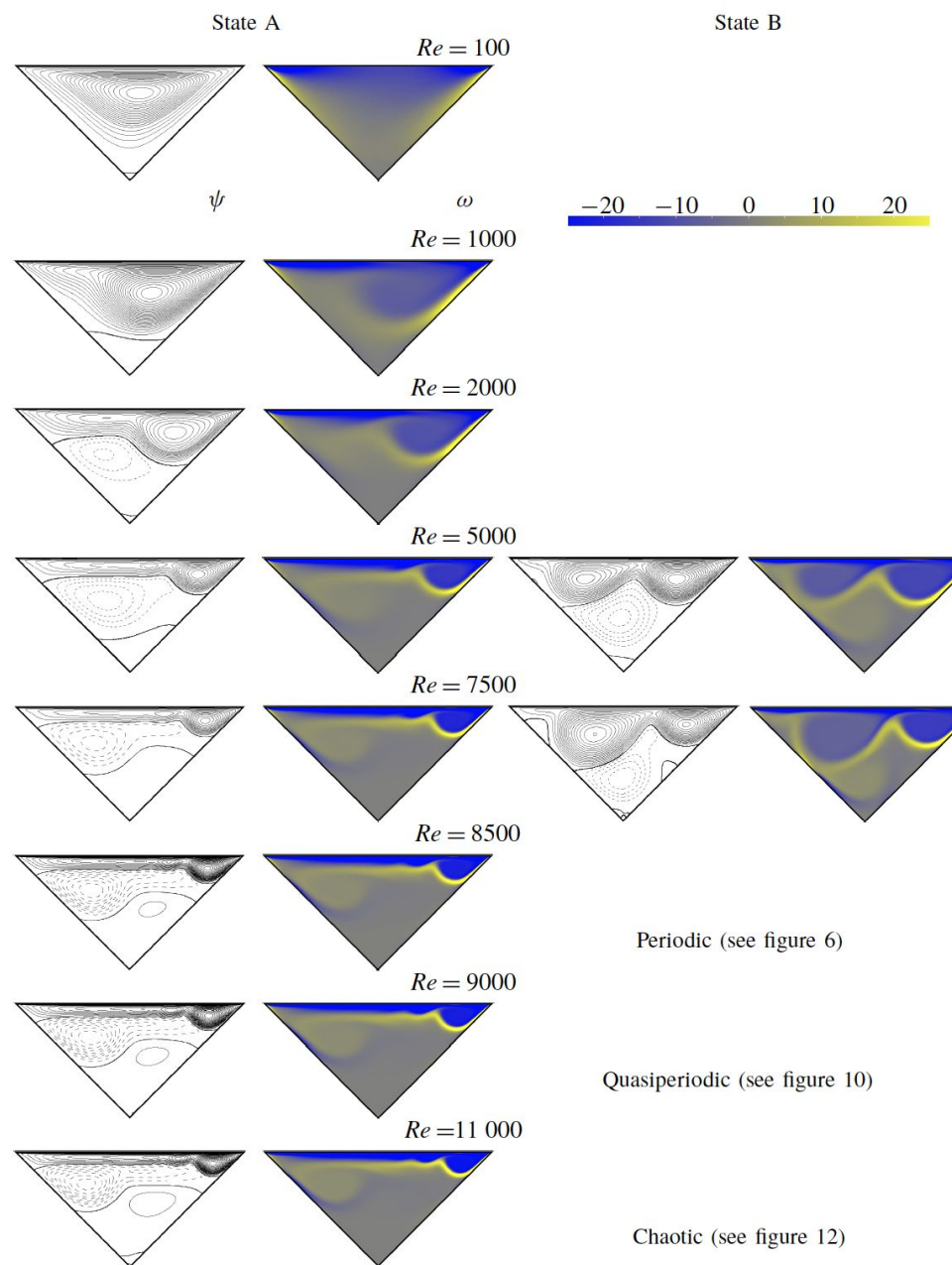


Figure 5. The solution branches of the lid-driven cavity flow problem. Round markers indicate fold bifurcations, a square marker denotes the first Hopf bifurcation of the main branch.

三角形キャビティ An et al. 2019



1次渦は小さい

正三角形領域における異なるタイプの数値解

以下は各論文に掲載されている流線



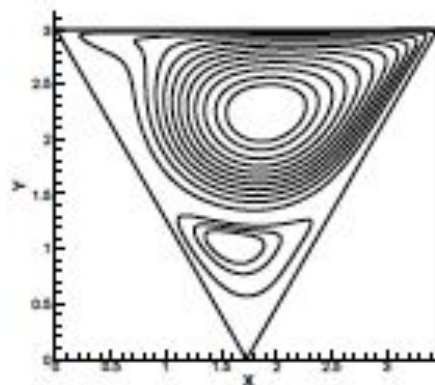
d) $Re=500$

$$Re = 500 \times 2\sqrt{3} \cong 1732$$

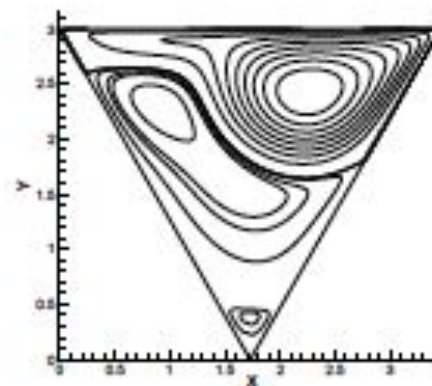


e) $Re=1000$

$$Re = 1000 \times 2\sqrt{3} \cong 3464$$



$Re = 2000$

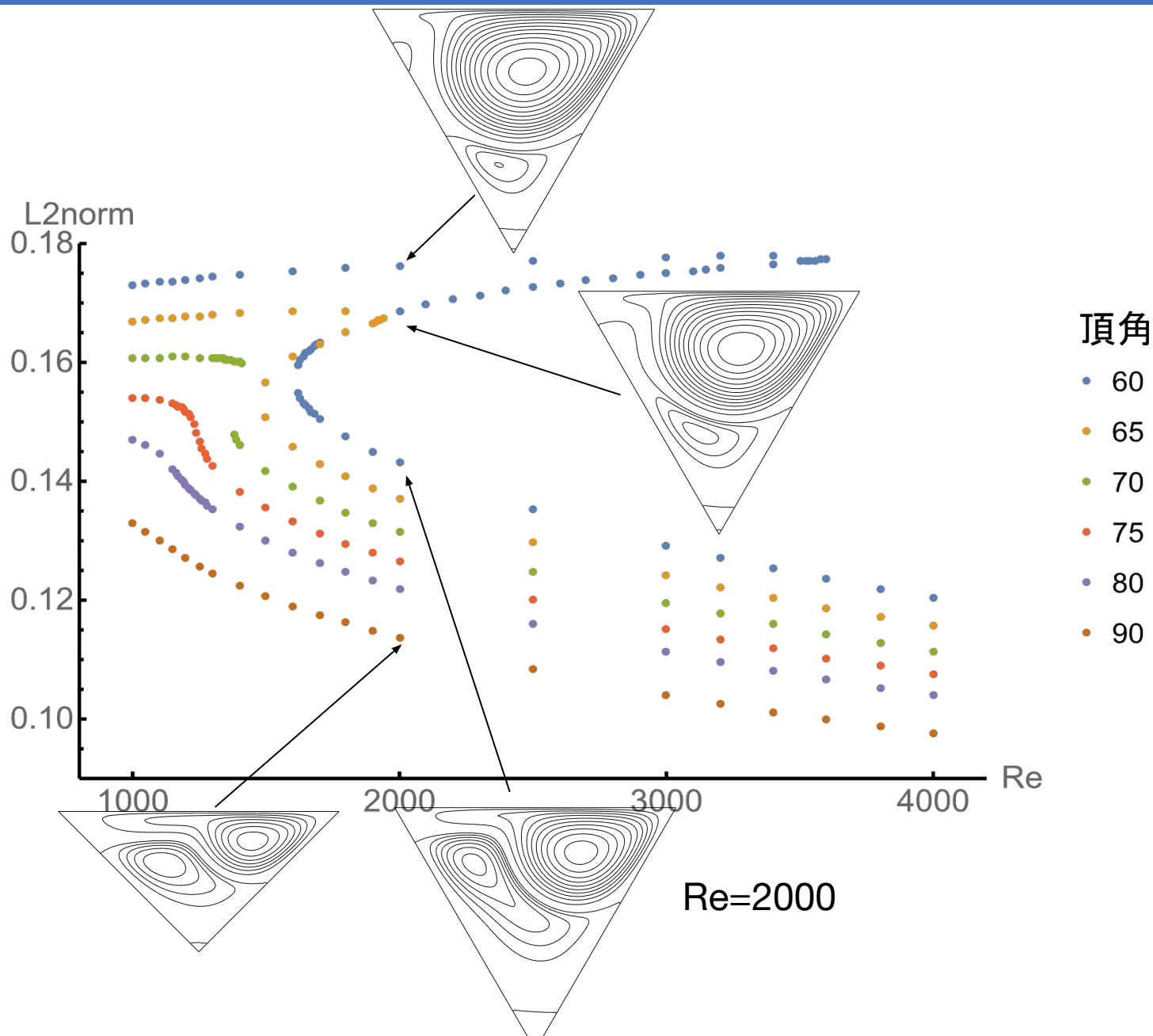


$Re = 3000$

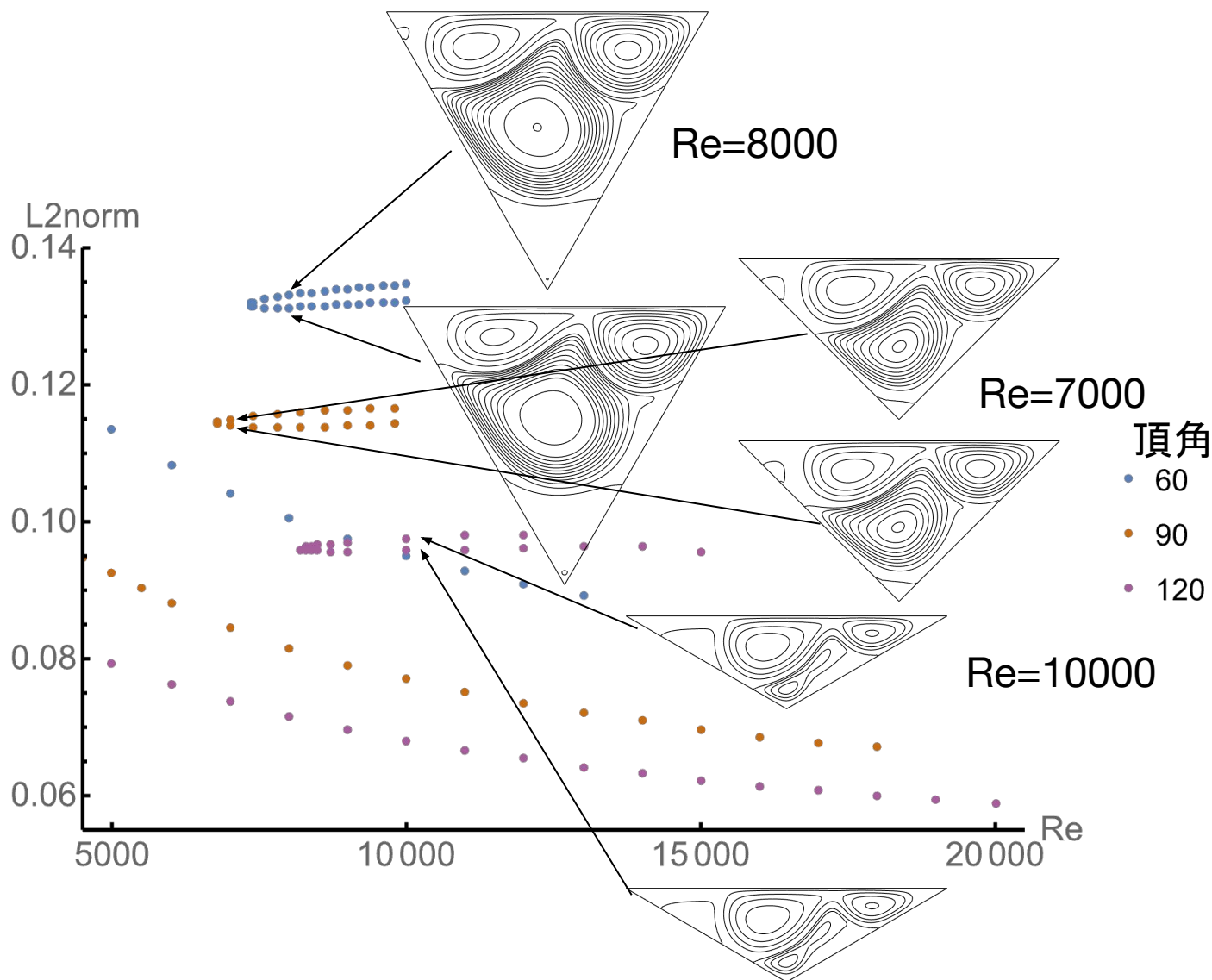
Erturk-Gokcol
2007

Pasquim-Mariani
2008

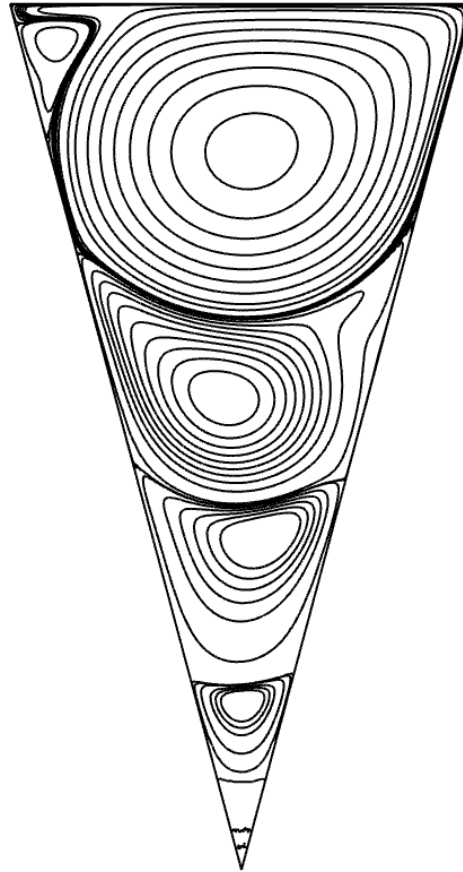
数値解のL2ノルムとレイノルズ数の図式 (U.)



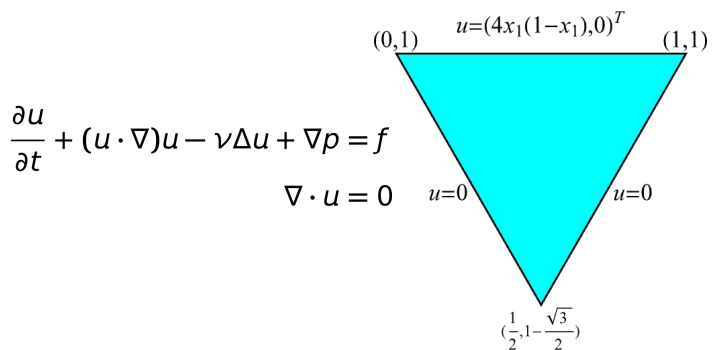
高レイノルズ数における図式 (U.)



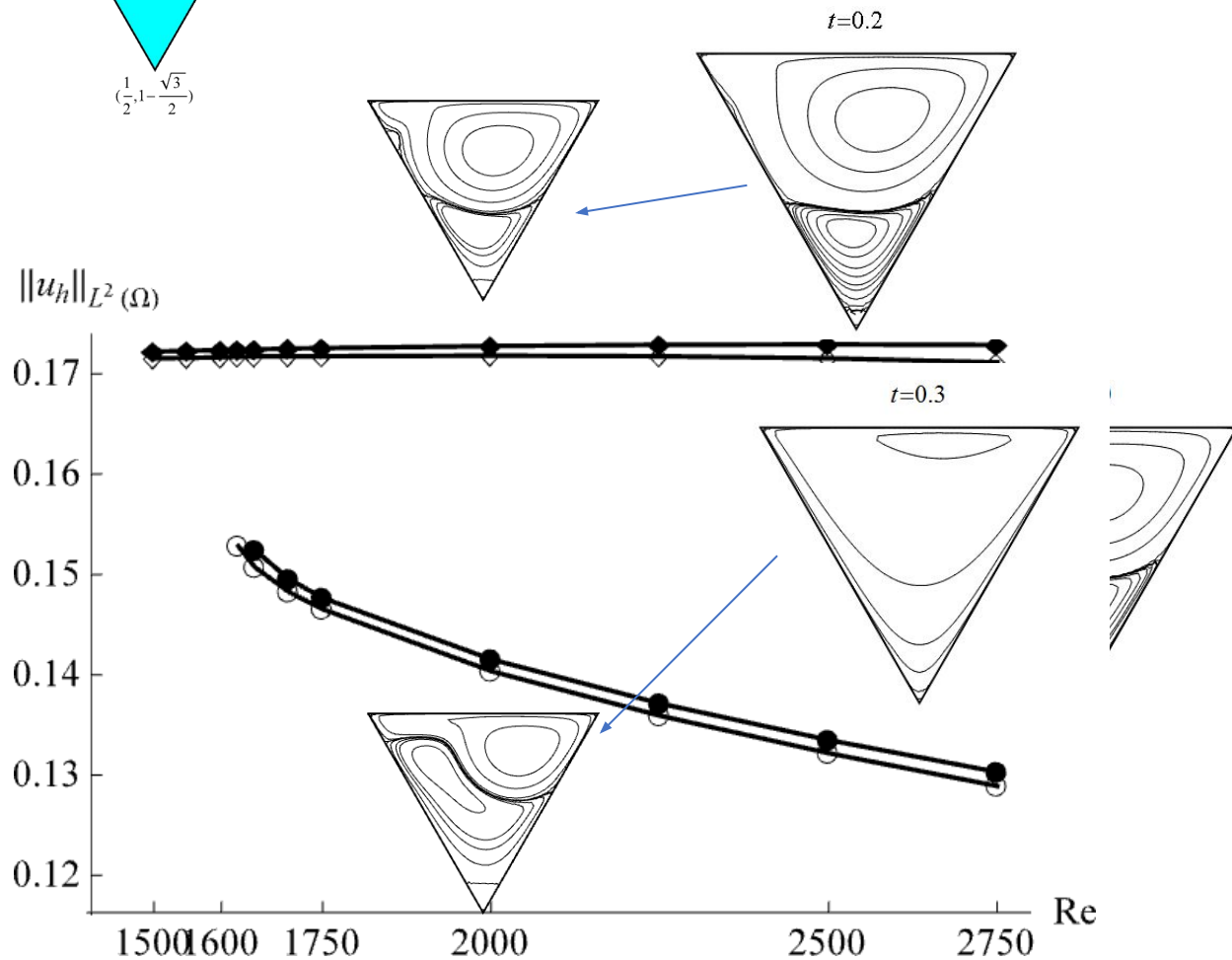
頂角が小さい場合の渦列



時間発展スキームによる定常解の計算 (Tabata-U. 2018)



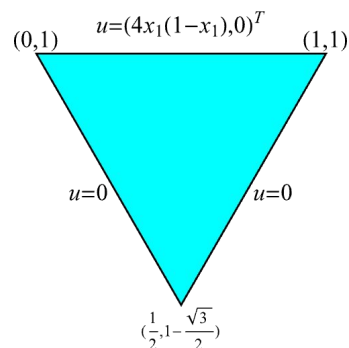
以下の図は流線



頂角が異なる三角形キャビティ問題における分岐図の比較

Tabata-U. 2018

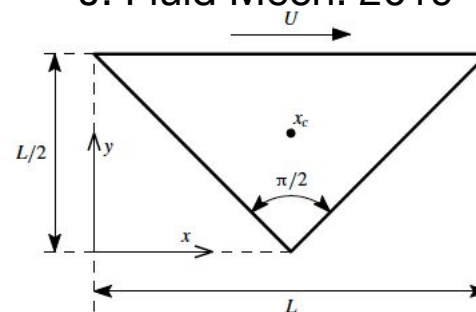
(再掲)



正三角形

An-Bergada-Mellibovsky

J. Fluid Mech. 2019



直角二等辺三角形

