27 January 2014 -- Computer Architectures -- part 1/2

Name, Matricola ....

Let us consider a linear system of two equations with two unknown terms x, y, i.e., $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$
where a, b, c, d, e, f are integer numbers. It is requested to write a program to compute the solutions x, and y by using determinants, i.e.,

$$\Delta = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = ae - bd; \quad \Delta_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = ce - bf; \quad \Delta_y = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = af - cd; \qquad x = \frac{\Delta_x}{\Delta}; \quad y = \frac{\Delta_y}{\Delta}$$

Please observe that $\Delta, \Delta_x, \Delta_y$ are signed numbers because each one is the result of a subtraction.
The coefficients, which are all integer numbers, are read from the keyboard and stored in variables:
COEFF_A DB ?   COEFF_B DB ?   COEFF_C DB ?   COEFF_D DB ?   COEFF_E DB ?   COEFF_F DB ?

As x and y are obtained by divisions, they have an integer and a fractional part. For sake of simplicity here it will be requested to compute their integer part only x_{INT} and y_{INT} , and to store them in the two variables X_INT and Y_INT respectively, chosen of suitable size by the Student.

Below please find a group of "Problems". **Only one in this group has to be chosen and solved.**
Additional points can be collected by solving supplementary "Features" later listed in the text.

- Problem 1: assuming that all coefficients are positive values on 5 bits and that $\Delta \neq 0$, compute x_{INT} and y_{INT} as signed integer values on 16 bits each one. (In fact, as $\Delta, \Delta_x, \Delta_y$ are signed numbers then also x_{INT} and y_{INT} are signed (in two's complement)).
- Problem 2: assuming that all coefficients are two's complement values on 5 bits and that $\Delta \neq 0$, compute x_{INT} and y_{INT} as signed integer values on 16 bits each one.
- Problem 3: assuming that all coefficients are two's complement values on 3 bits and that $\Delta \neq 0$, compute x_{INT} and y_{INT} as signed integer values **on the most suitable number of bits**.

The supplementary Features are:

- Feature A: Management of cases when $\Delta = 0$. Two possibilities exist when $\Delta = 0$, i.e. *impossible* and *undetermined* system. Students are requested to determine, when $\Delta = 0$ if the system is *impossible* or *undetermined*. An example is provided below.
$$impossible\ system: \begin{cases} 2x + y = 1 \\ 4x + 2y = 3 \end{cases} \qquad undetermined\ system: \begin{cases} 2x + y = 1 \\ 4x + 2y = 2 \end{cases}$$

- Feature B: Computation of the "overall squared error" $\theta(x_{INT}, y_{INT})$ due to truncation of x and y to their integer part only (i.e. x_{INT}, y_{INT}), defined as
$$\theta(x_{INT}, y_{INT}) = (ax_{INT} + by_{INT} - c)^2 + (dx_{INT} + ey_{INT} - f)^2$$
Please consider that $\theta(x_{INT}, y_{INT})$ is always positive and can be represented (for Problems above) on 16 bits. In fact, $\theta(x_{INT}, y_{INT}) < (|a| + |b|)^2 + (|d| + |e|)^2$. (Please do not demonstrate it, just trust!)).

- Feature C: Compute the pair X'_{INT}, Y'_{INT} minimizing "overall squared error" $\theta(X'_{INT}, Y'_{INT})$ on 16 bits; this can be done by checking which one of the following 9 combinations (where 0 or 1 is added/subtracted to the computed x_{INT}, y_{INT}) leads to the minimum $\theta(X'_{INT}, Y'_{INT})$;
$(X'_{INT} = x_{INT} - 1, Y'_{INT} = y_{INT} - 1)$; $(X'_{INT} = x_{INT} + 1, Y'_{INT} = y_{INT})$; $(X'_{INT} = x_{INT}, Y'_{INT} = y_{INT} + 1)$
$(X'_{INT} = x_{INT} + 1, Y'_{INT} = y_{INT} + 1)$; $(X'_{INT} = x_{INT}, Y'_{INT} = y_{INT} - 1)$; $(X'_{INT} = x_{INT} - 1, Y'_{INT} = y_{INT})$
$(X'_{INT} = x_{INT}, Y'_{INT} = y_{INT})$; $(X'_{INT} = x_{INT} + 1, Y'_{INT} = y_{INT} - 1)$; $(X'_{INT} = x_{INT} - 1, Y'_{INT} = y_{INT} + 1)$

The points related to writing correct 8086 assembly code for each completed item (as uncompleted items will not be evaluated)

- ONLY ONE → Problem 1: 19 points; Problem 2: 21 points; Problem 3: 25 points
- Feature A: 3 points
- Feature B: 4 points
- Feature C: 6 points

Please consider that a maximum of 33 points can be accounted here; larger values will be "cut" to 33.

27 January 2014 -- Computer Architectures -- part 1/2

Name, Matricola ....

Example $\begin{cases} -3x + 3y = -13 \\ 2x - 1y = 14 \end{cases}$ $\Delta = (-3) * (-1) - (3) * (2) = -3$

$$\Delta_x = (-13) * (-1) - (3) * (14) = -29; \quad \Delta_y = (-3) * (14) - (-13) * (2) = -16;$$
$$x = \frac{\Delta_x}{\Delta} = \frac{-29}{-3} = 9.6666; \quad y = \frac{\Delta_y}{\Delta} = \frac{-16}{-3} = 5.3333; \quad \rightarrow \quad x_{INT} = 9, y_{INT} = 5$$
$$\theta(x_{INT}, y_{INT}) = (-3 * (9) + 3 * (5) - (-13))^2 + (2 * (9) - 1 * (5) - (14))^2 = 1 + 1 = 2$$

HINTS

- Please observe that $\Delta, \Delta_x, \Delta_y$ are signed numbers because each one is the result of a subtraction.
- Problem 3 requires Students performing a prior analysis on the number of bits required to represent each term. For example, for Problem 1, we observe that $\Delta = ae - bd$. As *ae* and *bd* both require 10 bits it follows that Δ requires 11 bits in two's complement (in fact, the boundary case is *ae=0* and *bd* on 10 bits leading to a negative value on 10 bits, i.e. requiring 11 bits in two's complement); the same holds for Δ_x, Δ_y. For this reason as both x and y are computed by dividing two terms each on 11 bits (in two's complement), their integer parts require 11 bits.
- To solve Feature A, when $\Delta = 0$ to identify if the system is *impossible* or *undetermined* Students can have a look at the value of Δ_x or Δ_y (Students are invited to devise the rule on their own).

REQUIREMENTS (SHARP)

- It is not required to provide the optimal (shortest, most efficient, fastest…) solution, but a working and clear solution.
- It is required to write at class time a short and clear explanation of the algorithm used.
- It is required to write at class time significant comments to the instructions.
- The input-output part is not necessary in the class-developed solution, but its implementation is mandatory to be discussed at oral exam.
- Minimum score to "pass" this part is 15 (to be averaged with second part and to yield a value at least 18)

REQUIREMENTS ON THE I/O PART TO BE DONE AT HOME

- ~~The database (if any) has to be defined and initialized inside the code~~
- The values of the coefficients should be input by keyboard
- The program should present a menu with the choices corresponding only to the items developed during the written exam
- All inputs and outputs should be in readable ASCII form (no binary is permitted).

Please use carbon copy ONLY (NO PICTURES ARE ALLOWED) and retain one copy for home implementation and debug. At the end of the exam please give to professors <u>all</u> the sheets of your solution. Missing or late sheet will not be evaluated. Please provide your classroom submitted solution with several explanatory and significant comments. Please remember that only what has been developed at class time can and will be evaluated at oral time and that it is necessary to write the instructions of the program and not just the description of the algorithm.

When coming to oral discussion, please clearly mark <u>in red</u> on your "classroom" copy, <u>all modifications</u>. Please also provide an error-free and running release of the solution, as well as with its printed list of instructions. Please consider that the above are necessary but not sufficient requirements to success the exam, since the final evaluation will be based on a number of parameters.

FAILURE TO ACCOMPLISH ALL THE ABOVE NECESSARY REQUIREMENTS WILL CAUSE NO-QUESTION-ASKED AND IMMEDIATE REJECTION.