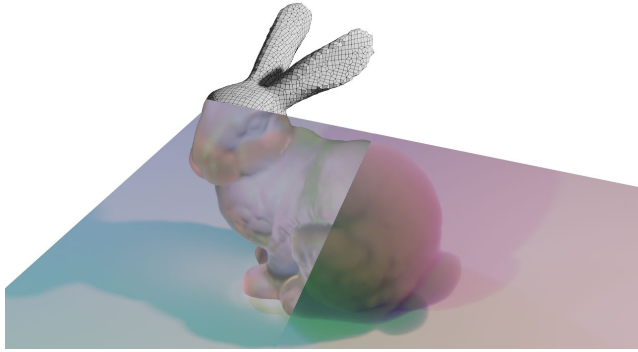
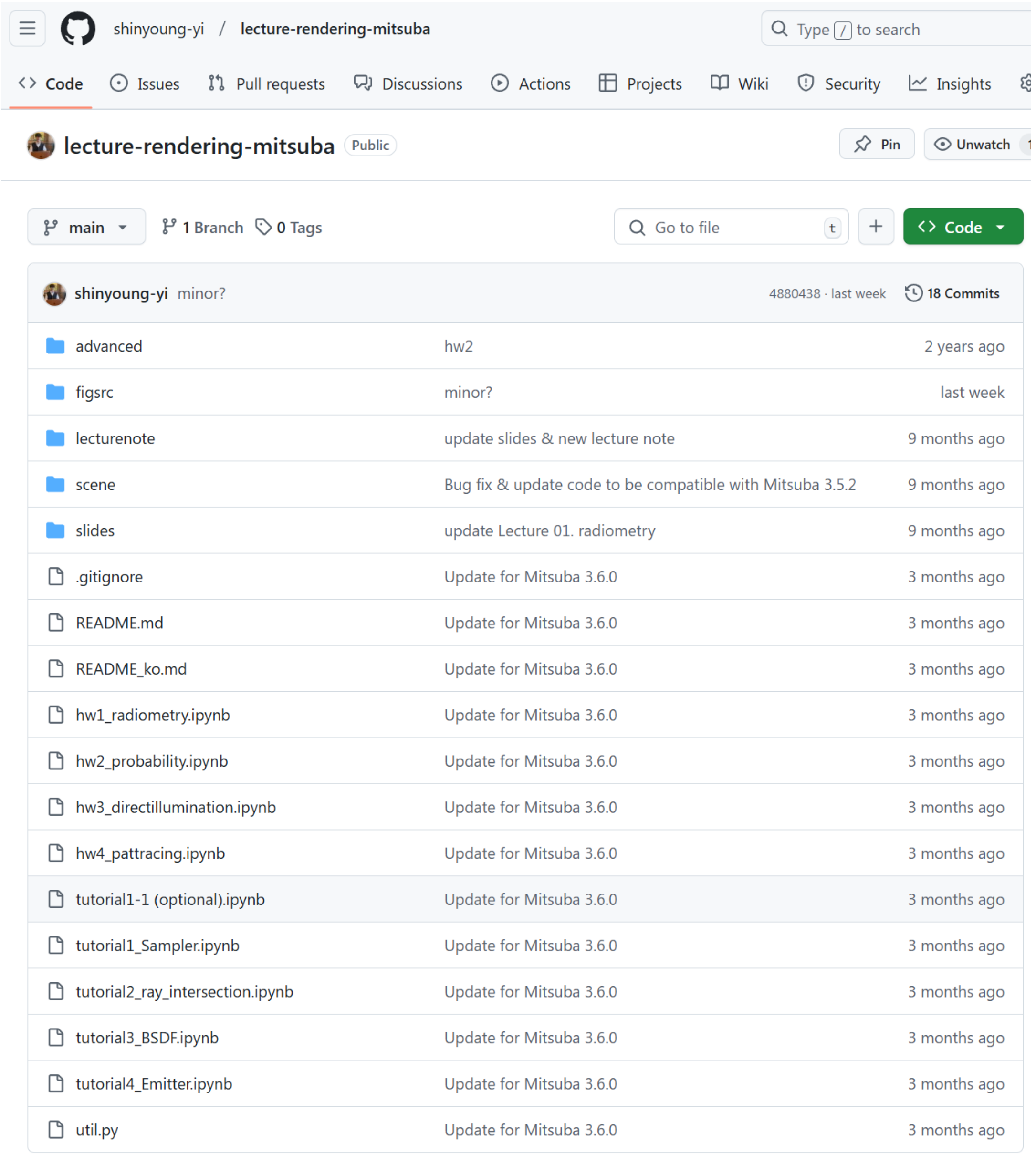


Path Tracing One Day Class

KAIST Visual Computing Laboratory

Ph. D. Shinyoung Yi (이신영)

See also…



https://github.com/shinyoung-yi/lecture-rendering-mitsuba

- Full version slides for each chapter:
  - 1. Radiometry and Light Transport
  - 2. Probability and Statistical Inference
  - 3. Path Tracing
- Tutorial for Mitsuba 3 API
- Python skeleton codes for homework

Why Should We Study Ray Tracing?

"I am only interested in computer vision"

2D Images → Rendering → 3D Objects

3D Objects → Inverse rendering (computer vision) → 2D Images

- Generate GT dataset for training
- Understand exact problem definitions
- Understanding GT forward methods

global illumination, participating media, etc.

Why Should We Study Ray Tracing?

"I only want to *use* rendering, not to understand"

https://www.youtube.com/watch?v=Lp4gqJb38h8

Why Should We Study Ray Tracing?

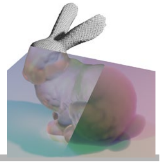
“I only want to *use* rendering, not to understand”

Why Should We Study Ray Tracing?

"I only want to *use* rendering, not to understand"

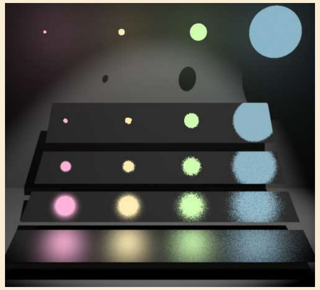
- Choosing ray tracing algorithms often requires understanding them
- We sometimes need to write special-purposed ray tracers

Why Should We Study Ray Tracing?



“Too hard and too many methods to study”

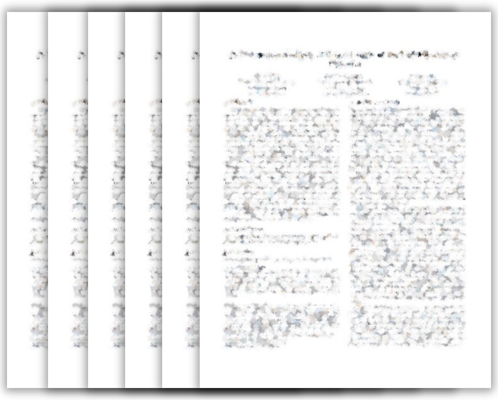
[Cook 1984]



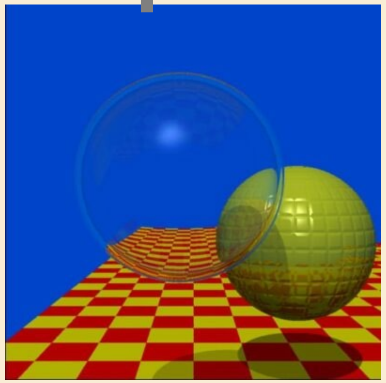
[Veach & Guibas 1995]

[Veach & Guibas 1995]

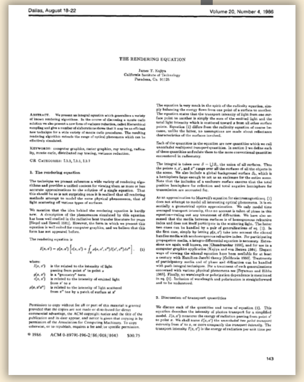
[Veach & Guibas 1995]



1980 1990 2000 2020

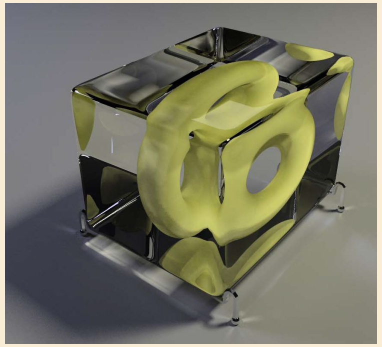


[Whitted 1980]

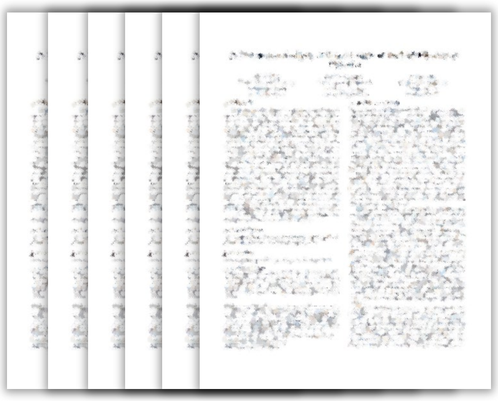


[Kajiya 1986]

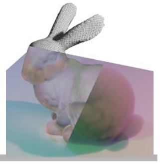
[Veach & Guibas 1997]



[Cline et al. 2005]



Why Should We Study Ray Tracing?

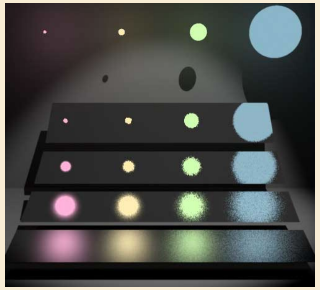


"Too hard and too many methods to study"

Early ray tracing

We can skip!

[Cook 1984]



[Veach & Guibas 1995]

[Veach & Guibas 1995]

[Veach & Guibas 1995]

1980

1990

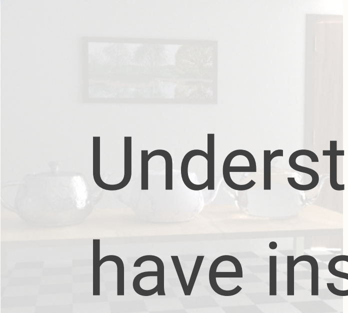
2000

2020

Path Tracing

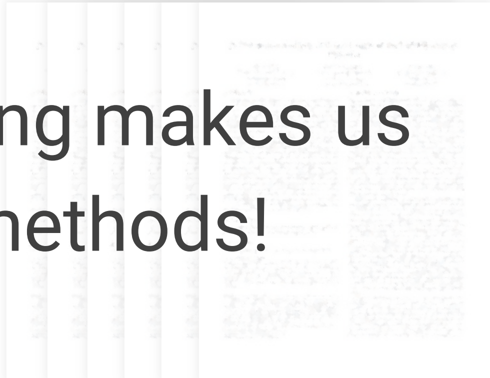
[Whitted 1980]

[Kajiya 1986]



[Veach & Guibas 1997]

[Cline et al. 2005]



Understanding path tracing makes us have insights of further methods!

Why Path Tracing?

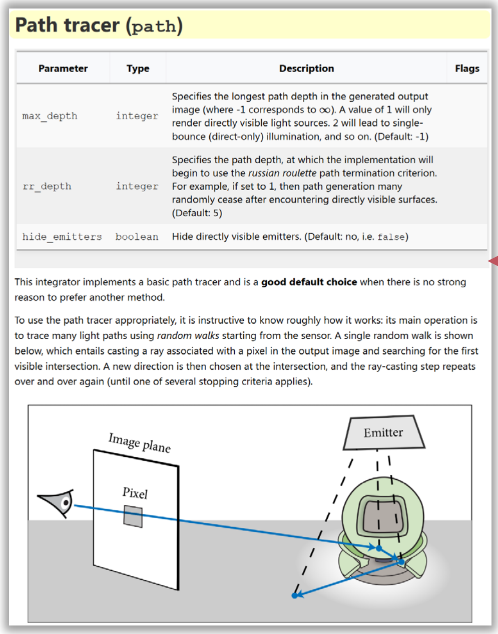
Path Tracing is the most popular standard method of ray tracing!

Q. How popular? How standard?

Path tracing in popular software

Mitsuba 3
Physically Based Renderer

blender®



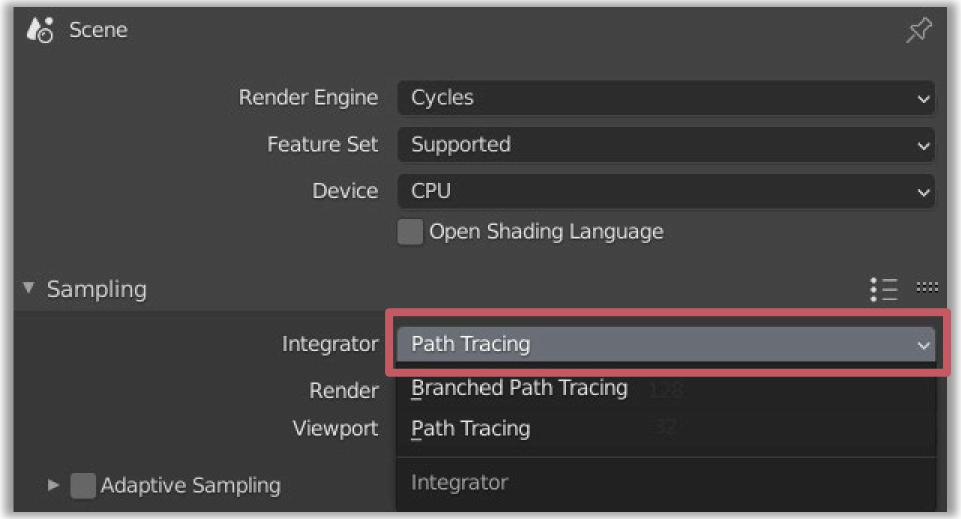
Path tracer (`path`)

Parameter	Type	Description	Flags
`max_depth`	`integer`	Specifies the longest path depth in the generated output image (where -1 corresponds to ∞). A value of 1 will only render directly visible light sources. 2 will lead to single-bounce (direct-only) illumination, and so on. (Default: -1)	
`rr_depth`	`integer`	Specifies the path depth, at which the implementation will begin to use the *russian roulette* path termination criterion. For example, if set to 1, then path generation many randomly cease after encountering directly visible surfaces. (Default: 5)	
`hide_emitters`	`boolean`	Hide directly visible emitters. (Default: no, i.e. `false`)	

This integrator implements a basic path tracer and is a **good default choice** when there is no strong reason to prefer another method.

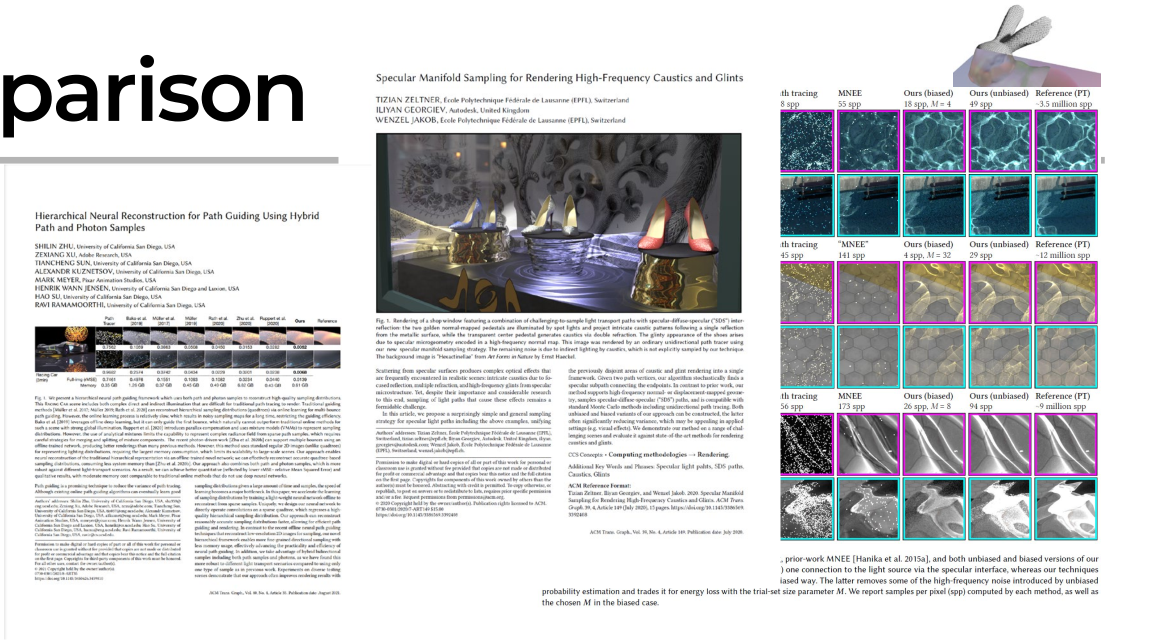
To use the path tracer appropriately, it is instructive to know roughly how it works: its main operation is to trace many light paths using *random walks* starting from the sensor. A single random walk is shown below, which entails casting a ray associated with a pixel in the output image and searching for the first visible intersection. A new direction is then chosen at the intersection, and the ray-casting step repeats over and over again (until one of several stopping criteria applies).

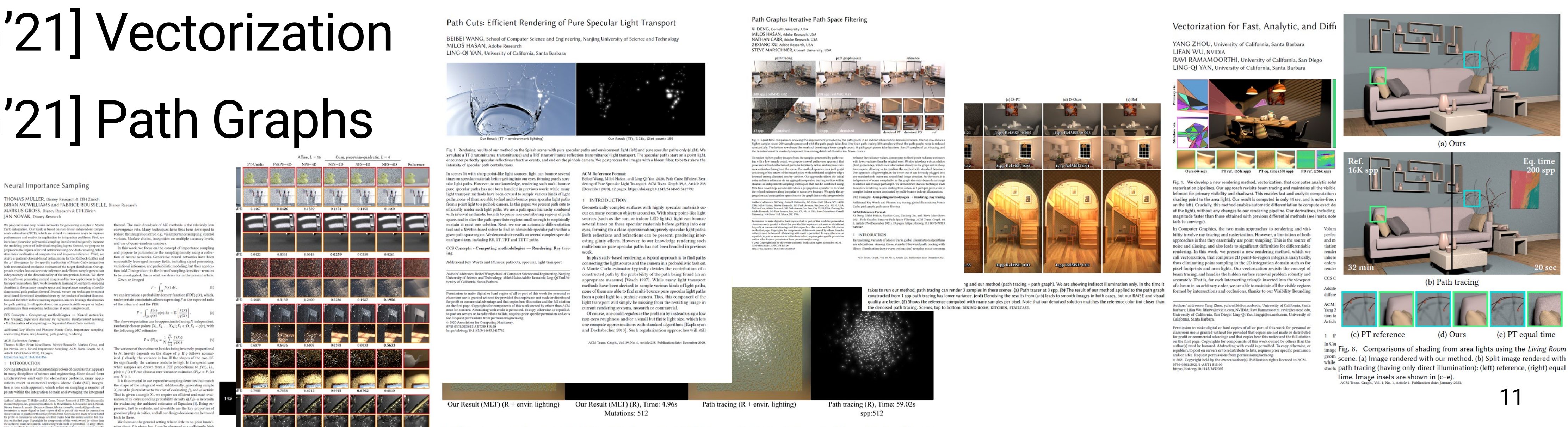
"a **good default choice**
when there is no strong reason
to prefer another method."



Path tracing is still used for comparison

- “PT” in comparison
- [TOG’19] Neural Importance Sampling
- [TOG’20] Path Cuts
- [TOG’20] Specular manifold sampling
- [TOG’21] Hierarchical Neural Reconstruction for Path Guiding
- [TOG’21] Vectorization
- [TOG’21] Path Graphs





“Too hard and too many methods to study”

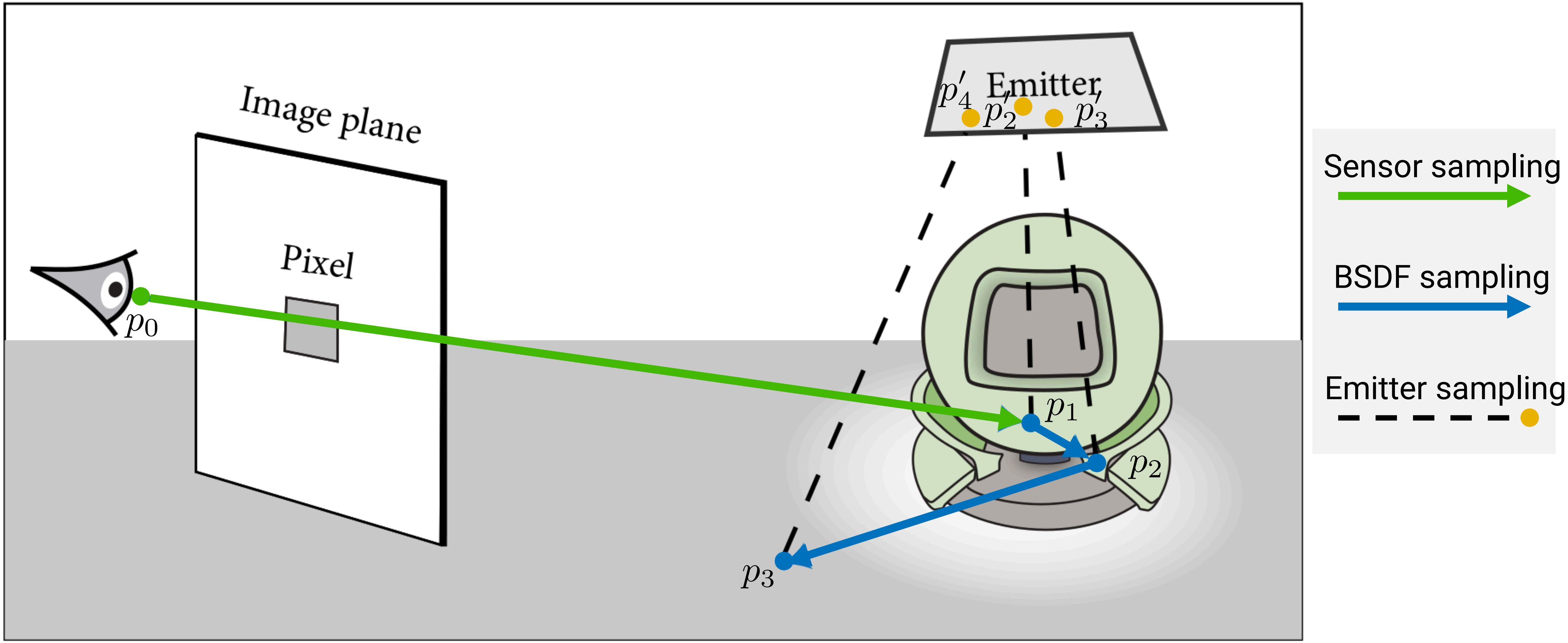
"Too hard and too many methods to study"

- We can learn **path tracing** in one day
- Only use **Python** based on **Mitsuba 3**

1. What should we compute?
   – Physics: radiometry

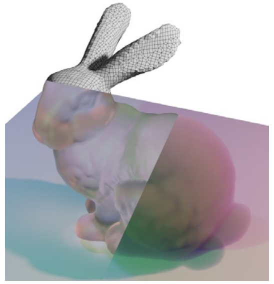
2. How can we compute?
   – Numerical method: Monte Carlo integration

3. Path Tracing Algorithm



[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

1. What should we compute?

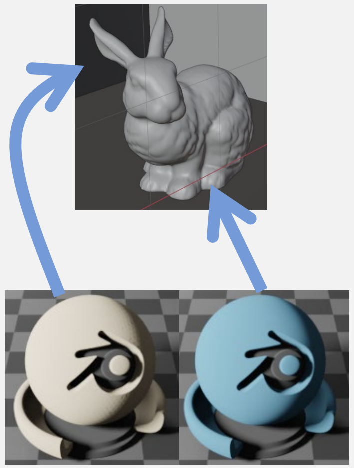


→ Radiometry

- What is radiance?

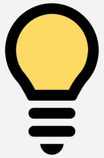
Rendering

Scene

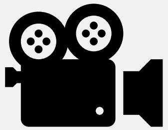


geometry

material



emitter



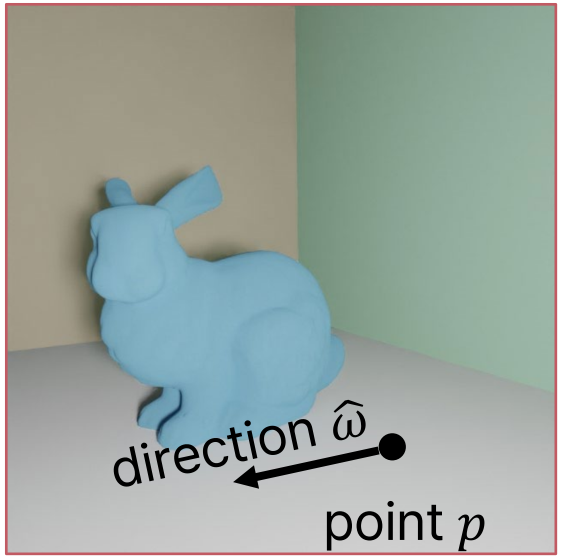
sensor

Which physical quantities?

Rendering

Which physical law?

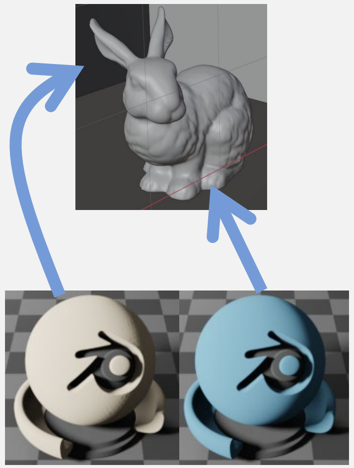
Image $\mathbf{I} \in \mathbb{R}^{H \times W}$



Which physical quantities?

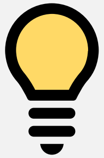
Rendering

Scene

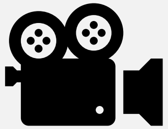


geometry

material
BRDF (BSDF)



emitter
Radiance



sensor
Sensor response

Which physical quantities?

Rendering

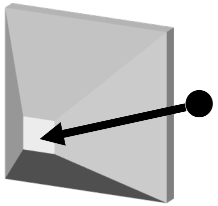
Which physical law?

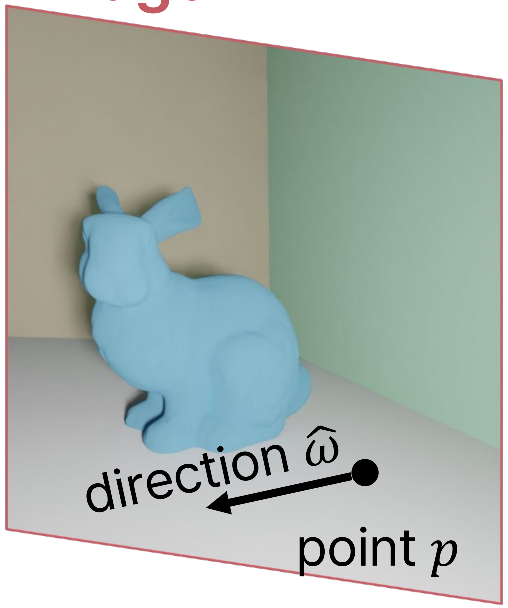
Rendering equation

(+ constant radiance along ray)

(+ measurement equation)

Image $\mathbf{I} \in \mathbb{R}^{H \times W}$

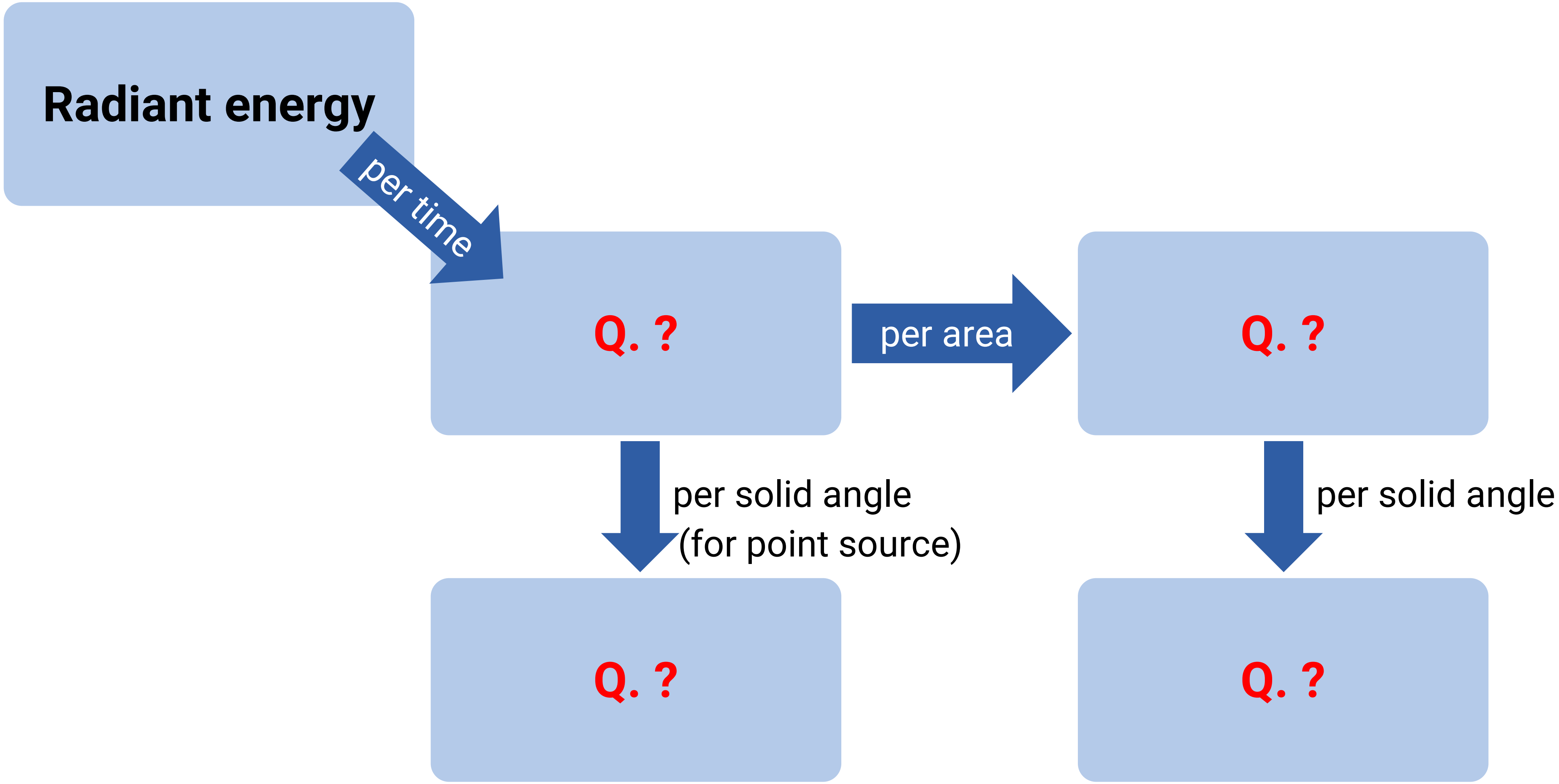




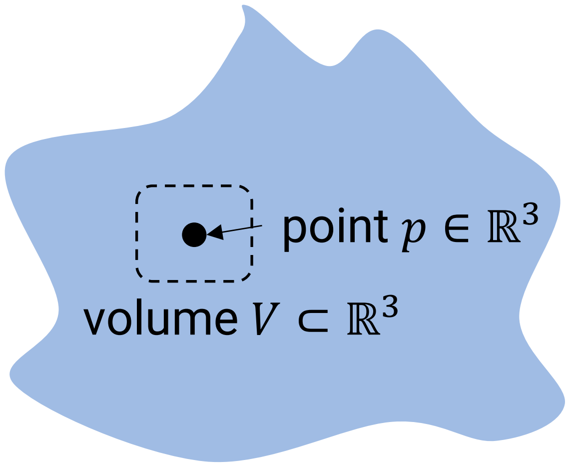
Which physical quantities?

Radiance at p along $\hat{\omega}$

Radiometric quantities



Concepts of mass vs. density

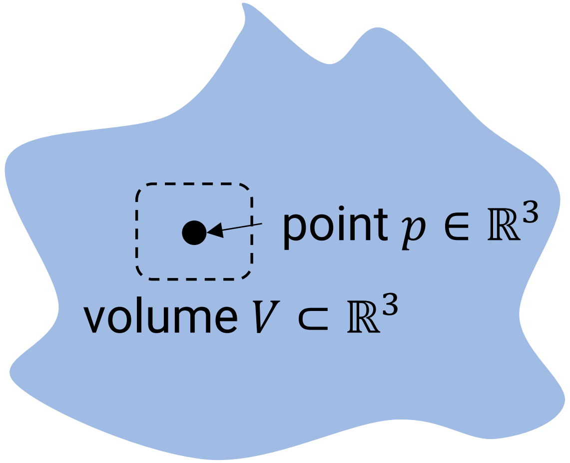


Object O

Mass → per volume → **Density**

Mass	Density
✓ Mass of the object O	✗ Density of the object O → illegal or "average density" of the object O
✓ Mass of some region (volume) V	✗ Density of some region (volume) V → illegal or "average density" of the volume V
✗ Mass at the point p → illegal or meaningless (always zero)	✓ Density at the point p

Concepts of mass vs. density



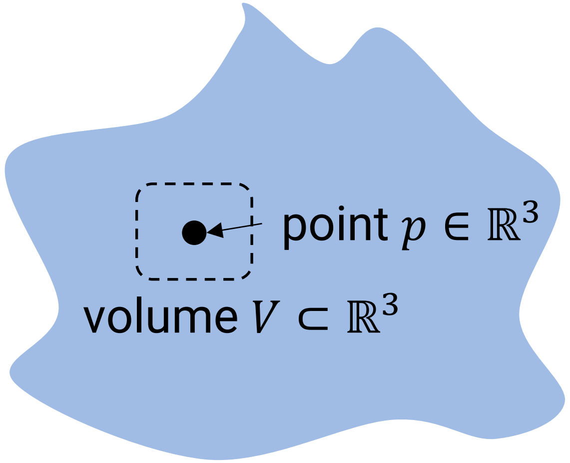
Mass of what? $V \subset \mathbb{R}^3$

Density of what? $p \in \mathbb{R}^3$

- “mass of an object O”

  = “mass of the volume of O”

Concepts of mass vs. density



Mass of $V \subset \mathbb{R}^3$

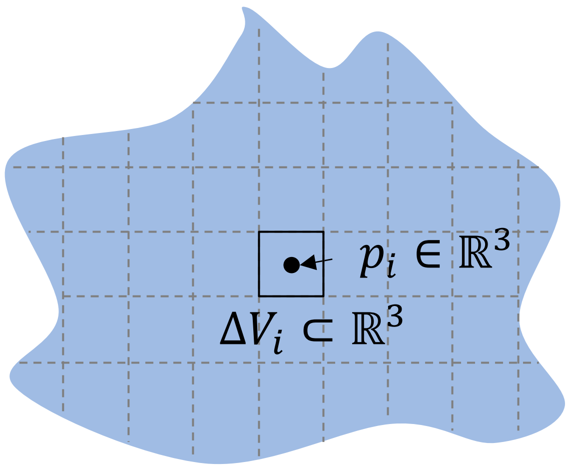
Density of $p \in \mathbb{R}^3$

$\mathrm{mass}(V)$

per volume

$$\mathrm{density}(p) = \lim_{\substack{\mathrm{vol}(V) \to 0 \\ p \in V}} \frac{\mathrm{mass}(V)}{\mathrm{vol}(V)}$$

Concepts of mass vs. density



Mass of $V \subset \mathbb{R}^3$

$$\text{mass}(V) = \sum_i \text{mass}(\Delta V_i)$$

$$\approx \sum_i \text{density}(p_i)\,\text{vol}(\Delta V_i)$$

$\text{vol}(\Delta V_i) \to 0$

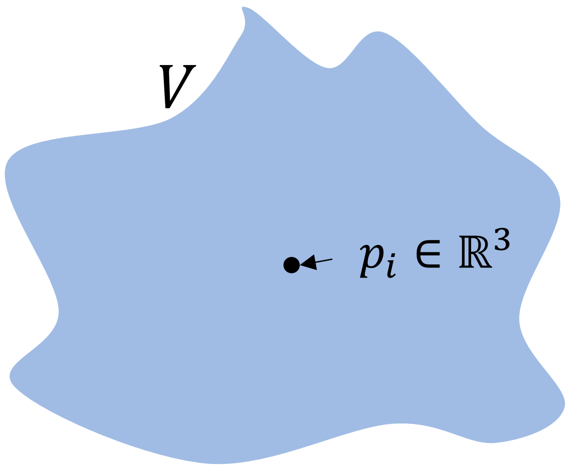
$$\int_V \text{density}(p)\,dp$$

Density of $p \in \mathbb{R}^3$

over volume

$$\boxed{\text{density}(p) = \lim_{\substack{\text{vol}(V)\to 0 \\ p\in V}} \frac{\text{mass}(V)}{\text{vol}(V)}}$$

Concepts of mass vs. density



Mass of $V \subset \mathbb{R}^3$ $[\,\mathrm{kg}\,]$

Density at $p \in \mathbb{R}^3$ $[\,\mathrm{kg/m^3}\,]$

Proposition

Relations between mass m of a volume $V \subset \mathbb{R}^3$ and density ρ at a point $p \in \mathbb{R}^3$ is that:

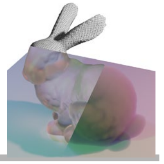
$$m(V) = \int_V \rho(p)\mathrm{d}p$$

per volume →

← over volume

$$\rho(p) = \lim_{\substack{|V|\to 0 \\ p\in V}} \frac{m(V)}{|V|}$$

Concepts of mass vs. density



Notation comparison

Other text often write $\frac{\mathrm{d}m}{\mathrm{d}V}$ instead of $\lim_{|V|\to 0, p\in V} \frac{m(V)}{|V|}$ but the former notation may give a misunderstanding that m is a function of a real number (volume measure) rather than one of a subset of $\mathbb{R}^3$ (volume region). The formula $\frac{\mathrm{d}m}{\mathrm{d}V}$ can be correctly understood only if it denoted a Radon-Nikodym derivative, which is dealt in *measure theory* (4th grade in Math. major).
We do not assume measure theory as a prerequisite, so we use the latter notation $\lim_{|V|\to 0, p\in V} \frac{m(V)}{|V|}$ for explicitness.

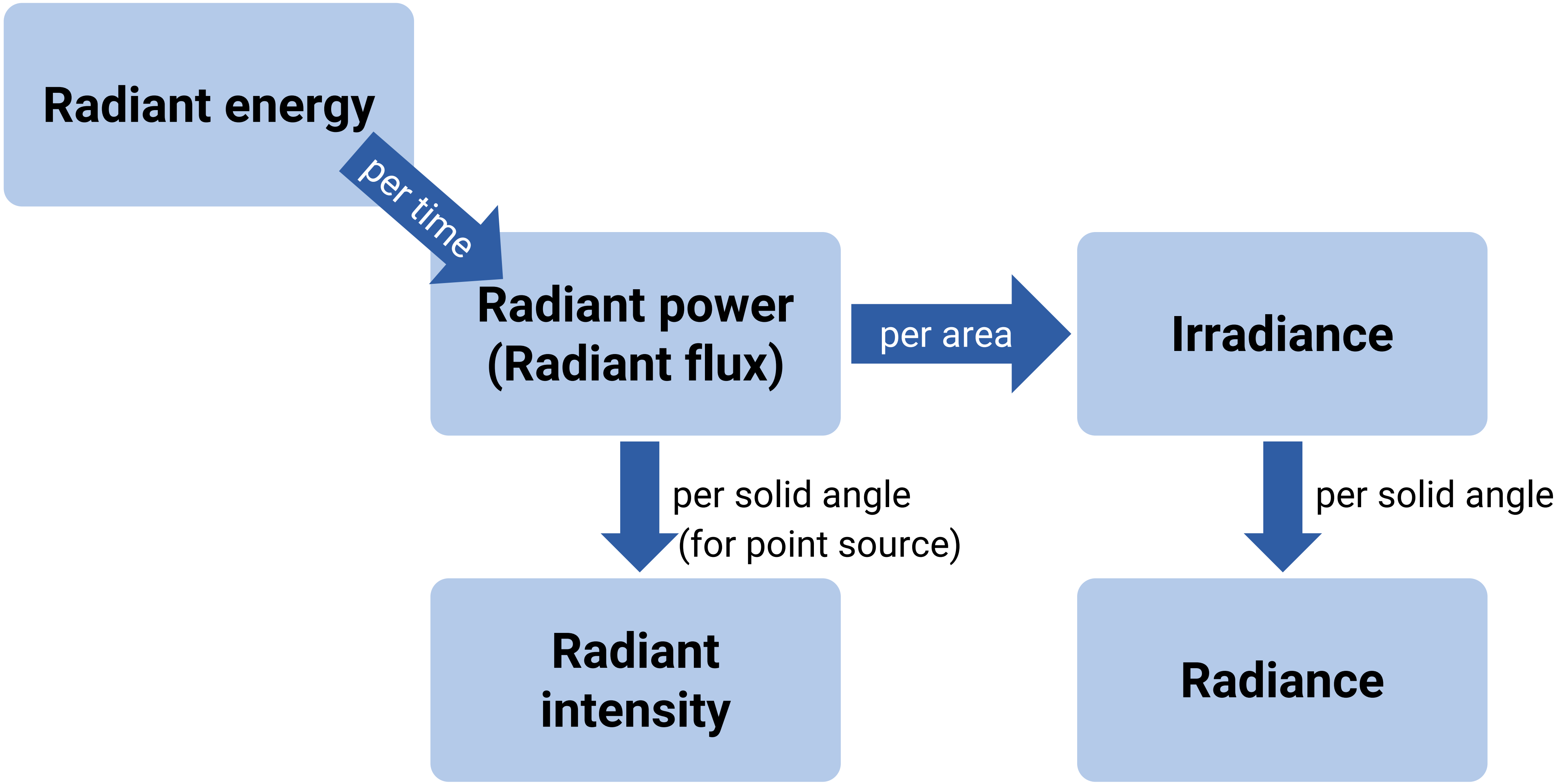
Mass of $V \subset \mathbb{R}^3$ $[\mathrm{kg}]$ **Density at** $p \in \mathbb{R}^3$ $[\mathrm{kg}/\mathrm{m}^3]$

Proposition

Relations between mass m of a volume $V \subset \mathbb{R}^3$ and density ρ at a point $p \in \mathbb{R}^3$ is that:

$$m(V) = \int_V \rho(p)\mathrm{d}p \quad \xrightarrow{\text{per volume}} \quad \xleftarrow{\text{over volume}} \quad \rho(p) = \lim_{\substack{|V|\to 0 \\ p\in V}} \frac{m(V)}{|V|}$$

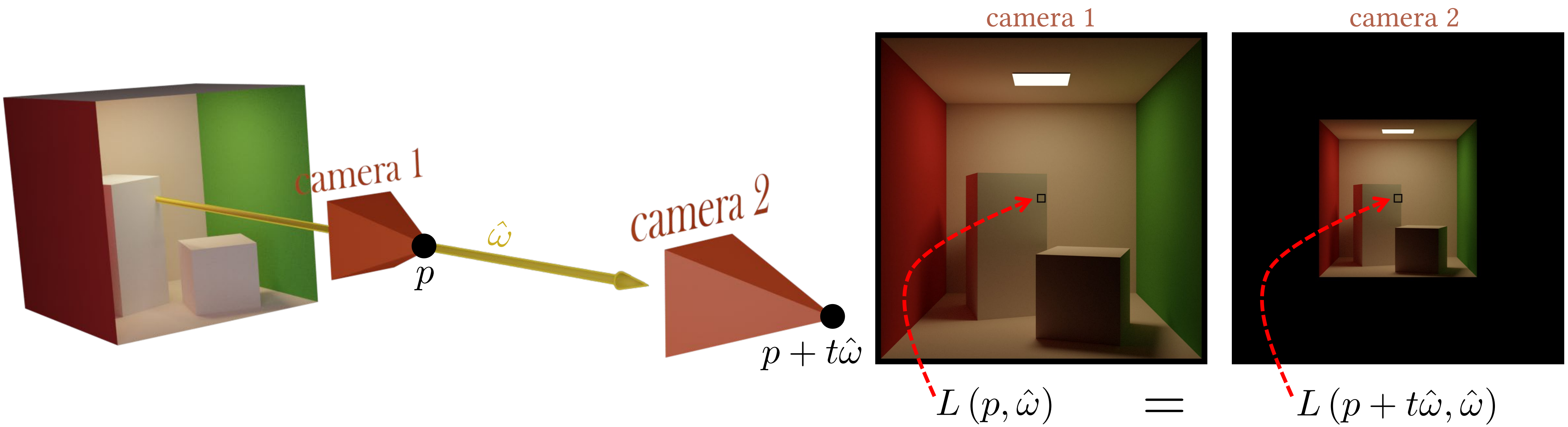
Radiometric quantities



See the full slides for more details:

- https://github.com/shinyoung-yi/lecture-rendering-mitsuba/blob/main/slides/01.%20radiometry%20and%20light%20transport.pdf

Radiance



* if the pixel footprints are sufficiently small

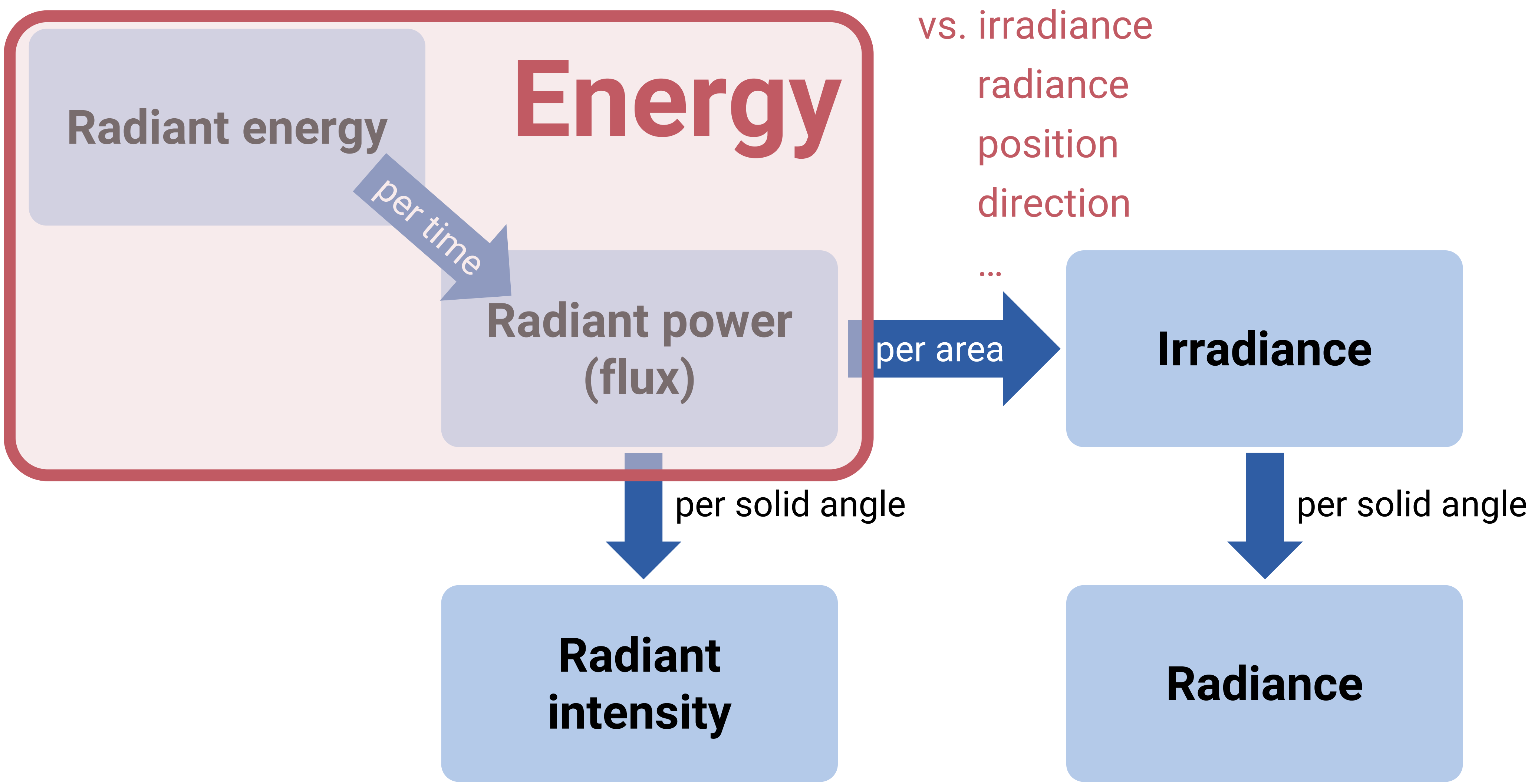
Proposition

Radiance is invariant along ray:

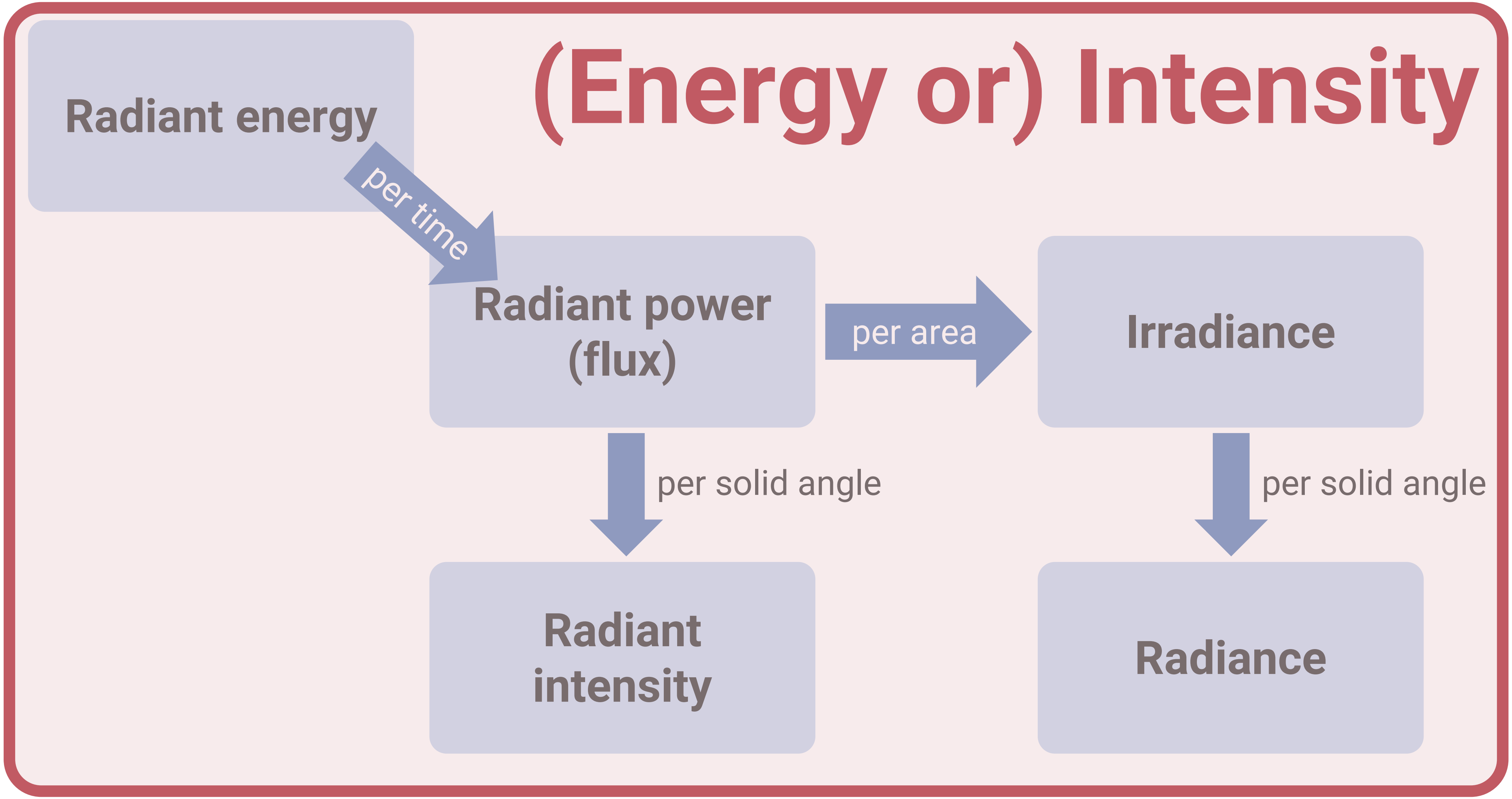
$$L(p, \widehat{\omega}) = L(p + t\widehat{\omega}, \widehat{\omega}) \; \forall t \in \mathbb{R}$$

whenever there is no material between p and $p + t\widehat{\omega}$

Slight abuse of terminology

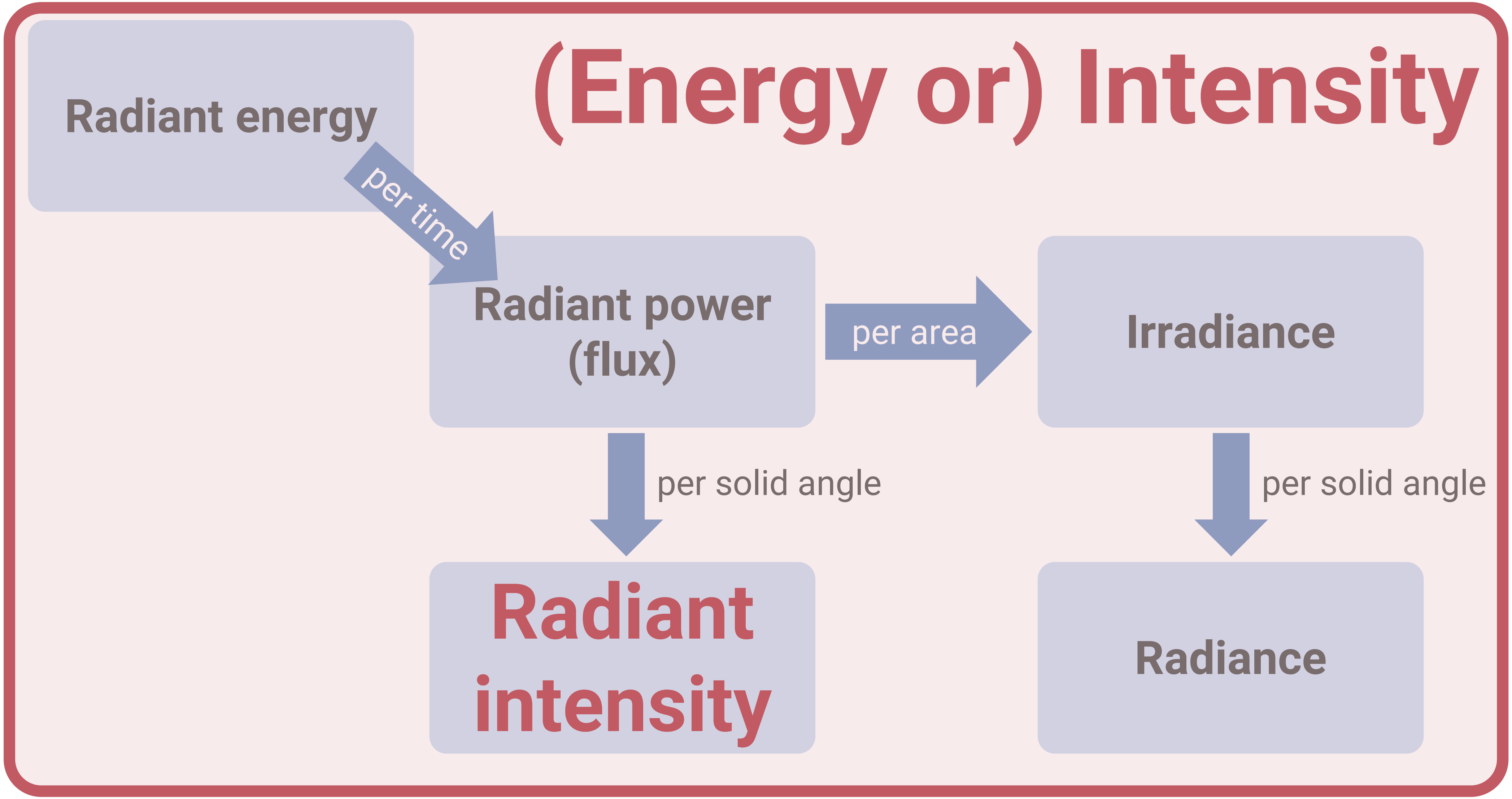


Slight abuse of terminology

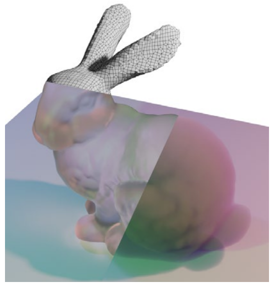


vs. position
direction
…

Unfortunate ambiguity



1. What should we compute?

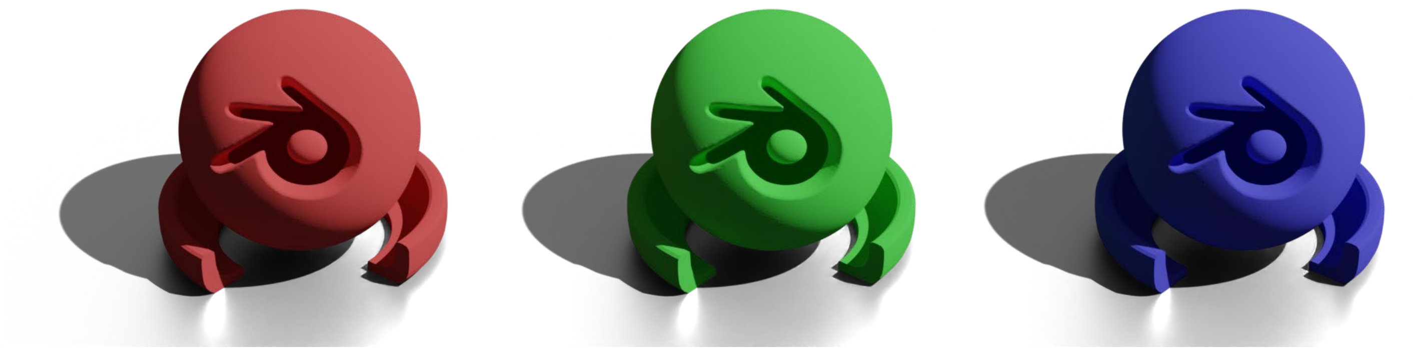


→ Radiometry

- What is radiance?
- BRDFs (BSDFs) and rendering equation

Material Appearance

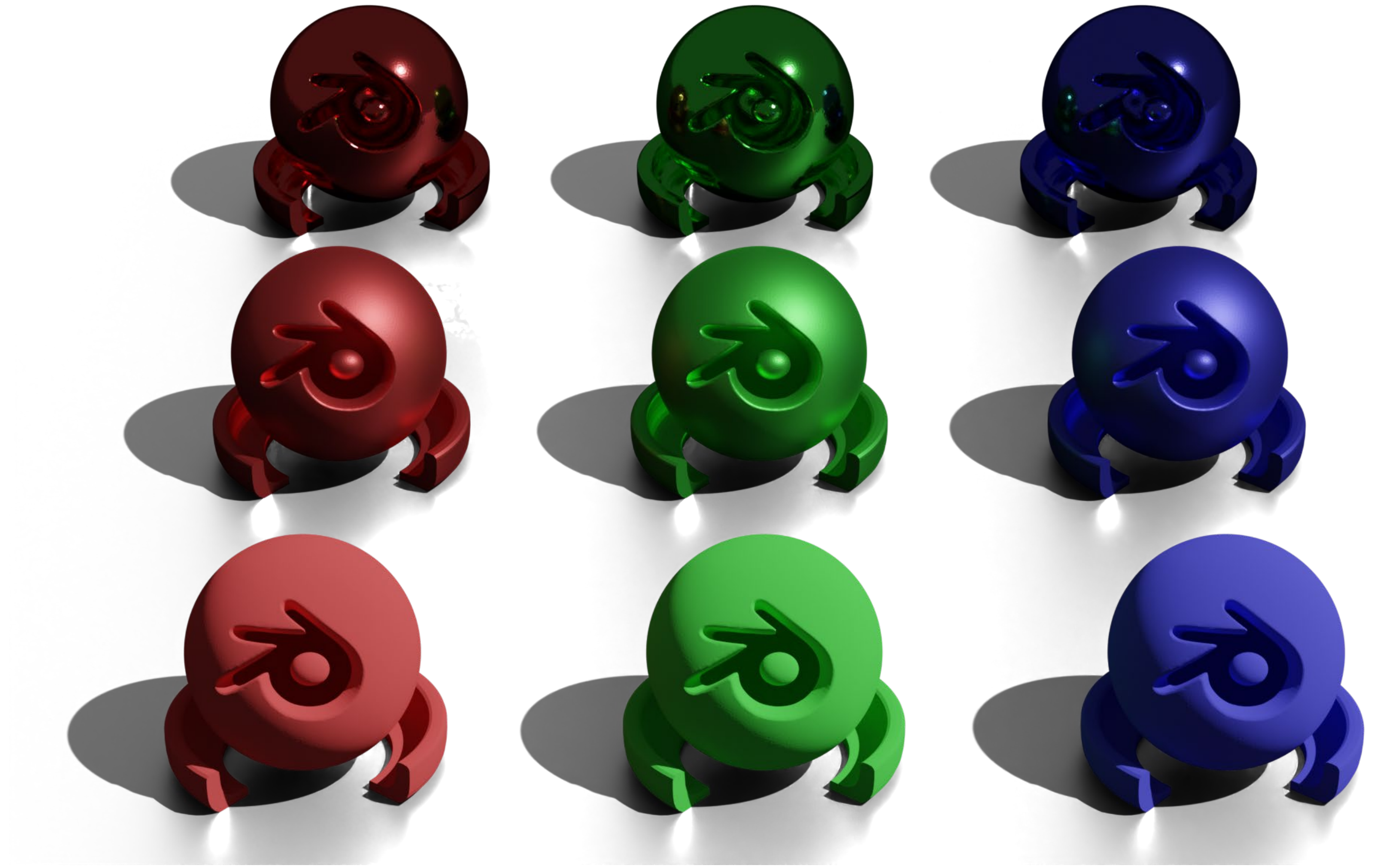
How can we characterize material appearance (refelctance)?



Is single numbers of reflectance per RGB channel (or wavelength) enough?

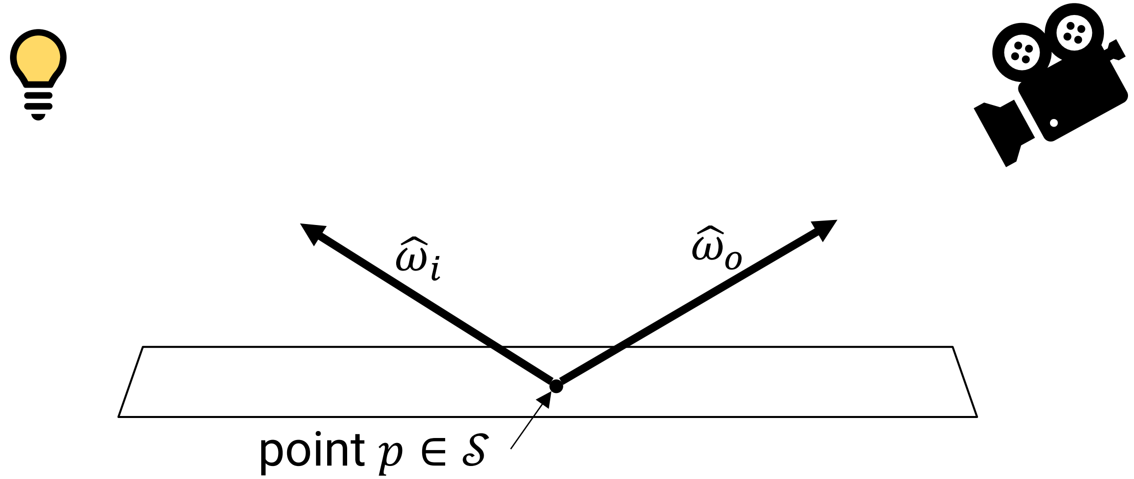
Material Appearance

How can we characterize material appearance (refelctance)?



Is single numbers of reflectance per RGB channel (or wavelength) enough?

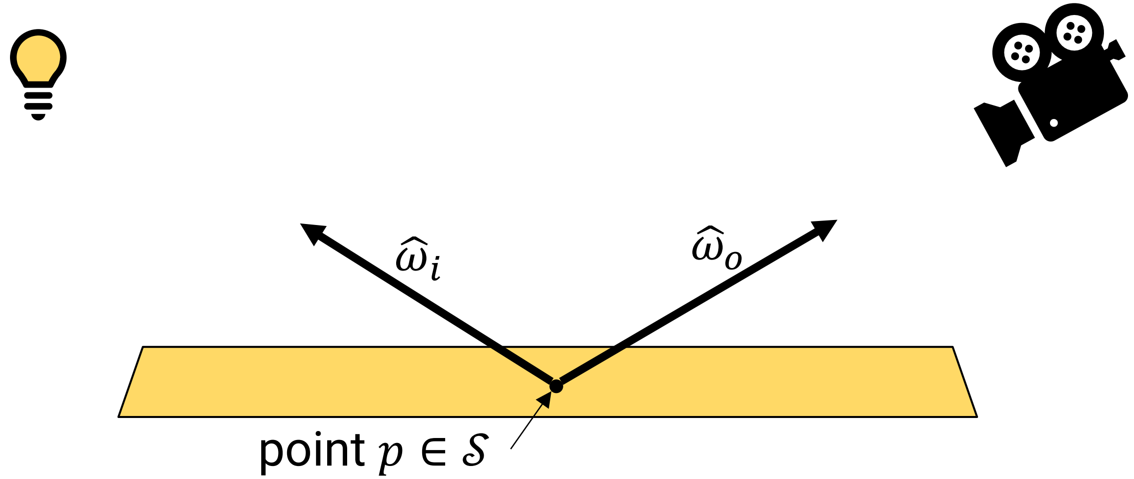
Rendering equation



From definition of BRDFs…

$$L^{(\mathrm{out})}(p, \widehat{\omega}_o) = \int_{\mathbb{S}^2} L^{(\mathrm{in})}(p, \widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Rendering equation



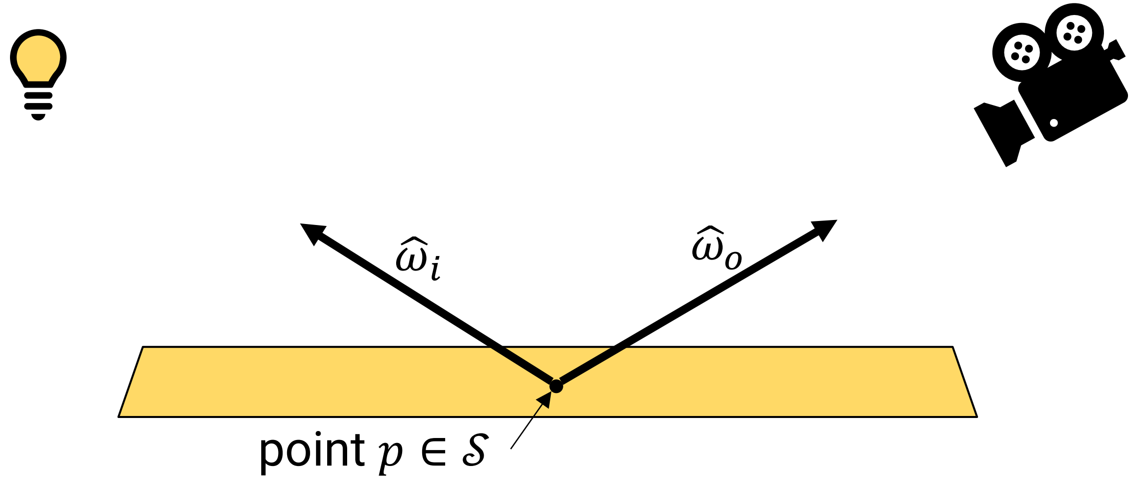
From definition of BRDFs…

+ Emission

Rendering equation
(light transport equation)

$$L^{(\mathrm{out})}(p, \hat{\omega}_o) = L_e(p, \hat{\omega}_o) + \int_{\mathbb{S}^2} L^{(\mathrm{in})}(p, \hat{\omega}_i) f_s(p, \hat{\omega}_i, \hat{\omega}_o) |\hat{n} \cdot \hat{\omega}_i| \mathrm{d}\hat{\omega}_i$$

Rendering equation

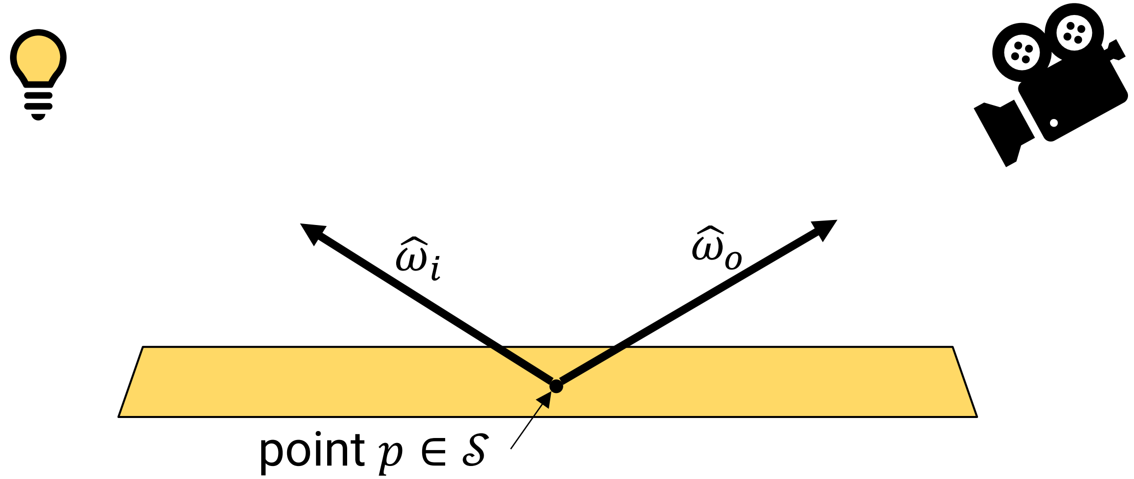


Q. What are knowns and unknowns?

Rendering equation
(light transport equation)

$$L^{(\mathrm{out})}(p,\hat{\omega}_o) = L_e(p,\hat{\omega}_o) + \int_{\mathbb{S}^2} L^{(\mathrm{in})}(p,\hat{\omega}_i) f_s(p,\hat{\omega}_i,\hat{\omega}_o)|\hat{n}\cdot\hat{\omega}_i|\mathrm{d}\hat{\omega}_i$$

Rendering equation



Q. What are knowns and unknowns?

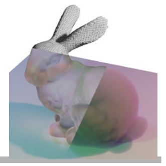
Rendering equation
(light transport equation)

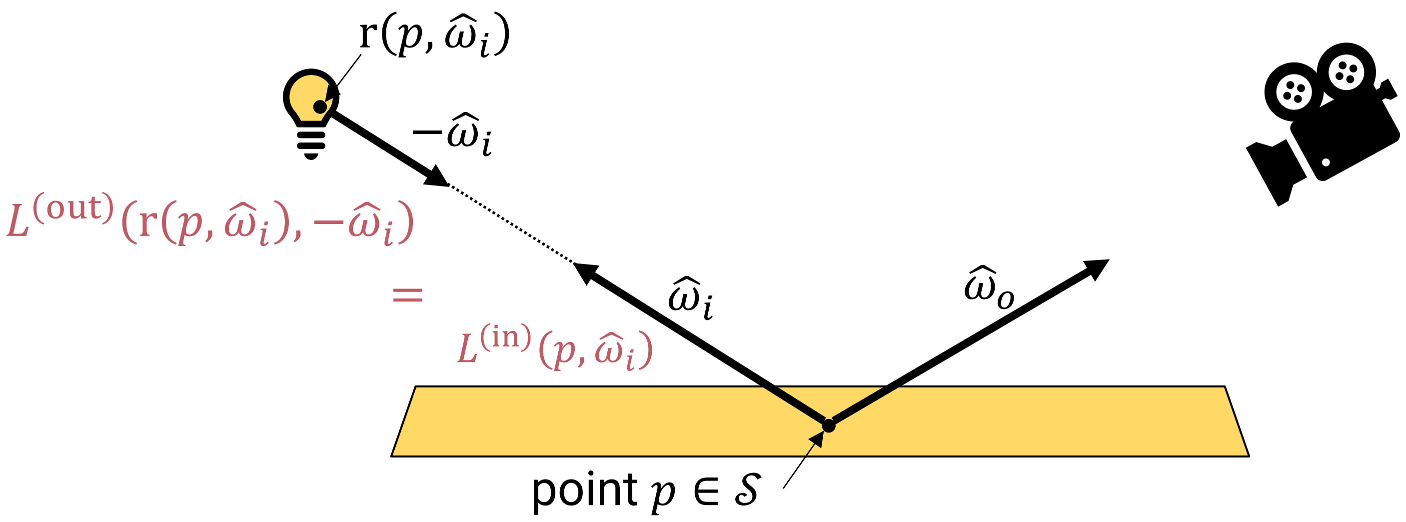
known

$$L^{(\text{out})}(p, \widehat{\omega}_o) = L_e(p, \widehat{\omega}_o) + \int_{\mathbb{S}^2} L^{(\text{in})}(p, \widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

unknown

Rendering equation



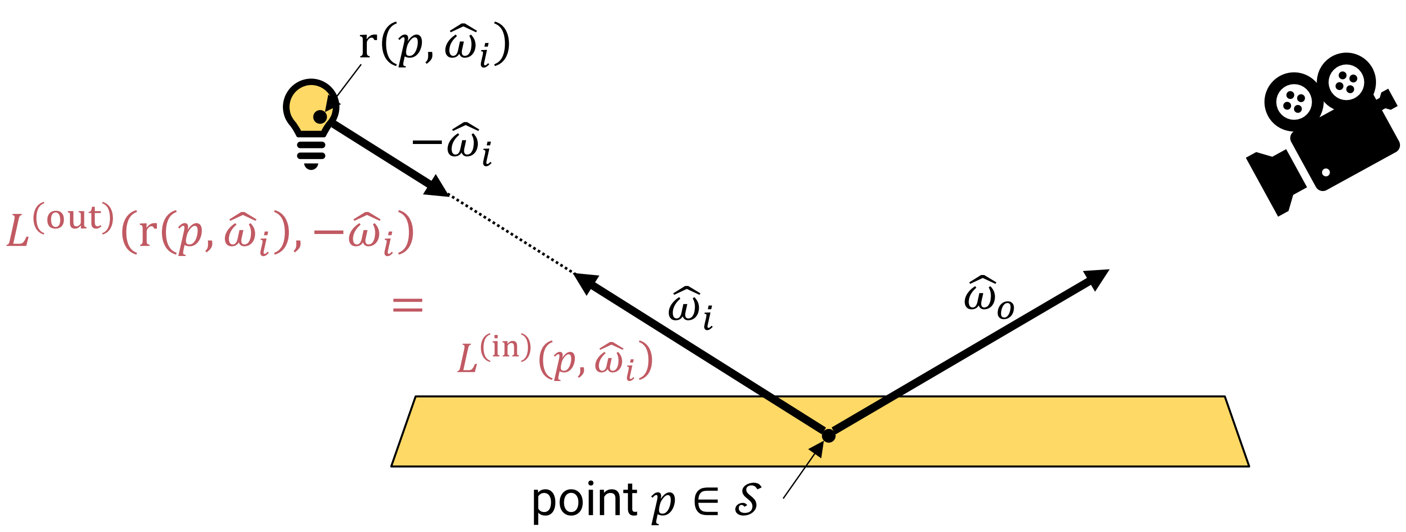


Q. Any relationship between $L^{(\mathrm{out})}$ and $L^{(\mathrm{in})}$?

Rendering equation
(light transport equation)

$$L^{(\mathrm{out})}(p, \widehat{\omega}_o) = L_e(p, \widehat{\omega}_o) + \int_{\mathbb{S}^2} L^{(\mathrm{in})}(p, \widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Rendering equation



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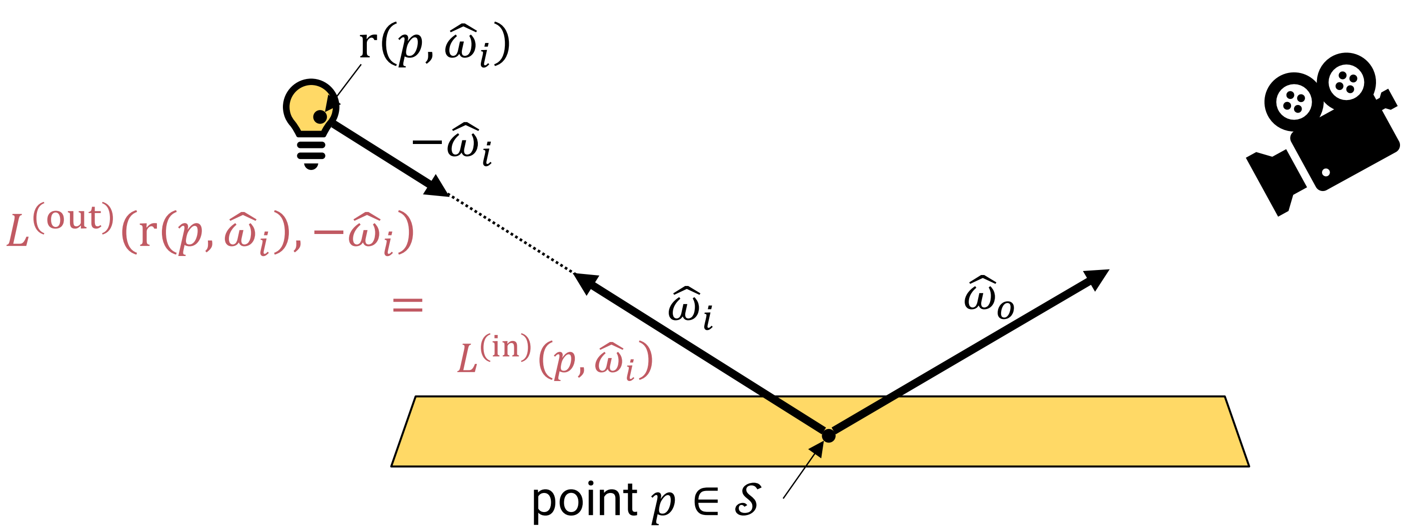
Rendering equation
(light transport equation)

$$L^{(\mathrm{out})}(p, \hat{\omega}_o) = L_e(p, \hat{\omega}_o) + \int_{\mathbb{S}^2} L^{(\mathrm{out})}(\mathrm{r}(p, \hat{\omega}_i), -\hat{\omega}_i) f_s(p, \hat{\omega}_i, \hat{\omega}_o) |\hat{n} \cdot \hat{\omega}_i| \mathrm{d}\hat{\omega}_i$$

Rendering equation



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Rendering equation

$$L^{(\mathrm{out})}(p, \widehat{\omega}_o) = L_e(p, \omega_{\hat{o}}) + \int_{\mathbb{S}^2} L^{(\mathrm{out})}(\mathrm{r}(p, \widehat{\omega}_i), -\widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Recursive!

$$L(p, \widehat{\omega}_o) = L_e + \int_{\mathbb{S}^2} L f_s |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Rendering equation

$$L^{(\mathrm{out})}(p, \widehat{\omega}_o) = L_e(p, \omega_{\hat{o}}) + \int_{\mathbb{S}^2} L^{(\mathrm{out})}(\mathrm{r}(p, \widehat{\omega}_i), -\widehat{\omega}_i) f_s(p, \widehat{\omega}_i, \widehat{\omega}_o) |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Recursive!

$$L(p, \widehat{\omega}_o) = L_e + \int_{\mathbb{S}^2} \left[L'_e + \int_{\mathbb{S}^2} L f_s' |\hat{n} \cdot \widehat{\omega}_i'| \mathrm{d}\widehat{\omega}_i' \right] f_s |\hat{n} \cdot \widehat{\omega}_i| \mathrm{d}\widehat{\omega}_i$$

Rendering equation

$$L^{(\mathrm{out})}(p,\widehat{\omega}_o) = L_e(p,\omega_{\hat{o}}) + \int_{\mathbb{S}^2} L^{(\mathrm{out})}(\mathrm{r}(p,\widehat{\omega}_i),-\widehat{\omega}_i) f_s(p,\widehat{\omega}_i,\widehat{\omega}_o)|\hat{n}\cdot\widehat{\omega}_i|\mathrm{d}\widehat{\omega}_i$$

Recursive!

$$L(p,\widehat{\omega}_o) = L_e + \int_{\mathbb{S}^2}\left[L'_e + \int_{\mathbb{S}^2}\left[L''_e + \int_{\mathbb{S}^2} L f''_s|\hat{n}\cdot\widehat{\omega}''_i|\mathrm{d}\widehat{\omega}''_i\right] f'_s|\hat{n}\cdot\widehat{\omega}'_i|\mathrm{d}\widehat{\omega}'_i\right] f_s|\hat{n}\cdot\widehat{\omega}_i|\mathrm{d}\widehat{\omega}_i$$

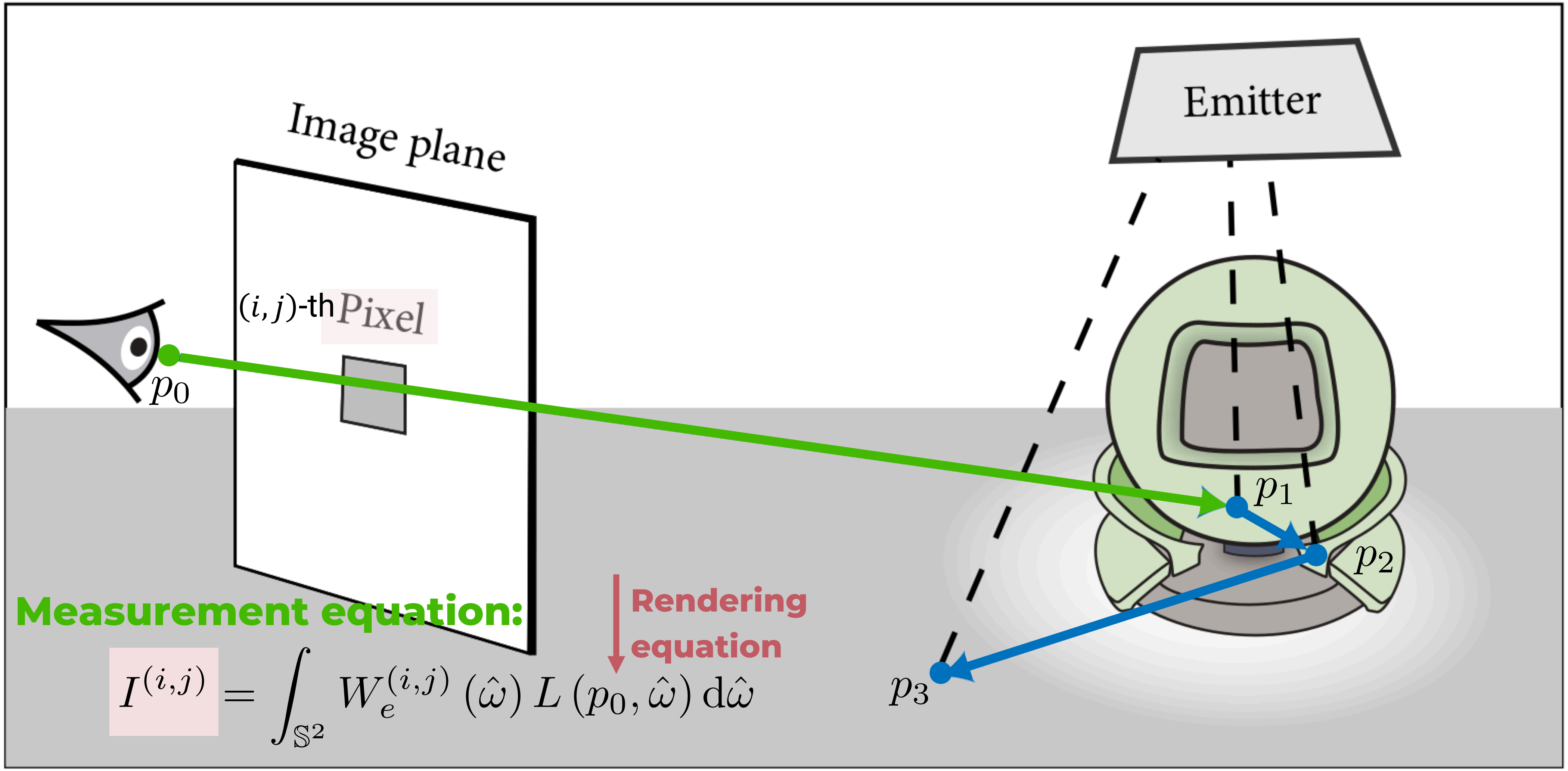
$$\cdots$$

$$\begin{aligned} L(p,\widehat{\omega}_o) &= L_e \\ &+ \int_{\mathbb{S}^2} L'_e f_s|\hat{n}\cdot\widehat{\omega}_i|\mathrm{d}\widehat{\omega}_i \\ &+ \int_{\mathbb{S}^2}\left[\int_{\mathbb{S}^2} L''_e f'_s|\hat{n}\cdot\widehat{\omega}'_i|\mathrm{d}\widehat{\omega}'_i\right] f_s|\hat{n}\cdot\widehat{\omega}_i|\mathrm{d}\widehat{\omega}_i \\ &+ \int_{\mathbb{S}^2}\left[\int_{\mathbb{S}^2}\left[\int_{\mathbb{S}^2} L'''_e f''_s|\hat{n}\cdot\widehat{\omega}''_i|\mathrm{d}\widehat{\omega}''_i\right] f'_s|\hat{n}\cdot\widehat{\omega}'_i|\mathrm{d}\widehat{\omega}'_i\right] f_s|\hat{n}\cdot\widehat{\omega}_i|\mathrm{d}\widehat{\omega}_i \end{aligned}$$

Rendering equation

$$
\begin{aligned}
L(p, \hat{\omega}_o) &= L_e \\
&+ \int_{\mathbb{S}^2} L'_e f_s |\hat{n} \cdot \hat{\omega}_i| \mathrm{d}\hat{\omega}_i \\
&+ \int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} L''_e f'_s |\hat{n} \cdot \hat{\omega}'_i| \mathrm{d}\hat{\omega}'_i \right] f_s |\hat{n} \cdot \hat{\omega}_i| \mathrm{d}\hat{\omega}_i \\
&+ \int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} L'''_e f''_s |\hat{n} \cdot \hat{\omega}''_i| \mathrm{d}\hat{\omega}''_i \right] f'_s |\hat{n} \cdot \hat{\omega}'_i| \mathrm{d}\hat{\omega}'_i \right] f_s |\hat{n} \cdot \hat{\omega}_i| \mathrm{d}\hat{\omega}_i
\end{aligned}
$$

Notation & Measurement Equation



$$I^{(i,j)} = \int_{\mathbb{S}^2} W_e^{(i,j)} \left(\hat{\omega}\right) L \left(p_0, \hat{\omega}\right) \mathrm{d}\hat{\omega}$$

[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

Rendering equation

$$
\begin{aligned}
L(p_1, \widehat{\omega}_{10}) = L_{e,10} \\
&+ \int_{\mathbb{S}^2} L_{e,21} f^{\perp}_{s,210} \mathrm{d}\widehat{\omega}_{12} \\
&+ \int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} L_{e,32} f^{\perp}_{s,321} \mathrm{d}\widehat{\omega}'_i \right] f^{\perp}_{s,210} \mathrm{d}\widehat{\omega}_{23} \\
&+ \int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} \left[\int_{\mathbb{S}^2} L_{e,43} f^{\perp}_{s,432} \mathrm{d}\widehat{\omega}''_i \right] f^{\perp}_{s,321} \mathrm{d}\widehat{\omega}'_i \right] f^{\perp}_{s,210} \mathrm{d}\widehat{\omega}_{34}
\end{aligned}
$$

Rendering equation

$$
\begin{aligned}
L(p_1, \widehat{\omega}_{10}) = L_{e,10} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} L_{e,21} \mathrm{d}\widehat{\omega}_{12} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} L_{e,32} \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12}
\end{aligned}
$$

Rendering equation

$$
\begin{aligned}
L(p_1, \widehat{\omega}_{10}) = L_{e,10} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} L_{e,21} \mathrm{d}\widehat{\omega}_{12} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} L_{e,32} \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \\
&+ \int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12}
\end{aligned}
$$

Rendering & Measurement equation

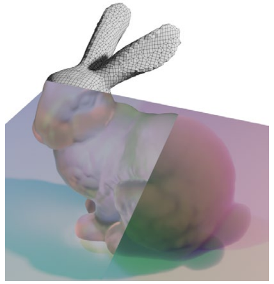
$$I = \int_{\mathbb{S}^2} W_e(\widehat{\omega}_{10}) L(p_1, \widehat{\omega}_{10}) \mathrm{d}\widehat{\omega}_{01}$$

$$\begin{aligned}
&= \int_{\mathbb{S}^2} W_e(\widehat{\omega}_{10}) L_{e,10} \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_e(\widehat{\omega}_{10}) \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} L_{e,21} \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_e(\widehat{\omega}_{10}) \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} L_{e,32} \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_e(\widehat{\omega}_{10}) \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}
\end{aligned}$$

Integration domain: all paths (tuples of vertices)

integrand: path contribution $f(p_0 \ldots . p_j)$

2. How can we compute?



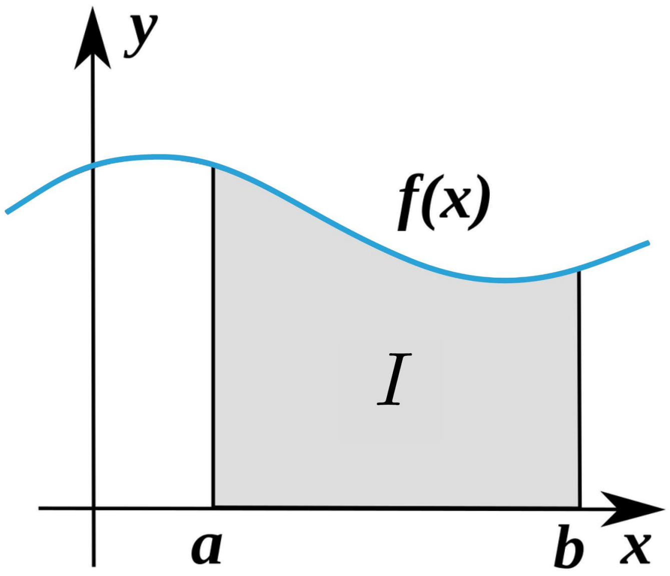
→ Monte Carlo Integration

- MC Integration and importance sampling

Integration

We have a function f on a domain $\mathcal{X}$.

Q. How can we compute $I = \int_{\mathcal{X}} f(x)\,\mathrm{d}x$?



$\mathcal{X} = [a, b] \subset \mathbb{R}$

source: User 4C / Wikipedia
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[Adam Celarek]

1. Analytic (symbolic) integration

- We can integrate only few functions analytically.
- Hardly predict difficulty of given problem

Try analytic integration for:

$\int e^{-x^2}\mathrm{d}x$ No

$\int x e^{-x^2}\mathrm{d}x$ Yes

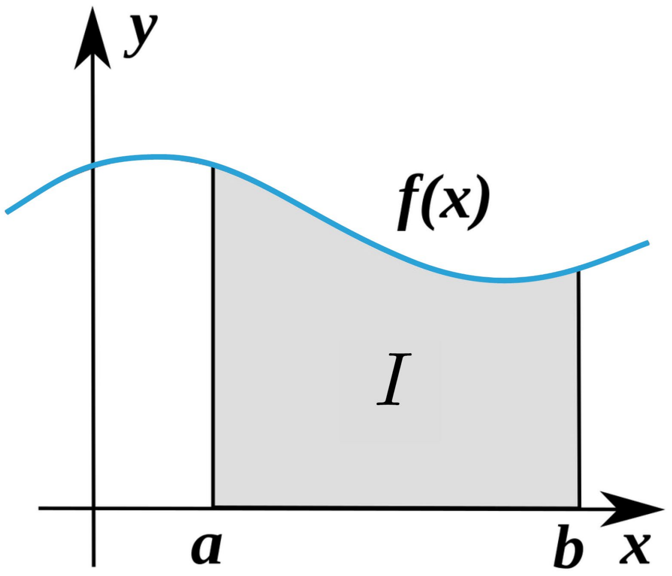
$\int x^2 e^{-x^2}\mathrm{d}x$ No

$\int x^3 e^{-x^2}\mathrm{d}x$ Yes

Integration

We have a function f on a domain $\mathcal{X}$.

Q. How can we compute $I = \int_{\mathcal{X}} f(x)\,\mathrm{d}x$?



$\mathcal{X} = [a, b] \subset \mathbb{R}$

source: User 4C / Wikipedia

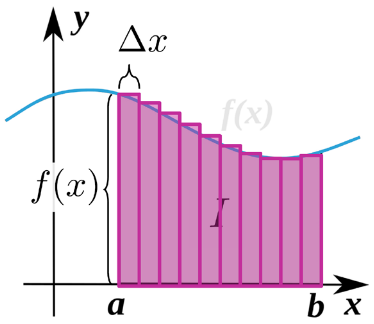
[Adam Celarek]

1. **Analytic (symbolic) integration**

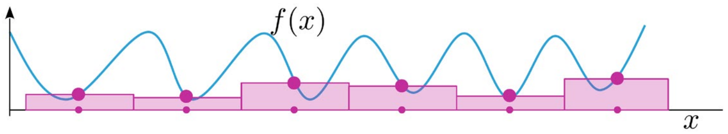
- We can integrate only few functions analytically.
- Hardly predict difficulty of given problem

2. **Numerical approximation by uniform grids**

- For a fixed error level, we need $N^{\dim(\mathcal{X})}$ samples!
- Aliasing!

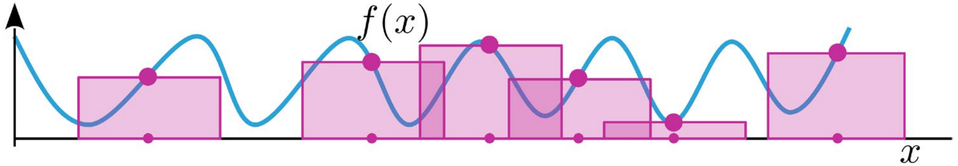


$$I \approx \sum_{n=1}^{N} f(x_n)\,\Delta x$$



3. **Monte Carlo estimation**: for a random variable $X \sim \mathcal{U}(\mathcal{X})$

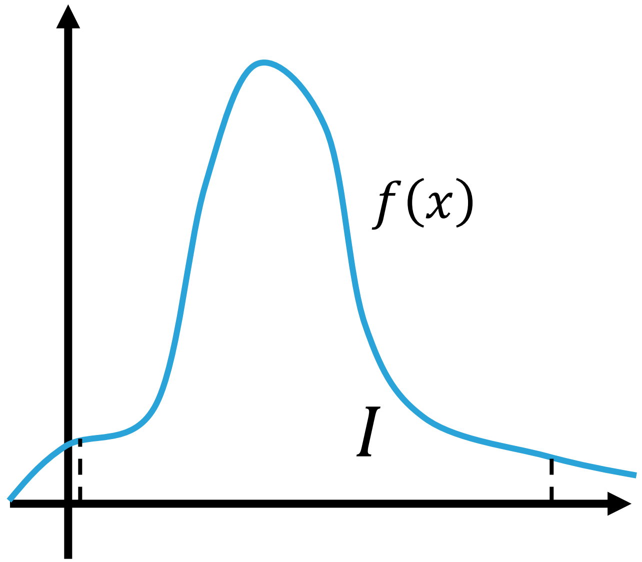
$$I \approx \hat{I} = \frac{|\mathcal{X}|}{N} \sum_{n=1}^{N} f(X_n)$$



Monte Carlo Integration

We have a function f on a domain $\mathcal{X}$.

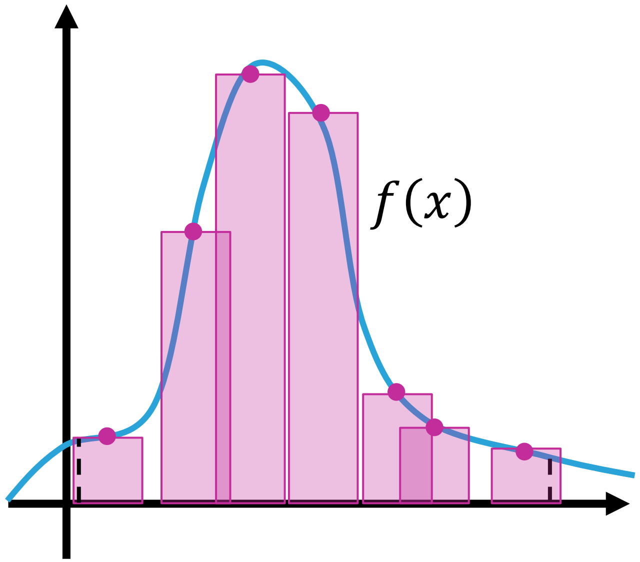
Q. How can we compute $I = \int_{\mathcal{X}} f(x)\,\mathrm{d}x$?



$\mathcal{X} = [a, b] \subset \mathbb{R}$

for a random variable $X \sim \mathcal{U}(\mathcal{X})$

$$I \approx \hat{I} = \frac{|\mathcal{X}|}{N} \sum_{n=1}^{N} f(X_n)$$

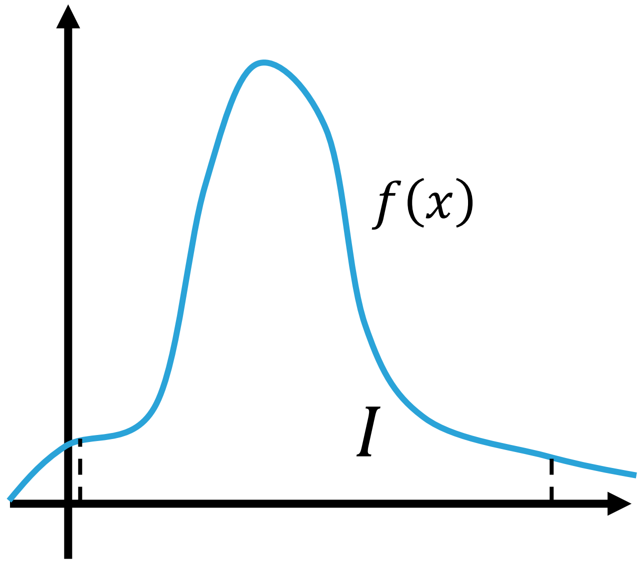


$$Var(\hat{I}) \propto Var(\quad)$$

Monte Carlo Integration: importance sampling

We have a function f on a domain $\mathcal{X}$.

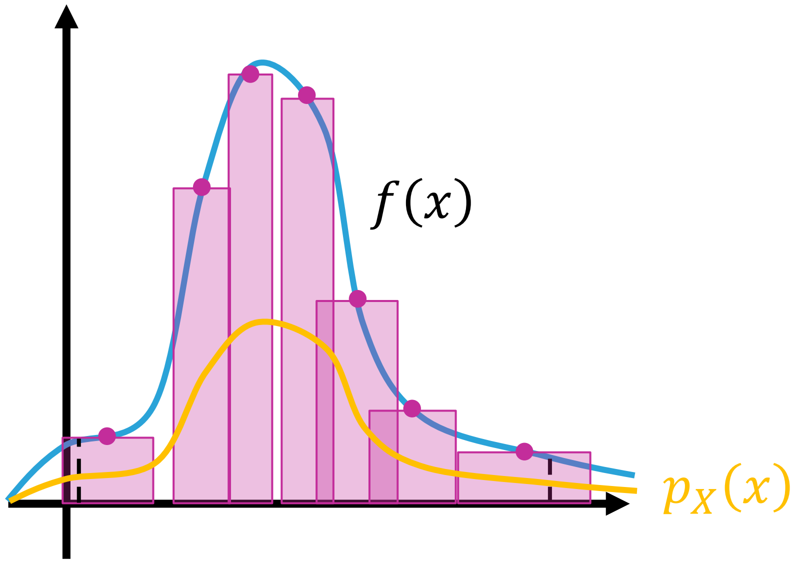
Q. How can we compute $I = \int_{\mathcal{X}} f(x)\,\mathrm{d}x$?



$\mathcal{X} = [a, b] \subset \mathbb{R}$

for a random variable X with PDF hopefully $\propto f$

$$I \approx \hat{I} = \frac{1}{N} \sum_{n=1}^{N} \frac{f(X_n)}{p_X(X_n)}$$



$$Var(\hat{I}) \propto Var(\square)$$

Monte Carlo Integration

When is the MC integration correct (unbiased)?

$$\int_{\mathcal{X}} f(x)\mathrm{d}x = I \approx \hat{I} = \frac{1}{N}\sum_{n=1}^{N}\frac{f(X_n)}{p_X(X_n)}$$

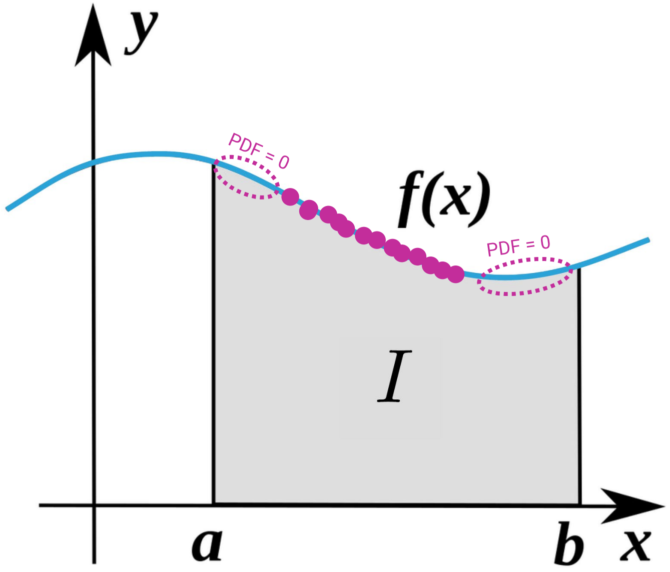
Monte Carlo Integration

When is the MC integration correct (unbiased)?

$$\int_{\mathcal{X}} f(x)\mathrm{d}x = I = \mathbb{E}[\hat{I}] = \mathbb{E}\left[\frac{1}{N}\sum_{n=1}^{N}\frac{f(X_n)}{p_X(X_n)}\right]$$

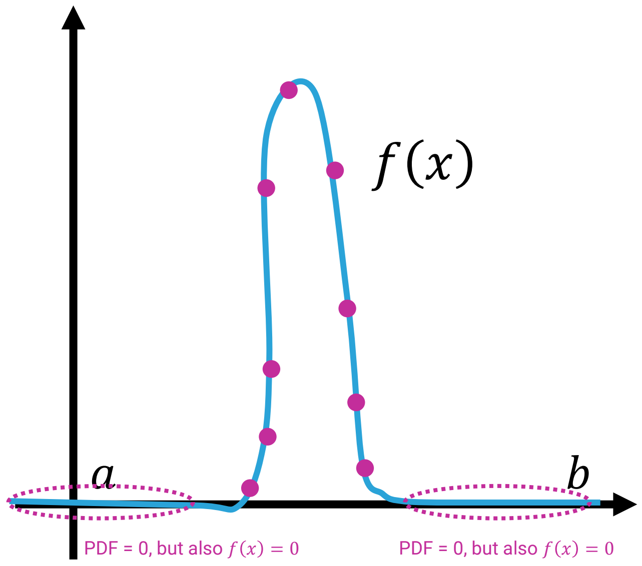
For any $x \in \mathcal{X}$, if $f(x) \neq 0$ then $p_X(x) > 0$

Failure case



$\mathcal{X} = [a, b] \subset \mathbb{R}$

Okay



$\mathcal{X} = [a, b] \subset \mathbb{R}$

source: User 4C / Wikipedia

[Adam Celarek]

Language of Monte Carlo Integration

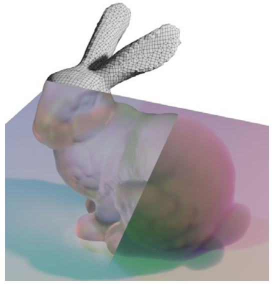
Sampling: generate a realization of a random variable

Which PDF do we choose?
How draw a sample of a r.v. with such PDF?

- *"This sampling strategy is good for given integrand f."*
- *"In path tracing, BSDF sampling and emitter samplings are used."*

Correct → **unbiased** (or **consistent**, at least)

Efficient (less noise) → **low variance**



3. Path Tracing

Rendering Methods

Rendering

Rasterization

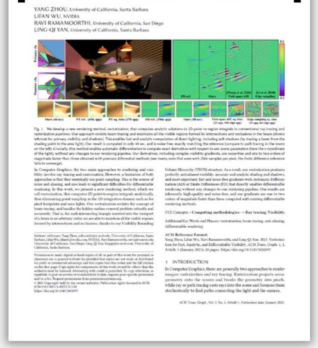
- OpenGL / DirectX
- Many techniques learned in undergrad-graphics courses

Precomputed radiance transfer

Radiosity

Vectorization

[Zhou et al. 2021]



Ray tracing

- Early ray tracing
  - Whitted [1980]
  - Cook [1984]
- **Path tracing**
- …
- …
- ….. many recent papers

…

Rasterization vs. Ray Tracing

Rasterization

- OpenGL / DirectX
- Many techniques learned in undergrad-graphics courses

Rasterization finds intersections of a set of rays with the same origin/direction

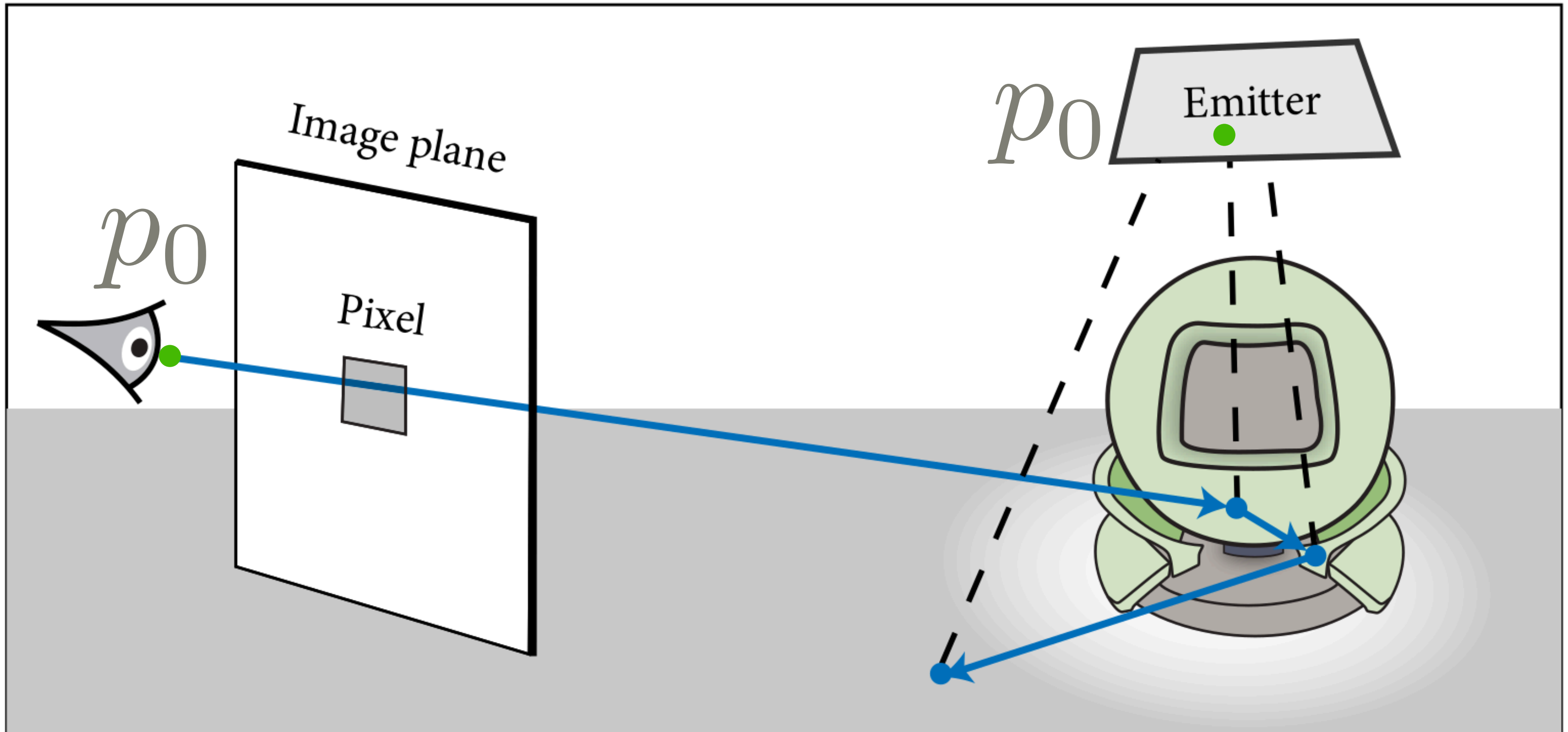
Finding a single ray intersection is inefficient

Ray tracing

- Early ray tracing
  - Whitted [1980]
  - Cook [1984]
- **Path tracing**
- …
- …
- ….. many recent papers

Recursively perform the ray intersection query

Path Tracing: Overview



[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

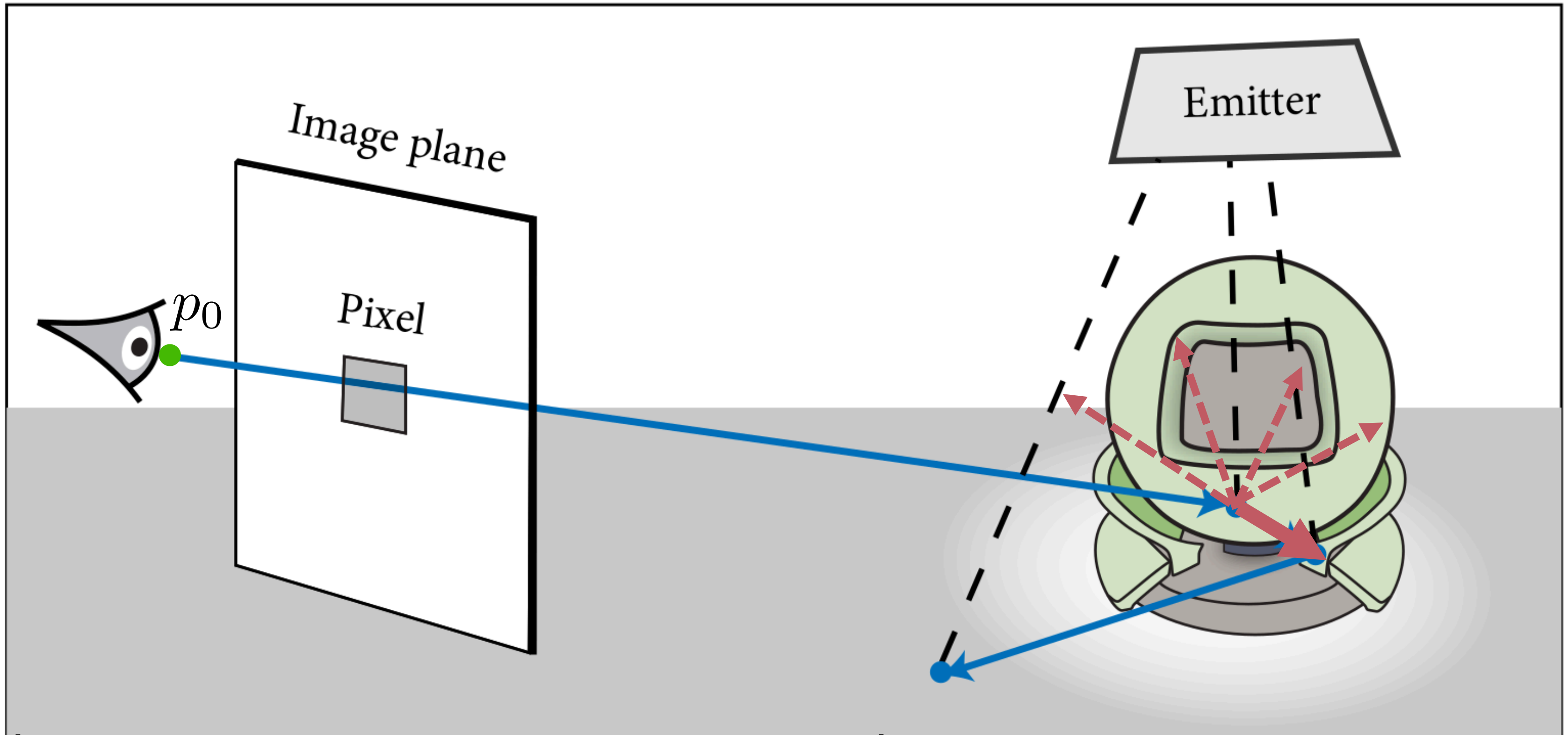
Q. Start from sensor vs. emitter?

(narrow meaning) path tracing

light tracing = particle tracing

→ sensor

Path Tracing: Overview



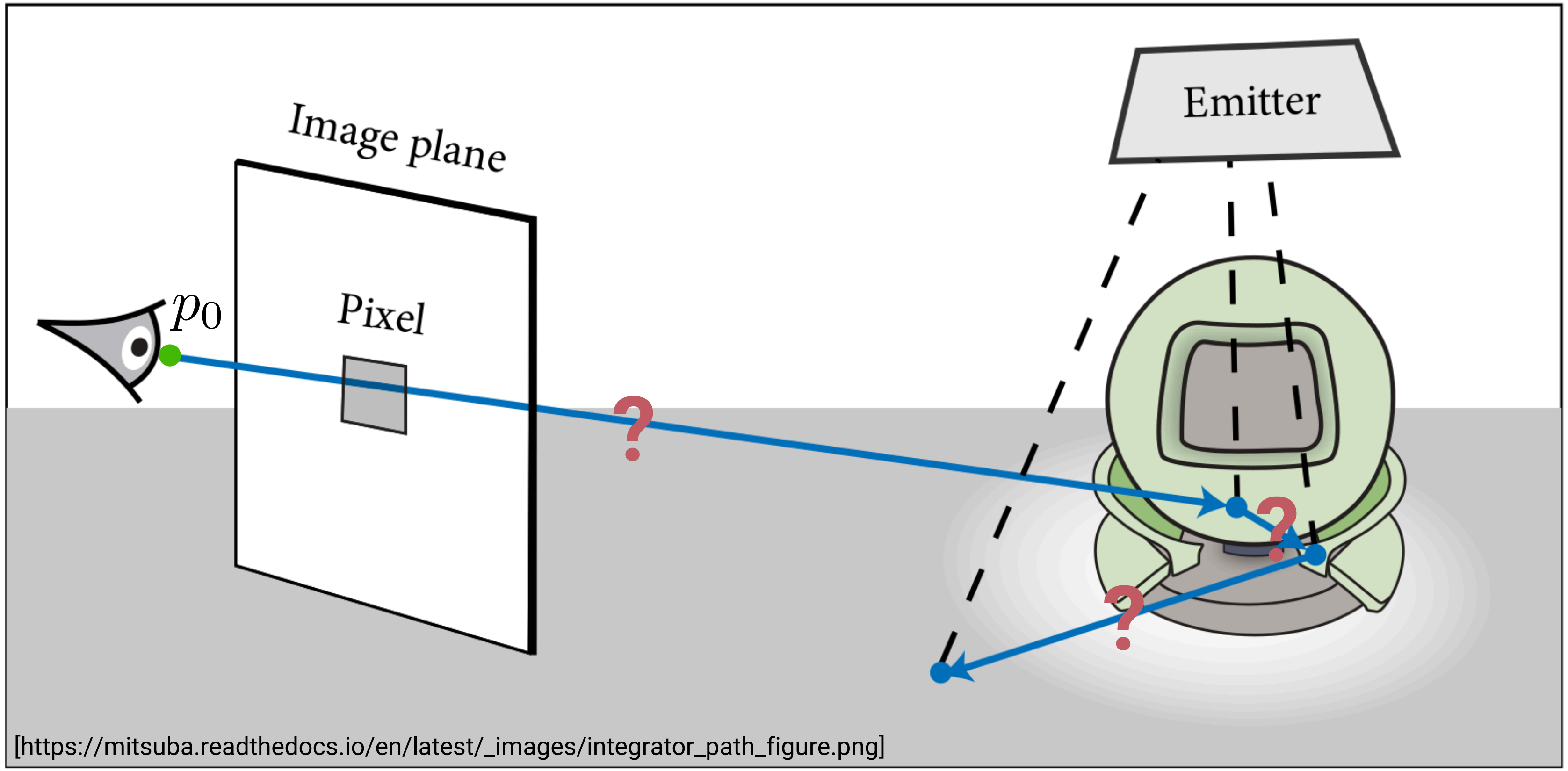
[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

Q. Multiple rays vs. single ray for each bounce?

[Cook 1984] “Distributed ray tracing” Path tracing

→ single ray

Path Tracing: Overview

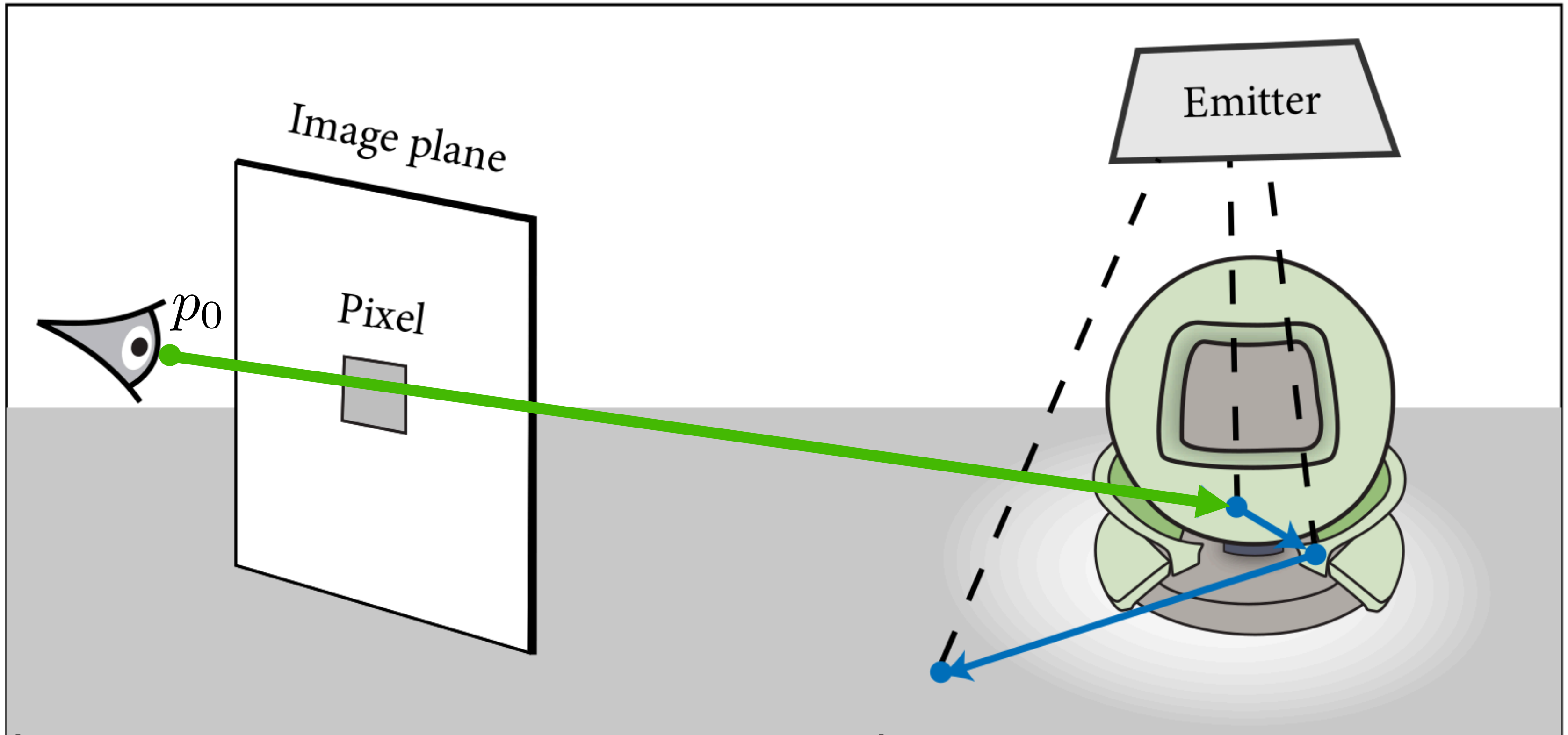


Q. How to sample the next direction from given vertex?

= Which probability distribution

→ Sensor sampling BSDF sampling ?

Path Tracing: Overview



[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

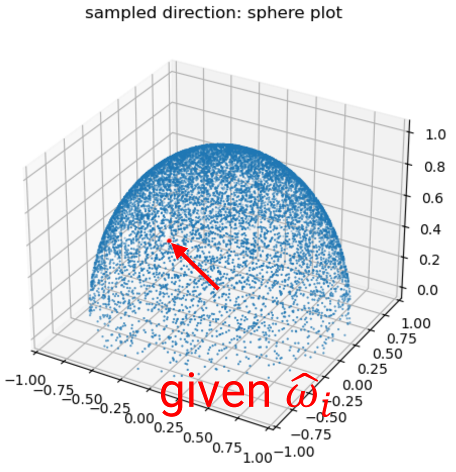
Q. How to sample the next direction from given vertex?

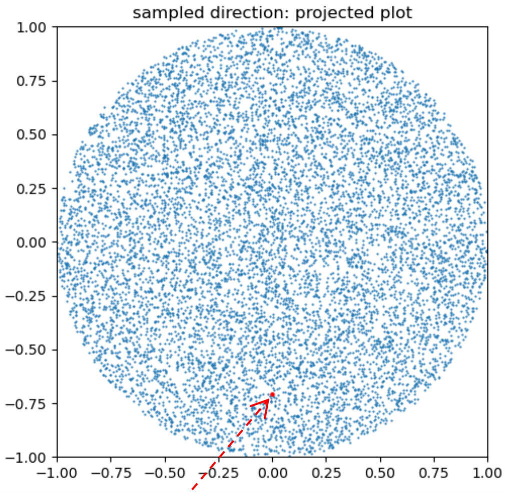
= Which probability distribution

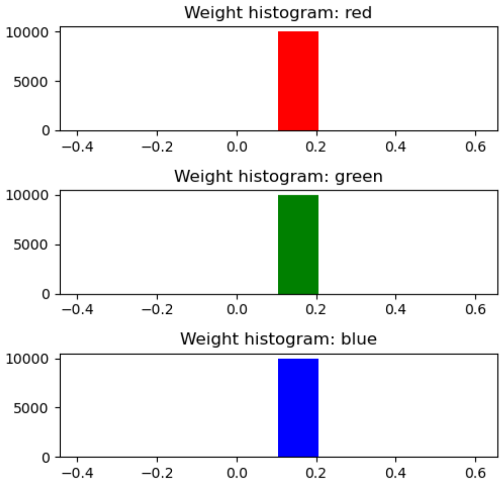
Path Tracing: Overview

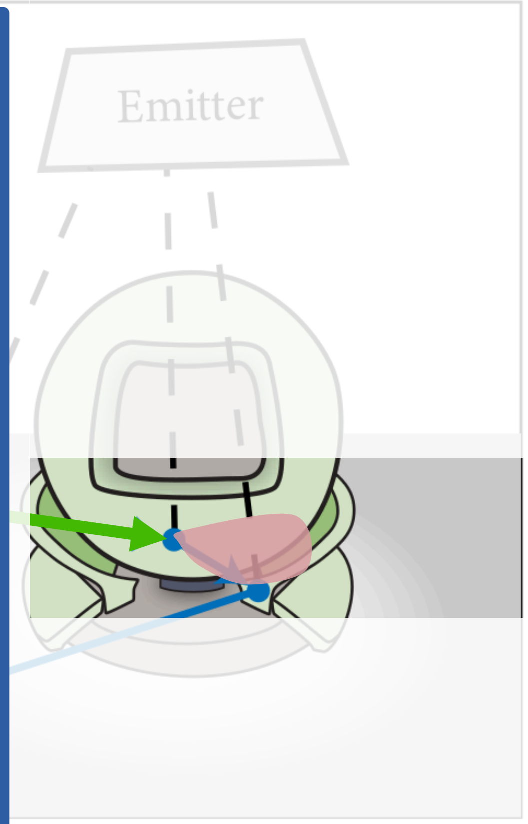
Diffuse BSDF

$$\frac{f^{\perp}(\widehat{\omega}_i, \widehat{\omega}_o)}{p(\widehat{\omega}_i)}$$









Q. How to sample the next direction from given vertex?

= Which probability distribution

Sensor sampling

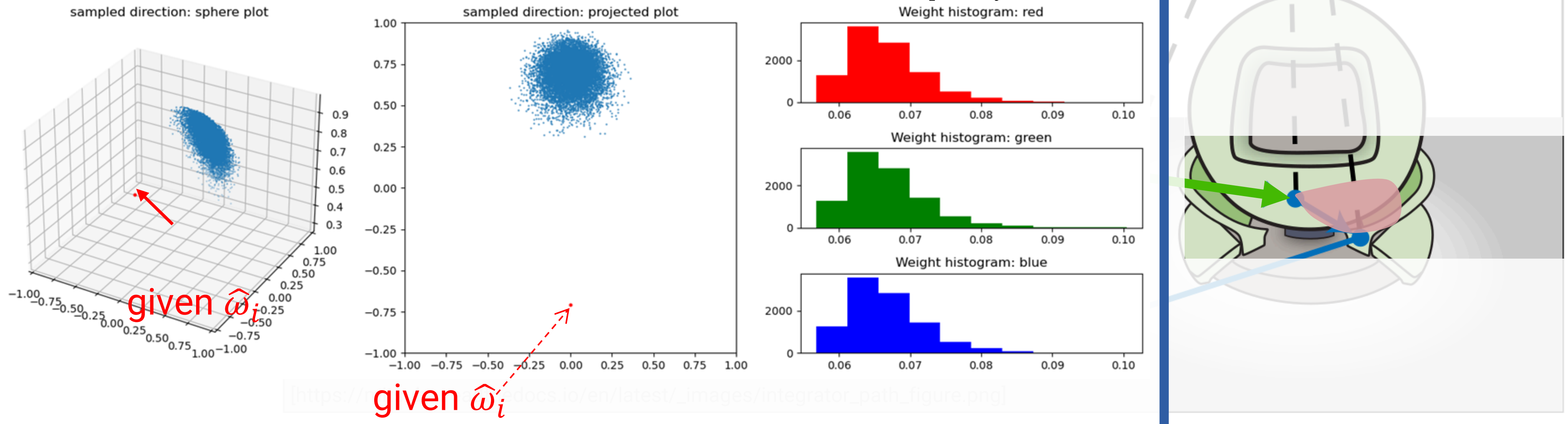
BSDF sampling

?

Path Tracing: Overview

Rough plastic BSDF

$$\frac{f^{\perp}(\hat{\omega}_i, \hat{\omega}_o)}{p(\hat{\omega}_i)}$$

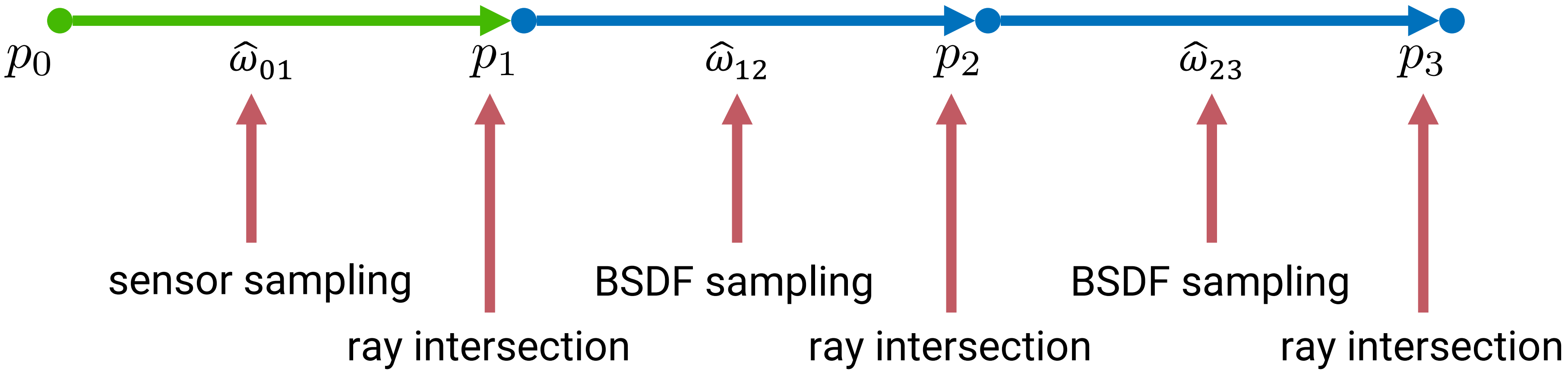


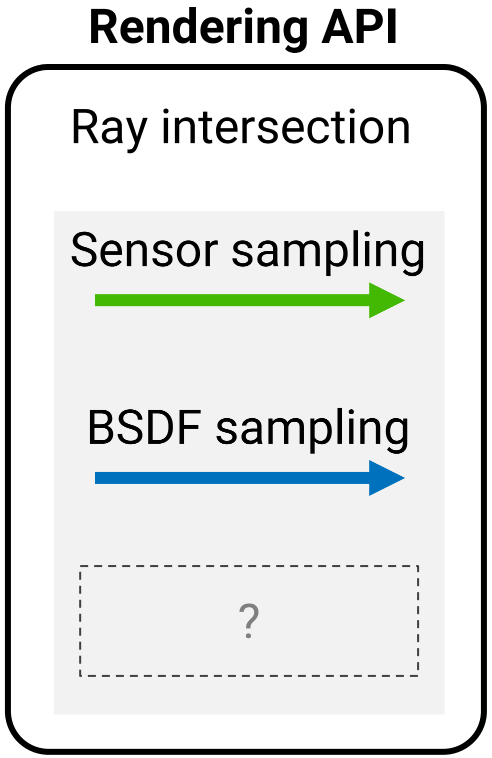
Q. How to sample the next direction from given vertex?

= Which probability distribution

Sensor sampling BSDF sampling ?

Path Tracing Overview





Rendering & Measurement equation

$$
\begin{aligned}
I = &\int_{\mathbb{S}^2} W_{e,10} L_{e,10} \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} L_{e,21} \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} L_{e,32} \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01} \\
&+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}
\end{aligned}
$$

Monte Carlo! $\int_{\mathbb{S}^2} f(\widehat{\omega}) \mathrm{d}\widehat{\omega} \rightarrow \frac{f(W)}{p(W)}$

Rendering & Measurement equation

$$\hat{I} = \frac{W_{e,10} L_{e,10}}{p(W_{01})}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} L_{e,21} \, \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} L_{e,32} \, \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \, \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

Monte Carlo! $\displaystyle \int_{\mathbb{S}^2} f(\widehat{\omega}) \mathrm{d}\widehat{\omega} \rightarrow \frac{f(W)}{p(W)}$

Rendering & Measurement equation

$$\hat{I} = \frac{W_{e,10} L_{e,10}}{p(W_{01})} + \frac{W_{e,10} f_{s,210}^{\perp} L_{e,21}}{p(W_{01}) p(W_{12})}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f_{s,210}^{\perp} \left[\int_{\mathbb{S}^2} f_{s,321}^{\perp} L_{e,32} \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f_{s,210}^{\perp} \left[\int_{\mathbb{S}^2} f_{s,321}^{\perp} \left[\int_{\mathbb{S}^2} f_{s,432}^{\perp} L_{e,43} \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

Monte Carlo! $\int_{\mathbb{S}^2} f(\widehat{\omega}) \mathrm{d}\widehat{\omega} \rightarrow \frac{f(W)}{p(W)}$

Rendering & Measurement equation

$$\hat{I} = \frac{W_{e,10} L_{e,10}}{p(W_{01})}$$

$$+ \frac{W_{e,10} f^{\perp}_{s,210} L_{e,21}}{p(W_{01}) p(W_{12})}$$

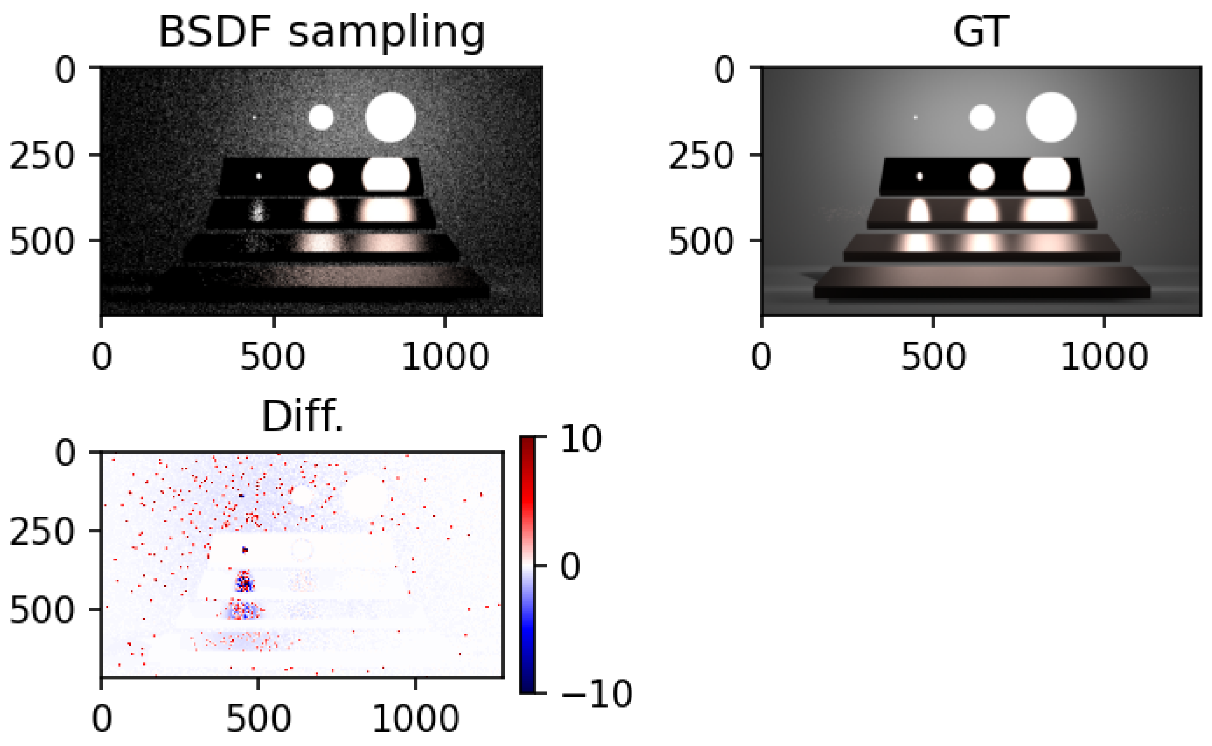
$$+ \frac{W_{e,10} f^{\perp}_{s,210} f^{\perp}_{s,321} L_{e,32}}{p(W_{01}) p(W_{12}) p(W_{23})}$$

$$+ \int_{\mathbb{S}^2} W_{e,10} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,210} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,321} \left[\int_{\mathbb{S}^2} f^{\perp}_{s,432} L_{e,43} \, \mathrm{d}\widehat{\omega}_{34} \right] \mathrm{d}\widehat{\omega}_{23} \right] \mathrm{d}\widehat{\omega}_{12} \right] \mathrm{d}\widehat{\omega}_{01}$$

Monte Carlo! $\int_{\mathbb{S}^2} f(\widehat{\omega}) \mathrm{d}\widehat{\omega} \quad \to \quad \frac{f(W)}{p(W)}$

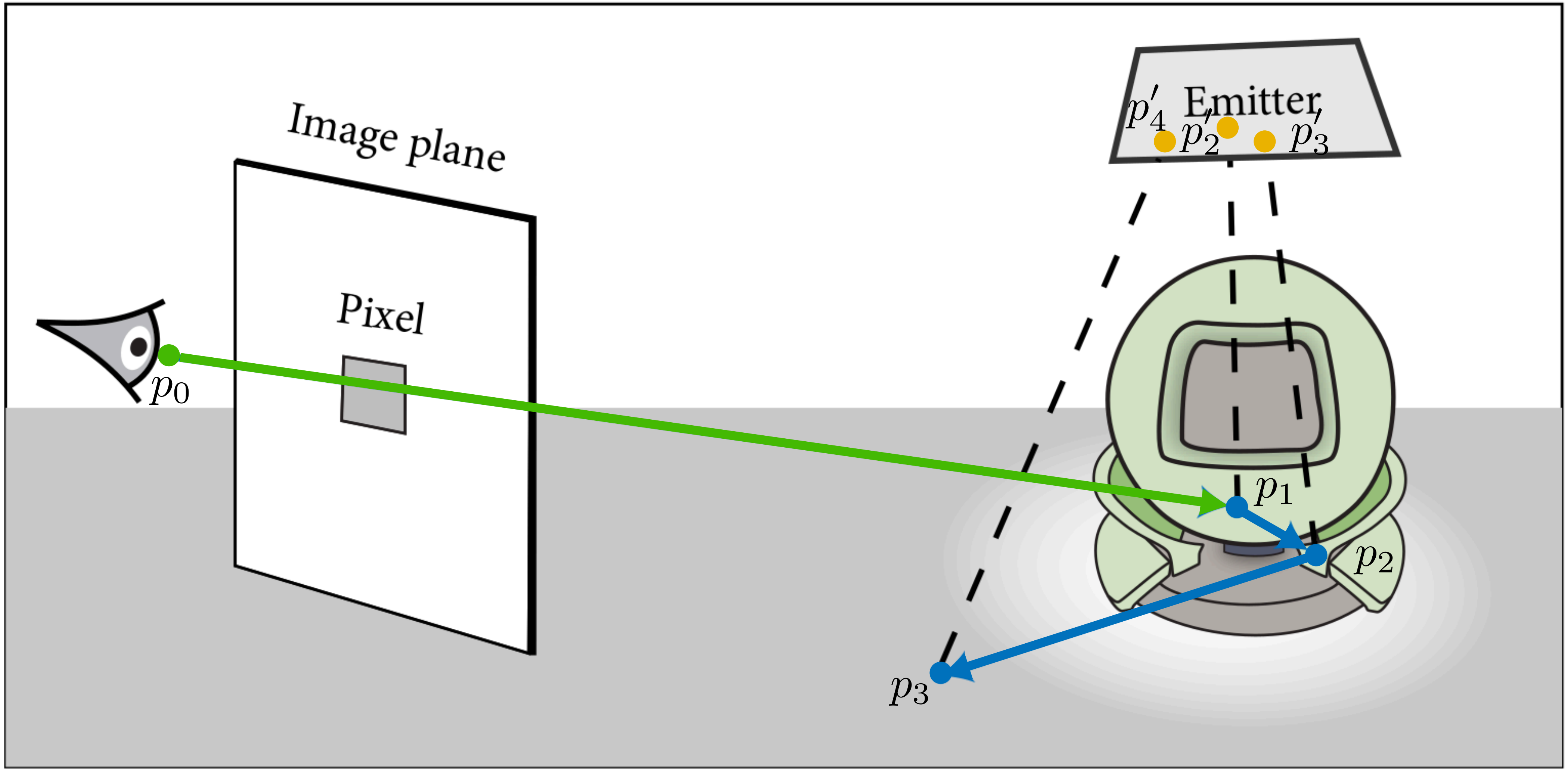
BSDF Sampling

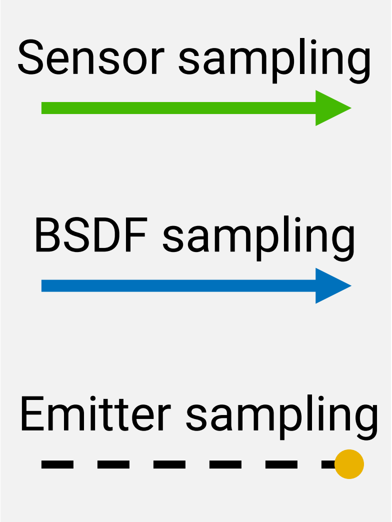
$$\hat{I} = \frac{W_{e,10} L_{e,10}}{p(W_{01})} + \frac{W_{e,10} f^{\perp}_{s,210} L_{e,21}}{p(W_{01}) p(W_{12})} + \frac{W_{e,10} f^{\perp}_{s,210} f^{\perp}_{s,321} L_{e,32}}{p(W_{01}) p(W_{12}) p(W_{23})} + \frac{W_{e,10} f^{\perp}_{s,210} f^{\perp}_{s,321} f^{\perp}_{s,432} L_{e,43}}{p(W_{01}) p(W_{12}) p(W_{23}) p(W_{34})}$$



Monte Carlo! $\int_{\mathbb{S}^2} f(\hat{\omega}) \mathrm{d}\hat{\omega} \rightarrow \frac{f(W)}{p(W)}$

Emitter Sampling





[https://mitsuba.readthedocs.io/en/latest/_images/integrator_path_figure.png]

Rendering & Measurement equation

$$\hat{I} = \frac{W_{e,10} L_{e,10}}{p(W_{01})} + \frac{W_{e,10} f_{s,210}^{\perp} L_{e,21}}{p(W_{01}) p(W_{12})} + \frac{W_{e,10} f_{s,210}^{\perp} f_{s,321}^{\perp} L_{e,32}}{p(W_{01}) p(W_{12}) p(W_{23})} + \frac{W_{e,10} f_{s,210}^{\perp} f_{s,321}^{\perp} f_{s,432}^{\perp} L_{e,43}}{p(W_{01}) p(W_{12}) p(W_{23}) p(W_{34})}$$

Rendering Equation

$$\hat{L} = L_{e,10} + \frac{f_{s,210}^{\perp} L_{e,21}}{p_B\left(W_{12,B}\right)} + \frac{f_{s,210}^{\perp} f_{s,321}^{\perp} L_{e,32}}{p_B\left(W_{12,B}\right) p_B\left(W_{23,B}\right)} + \frac{f_{s,210}^{\perp} f_{s,321}^{\perp} f_{s,432}^{\perp} L_{e,43}}{p_B\left(W_{12,B}\right) p_B\left(W_{23,B}\right) p_B\left(W_{34,B}\right)}$$

BSDF sampling

Emitter sampling

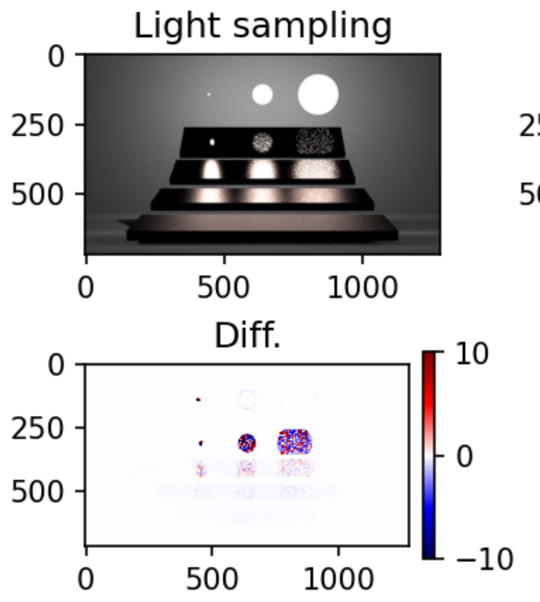
Rendering Equation with Emitter Sampling

$$\hat{L} = L_{e,10} + \frac{f^{\perp}_{s,210} L_{e,21}}{p_B\left(W_{12,B}\right)}$$

$$+ \frac{f^{\perp}_{s,210} L_{e,21}}{p_E\left(W_{12,E}\right)} + \frac{f^{\perp}_{s,210} f^{\perp}_{s,321} L_{e,32}}{p_B\left(W_{12,B}\right) p_E\left(W_{23,E}\right)} + \frac{f^{\perp}_{s,210} f^{\perp}_{s,321} f^{\perp}_{s,432} L_{e,43}}{p_B\left(W_{12,B}\right) p_B\left(W_{23,B}\right) p_E\left(W_{34,E}\right)}$$

BSDF sampling

Emitter sampling



Which Sampling Strategy?

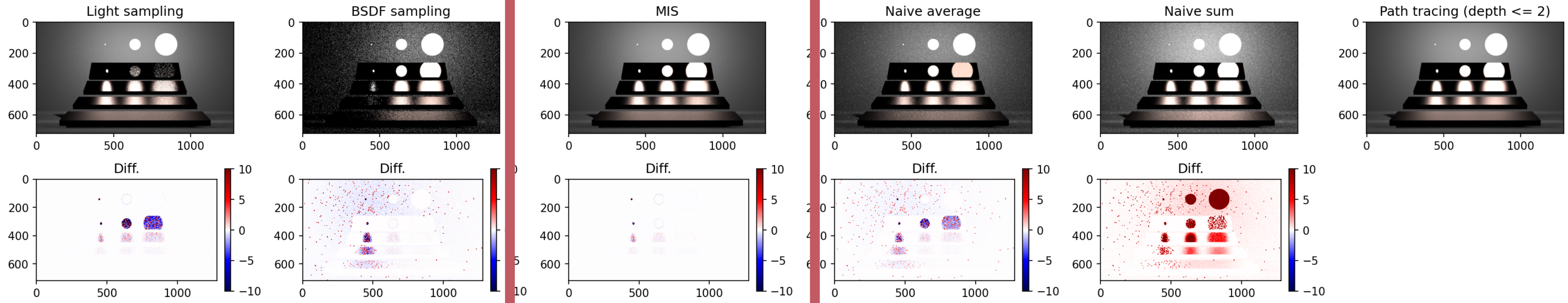
$$\hat{L} = L_{e,10} + \frac{f^{\perp}_{s,210} L_{e,21}}{p_B\left(W_{12,B}\right)} \quad \text{vs.} \quad + \frac{f^{\perp}_{s,210} L_{e,21}}{p_E\left(W_{12,E}\right)}$$

BSDF sampling

Emitter sampling

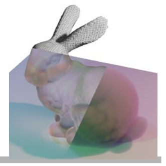
Which Sampling Strategy?

$$\frac{f^{\perp}_{s,210} L_{e,21}}{p_E(W_{12,E})} \qquad \frac{f^{\perp}_{s,210} L_{e,21}}{p_B(W_{12,B})} \qquad \frac{f^{\perp}_{s,210} L_{e,21}}{2p_E(W_{12,E})} + \frac{f^{\perp}_{s,210} L_{e,21}}{2p_B(W_{12,B})} \qquad \frac{f^{\perp}_{s,210} L_{e,21}}{p_E(W_{12,E})} + \frac{f^{\perp}_{s,210} L_{e,21}}{p_B(W_{12,B})}$$



Multiple Importance Sampling (MIS)

MIS for BSDF and Emitter Sampling Methods



$$\hat{L} = L_{e,10} + w_B(W_{12,B})\frac{f^{\perp}_{s,210}L_{e,21}}{p_B(W_{12,B})} + w_E(W_{12,E})\frac{f^{\perp}_{s,210}L_{e,21}}{p_E(W_{12,E})}$$
$$+ w_B(W_{23,B})\frac{f^{\perp}_{s,210}f^{\perp}_{s,321}L_{e,32}}{p_B(W_{12,B})p_B(W_{23,B})} + w_E(W_{23,E})\frac{f^{\perp}_{s,210}f^{\perp}_{s,321}L_{e,32}}{p_B(W_{12,B})p_E(W_{23,E})}$$
$$+ w_B(W_{34,B})\frac{f^{\perp}_{s,210}f^{\perp}_{s,321}f^{\perp}_{s,432}L_{e,43}}{p_B(W_{12,B})p_B(W_{23,B})p_B(W_{34,B})} + w_E(W_{34,E})\frac{f^{\perp}_{s,210}f^{\perp}_{s,321}f^{\perp}_{s,432}L_{e,43}}{p_B(W_{12,B})p_B(W_{23,B})p_E(W_{34,E})}$$

BSDF sampling

Emitter sampling

MIS weights

(See the full version slide from our repo)

MIS for BSDF and Emitter Sampling Methods

$$\hat{L} = L_{e,10}$$

$$+ w_B(W_{12,B}) \frac{f^{\perp}_{s,210} L_{e,21}}{p_B(W_{12,B})} + w_E(W_{12,E}) \frac{f^{\perp}_{s,210} L_{e,21}}{p_E(W_{12,E})}$$

$$+ \frac{f^{\perp}_{s,210}}{p_B(W_{12,B})} \left[w_B(W_{23,B}) \frac{f^{\perp}_{s,321} L_{e,32}}{p_B(W_{23,B})} + w_E(W_{23,E}) \frac{f^{\perp}_{s,321} L_{e,32}}{p_E(W_{23,E})} \right]$$

$$+ \frac{f^{\perp}_{s,210} f^{\perp}_{s,321}}{p_B(W_{12,B}) p_B(W_{23,B})} \left[w_B(W_{34,B}) \frac{f^{\perp}_{s,432} L_{e,43}}{p_B(W_{34,B})} + w_E(W_{34,E}) \frac{f^{\perp}_{s,432} L_{e,43}}{p_E(W_{34,E})} \right]$$

throughput variable β

BSDF sampling

Emitter sampling

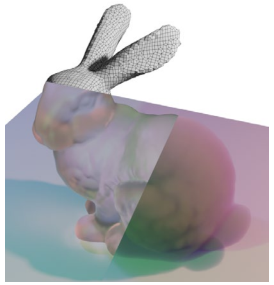
Russian roulette

finite computation infinite sum???

$$\sum_{i=1}^{\infty} a_i$$

$$A_i, \mathbb{E}[A_i] = a_i$$

$$var \leftarrow var + \begin{cases} & \dfrac{A_i}{p(\quad)} \\ 0, & \end{cases}$$



Thank you

https://syyi.graphics

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https://github.com/shinyoung-yi/lecture-rendering-mitsuba

Feel free to contact to me!