F.E. SEM-II

ENGINEERING PHYSICS

INTERFERENCE IN THIN FILMS.

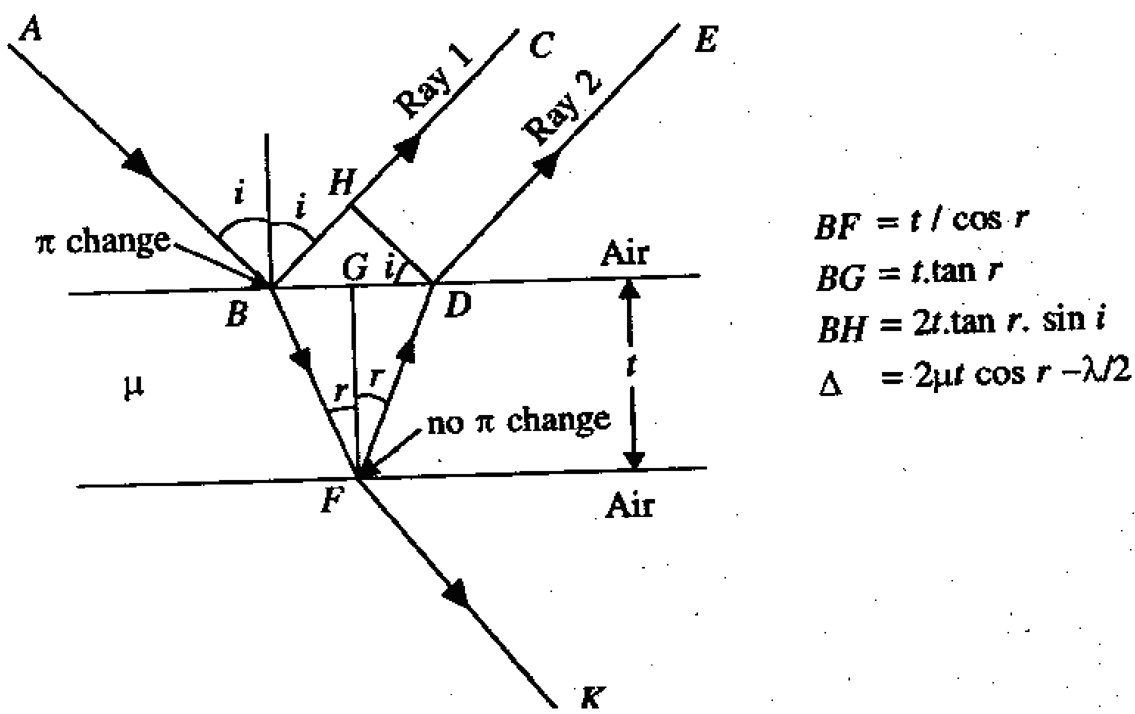
Q.1] Derive the expression for brightness and darkness for a monochromatic light beam reflected from a thin parallel film of transparent material. (98,2002,2000)
Ans.
Thin film: A film is called thin when its thickness is of the order of one wavelength of visible light $(\lambda = 55000A^0)$ thickness is about $10\mu m$ is a thick film. e.g .Air film enclosed between two transparent sheets.

Need of thin film in interference: In thin films two bounding surfaces strongly transmit light and weakly reflect it.

Two Methods of producing interference:

1] Division of wave front: As in case of Young's double slit experiment already studied at lower standards.
2] Division of Amplitude: That occurs in thin film due to successive reflection from upper & lower surface. These incident rays travel different paths and if reunited produce interference.

Plane parallel thin film:
Consider a transparent film of uniform thickness t bounded by two plane (l,m) surfaces. Let μ be the refractive index of material of the film. Let a broad monochromatic source illuminate film at an angle 'i' with a normal (N) to one of the surfaces.



Ray 1 travels paths ABF
Ray 2 travels ABCDE
$\therefore$ Path difference = (BC+DC) – BH

Path difference between the two Rays be denoted by Δ

In $\Delta BGC \cong \Delta DGC$

So $\angle GBC \cong \angle GDC \Rightarrow BC = DC$

$$\therefore \Delta = 2BC\text{-}BH \text{-----------} \quad (1)$$

Now, $\cos r = \frac{GC}{BC} = \frac{t}{BC} \Rightarrow BC = \frac{t}{\cos r}$ ---------- (a)

Now, BH= 2BG from { $\Delta BGC \cong \Delta DGC$}

BG= $t \tan r \Rightarrow BD = 2t \tan r$ ------(a)

Now, BH= BD sin $i \Rightarrow$ BH = 2t $\tan r \sin i$ ------ (b)

Putting (a) & (b) in (1) we get

Geometrical path difference

$$\Delta = \frac{2t}{\cos r} - 2t \tan r \sin i$$

So the optical path difference is

$$\Delta_{opd} = \mu(\frac{2t}{\cos r}) - (2 \tan r \sin i) \text{-----------} (2)$$

as BC is travelled in medium with R.I [Refractive index] μ and BH is traveled in air as we assume medium outside film to be air with R.I. = 1.

According to Snell's law of refraction $\mu = \frac{\sin i}{\sin r}$

Putting in (2) we have

We know that the phase change of Π radian is equivalent to path difference of $\lambda/2$.

Also at the boundary of denser medium phase change of Π occurs at BH so the total optical path difference is Δt i.e.

$$\Delta t = \Delta a - \lambda/2.$$

$$\Delta t = 2\mu t \cos r - \lambda/2.$$

Condition for constructive interference:

If the total path difference is equal to integral number of full waves then Ray (1) & (2) assumed to meet in phase and undergo constructive interference.
The condition for constructive interference is
$\Delta = m\lambda$

$$\therefore 2\mu t \cos r - \lambda / 2 = m\lambda$$

$$\therefore 2\mu t \cos r = (2m+1)\lambda / 2 \qquad \text{Brightness } m= 0,1,2,3 \ldots\ldots$$

Condition for destructive interference :

If the total path difference is equal to integral number of half waves , then Ray (1) & (2) meet in opposite phase and undergo destructive interference . The condition for destructive interference is

$\Delta = (2m+1)\frac{\lambda}{2}$ (odd $\div 2$ gives us ½ multiple of λ)

$2\mu t \cos r - \lambda/2 = (1+2m)\lambda/2$

$2\mu t \cos r = (m+1)\lambda$

We can write m=m+1 as addition of 1 full wave does not change the phase relation of the two interfering waves.

$\therefore 2\mu + \cos r = m\lambda \rightarrow$ darkness m=0,1,2,3,….

m is called order of interference.

Interfering waves are parallel to each other so interference occurs at infinity. So we say interference fringes are localizing at infinity.

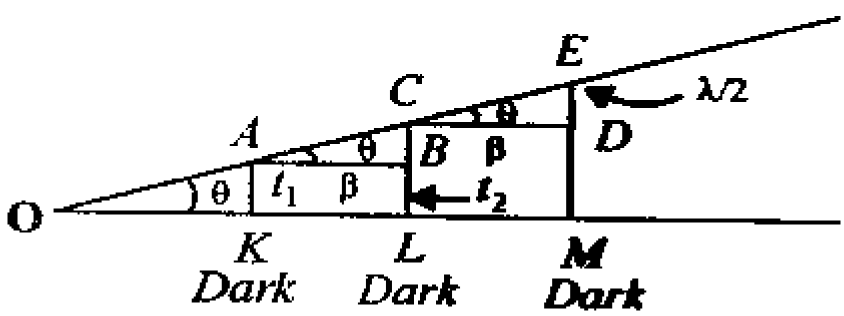
Q.2] Obtain an expression for fringe width in wedge shaped thin film, How it is used to test optically flat surface? Enlist silent features of such interference ?

Ans.

Wedge shaped thin film: A thin film of varying thickness is called wedge shaped film. The two surfaces of this film are inclined to each other at an angle. It has zero thickness at one end progressively increasing to a particular thickness at the other end.

e.g. wedge air film is formed between two micro sides resting on each other separated by a spacer.

Consider a wedge film of very small angle θ of a material of refractive index μ [say] When such a wedge is illuminated by a monochromatic source the reflected rays from two surfaces are not parallel they appear to diverge . But due to path difference between reflected rays from two surfaces interference pattern of bright and dark bands is obtained on the surface of the film.



Light incident is partially reflected and partially transmitted. The transmitted light is reflected back from bottom and passes out after refraction to interfere with reflected ray (BC) (1) &(2) ray are both derived from AB hence are coherent and on overlapping produce interference.

Expression for fringe width:

Optical path difference = $2\mu t \cos r - \lambda/2$ --------- (1)

as in case of thin film (symbols with usual meaning)

Condition for darkness $2\mu t \cos r = m\lambda$ ----------- (2)

Say if light is normally incident then $r = 0 \therefore \cos r = 1$ at thickness t_1 we have $2\mu t_1 = m\lambda$---- (3) from (2)

Next dark fringe occurs at C there thickness say $2\mu t_1 = (m+1)\lambda$------ (4) from (2)

Subtracting (3) from (4) we have $2\mu(t_2 - t_1) = \lambda = 2\mu(BC)$

In ΔABC we see AB=$\beta = \frac{BC}{\tan\theta} = \frac{\lambda}{2\mu\tan\theta}$

θ is taken small as assumed $\therefore \tan\theta \div \theta$

$\therefore \beta = \dfrac{\lambda}{2\mu\theta}$ this is expression for fringe width.

Salient features of such interference:

1] Fringes obtained are equidistant
2] Fringes at apex (zero thickness is dark)
3] Fringes are straight and parallel
4] Fringes are of equal thickness
5] Fringes are localized
6] Wedge (θ)angle can be determined if β is known
7] We can also find spacer thickness (t)

$\theta = \dfrac{N}{(X_2 - x_1)} \times \dfrac{\lambda}{2\mu}$ N= no of fringes between X_2 to X_1 $t = \dfrac{NL}{(X \ - X_1)} \times \dfrac{\lambda}{2\mu}$

$\therefore$ L= length of air wedge

Q.3] Explain the formation of Newton's Rings. Why are circular fringes o obtained in Newton's Rings . Why these fringes are called fringes of equal thickness? Is the Central spot dark or bright?

Ans:

Newton's rings:

the phenomenon of Newton's rings, named after Isaac Newton, is an interface pattern caused by the reflection of light between two surfaces-a spherical surface and an adjacent flat surface. The resulting interface known as Newton's Rings. For e.g.: Fringes observed when light is reflected from a Plano-convex lens of a long local length placed in contact with a place glass plate.

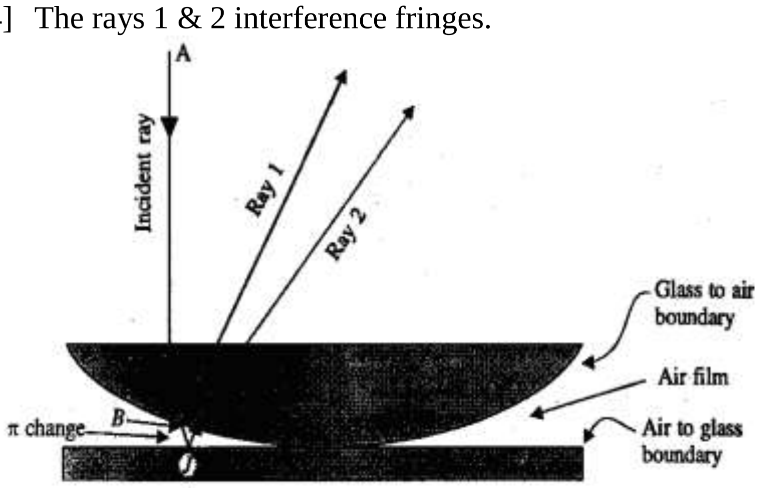
Conditions for formation Newton's rings:

1] Plano- convex lens must be of long focal length
2] Interference must be produced by the method of division of amplitude.

Formation of Newton's rings:

Newton's rings are formed as a result of interference between the waves reflected from the top & bottom surfaces of the air film formed between the lens & the plate

1] Let AB be a beam of monochromatic light of wavelength λ incident on the system.
2] This beam of light gets partially reflected at the bottom curved surface of the lens (ray 1) & part of the transmitted ray is partially reflected(ray 2) from the top surface of plane glass sheet.
3] The rays 1 & 2 are derived from the same incident ray by division of amplitude & thus are called coherent beams.
4] The rays 1 & 2 interference fringes.



5] Ray 2 undergoes a phase change of π i.e. has a path difference of $\dfrac{\lambda}{2}$ upon reflection since it is reflected from air to glass boundary.
6] When viewed with a monochromatic light it appears as a series of concentric, alternating light & dark rings centered at the point of contact between the two surfaces.
7] When viewed with white light, it forms a concentric ring pattern of rainbow colors because the different wavelengths of light interfere at different thickness of air layer between the surfaces.

8] The light rings are caused by constructive interference between the light rays from both surfaces, while the dark rings are caused by destructive interference.

Reason for obtaining circular fringes:

The circular ring wedge of air film between the two glass surfaces can be regarded as formed by an air- wedge rotated around an axis passing through the point of contact. Since all the fringes are of equal thickness there locus will fall on a circle having its centre at the point of contact. Thus the thickness of the film is same at all points on any circle having 'O'as the centre. The fringes are thus circular.

fringes are of equal thickness:

The equation for Bright fringe is given by

$$2\mu t\cos r = (2n-1)\lambda/2$$

where,n = n= 1, 2, 3 ---------

The equation for dark fringes is given by

$$2\mu t\cos r = n\lambda$$

Where,m=0,1,2,3---------

If incident light is normal, μ = O $\therefore \cos\mu$ =1 and for air film $\mu = 1$

Thus

$2t = (Zm + 1)\ \lambda/2$ ......Bright

$2t = m\lambda$ ......Dark

Hence, from the above equations it can be seen that the occurrence of alternate maxima & minima is due to the variation in the thickness $\pm$ of the film. Each maximum or minimum is locus of constant film thickness. Hence the fringes are called fringes of equal thickness.

Dark central spot:

Central spot is dark. At the point of contact of the lens with the glass plate, the thickness of The air film is only a few molecules thick & is very small compared to a wavelength. Due to reflection a path difference of $\lambda/2$ is introduced for the wave reflected from the glass plate. Consequently, the interfering waves at the centre are opposite in phase & interfere destructively. Thus in the area of contact of lens & plate, a dark spot is produced.

Q.4] Show that radius of Nth dark ring is proportional to the square root of natural number. TO PROVE: $r_n \alpha \sqrt{n}$

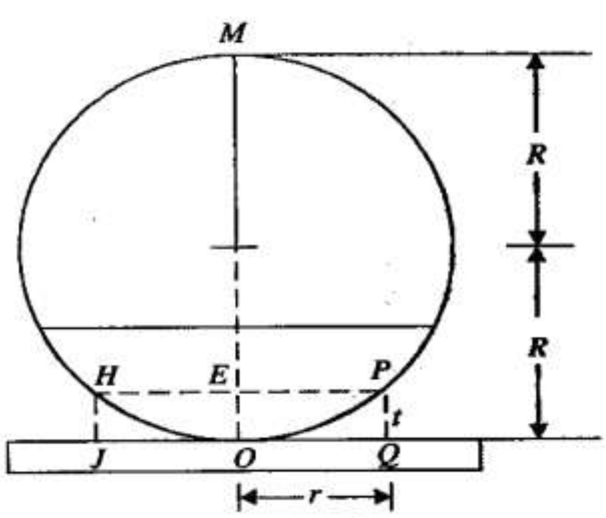


Fig. 7.15. Determination of the relationship between the radius of the dark ring and air film thickness.

ASSUMPTION:

Let,R be the radius of curvature of the lens $\pm$ be the thickness of air film at point Q HOP be a Plano-convex lens placed on glass plate JQ ,R be the radius of curvature of the lens surface in contact with the plate.

PROOF:

The radius of n[th] ring at Q= OQ = r_n

In $\Delta\ O^1AB$,

$O^1A^2 + AB^2 = O^1P$ {Pythagorus theorem}

$$(R - t)^2 + r^2 = R^2$$

$$r^2 = 2Rt - t^2$$

As 2 Rt >> t^2, t^2 can be neglected,

$$\therefore r^2 = 2Rt$$

For n^{th} ring, radius will be

$$r_n^{\ 2} = 2Rt \qquad \ldots\ldots\ldots \qquad (1)$$

Condition of dark ring is given by:

$$2t = n\lambda \qquad \ldots\ldots.. \qquad (2)$$

When light ray is at normal incidence i.e. cos $\pi = 1$ and for air film $\mu = 1$.

e.q. (i.e) can be rewritten as,

$$\frac{r_n^{\ 2}}{R} = 2t \qquad \ldots\ldots. \qquad (3)$$

Comparing equations (2) & (3)

$$\frac{r_n^{\ 2}}{R} = n\lambda \ ; \ r_n^{\ 2} = Rn\lambda \ ; \therefore r_n = \sqrt{Rn\lambda} \qquad \ldots\ldots \qquad (4)$$

$\therefore r_n \propto \sqrt{n}$, when R and λ are constant

Hence, the radius of the dark rings are proportional to the square root of the natural numbers.

Q.5] Show that radius of n^{th} dark ring is proportional to square root of the wavelength.
Ans.
From eq (4) ,

$$r_n = \sqrt{Rn\lambda}$$

When, R and n are constant,

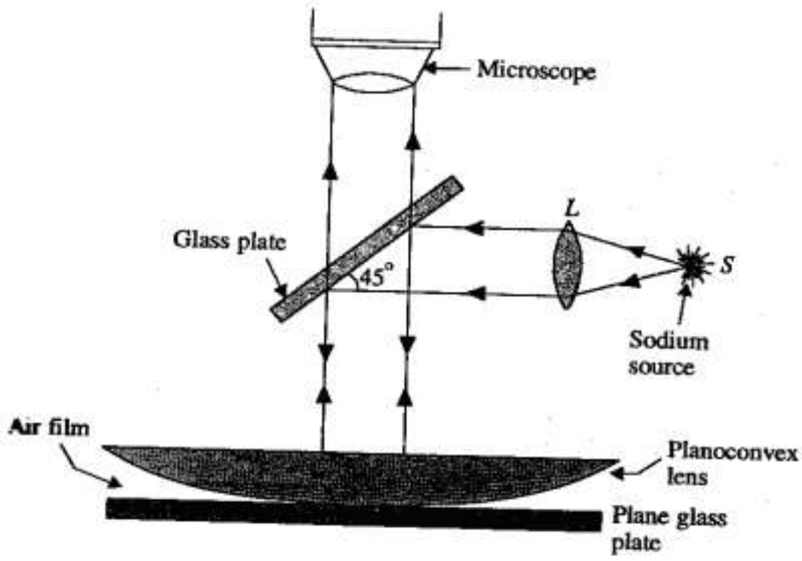
$$\therefore r_n \propto \sqrt{\lambda}$$

Hence the radius of nth dark ring is proportional to square root of the wavelength.

Q.6] Describe the experimental set up for Newton's Rings formation and draw a neat diagram for the same. How is the ring diameter and film thickness related? Why the rings are not evenly spaced?
Ans.
Apparatus:
1] Plano-convex lens of large focal length.
2] Plane – glass plate.
3] Monochromatic source of light e.g. sodium lamp.
4] Glass plate held suitably at angle 45^0 with the vertical.
5] Condensing lens
6] Microscope.

Experimental arrangement:
1] A carefully cleaned convex surface of a Plano-convex lens L of large radius of curvature is placed on a plane glass plate P.
2] A monochromatic source of light such as sodium lamp is used as light source.
3] A glass plate G is held at a suitable distance above at an angle 45^0 with the vertical to make the normal incidence of the light coming from source on the film.
4] A condensing lens is placed in front of the light source so that maximum light from source falls on G.
5] Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.

6] A low power microscope is focused on the air film to see the interference pattern obtained.



Relationship between ring diameter and film thickness:

Relationship between the ring radius and film thickness is given by:

$$r^2 = 2Rt$$

$$\therefore r = \sqrt{2Rt}$$

Multiplying both sides with 2, we get:

$$2r = 2\sqrt{2Rt}$$

$$\therefore D = 2\sqrt{2Rt} \qquad \therefore 2r = D$$

Newton's rings are not evenly spaced:

There are two reasons that rings are not evenly spaced. They are as follows.

1) The diameters of the dark rings are proportional to the square root of the natural numbers. But the diameters of the bright rings are proportional to the square root of odd natural numbers.

   $D_n \propto \sqrt{n}$ …… for dark rings

   $D_n \propto \sqrt{2n-1}$ …… for bright rings

   Where, n=1, 2, 3…….

   As the order of the ring increases i.e. n increases the diameter does not increase in the same proportion as a result the rings get closer and closer at an increasing rate.
2) The angle of air wedge increases as one move away from the centre.
   The fringe separation decreases with increasing wedge angle. Therefore, the rings draw closer together with the increase in their radii.

Q.7] What are the applications of interference?
Ans.
The following are the applications of interference:

1] Testing the optical flatness of surfaces:
The phenomenon of interference is used in testing the planners of the surfaces. To test the optical flatness of a surface, the specimen surface to be tested (OB) is placed over an optically plane surface(OA). If the fringes obtained are of equal thickness the surface OB is plane. If not than surface OB is not plane. The surface OB is polished and the process is repeated. When fringes observed are of equal thickness then surface OB is plane.

2] Production of non reflecting coating:
Production of non reflecting coatings on the surfaces of camera lenses and optical instruments with the help of interference minimizes the loss of light due to reflection and thereby increases the amount of transmitted light.

3] Measurement of wavelength of light by Newton's Rings:
Wavelength of light can be calculated by using following relation

$$r_m = \sqrt{m\lambda R}\ ;\ r_m^{\ 2} = m\lambda R\ ;\ \lambda = \frac{r_m^{\ 2}}{mR} \text{ where, m=0,1,2,3….}$$

r_m = radius of the mth dark ring
R= Radius of Curvature
λ = wavelength of light

By measuring the radius of dark rings and radius of curvature R, the wavelength λ can be calculated for different values of m.The radius of rings is measured with the traveling microscope and the radius of curvature can be determined by using spherometer. Thus interference is used for making precision measurements.

4] Determination of refractive index of liquid by Newton's Rings:
Refractive index and diameter of (n+p) the ring and nth ring is given by the relation:

$$\mu = \frac{\left[D^2{}_{n+p} - D^2{}_n\right]}{\left[D^2{}_{n+p} - D^2{}_n\right]}$$

To find μ, Newton's rings are formed with air film. The diameter of nth and (n+p)th dark rings are measured for air film. Then the transparent liquid is introduced between lens and plate. The diameters of nth and (n + p) th dark rings are measured with liquid film.

5] Determination of Young's Modulus:
Interference method is used for measuring small displacements such as those produced by compression or elongation of a metal rod, crystals etc. From these measurements the Young's Modulus and Poisson's ratio can be accurately determined.

6] Astronomical applications :
Interference phenomenon is used in astronomy where double slit interference method is used to determine the angular separation of doubles stars and the diameter of fixed stars.

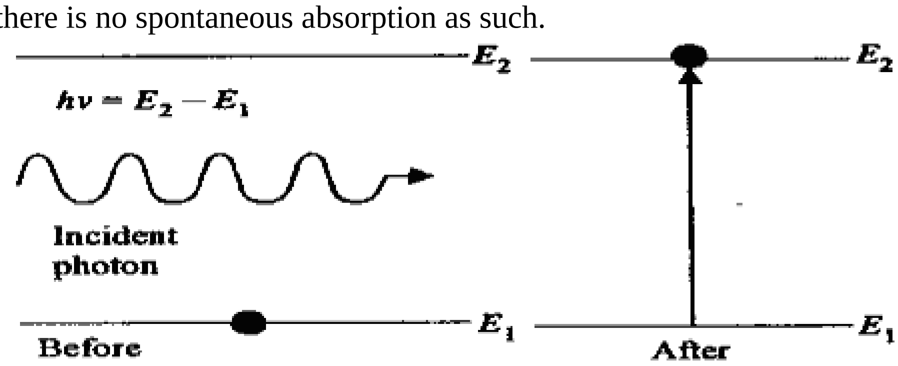
LASERS

Q.1] How does radiation interact with matter , name and explain the interaction processes.
Ans.
Interaction of Radiation with Matter

Radiation consists of photons and matter consists of atoms and molecules. Obviously, interaction of radiation with matter merely means interaction of photons with atoms and molecules. The interaction can be as simple as light illuminated on a material or light emitted from a material. There are three processes, which coexist and constantly occur at all temperatures whenever there is an interaction between radiation and matter. Their proportion may change depending upon the material and its temperature. They are Absorption, spontaneous and stimulated emission.

Absorption

Consider an atom residing in the lower energy state E_1 and a radiation of certain frequency is incident. The atom may absorb the incident energy (if sufficient) and jump to the next energy state E_2 . This process is called as induced or stimulated absorption because we have to externally supply energy. The atoms will not automatically seek for photons and go into the excited states. This is against their natural tendency. This in reflected in the fact that there is no spontaneous absorption as such.

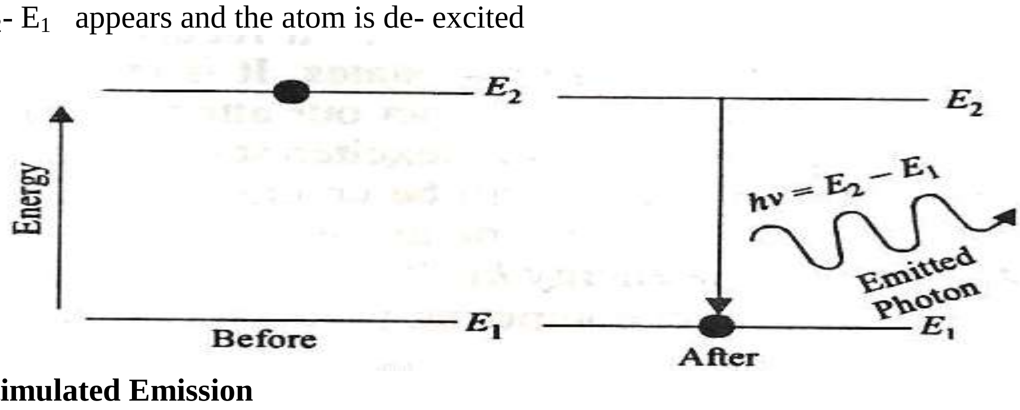


Corresponding to each transition from E_1 to E_2 , one photon of energy $h\nu = E_2 - E_1$ disappears and the atom is excited.

Spontaneous Emission

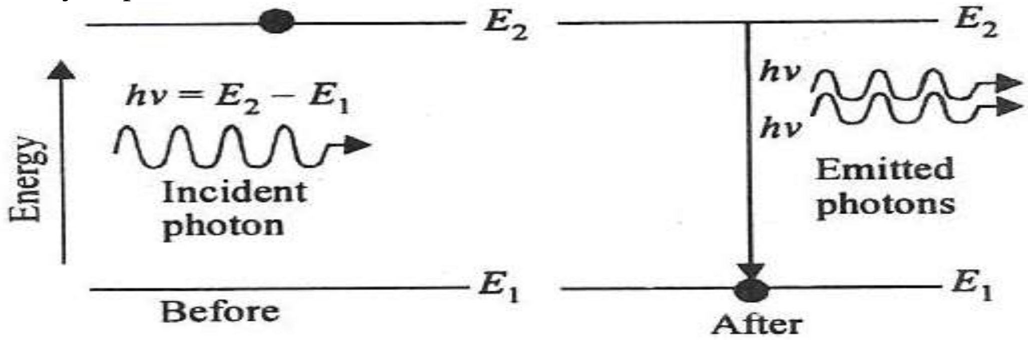
An atom raised to its excited state does not stay there for a longer time because the exited state is inherently unstable. Excited atoms rapidly de-excite due to the natural tendency of atoms to seek for the stable configuration.

This type of process in which, atom on its own releases a photon is called as spontaneous emission.Corresponding to each transition E_2 to E_1, one photon of energy $h\nu = E_2 - E_1$ appears and the atom is de- excited



Stimulated Emission

This type of process also exists along with the above two processes but at room temperature, spontaneous emission dominates stimulated emission and the latter is generally negligible. The idea* behind stimulated emission is that an atom need not wait in the excited state for some spontaneous emission event to occur. That is the process of emission need not be probabilistic but we can make some transition to occur. A photon of energyν hν= E_2- E_1 or greater can induce the excited atom to de-excite by imparting some momentum to release energy in the form of another photon. The released photon is identical to the incident photon in every respect. This is called stimulated emission.



Corresponding to every induced transition E_2 to E_1 due to a single photon, two indentical photons of energy hν= E_2- E_1 appear.

Q.2] Explain the terms metastable state , population inversion , Pumping and active medium .
Ans.
Metastable state

A state where excited atom can remain for longer time then normal excited state is called metastable state.The lifetime of an atom in metastable state is 10^{-6} to 10^{-3} sec. Metastable state allows accumulation of a large no. of excited atoms in that level. The metastable state population can exceed population at a lower level and establish the condition of population inversion in lasing medium.

Population invertion

When the number of atoms present in excited state are more as compared to the ground state. The medium is said to have gone into the state of population inversion. In the state of thermal equilibrium there are more atoms in lower level than in upper level.There must be more atoms in the upper level than in lower level in order to achieve stimulated emission exclusively. Therefore, a non-equilibrium state is produced in which population of upper excited level exceeds to a large extent than population of lower energy level..

Pumping:
For realising and maintaining the condition of population inversion , the atoms have to be raised continuously to the excited state . It requires energy to be supplied to the system . The process of supplying energy to to a medium with a view to transfer it into the state of population inversion is known as pumping.

There are number of techniques for pumping a collection of atoms to an inverted state . they are
1] Optical pumping : In this type of pumping a light source such as flash discharge tube is used . This method is adopted mostly in solid state lasers . eg. Nd-YAG laser.
2] Electrical pumping : IN this type of pumping the electric field causes the ionization of the medium and raises it to the excited state . This technique is used in gas lasers .eg. He Ne laser.
3] Direct conversiton : In this method the electrical energy is directly converted to light energy as in case of semiconductor diode laser .

Active medium.
A medium in which light gets amplified is called an active medium . The said medium may be solid , liquid or gas.Only a small fraction of the total are responsible for stimulated emission and consiquent light amplification . They are called active centres.the remaining bulk plays role of host and supports the active centres.

Q.3] Obtain expression for Einsten's coefficients.
Ans.

Consider a two level laser system . let E_1 be the lower level E_2 be the upper excited state let N_1 and N_2 be the number of atoms in the lower and upper level respectively and let Q be the density of radiation incident on the system which is capable of exciting the atoms to the upper excited state . Let probability of atom to go from E_1 to E_2 be P_{12} and the corresponding probability of atom to go from E_2 to E_1 be $P_{21.}$

The probability of stimulated absorption i.e probability of atom to go from E_1 to E_2 depends on the number of atoms in E_1 [i.e N_1] and also on the density of incident radiation [i.e Q].

$P_{12} \propto N_1 Q$

$P_{12} = B_{12} N_1 Q$----------[1] where B_{12} is proportionality constant

In 1917 Einstein predicted that there are two types of emission possible spontaneous emission and stimulated emisssion

The probability spontaneous emission emission[i.e atom to go from E_2 toE_1], depends on the number of atoms in E_2 [i.e N_2] only as it does not need any incident radiation to occur.

$P_{21sp} \propto N_2$

$P_{21sp}=A_{21} N_2$----------[2] where A_{21} is proportionality constant

The probability stimulated emission emission [i.e atom to go from E_2 to E_1], depends on the number of atoms in E_2 [i.e N_2] and also on the density of incident radiation [i.e. Q].

$P_{12st} \propto N_1 Q$

$P_{12st}= B_{21} N_1 Q$----------[3] where B_{21} is proportionality constant

The total probability of emission is sum of spontaneous and stimulated emission and can be written as follows

$P_{21}= P_{21sp}+ P_{12st}$

Hence $P_{21} = N_2 A_{21} + N_2 B_{21} Q$ ------from[2&3]-----[4]

$P_{21} = N_2 [A_{21} + B_{21} Q]$

During the state of equilibrium

$P_{12} = P_{21}$

$N_2 [A_{21}+B_{21}Q] = B_{12} N_1Q$------From[1&4]

$N_2 A_{21}+ N_2 B_{21} Q = N_1 B_{12} Q$

$N_2 A_{21} = Q [N_1 B_{12}-N_2 B_{21}]$

$$Q = \frac{N_2A_{21}}{N_1B_{12} - N_2B_{21}}$$

$$Q = \frac{N_2A_{21}}{N_2B_{21}\left[\frac{N_1B_{12}}{N_2B_{21}} - 1\right]}$$

$$Q = \frac{A_{21}}{B_{12}\left[\frac{N_1B_{12}}{N_2B_{21}} - 1\right]}$$

From Max well's Bothmann equation we have

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}}$$ ----------------where $E_2 - E_1= h\nu$

$$\therefore Q = \frac{A_{21}}{B_{21}\left[e^{\frac{h\nu}{kT}} x \frac{B_{12}}{B_{21}} - 1\right]} \text{ --------------[5]}$$

From Plancks law we have that $Q = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right]$ --------[6]

Comparing [5] and [6] we have

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \text{ and } \frac{B_{12}}{B_{21}} = 1$$

This is the expression for Einsteins coefficients .

Q.4] Describe the construction and action of Optical Resonator

Ans.

Light can be amplified by an active medium taken into the state of population inversion . But the spontaneous photons are emitted by atoms independently in various directions which produce stimulated emission in different directions . The resultant efffect would be production of incoherent light . To avoid this from happening we for laser production place active medium in optical resonator cavity .

Construction:-

The optical resonator consist of two opposing plane parallel mirrors, with active material placed in between them.One of the mirrors its partially reflecting while the other one is made 100% reflecting.Mirrors are placed normal to the optic axis of the material.Such a structure is also called Fabry –Perot optical resoantor .

Fabry-Perot optical resonator.

ACTION :-

Light amplification and oscillations due to the action of optical resonator occur as following steps

Non Excited state:-

Active centers in active medium are initially in the ground state.

Optical Pumping :-

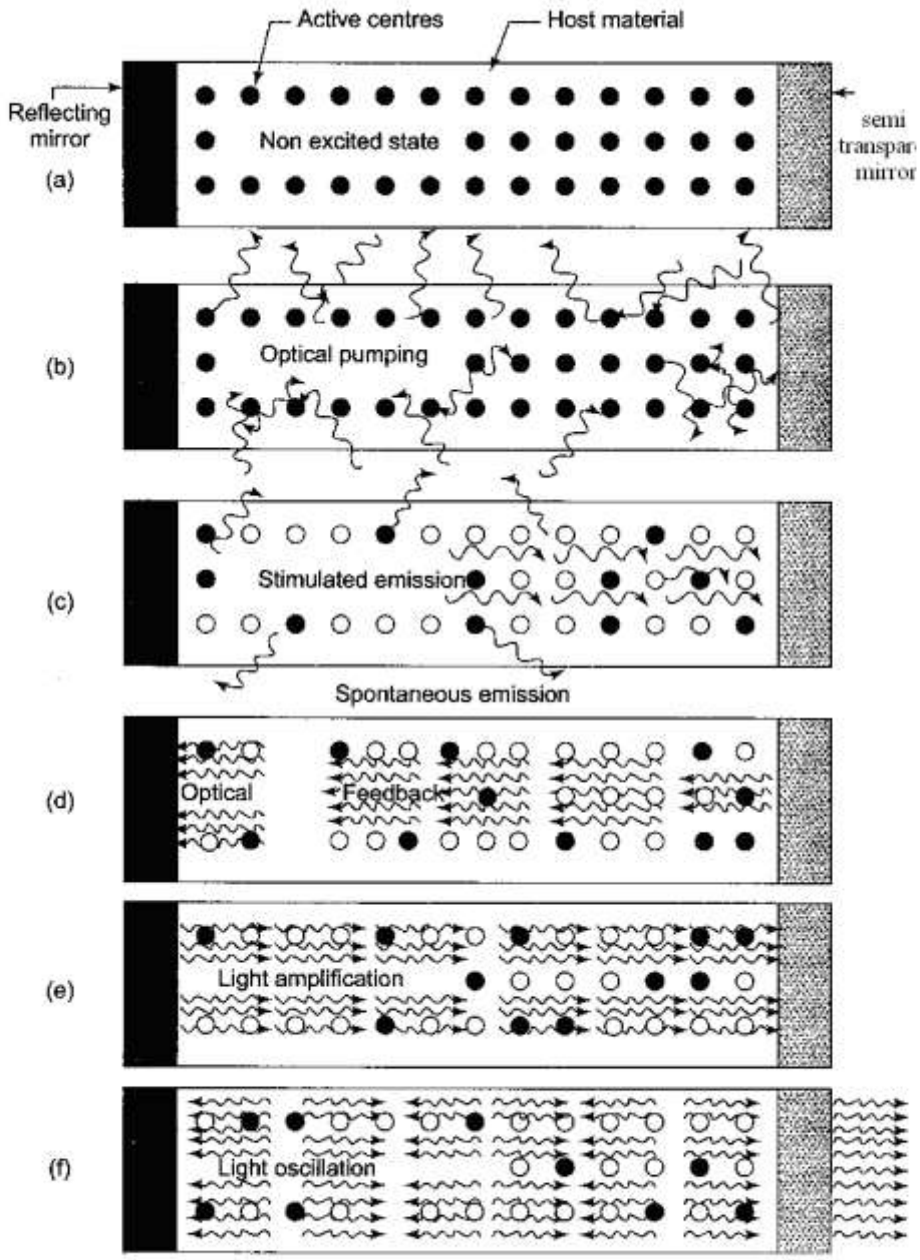
Laser production starts with pumping through suitable pumping mechanism . The material is taken into a state of population invertion.

Spontaneous & stimulated emission :-

Spontaneous photons are emitted in the initial stage in every direction . To generate a light o/p photons with a specific direction are selected while others are rejected. Also for maximum amplification , stimulated photons are made to pass through the medium number of times . Photons emitted in the direction parallel to the optic axis are selected..

Optical feedback :-

While propagating in opposite direction photons de- excite more & more atoms & build up their strength . At 100% Reflecting Mirror, some of the photons are absorbed but a major part is reflected back. this is called optical feedback.



(a) (b) (c) (d) (e) (f)

Light amplification :-

The beam travels along the same path as that of starting photon & undergoes multiple reflections at the mirrors & gains strength .At each reflection at front mirror, the beam is partially transmitted through it & partially fed into the medium.

Light oscillation :-

Begins when the amount of amplified light becomes equal to the amount of light lost through the sides of resonator, through the mirrors & through absorption by the medium . After enough intensity is built up a highly collimated intense beam comes out.

The waves propagating within the cavity should make a standing wave pattern . The optical path length traveled by a wave between consecutive reflection should be integral multiple of wavelength.

$$2L=m\lambda$$

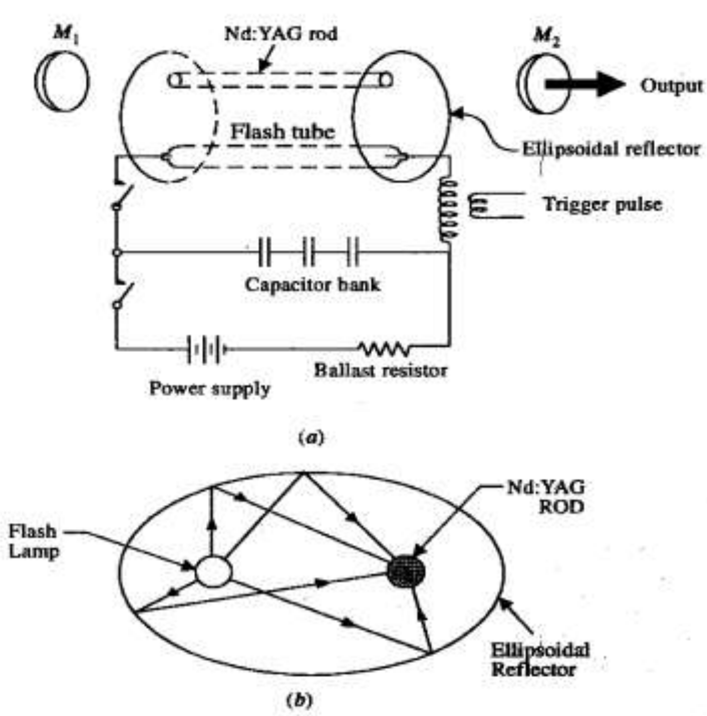
The resonator may support several standing waves with slightly different λ they are called longitudinal modes and m is the mode number.

Q.5]Describe the construction and working of the Nd – YAG laser and state its characteristics .

Ans.

Nd-YAG laser is one of the most popular **Solid** state laser . it is a four level laser

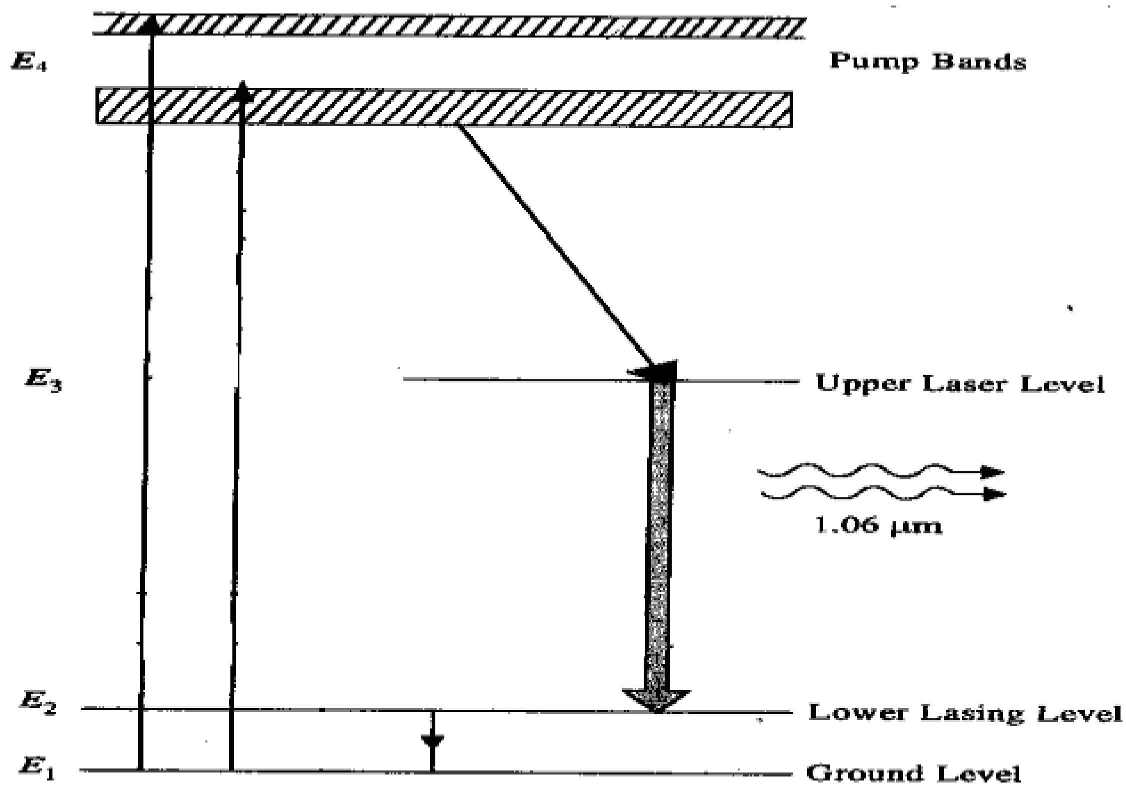
Yttrium aluminum garnet is commonly called YAG($Y_3A_{15}O_{12}$) and Nd represents Neodymium (Nd $^{3+}$ ions are used). Some of Y^{+3} ions are replaced by Nd^{+3}. The crystal atoms of YAG do not take part into lasing action, but serve as horizontal lattice in which Nd^{+3} ions reside.



Construction:-

Fig. shows typical design of the laser .It consists of an elliptically cylindrical reflector having the laser rod along one of its focus lines & a flash lamp along the other focus line. The light from one focus of ellipse passes through the other focus after reflection from the silvered surface of the reflector .The entire flash lamp radiation gets focused on the lasers rod. The two ends of laser rod are polished & silvered & constitute the optical resonator, necessary to produce amplification. The krypton flash lamp is provided with the necessary power supply arrangement.

Energy level diagram :-



Working :-
Energy levels of neodymium ion in YAG crystal are shown in fig .

Non-Exited State :-

E_1 , E_2 & E_3 are the energy level of Nd along with many other levels of YAG. E_1 is the ground state.E_2 is the lower metastable state on lower laser level. E_3 is the upper metastable state on upper laser level.

Optical Pumping :-

The pumping of Nd^{3+} ions to upper states is done by krypton are lamp.Optical pumping with light of wavelength range to 5000 to 8000A^0 exites the ground state Nd^{3+} ions to higher state .

Population Inversion:-

The upper laser level E_3 is rapidly populated, as the excited from the upper energy bands.The lower laser level E_2 is far above the ground level & hence it cannot be population by Nd^{3+} ions through thermal excitation from the ground level.Thus, population invention is readily achieved between E_3 level & E_2 level.

Laser Emission:-

The laser emission occurs in the infrared region at a wavelength of 10,600A^0.As the laser is a four level laser the population inversion can be maintained in the face of continuous laser emission.laser can be obtained in the continuous wave mode .Efficiency of better than 1% is achieved.

Application:-

YAG laser have industrial application such as machining of eration like welding etc. They are also used in surgery.

Q.6]Describe the construction and working of the He- Ne Laser and state its characteristics .

Ans.

He- Ne laser was the first successful gas laser. It was build by Ali Javan , W. Benett & D Herriot.

Principle:-

Gas lasers are employed with a mixture of two gases A & B where atoms of A are initiallyexcitedbyelectronimpact. Atom of A intern transfer their energy to atoms of type B which are actual active centers .Energy transfer is done by atomic collision betwwe A & B where two of their energy levels are equal. This type of energy transfer is known as resonance transfer energy.

$$e_1 + A \rightarrow e_2 + A^*$$

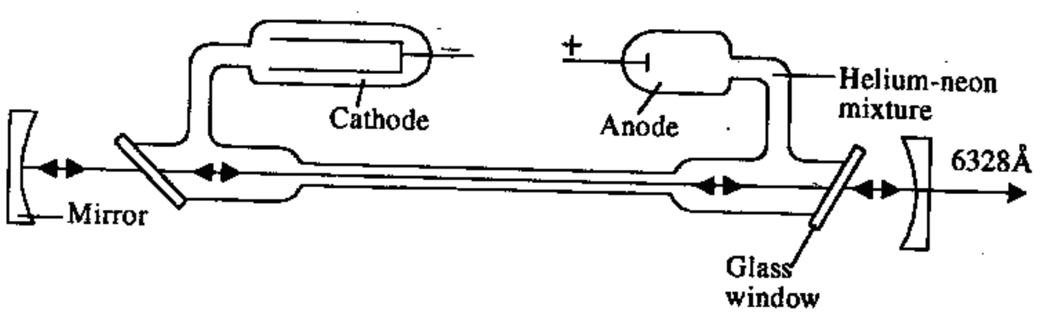
(electron) (non-excitation) (excited atom)

$$A^* + B \rightarrow A + B^*$$

(E) (NE) (NE) (E)

A^* - Energy value of atom A in metastable state. B^* - EV in ES.
A - Energy value of atom A in ground state. B - E.V. in G.S.

Diagram:-



(*a*)

Construction :-

Discharge Tube:-

1] Long & narrow.
2] Length 50 cm & diameter 1 cm.
3] Tube is filled with a mixture of helium & neon gases in the ratio 10:1

Electrodes :-

1] These are to produce a discharge in the gas .
2] They are connected to high voltage supply.

Glass quartz plates:-

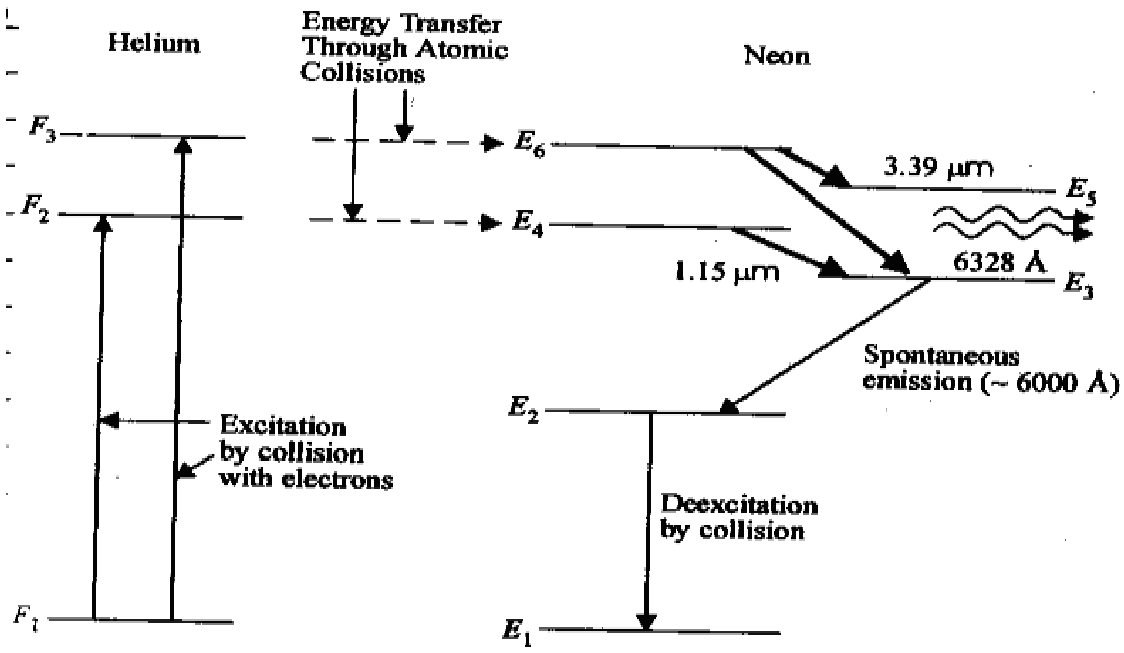
1] These function as Brewster windows.
2] The tube is hermetically sealed by these inclined windows at its two ends.

Mirrors:-

1] On the axis of the tube, two reflectors are fixed which from the resonator.
2] One of the mirror is silvered with 100% reflectivity whereas the other is slightly less so that 1 % of the incident beam could be trapped by transmission.

Working :-

He Ne laser employs a four level pumping scheme .When the power is switched on, the electric field ionizes some of the atoms in the in the mixture of helium – neon gases. Due to electric field the e^- & ions will be accelerated towards anode & cathode . since the e^{-1} has smaller mass they have higher velocity. The helium atoms are more readily excitable than atoms therefore they are lighter.Energy level scheme of atom the energetic e^{-1} excite the atoms through collisions to excited levels F_2 lies at 19.81 ev above the ground state .F_3 lies at 20.61 eV above the ground state.F_2 and F_3 are metastable states.with the passage of current through the discharge tube more He atoms accumulate in the excited states



The E_6 & E_4 levels of neon atoms are metastable states. Therefore, as collision go on, neon atoms accumulate these two states.A state of population inversion is achieved between E_6 & E_3, E_4 & E_3 levels.

Following are the transitions being produced :

1] E6-E3: this trnsition gnerated a laser beam of red colour of wavelength 6328 A^0.

2] E4-E3:this produces infrared laser beam at wavelength 11500 A^0.

3] E6-E5:A laser beam of wavelength 33900 A^0 in the far infrared region .

From E3 and E5 lasing transitions take neon atoms to E2 level . As lower level depopulate easily it is easier to maintian population inversionbetween lasing levels.Diameter of He- Ne tube is narrow to increase the probability of collision. He -Ne Laser operates in CW mode.

Q.7]Describe the construction and working of the Semiconductor Diode Laser and state its characteristics .

Ans

Semiconductor diode laser is a solid state laser with a two level homojunction laser system or a fourlevel hetrojunction laser system . It was first designed by R.N. Hawlin 1962. It is a specially designed P-N junction device which emits coherent radiation when forward biased.

Principle :-

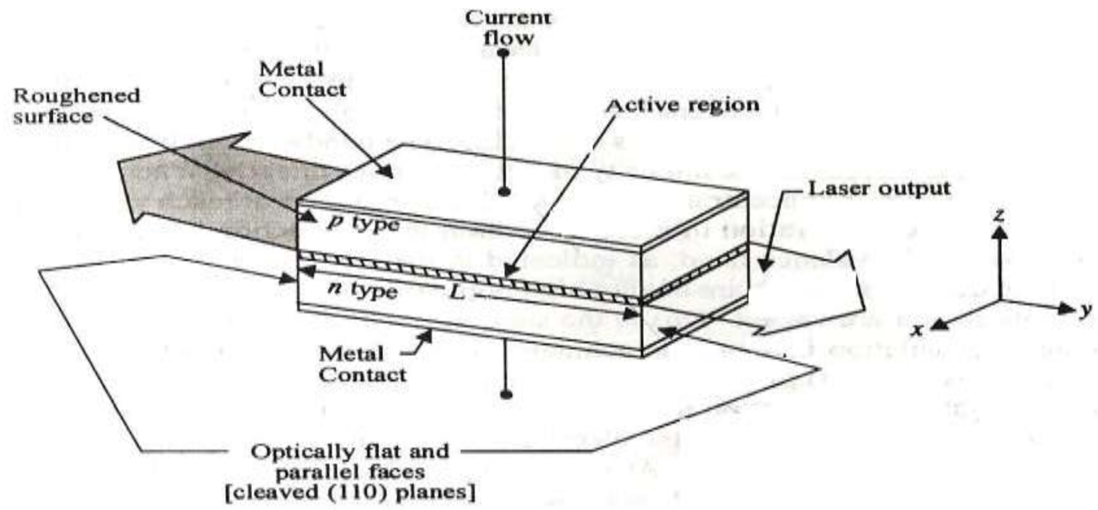
The band structure of an intrinsic semiconductor consists of a V.B. & a C.B. separated by energy gap Eg.. In the normal condition , the V.B. at lower energy EV contains a large no. of e^{-s} &few no. of holes ; while the C.B. at higher energy E_C has few no.of e^{-s} . if a transition of e^- occurs from C.B. to V.B. so that e^{-1} recombines with the hole in V.B., the energy $(E_C - E_V) = E_{.g..}$ is given out in the form of photons. This e^- hole recombination is the basic mechanism involved in the emission of radiation .

Construction :- Homosturecture P-N iunction laser:-

Diode :- Extremly small in size with sides of order 1 mm.

Junction :- lies in a horizontal plane through the centre. Top & bottom faces are metallised & ohmic contacts a provided to pass current through the diode. The front & near are polished parallel to each other & perpendicular to the plane of the junction .These polished surfaces constitute resonator. The other two opposite faces are roughened to prevent losing action in that direction. Active region is about 1μm in thickness .

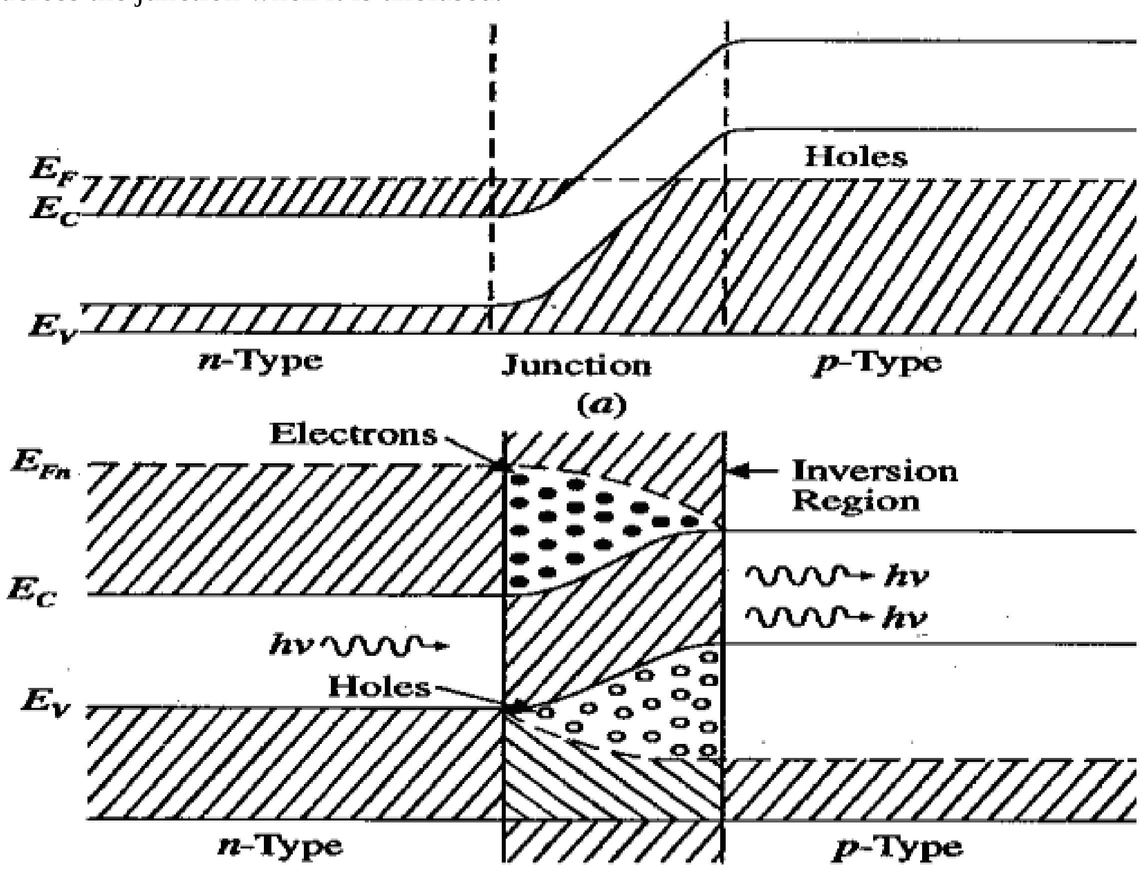
Diagram:-



Working:

To achieve required P.I. in semiconductor we use heavily doped P-N Junction and forward bias it. Due to heavy doping on N-side , the donor levels as well as a portion of C.B. are occupied by e^{-s} & terminal level lies within the C.B.

Due to heavy doping p-side , holes exists in V.B. & acceptor levels are unoccupied so bthat terminal level lies within the V.B.. At thermal equilibrium the terminal level is uniform across the junction when it is unbiased.



Junction is forward biased :-

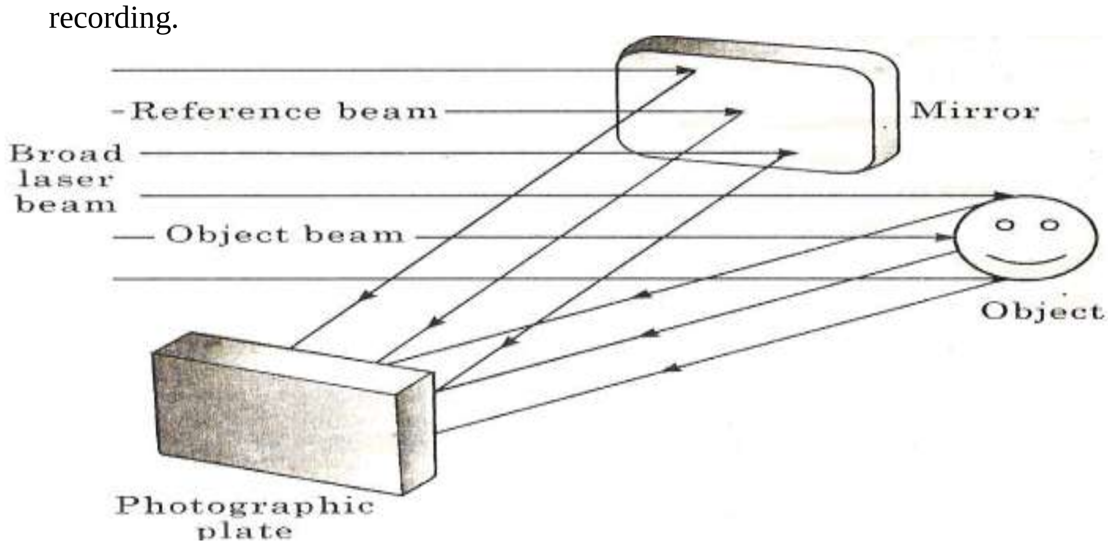
When the junction is forward biased the energy levels shift due to movement of electrons & holes into the depletion region its width reduces.

At low forward current level the e = hole recombination's result in spontaneous emission of photons & junction acts as LED , with large bandwidth of emitted light. With the increase in current, the intensity of light increase & for a threshould current, the carrier concentration in the depletion region reaches very high value.The depletion region now contains large no. of e^{-s} in C.B. & a large no. of holes in V.B. The upper energy levels in the depletion region have a high population density of e^{-1}s while lower energy levels in the same region are vacant.Thus, necessary P.I. is achieved.The narrow region in which P.I. is achieved is called inversion region or active region Thus, P-N junction , the forward bias plays the role of pumping agent.The spontaneously generated photons induce the conduction electrons to jump into the vacant state therefore of the stimulate electrons hole recombination , wherent radiation of very narrow bandwidth is emitted .

Q. 8]Write a note on Holography

Ans.

1st introduced by Denniel's Gabor in 1947.Holography is a technique for obtaining three dimential optical images of object & it Is based on the interference of light.This three dimensional optical image is called a hologram which is a Photograph consisting of compute information of the real object are visible in the Absence of the object. The word holography is derived from the Greek world where holes means entire & graphing means recording/ writing. Thus holography means complete recording.



Principle:-

A weak but broad beam of laser lights its split into two beams.

1] Reference beam

2] object beam

Reference beam:-

It is allowed to mean the photographic plate directly.

Object beam:-

It illuminates the object.Part of the light scattered by the object travels towards the photographic plate & interfaces with the reference beam & produces an interference pattern on the photographic plate.

The photographic plate carrying the interference pattern is called "hologram".

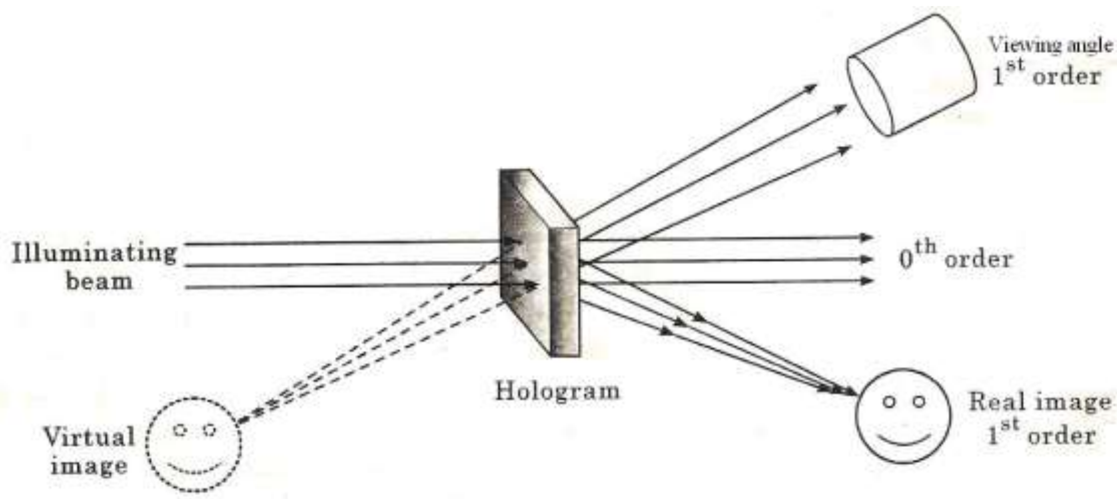
Reproduction of image:-

Like the ordinary photographic plate a hologram also needs to be developed fixed & stored a photography contains direct details of the object where consist of interference pattern formed by the superposition of two coherent light beam. A laser beam is identical to the reference beam & is used for reconstruction of the object. The laser beam illuminates the hologram at the same angle as the reference beam.

The hologram acts as a diffraction grating & secondary waves from the hologram interference constructively in certain dir. & destructively in other direction. An image of the object appears where the object once thus image is 3-D.Thus holography is a two state produces:-

Hologram is recorded in the form of IP .

Hologram acts as diffraction grating for the reconstruction beam & the image of the object its reconstructed from the hologram.



Applications:-

In an ordinary photo each region contains a separated & individual part of the original object destruction of a portion of a photographic image leads to an irseparated loss of into on the other hand, in a hologram each part contains into about entire object.

Destruction of a part of hologram does not cause a loss of into. About the object . from even a small part of hologram the entire image of can be reconstructed .

High into holding capacity:-

It is not useful to record several images on a single photo film. Such a record cannot give into about any of the individual images. On the other hand, several images can be recorded on a hologram.

Holograms can be viewed with white light. Thus, necessity for reconstruction of laser beam is dispensed away.

Q. 9] Give important characteristics of a laser beam.
Ans.

Fullform of LASER is light amplification by stimulated emission of radiations . The important characteristics of laser beam are as follows :

1] Coherence : The waves emitted by a laser source will be in phase and are of same frequency . The laser beam is spatially and temporally coherent to an extraordinary degree.

2] Directionality : Laser emit light only of one direction as only photons travelling along optic axis of system are selected and amplified.

3] Divergence : the divergence or the angular spread of laser beam is extremely samll 10^{-5} to 10^{-6}.

4] Intensity : A laer emits light in form of narrow beam propagating as plane waves .as energy is concentrated in the narrow region its intensity is tremendously high . eg. Light

WAVE MECHANICS

Q.1] Derive expression for De- Broglie's Relation.
Ans.
Consider a wave of frequency υ .
Now, according to quantum theory of radiation,
Energy of a photon , E=hυ ----------------[1]
According to Einstein's relation of mass- energy

$$E= mc^2 \text{ -------------- [2]}$$

Comparing [1] & [2]

$$h\upsilon = mc^2 \text{ ----------- [3]}$$

P= momentum associated with a photon which travels in free space.

$$P= mc$$

$$\upsilon = \frac{c}{\lambda}$$

$\therefore$ Equation [3] becomes

$$h \text{ x } \frac{1}{\lambda} = P$$

$$\frac{h}{\lambda} = P$$

$$\therefore \lambda = \frac{h}{p}$$

λ = wavelength of the particle.

h= Planck's constant.

P= momentum of the particle.

Q.2] Derive expression for De- Broglie's Relation in terms of K.E.
Ans.
Expression for de- broglie wavelength in terms of K.E.
Kinetic energy is given by

$$E=\frac{1}{2}m\upsilon^2$$

$$\upsilon = \sqrt{\frac{2E}{m}}$$

De- broglie wavelength of an e is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2E}{m}}}$$

$$= \frac{h}{\sqrt{\frac{2Em^2}{m}}}$$

$$\boxed{\therefore \lambda = \frac{h}{\sqrt{2mE}}}$$

Q.3] Derive expression for De- Broglie's Relation in terms of **accelerating potential**
Ans.
Expression for de- Broglie in terms of accelerating potential:-

If a particle of mass m & charge e is accelerated through a potential difference V. Then energy acquired by the particle is given by,

E= eV-------------------[i]

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

de- Broglie λ associated with m= 9.1x 10^{-31} kg

e = 1.6 x 10^{-19} C

h= 6.6 x 10^{-34} J

$$\lambda = \boxed{\frac{6.6 \text{x} 10^{-34}}{\sqrt{2 \text{x} 9.1 \text{x} 10^{-31} \text{x} 1.6 \text{x} 10^{-19} V}}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}} A^0}$$

Q.4] Explain the terms Phase velocity and group velocity .
Ans.
Phase velocity

If a point is imagined to be marked on a traveling wave , then it becomes a representative point for a particular phase of the wave & the velocity with which it is transported owing to the motion of the wave is known as phase velocity.

Considering a spring producing SHM on y-axis.

Now at t=t, displacement on y-axis when mass is at at M is given by

y=Asinωt

Between M& N

y=Asin[ω (t-t_0)]

As distance between M & N & time table is to

$$\therefore t_0 = \frac{x}{\upsilon}$$

$$\therefore y = A\sin\left[\omega\left(t - \frac{x}{\upsilon}\right)\right]$$

$$\therefore y = A\sin\left[\left(\omega t - \frac{wx}{\upsilon}\right)\right]$$

$$\therefore y = A\sin[(\omega t - kx)]$$

$$K = \frac{\omega}{\upsilon} = \text{wave number}$$

$$\upsilon = \frac{\omega}{k}$$

Where , υ is the velocity with which wave disturbance is carried out & is phase velocity.

$$\boxed{\upsilon_{phase} = \frac{\omega}{k}}$$

Group velocity :-

When two or more waves with slightly different velocities are superimposed together , the resultant patterns emerges in the shape of variation in amplitude & is called wave packet or wave group.

Group velocity is the velocity with which the envelope enclosing wave group is transported. it is the velocity with which energy transmission takes place in a wave.

Q.5] State the uncertainty Principle give its characteristics, significance and applications .

Ans.

Uncertainty Principle

It states that it is impossible to determine simultaneously both the position & velocity of a sub atomic particle with absolute accuracy or certainty.

Mathematically,

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

$$\Delta x \cdot m\Delta v \geq \frac{h}{2\pi}$$

$$\boxed{\Delta x \cdot \Delta v \geq \frac{h}{2\pi m}}$$

Δx = uncertainty in the measured values of position.
Δv = uncertainty in the measured values of velocity.
h= Planck's constant.

Important Characterstics:-

1] Uncertainty principal is direct consequence of the wave nature of particle.
2] The limit on measurement is independent of measuring procedure or Sophistication of instruments.
2] It Is fundamental Property of nature.
3] It is applicable for conjugate variables , like energy & time , as

$$\Delta E \cdot \Delta t \geq h$$

Also to angle & angular momentum .

$$\Delta L \cdot \Delta \theta \geq h$$

Significance:-

Heisenberg's uncertainity principle is, significant only for motion of microscope objects & is negligible for that of microscopic objects.
It can be seen from e.g..:-
If uncertainty principle is applied to an object of mass, say about a milligram (10^{-6} kg), then

$$\Delta V \cdot \Delta X = \frac{h}{2\pi m} = \frac{6.626 \, x 10^{-34} \, J}{2X3.1416 \, X 10^{-6} \, kg} \approx 1X10^{-28} \, m^2 s^{-1}$$

The value of $\Delta V \cdot \Delta X$ obtained is extremely small & therefore one can say that in dealing with milligram sized or heavier objects, uncertainties of such small dimensions are hardly of any real consequence .

If principle is applied to an object like an e whose mass is 9.11 x 10^{-31} kg , then

$$\Delta V \cdot \Delta X = \frac{h}{2\pi m} = \frac{6.626 \, x 10^{-34} \, J}{2X3.1416 \, X \, 9.11 \, x 10^{-31}} \approx 10^{-4} \, m^2 s^{-1}$$

The value of $\Delta V \cdot \Delta X$ obtained is much larger & such uncertainties are of real consequence.

Application :-

1] **Absence Of Electron In Nucleus**:-

The radius of the nucleus is about 5 x 10^{-15} m. Now if the e were to exist within the nucleus then the maximum uncertainty inits position would have been

5 x 10^{-15} m.

$$\Delta x = 5 \text{ x } 10^{-15} \text{ m.}$$

$$\Delta x \cdot \Delta p = \frac{h}{2\pi}$$

$$\Delta x \cdot m\Delta \upsilon = \frac{h}{2\pi}$$

$$\Delta \upsilon = \frac{h}{2\pi m \Delta x}$$

Now, $\Delta x = 5 \text{ x } 10^{-15}$ m.

m= 9.11 x 10^{-31} kg

$$\therefore \Delta \upsilon = \frac{6.626 \, x 10^{-34} \, J}{2X3.1416 \, X \, 9.11 \, x 10^{-31} \, x5x10^{-15}}$$

Δv =2.315x 10^{10}ms^{-1}

The value of uncertainty in velocity Δv is larger than the velocity of light of light (3 x 10^8 ms^{-1}) & therefore , it is not possible . hence an electron cannot be found within the atomic nucleus.

Q.6] Derive One- Dimentional Time Dependent Schrodinger Equation.
Ans.
One- Dimentional Time Dependent Schrodinger Equation,

Let us consider a microparticle . let 'p' be the wave junction associated with the motion of this microparticle. 'p' junction represents the wavefield of the particle.

For one dimensional case, the classical wave eq has the following

$$\frac{d^2y}{dx^2} = \frac{1}{\upsilon^2} \cdot \frac{d^2y}{dt^2} \text{ -------------[1]}$$

The solution of the above eqution is,

$$Y(x,t) = Ae^{-I\,(wt-kx)} \text{ ----------[2]}$$

Where $w = \upsilon k$

& υ = phase velocity

We know, for microparticle

$$E = h\upsilon = \frac{hw}{2\pi} = h \quad \therefore w = \frac{E}{h} \text{ ----------------------[3]}$$

Wave number,

$$K = \frac{w}{v} = \frac{2\pi\upsilon}{\upsilon\lambda} = \frac{2\pi}{\lambda} = \frac{2\pi x p}{h}$$

$$\therefore k = \frac{p}{h} \text{ ----------------------[4]}$$

Substituting eq. (3) & (4) is (2) & replacing Y(x,t) with wave function $\psi(x,t)$

$$\psi(x,t) = Ae^{-i\,(Et-px)/h} \text{ ---------[5]}$$

Differtiating eq.[5] w.r.t. 't'

$$\frac{d\psi}{dt} = Ae^{-i\,(Et-px)/h} \cdot \frac{iE}{h}$$

$$\boxed{\frac{d\psi}{dt} = \frac{i}{h} E\,\psi} \text{ ---------------[6]}$$

Differsiating eq. [5] w.r.t.'x'

$$\frac{d\psi}{dx} = Ae^{-i\,(Et-px)/h} \cdot \frac{ip}{h}$$

$$\boxed{\frac{d\psi}{dx} = \frac{i}{h} p\,\psi}$$

Differtiatng again w.r.t. x,

$$\frac{d^2\psi}{dx^2} = \frac{-p^2}{h^2}\psi \text{ ------------------[7]}$$

The kinetic energy & momentum of a free particle are related by the expression ,

$$K.E. = \frac{P^2}{2m}$$

The total energy of a particle is

T.E.= K.E. + P. E

$$\boxed{E = \frac{P^2}{2m} + V}$$

Multiplying the above eq with ' ψ ',

$$E\psi = \frac{P^2}{2m}\psi + V\psi \text{ ------------[8]}$$

Substituting $E\psi$ & $P^2\psi$ using eq. [6] & [7] in eq [8]

$$-\frac{h}{i}\frac{d\psi}{dt} = -\frac{h^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$$

$$\boxed{-\frac{h^2}{2m}\frac{d^2\psi}{dx^2}+V\psi=ih\frac{d\psi}{dt}}$$

This is 1-D time dependent schrodinger's eqution.

Q.7] From Schrödinger's' Time dependent equation derive a time Independent form
Ans.
Reduction of S.E. to Time Independent form :-

we can separate the variables of schrodingers wave eqution as

$$\psi\ (x,t) = \psi\,(x)\,\phi\,(t)$$

$\therefore$ schrodingers time dependent eq becomes ,

$$-\frac{h^2}{2m}\phi\,(t)\ \frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)\phi(t)=ih\psi(x)\frac{d\phi(t)}{dt}$$

Dividing both sides by $\psi(x)\phi(t)$, we get

$$\therefore -\frac{h^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}+V(x)=ih\frac{1}{\phi}\frac{d\phi}{dt}$$

To make the eq time independent we ignore the R.H.S. since it is a function of t.

$$\therefore -\frac{h^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}+V(x)=E$$

Q.8] Write Schrödinger's equation for a particle in a box and solve it to obtain eigen values and eigen functions?
Ans.
For a partical inside box.

$$\phi_n=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

Where a is the height of box.

$$\frac{8\pi^2 m}{h}E=KL\ , \qquad E=\frac{K^2h^2}{8\pi^2 m}$$

$$\frac{d^2\phi}{dx^2}+k^2\psi=0$$

Q.9] Obtain Schrödinger's equation for Free Particle .
Ans.
Consider an electron moving freely in the space in the positive direction of X axis and not acted upon by any force . Since no force is acting on the electron and its potential energy is zero .

Consider Schrödinger's Time Independent eq.

$$\therefore -\frac{h^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}+V(x)=E\ (x)$$

$\because$ The particle is traveling along the x- direct only.

$$\therefore -\frac{h^2}{2m}\frac{d^2\psi}{dx^2}+V(x)=E\ (x)$$

$$\therefore -\frac{h^2}{2m}\frac{d^2\psi}{dx^2} = (E - V)\psi$$

$$\frac{d^2\psi}{dx^2} = (E - V)\psi \ X \ \frac{2m}{h}\text{------------[1]}$$

We know that, $h = -\frac{h}{2\pi}$

Substituting h in eq [1]

$$\frac{d^2\psi}{dx^2} = (E - V)\psi \ X \ \frac{2m}{\left(\frac{h}{2\pi}\right)^2}$$

$$\frac{d^2\psi}{dx^2} = (E - V)\psi \ X \ \frac{8\pi^2 m}{h^2}$$

$$(E - V)\psi \ X \ \frac{8\pi^2 m}{h^2} + \frac{d^2\psi}{dx^2} = 0$$

Since V=0

$\therefore eq^n$ becomes ,

$$E\psi \frac{8\pi^2 m}{h^2} + \frac{d^2\psi}{dx^2} = 0$$

Let

$E = k^2$

$$E = \frac{K^2 h^2}{8\pi^2 m} \text{----------------------[3]}$$

Sub. $\frac{8\pi^2 m}{h^2}$ E in eq [2]

$$\boxed{\frac{d^2\psi}{dx^2} + k^2\psi = 0}$$

Also from eq[3] $E \alpha k^2$

Thus, it a particle is free energy has any value.

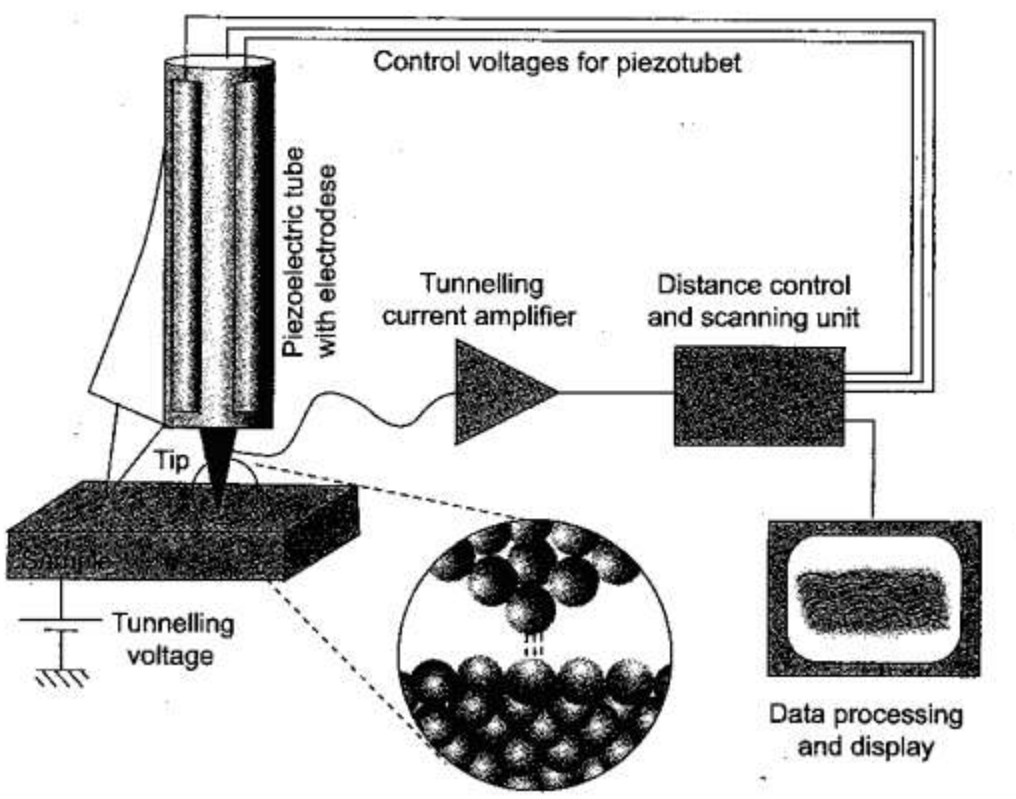
Describe the scanning Tunnelling microscope (STM)

Ans.

STM is considered as a type of electron microscope. STM is a non-optical microscope that scans an electrical probe over a surface to be imaged to detect a weak electric current flowing between the tip and the surface. The STM was invented in 1981 by Gred Binning & Heinrich Rohrer who were awarded Nobel prize in physics (1986).

STM allows scientists to visualize regions of high electron density and hence infer the position of individual atoms and molecules on the surface of a lattice. The STM can obtain images of conductive surfaces at an atomic scale 2 x 10^{-10} m or 0.2 nm and also can be used to manipulate individual atoms, trigger chemical reactions or reversibly produce ions by removing or adding individual electrons from atoms or molecules.

The STM is a non-optical microscope which employs principle of electron tunneling of quantum mechanics. The equations of quantum mechanics assign a finite non-zero probability for an electron to go from one region of low potential energy to another such region, even if it does not possess sufficient energy to overcome the barrier between two regions. This phenomenon may be visualized as a football going from one well into which it has been dropped into another one nearby, without coming to the surface or making a hole between the two wells. In STM, this phenomenon is used in the following way. A very sharp needle is brought close to the surface to be imaged. The distance is of the order of a few Angstroms. At such a distance electrons from the surface will tunnel across the gap and set up a measurable tunneling current in the needle. This current is a precise indicator of the distance between the tip and the surface. The needle can be moved (in a raster pattern) across the surface, at each point its height being adjusted to maintain a tunnellling current at a constant value. Thus, a record of the vertical portion of the needle at each position because equivalent to a record of surface tomography and can be converted into an observable picture.



Schematic view of an STM.

High quality STM can reach sufficient resolution to show single atoms. Its is widely used in both industrial and fundamental research to obtain atomic scale images of metal surfaces. It provides a three-dimensional profile of the surface which is very useful for characterizing surface roughness, observing surface defects, and determining the size and conformation of the molecules and aggregates on the surface. The use of this microscope in the sudy of biological samples is still or semi conducting materials have to be stained with heavy metals before they can be imaged, leading to a loss of resolution. The preparation of the tip and its positioning above the surface also has to be more precise and controlled in case of biological materials.

PROBLEMS FROM ALL UNITS

Interference in thin films

FORMULA LIST :

1	$2\mu t \cos r = n\lambda$ for n^{th} dark band (Reflected system)	r = $\angle$ of retrac t = thickness μ = R. I. n = number λ = wave length
2	$2\mu t \cos r = (2n - 1)\lambda/2$ for n^{th} bright ring. (Reflected system)	
3	$\beta = \dfrac{\lambda}{2\mu\theta}$	
4	$\mu_f \approx \sqrt{\mu g}$	Antireflecting

5 $t = \dfrac{\lambda}{4\mu_f}$

Interference in thin films

1. A soap film of R-I 1.43 is illuminated by white light incident at a 30^0. The refracted light is examined by a spectroscope in which <u>dark</u> band corresponding to wavelength 6×10^{-7} m is observed. Calculate the thickness of the films.

Sol^n: DATA :

$\mu = 1.43$ $\quad$ $t = ?$

$\theta = 30^0$

$\lambda = 6 \times 10^{-7}$ $\quad$ * Transmitted system *

FORMULA : $t = \dfrac{(2n - 1)\,\lambda/2}{2\mu \cos r}$

FORMULA : $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 30}{1.43} = 0.349$

$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{0.877} = 0.936$

Minimum thickness n = 1

$$t = \frac{(2\text{x}\ 1-1)6 \text{ x } 10^{-7}}{2 \text{ x } 2 \text{ x } 1.43 \text{ x } 0.936}$$

$$\boxed{t = 1.12 \text{ x } 10{-}7 \text{ m}}$$

2 Find t when μ = 1.46 that results in constructive interference in reflected light, if film is illuminated with light whose wavelength is 6000 A^0.

Sol^n: DATA : * reflected system *

μ = 1.46 t min then n = 1

$\lambda = 6000 \times 10^{-8}$ cm

γ = 0 normal incidence.

FORMULA : $$t = \frac{(2n-1)\,\lambda/2}{2\mu \cos r}$$

3 Two glass surface in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in Na light at normal incidence. What is the thickness of wire? (λ = 5893 A^0)

Sol^n: DATA:

μ = 1air film

$\lambda = 5893 \times 10^{-10}$ m

n = 20

FORMULA: $t = \ell \tan\theta \approx \ell\,\theta$ for small θ

$= 20 \times \beta \times \theta$

$= 20 \times \frac{1\lambda}{2\mu\theta} \times \theta$

$t = 10 \times 5893 \times 10^{-10}$

$$\boxed{t = 5.8 \times 10^{-6} \text{ m}}$$

4 Can a thin film of water (μ_f = 1.33) formed on a glass window act (1.52) as anti reflecting film? If so, how thick should be water film?

Sol^n: μ_f = 1.33

μ_g = 1.52

t = ?

FORMULA: Antireflecting if $\mu_f \approx \sqrt{\mu g}$

$\sqrt{\mu g} = \sqrt{1.52} = 1.233 \approx 1.33$ $\therefore$ Antireflecting

visible assumption

FORMULA: $t = \dfrac{\lambda}{4\mu_f} = \dfrac{5500A^0}{4 \times 1.33} = 1034\ A^0$

$$\boxed{t = 0.10\ \mu_m}$$

Newton's Rings

1. Newton's rings are observed in reflected light of wavelength 6000 A0. Diameter of 10^{th} ring is 0.5 cm, find the radius of curvature of lens and thickness of corresponding film.

Sol^n: DATA: $\lambda = 6 \times 10^{-7}$ m R = ?

$Dn = 0.5 \times 10^{-2}$ m t = ?

n = 10

FORMULA : $R = \frac{Dn^2}{4n\lambda} = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 6 \times 10^{-7}} = 10.4$ cm

FORMULA : $t = \frac{D^2}{8R} = \frac{(0.5 \times 10^{-2})^2}{8 \times 104} = 3 \times 10^{-6}$ m

2. In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark ring are 0.4 cm and 0.7 cm respectively. Deduce the diameter of 20^{th} ring.

Sol^n: DATA: $D_4 = 0.4 \times 10^{-2}$ m

$D_{12} = 0.7 \times 10^{-2}$ m

$D_{20} = ?$

FORMULA : $\frac{Dm + n^2 - Dn^2}{4m\lambda} = R$

$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda R$ — 1

$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda R$ —2

Dividing 1&2 & cross multiplying.

$$2 \text{ x } (D_{12}^2 - D_4^2) = D_{20}^2 - D_4^2$$

$$\therefore D_{20}^2 = 2D_{12}^2 - D_4^2$$

$$= 2(0.7)^2 - (0.4)^2$$

$$= 0.98 - 0.16 = 0.82$$

$$D_{20} = \sqrt{0.82} = 0.9055\text{cm.}$$

3. Newton's rings are formed in reflected light of light of wavelength 6000^0A with a liquid between a plane and curved surfaces. If the diameter of the 6^{th} bright ring is 3.1mm and the radius of curvature of curved surface is 100 cm, calculate the R.I of the liquid.

Sol^n: DATA : $\lambda = 6000 \text{ A}^0 = 6 \text{ x } 10^{-7}\text{m} = 6 \text{ x } 10^{-5}$ cm

$R = 100$ cm

$D_6 = 0.31$ cm

FORMULA : $rn^2 = 2Rt$ □ $Dn^2 = 8 \text{ Rt}$ □ $t = \frac{Dn^2}{8R}$

Normal incidence bright ring $2\mu t = \frac{(2n-1)\lambda}{2}$

$$\therefore 2\mu = \frac{Dn^2}{8R} = \frac{(2n-1)\lambda}{2}$$

$$\therefore \mu = \frac{2(2n-1)\lambda R}{Dn^2}$$

$$= \frac{2(2 \text{ x } 6-1) \text{ } 6000 \text{ x } 10^{-8} \text{ x } 100}{(0.31)^2}$$

$$\boxed{\mu = 1.373}$$

4. A film of R.I μ is illuminated by white light at an angle of incidence i. In reflected light two consecutive bright fringes of wavelength λ_1 and λ_2 are found overlapping. Obtain expression for thickness of film.

Sol^n: for n^{th} band $2\mu t \cos r = (2n - 1)\, \lambda_{1/2}$ — (1)

for $n+1^{th}$ band $2\mu t \cos r = 2(n^{-1}_{+1})\, \lambda_{2/2}$ — (2)

from (1) & (2) $(2n - 1)\, \lambda_{1/2} = [2(n + 1) - 1]\, \lambda_{2/2}$

$$2n(\lambda_1 - \lambda_2) = (\lambda_2 + \lambda_1) \Rightarrow 2n = \frac{\lambda_2+\lambda_1}{\lambda_1-\lambda_2} \quad \text{— (3)}$$

w.k.t $2\mu t \cos r = (2n - 1)\, \lambda_{1/2}$

$$2\mu t \cos r = \left\{\left(\frac{\lambda_2+\lambda_1}{\lambda_1-\lambda_2}\right) - 1\right\} \frac{\lambda_1}{2}$$

$$\boxed{t = \frac{\lambda_2+\lambda_1}{2\mu \cos r\,(\lambda_1-\lambda_2)}}$$

5. Show with example that separation between two consecutive rings, In NR expt. goes on reducing as the serial number of ring increases.

Sol^n: In NR expt. $D_n = \sqrt{4nR\lambda} \Rightarrow D_n \,\alpha \sqrt{n}$

e.g Calculate separation of 5th & 4th dark ring.

$D_5 \alpha \sqrt{5} = 2.236$

$D_4 \alpha \sqrt{5} = 2$

$\therefore D_5 - D_4 \,\alpha\, 0.236$ SI units — (1)

||y $D_{80} \,\alpha\, \sqrt{80} = 8.9442$

$D_{79} \,\alpha\, \sqrt{79} = 8.8881$

$\therefore D_{80} - D_{79} \,\alpha\, 0.0560$ SI units — (2)

$\therefore D_{80} - D_{79} < D_5 - D_4$ hence proved.

6 In NR expt. Source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}$ cm and $\lambda_1 = 4.5 \times 10^{-5}$ cm. It is found that the nth dark ring due to λ_1 coincides with $(n + 1)^{th}$ dark ring of λ_2. If the radius of curved surface is 90 cm, find the diameters of nth dark ring of λ_1.

Sol^n: DATA : $\lambda_1 = 6 \times 10^{-5}$ cm $(D_n)\, \lambda_2 = (D_n + 1)$

$\lambda_2 = 4.5 \times 10^{-5}$ cm

$R = 90$ cm

FORMULA : $D_n^2 = 4nR\lambda$

$\therefore 4nR\,\lambda_2 = 4(n + 1)\, R\, \lambda_1 \Rightarrow n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = 3$

$D_3^2 = \frac{4 \times 3 \times 90 \times 6 \times 10^{-5}}{\sqrt{4 \times 3 \times 90 \times 6 \times 10^{-5}}}$ 3rd ring of λ_1

$\therefore \boxed{D_3 = 0.2545 \text{ cm}}$

- **Problems for practice :**

1. Newton's rings are formed by light reflected normally from a convex lens of radius of curvature 90 cm and a glass plate with a liquid between them. The diameter of nth dark ring is 2.25 mm and that of (n + 9)th dark ring is 4.5 mm. Calculate R. I. of liquid (λ = 6000 A^0).
2. NR are obtained with light of λ = 5500 A^0. The diameter of 10th dark ring is 5 mm. Now the space between the lens and the plate is filled with a liquid of refractive index 1.25. What is the diameter of 10th ring now?
3. In NR expt. The diameter of 4th and 12th dark rings are 0.4 cm and 0.7 cm respectively. Find the diameter of 20th dark ring.
4. Light containing the wavelength λ_1 and λ_2 falls normally on a convex lens of radius of curvature R resting on a glass plate. Now if the nth dark ring due to λ_1 coincides with (n + 1)th dark ring due to λ_2 then prove that the radius of the nth dark ring is $\sqrt{\frac{\lambda_1\lambda_2}{\sqrt{\lambda_1 - \lambda_2}}}$ R

Quantum Mechanics

FORMULA LIST :

1 $\lambda = \frac{h}{p}$

2 $P = \sqrt{2mE} = \sqrt{2meV}$

3 $\Delta x \ \Delta P \geq \frac{h}{2\pi}$

4 $V \text{ group} = \frac{p}{m} = \frac{h}{\lambda m}$

5 V group x V phase = C^2

6 For particle in a box

$$E = \frac{n^2h^2}{8ma^2}$$

7. Probability = $\int_a^b |\psi|^2 \, dx$

Quantum Mechanics

- Problems based on De Broglie's hypothesis.

1 Calculate the De Broglie's λ associated with an α particle accelerated by a potential difference of 30 KV.
(Mα = 6.68 x 10–27 kg).

Sol^n: DATA : $M\alpha = 6.68 \times 10^{-27}$ kg

$V = 30 \times 10^3$ Volts

$q = 2 \times 1.6 \times 10^{-19}$ C ≈ 2e

FORMULA : $\lambda = \frac{h}{\sqrt{2M\alpha qV}}$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 30 \times 10^3}}$$

$$\boxed{\lambda = 5.83 \times 10^{-14} \text{ m}}$$

2 Calculate the wavelength of de = Broglie's waves associated with mass 1 kg moving with speed of 10^3m/sec.
Comment on your answer.

Sol^n: DATA : $M = 1$ kg

$\nu = 10^3$ m/sec

FORMULA : $\lambda = \frac{h}{p} = \frac{h}{m\nu}$

$$= \frac{6.63 \times 10^{-34}}{1 \times 10^3}$$

$$\boxed{\lambda = 6.63 \times 10^{-37}\ \text{m}}$$

3 Find the energy of the neutron in eV, whose De Broglie $\lambda = 1A^0$ also $m\cap = 1.674 \times 10^{-27}$ kg

Sol^n: DATA : $\lambda = 1\ A^0 \times 10^{-10}$ m

$m\cap = 1.674 \times 10^{-27}$ kg

FORMULA : $\lambda = \frac{h}{\sqrt{2mE}} \longrightarrow E \frac{h^2}{\sqrt{2m a\lambda^2}}$

$$E = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 6.67 \times 10^{-24} (1 \times 10^{-10})^2}}$$

$E = 1.3\ 10^{-20}$ J

E in eV = $\frac{1.3 \times 10^{-20}}{1.6 \times 10^{-19}}$ eV [$1eV=1.6 \times 10^{-19}$J]

- **Problem for practice :**

1 A bullet of mass 40g and e travel with velocity 1100m/s. what can be associated with them? Why wave nature of bullet not revealed by diffraction.

2 Electron accelerated through 100y are reflected from crystal. What is glancing angle of which the first reflection occurs? Lattice spacing =2.15A

3 An e and proton have each of 2 A0. Find their energies and momentum ?

4 Calculate the De Braglie λ for a 2 Ke V electron.

5 Which practical has shortest λ if e^-, α particle and neutron have same energy?

- **Problems based on uncertainity:**

1 An electron has a speed of 400 m/s with uncertainity 0.01%. Find accuracy in its position.

Sol^n: DATA : $M = 9.1 \times 10^{-31}$ kg we know

$V = 400$ m/sec

$\Delta V\% = 0.01\%$

FORMULA : $\Delta x \, \Delta px \geq \frac{h}{2\pi} \Rightarrow \Delta x \geq \frac{h}{2\pi m \Delta v}$

$$\Delta v = \frac{\Delta v\%}{100} \times V$$

$$= \frac{0.01}{100} \times 400 = 0.04$$

$$\therefore \Delta x \geq \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.04}$$

$$\boxed{\Delta x \geq 2.9 \times 10^{-3} \text{ m}}$$

2 Uncertainity in measurement of time spent by nucleus in excited state is 1.4×10^{-10} sec. Estimate uncertainity that results in its energy in that state.

Sol^n: DATA : $\Delta t = 1.4 \times 10^{-10}$ sec.

FORMULA : $\Delta E \times \Delta t \geq \frac{h}{2\pi} \Rightarrow \Delta E \geq \frac{h}{2\pi \Delta t}$

$$\Delta E \geq \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 1.4 \times 10^{-10}}$$

$$\boxed{\Delta E \geq 7.5 \times 10^{-25} \text{ J}}$$

Problems for practice:

1. An electron is confined in a box of length 10^{-8} m. Calculate minimum uncertainity in its velocity.
2. The speed of electron is measured to within an uncertainity of 2 x 104 m/sec. What is the minimum space required by the electron to be confined in an atom?

3. An electron and a 150 gm baseball are travelling at 220 m/s measured to an accuracy of 0.065%. Calculate uncertainity in position of each.
4. In jumping from an excited state to a stationary state, an atom takes 10^{-8} sec. What is the uncertainity in the energy of the emitted radiation?
5. Using uncertainity relation calculate time required for the atomic system to retain rotation energy for a line of wave length 6000 A^0 and width 10^{-4} A^0.

$$\left(\text{Hint} : \Delta E = \frac{hc}{\lambda^2}\Delta\lambda\right)$$

6. Find the percentage accuracy in determination at momentum of an electron if it is located within 0.5 A^0 having an energy of 10 keV.

   [$\Delta p\% = 1.96\%$]
7. Calculate the time required to measure the kinetic energy of an electron moving with speed 10^6 m/s having an uncertainity of 0.1%. Also determine distance covered by e^- in this time.

- **Problems based on group and phase velocity.**

1. Practical has mass 1.157 x 10–30 kg KE = 80 eV. Find De Broglie λ, group and phase velocity.

Sol^n: DATA : $m = 1.157 \times 10^{-30}$ kg.

$E = 80 \text{ eV} = 1.28 \times 10^{-17}$ J

FORMULA : $\lambda = \frac{h}{\sqrt{2mE}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.157 \times 10^{-30} \times 1.28 \times 10^{-17}}}$$

$$\boxed{\lambda = 1.218 \times 10^{-10} \text{ m}}$$

FORMULA : $V \text{ group} = \frac{P}{m} = \frac{h}{\lambda m}$

$$= \frac{6.63 \times 10^{-34}}{1.218 \times 10^{-10} \times 1.157 \times 10^{-30}}$$

$$\boxed{Vg = 4.7 \text{ x } 10^{-6} \text{ m/s}}$$

FORMULA : $Vg\ Vp = C2 \Rightarrow Vp = \frac{c^2}{Vg}$

$$Vp = \frac{(3 \text{ x } 10^8)^2}{4.7 \text{ x } 10^6}$$

$$\boxed{Vp = 1.913 \text{ x } 10^{-10} \text{ m/s}}$$

- **Problem to practice :**

1. Calculate group velocity and phase velocity of an electron whose De Broglie wavelength is 2 x 10^{-10} m.

- Problem on practice in a box.

1. e– is bound in an 1–D potential well of width 2A^0, but of infinite height. Find its E_0, E_1, DE_2.

Sol^n: DATA : $a = 2 \text{ x } 10^{-10}$ m

FORMULA : $E = \frac{n^2h^2}{8ma^2} = \frac{h}{\lambda m}$

$$E_0 = \frac{(1)^{-2}(6.63 \ x\ 10^{-34})^{\ 2}}{8\ x\ 9.1\ x\ 10^{-31}\ x\ (2\ x\ 10^{-10})^{\ 2}} = 1.5x10^{-18}J$$

$$E_1 = \frac{(2)^{-2}(6.63 \ x\ 10^{-34})^{\ 2}}{8\ x\ 9.1\ x\ 10^{-31}\ x\ (2\ x\ 10^{-10})^{\ 2}} = 6\ x10^{-18}J$$

$$E_2 = \frac{(3)^{-2}(6.63 \ x\ 10^{-34})^{\ 2}}{8\ x\ 9.1\ x\ 10^{-31}\ x\ (2\ x\ 10^{-10})^{\ 2}} = 1.35x10^{-17}J$$

2. Find the probability that a particle confined in well of size 0 is found within 0.25 a to 0.75 a n=1.

Sol^n: FORMULA : $P = \int_{0.25a}^{0.75a} |\ \psi\ |^2 \ dx$

Wkt $\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi n x}{a}\right) \Rightarrow \psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$

$$\therefore\ P = \int_{0.25a}^{0.75a} \left(\frac{2}{a}\right) \sin^2\left(\frac{2\pi x}{a}\right) dx$$

$$P = \frac{2}{2a}\left\{\int_{0.25a}^{0.75a} 1\, dx - \int_{0.25a}^{0.75a} \cos\left(\frac{4\pi x}{a}\right) dx\right\}$$

$$= \frac{1}{a}\left\{0.5a - \frac{a}{4\pi}\ (\sin 3\pi - \sin \pi)\ = 0.5\right\}$$

$$\boxed{\therefore\ P = 0.5 \text{ or } 50\%}$$

3. The wave f^n of particles confined to infinite square well is given

$$\psi_n(x) = A \sin \frac{n\pi x}{a} \quad \text{i} \; -a/2 < x < a/2$$
$$= 0 \quad \text{i} \; |x| \geq a/2$$

Determine A constant using normalization condition.

Sol^n: Normalization condition : $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

Here : $\int_{-a/2}^{a/2} A^2 \sin^2 \frac{\pi n x}{a}\ dx = 1$

$$A^2 \int_{-a/2}^{a/2} \left(\frac{1 - \cos\left(2\frac{\pi n x}{a}\right)}{2}\right) dx = 1$$

$$\frac{A^2}{2}\left\{[x]_{-a/2}^{a/2} \left(\frac{a \sin}{2n\pi}\left(\frac{2n\pi x}{a}\right)\right)_{-a/2}^{a/2}\right\} = 1$$

$$\frac{A^2}{2}\left(a - 0\right) = 1 \Rightarrow A^2 = \frac{2}{a} \Rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$

- Problems for practice

1. Calculate energy difference between first two states for one e– confined within distance of 0.1 nm.
2. A marble of mass 10 gm is confined to a box of width 10 cm. Calculate it's allowed energy level. Comment on results.

[Ans.: energy continuous]

3. ψ is given as ψ (k) $= \sqrt{\frac{\pi}{2}}\, x$; $0 \leq x \leq 1$

$= 0$; $x < 0$, $x > 1$

Find probability that particle found bet^{n} 0.35– 0.65.

4. A particle moving in 1–D potential box of width 25 A^0. Calculate probability of finding the particle within an interval of 5A^0 at the centre of box in Eo state.

[Hint: p = | ψ |2Δx and $\psi = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$ Put x = $a/2$ centre]