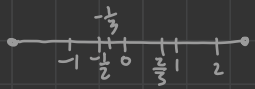
TASK 1.1

1.

x	f(x)		
1	2	=	$\frac{2}{1}$
2	-1	=	$\frac{-1}{1}$
3	1	=	$\frac{2}{2}$
4	$-\frac{1}{2}$	=	$\frac{-1}{2}$
5	$\frac{2}{5}$	=	$\frac{2}{5}$
6	$-\frac{1}{3}$	=	$\frac{-1}{3}$

⋮



$$f(n) = \begin{cases} \frac{2}{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ \frac{-1}{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases} = \begin{cases} \frac{4}{n+1} \\ \frac{-2}{n} \end{cases}$$

$\Rightarrow$ cannot be piecewise

$$\text{numerator}: \begin{cases} 4 & \text{if } n \text{ is odd} \\ -2 & \text{if } n \text{ is even} \end{cases} \Rightarrow 1-3(-1)^n$$

$$\text{denominator}: \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases} \Rightarrow n + \frac{1-(-1)^n}{2}$$

$$\Rightarrow f(n) = \frac{1-3(-1)^n}{n + \frac{1-(-1)^n}{2}} \longrightarrow \text{defined } \forall x \in \mathbb{R} \setminus \{0\}$$

check:

n	1	2	3	4	5	6
f(n)	$\frac{4}{1+1}$	$\frac{-2}{2+0}$	$\frac{4}{3+1}$	$\frac{-2}{4+0}$	$\frac{4}{5+1}$	$\frac{-2}{6+0}$
	$=2$	$=-1$	$=1$	$=-\frac{1}{2}$	$=\frac{2}{3}$	$=-\frac{1}{3}$

2. to find summation, rewrite f(n) as $f(n) = \begin{cases} \frac{2}{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ \frac{-1}{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$

for even n s.t. $1 \le n \le 1000$, $\frac{2}{\frac{n+1}{2}} = \frac{2}{x}$ $\forall x$ s.t. $1 \le x \le 500$

for odd n s.t. $1 \le n \le 1000$, $\frac{-1}{\frac{n}{2}} = \frac{-1}{x}$ $\forall x$ s.t. $1 \le x \le 500$

$$\Rightarrow \sum_{i=1}^{1000} f(i) = \sum_{i=1}^{500} \frac{2}{i} + \sum_{i=1}^{500} \frac{-1}{i} = \sum_{i=1}^{500} \frac{1}{i}$$

$\Rightarrow \sum_{i=1}^{500} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \dots = 6.7928$ (using wolfram alpha)

3. $f(-2.7) = \dfrac{1-3(-1)^{-2.7}}{-2.7 + 1\frac{-(-1)^{-2.7}}{2}} = \dfrac{2.763385}{-2.7 + 0.7939} = -1.4497\dots$ (wolfram alpha)

$(-1)^{-2.7}$ is a complex # $\Rightarrow$ can be approx. using the real part

$(-1)^{-2.7} \approx -0.587785 - 0.809017i$ (wolfram alpha)

≈ -0.587785

$\forall x$ s.t. $s \notin \mathbb{N}$, $f(x)$ is complex since $f(x)$ includes $(-1)^x$, so taking any root of it results in imaginary #

from professor's answer, $f(-2.7) = -1.069 - 1.472i$

↳ should get this if plugged into calculator