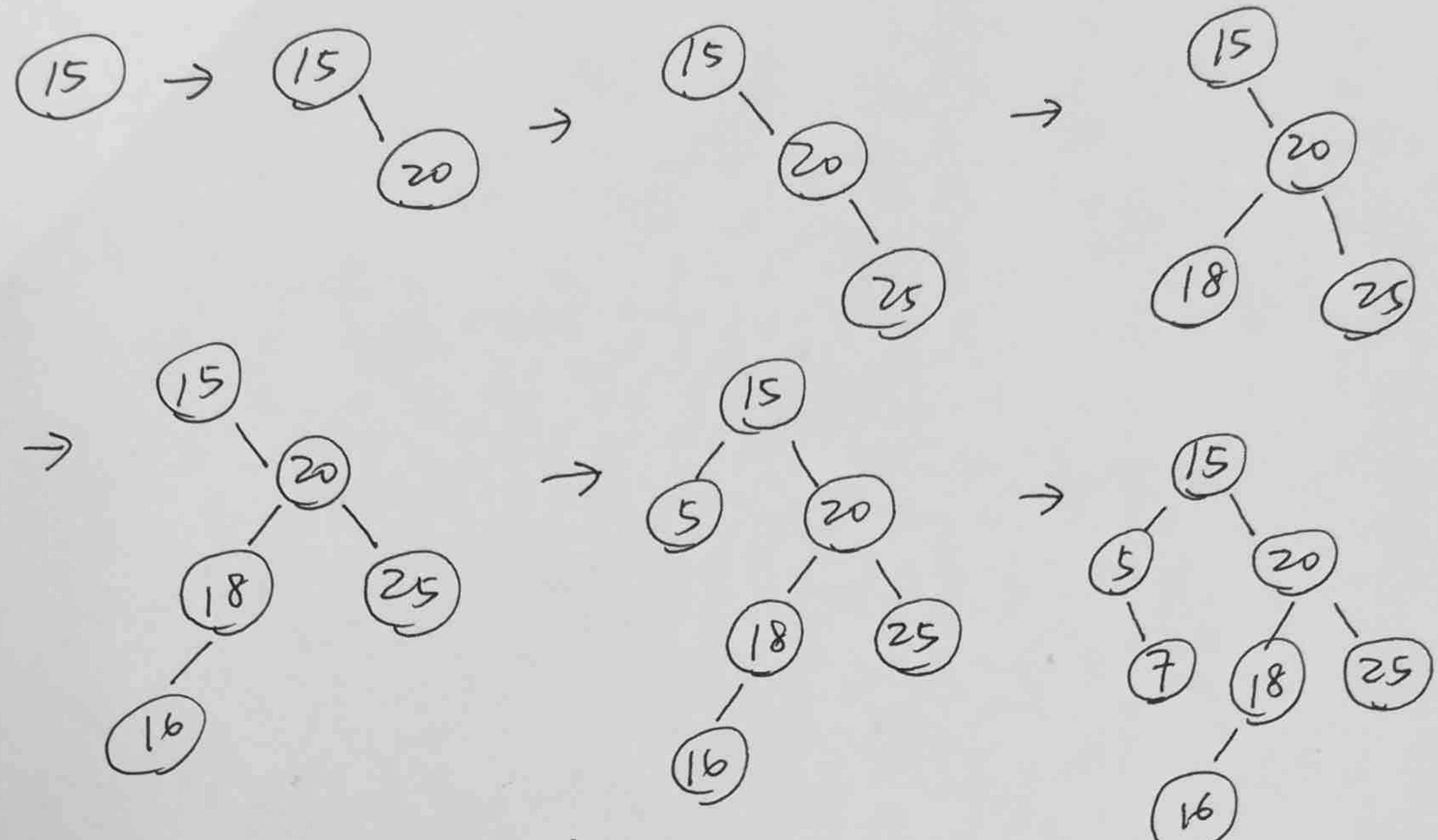
Exercise 5.15(a)



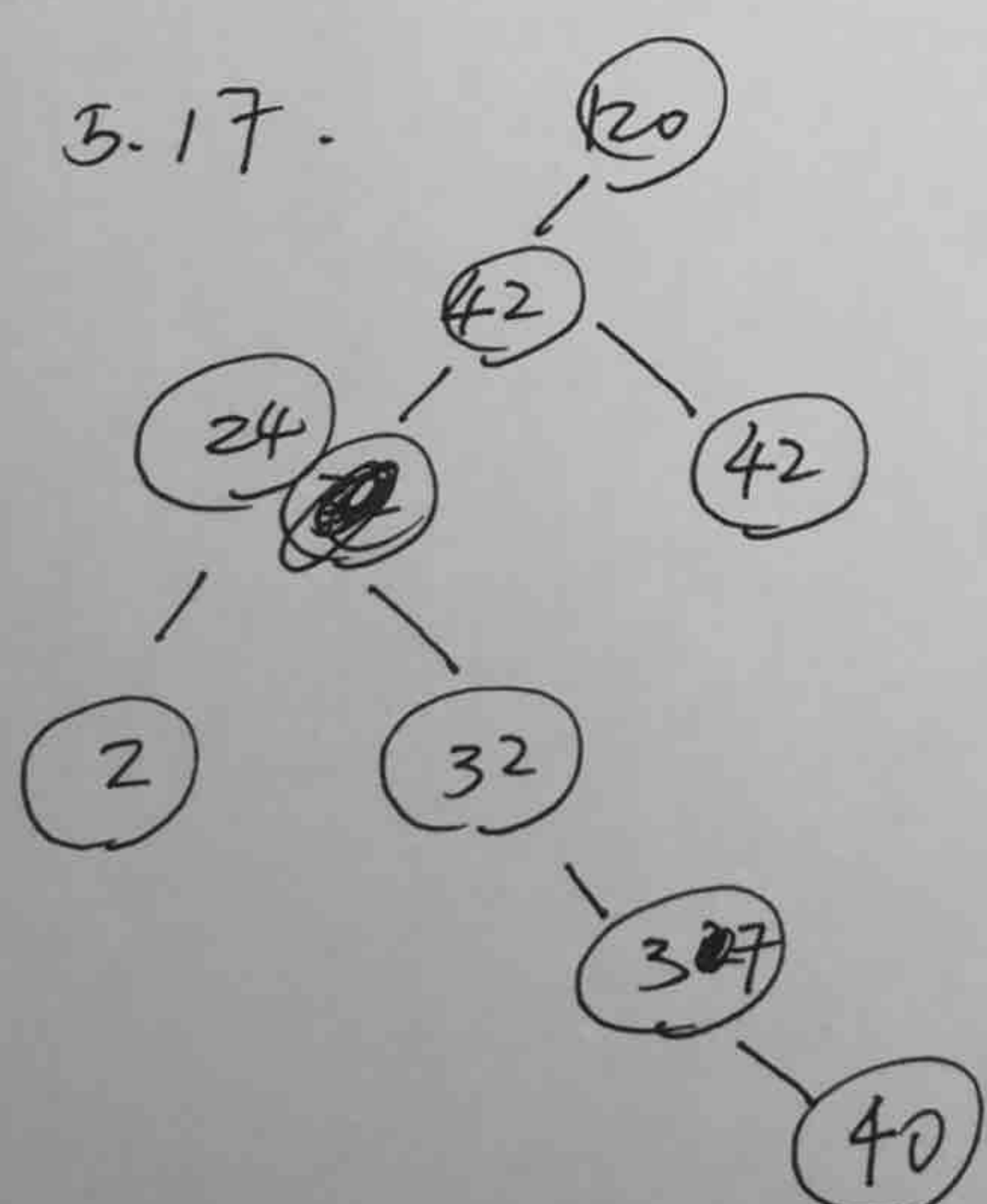
~~5.17 20~~

(b) ~~pre~~inorder: 5 7 15 16 18 20 25

preorder: 15 5 20 7 18 25 16

postorder: 7 5 16 18 25 20 15.

5.17.



Exercise 3.8

(a) $T(n) = C_1 \cdot n \in O(n)$

$C_1 \cdot n \le C_1 \cdot n$, So $T(n)$ is in $O(n)$ for $n_0 = 1$ and $C = C_3$

$T(n) = C_1 \cdot n$

$g(n) = n$

$T(n) \geq \Omega\, g(n)$

so $C_1 \cdot n \geq C \cdot n$ when $C = C_1$ & $n_0 = 1$

(b) $T(n) = C_2 n^3 + C_3 \in O(n^3)$

Then, $C_2 n^3 + C_3 \le C_2 n^3 + C_3 n^3 \le (C_2 + C_3) n^3$ for all $n \geq 1$

so $T(n) = C_2 n^3 + C_3 \le C n^3$ for $C = C_2 + C_3$ & $n_0 = 1$

$T(n) = C_2 n^3 + C_3$

Then, $C_2 n^3 + C_3 \geq C_2 n^3$ for all $n \geq 1$

so, $T(n) \geq C \cdot n^3$ when $C = C_2$ & $n_0 = 1$

Therefore, $T(n)$ is in $\Omega(n^3)$

(c). $T(n) = C_4 n \log_n + C_5 n$

$C_4 n \log_n + C_5 n \le C_4 \cdot n \log_n + C_5 \cdot n \cdot \log_n$

$\le (n \cdot \log_n)(C_4 + C_5)$

$\le C \cdot (n \cdot \log_n)$ for $n > 0$ & $C > 0$

Therefore, $T(n) \in O(n \cdot \log_n)$ when $C = C_4 + C_5$ & $n_0 = 0$.

$T(n) = C_4 n \log_n + C_5 n$

$C_4 n \log n + C_5 n \geq C_4 n \log_n \geq C \cdot n \cdot \log_n$ for $C > 0$ & $n > 0$

Therefore, $T(n) \in \Omega\,(n \cdot \log_n)$ when $C = C_4$

(d) $T(n) = n^6 2^n + C_7 n^6$

$C_6 2^n + C_7 \cdot n^6 \le C_6 \cdot 2^n \cdot n^6 + C_7 \cdot 2^n \cdot n^6$

$\le (C_6 + C_7) \cdot (2^n \cdot n^6)$

$\le C \cdot (2^n \cdot n^6)$ for $C > 0$ & $n > 0$

Therefore, $T(n) \in O(2^n \cdot n^6)$ when $C = C_6 + C_7$

$T(n) = C_6 2^n + C_7 n^6$

$C_6 2^n + C_7 n^6 \geq C_6 \cdot 2^n \geq C \cdot 2^n$ for $C > 0$ & $n > 0$

Therefore, $T(n) \in \Omega(2^n)$ when $C = C_6$.

3.10 (a) $T(n) = 2n$, $g(n) = 3n$

$\lim_{n \to \infty} \frac{T(n)}{g(n)} = \lim_{n \to \infty} \frac{2n}{3n} = \frac{2}{3}$, which is a constant number.

So, $2n = \Theta(3n)$

(b) $T(n) = 2^n$ $g(n) = 3^n$

$\lim_{n \to \infty} \frac{T(n)}{g(n)} = \lim_{n \to \infty} \frac{2^n}{3^n} = \lim_{n \to \infty} \left(\frac{2}{3}\right)^n = 0$

So, $g(n)$ goes faster than $T(n)$

Therefore, $2^n \in O(3^n)$

and $2^n \notin \Theta(3^n)$

3.12 (a) $\Theta(1)$

$\Theta(1)$

(b) $\Theta(C_1 + C_2 \cdot 3n)$ is simply $\Theta(n)$

(c) $\Theta(C_1 + C_2 \cdot n \cdot n)$ is simply $\Theta(n^2)$

(d) $\Theta\left(\sum_{i=1}^{n-1}\right) = \Theta\left(\frac{(n-1) \cdot n}{2}\right)$

$= \Theta\left(\frac{n^2 - n}{2}\right) = \Theta(n^2)$

(e) $\Theta(C_1 + C_2 \cdot n \cdot n) = \Theta(n^2)$

(f) $\Theta(C_1 + C_2 \cdot n \cdot \lg n) = \Theta(\cancel{n^2} \log n \cdot n)$

(g) $\theta(n^2 \cdot (max(c_1, n\log n)))$
$= \theta(n^2 \cdot n \cdot \log n) = \theta(n^2 \cdot \log n)$

(h) $\theta(c_1 + (n \cdot \frac{n}{2}) \cdot c_2)$
$= \theta(n^2)$

(i) $\theta(c_1 + \frac{c_2}{2} \cdot n + \frac{c_3}{2}) = \theta(\frac{n+1}{2}) = \theta(n)$

3.13

proof: a. $f(n) = \theta(f(n))$ for every n since $1 \cdot f(n) \le f(n) \le 1 \cdot f(n)$.
So it's reflective

b. Assume $f(n) = \theta(g(n))$
There exists $c_1, c_2 > 0$ that $c_1 \cdot g(n) \le f(n) \le c_2 g(n)$
Since $f(n) \le c_2\, g(n)$ & $f(n) \ge c_1 \cdot g(n)$
so $(\frac{1}{c_2}) \cdot g(n) \le g(n) \le (\frac{1}{c_1}) \cdot g(n)$
Thus, $g(n) = \theta(f(n))$
Therefore, it's symmetry.

c. Assume $f(n) = \theta(g(n))$
$g(n) = \theta(h(n))$
so, $c_1 \cdot g(n) \le f(n) \le c_1 \cdot g(n)$
$c_3\, h(n) \le g(n) \le c_4 \cdot h(n)$ when $c_1, c_2, c_3, c_4 > 0$
so, ~~f(n)~~ $c_1 \cdot c_3 \cdot h(n) \le c_1 g(n) \le f(n)$
$c_2 \cdot c_4 \cdot h(n) \le c_2 \cdot c_4 \cdot h(n) \le f(n)$
Thus, $f(n) = \theta(h(n))$
Therefore, it's transitive
$\square$