Exercise 2.2 ~~(a)~~

(a). proof: Relation is reflective, because if n is even, then $n+n$ is even, $n \equiv n$. so nRn.

Relation is symmetric, because if m, n are even, then $m \equiv n$, so mRn

Relation is transitive, because if m, n, p are even, such that mRn, nRp, ~~then $m \equiv n \equiv p$, so mRp.~~

So it's a equivalence relation.

(d) proof: Relation is reflective because if n is a nonzero rational number, then $n/n = 1$ is an integer

Relation is symmetric because if m, n are nonzero rational number such that mRn if $m = n$, then m/n is an integer

Relation is not transitive

So it's not a equivalence relation

2.3. (a) False, because it's neither antisymmetric or transitive

(b) True, because it's both antisymmetric and transitive

(c) True

(d) False

(e) False

(f) True

2.19 (a) Proof: Base case let $p(n) = n^2 - n$

So $P(1) = 1 - 1 = 0$, which is even.

Assume $n^2 - n = 2k$, $k \in \mathbb{Z}$, we have

Induction: True for $n = k+1$

$(k+1)^2 - (k+1) = k^2 + k = k(k+1)$, which is always even proof:

(b). Assume n is even, then $n=2k$, $k \in \mathbb{Z}$

Then $n^2 = (2k)^2 = 4k^2$, which is even

Then $n^2 - n = 4k^2 - 2k = 2k(2k-1)$

Since $k \in \mathbb{Z}$ and $2k-1 \in \mathbb{Z}$, so $2k(2k-1)$ is even.

So $n^2 - n$ is even

$\square$.

(c) proof: Base step: Let $P(n) = n^3 - n$

For $n=1$, $P(1) = 1^3 - 1 = 0$, which is divisible by 3.

Induction: $(n+1)^3 - (n+1) = n^3 + 3n^3 + 3n + 1 - n - 1$

$= (n^3 - n) + 3(n^2 + n)$

which is $n^3 - n$ plus a multiple of 3

Since we assumed $n^3 - n$ is divisible by 3, then $(n+1)^3 - (n+1)$ is also divisible by 3.

So, Since $n^3 - n$ is divisble by 3 is true for 1 and $n+1$, the statement is true for all whole number n. $\square$

(d) proof: Base step: for $n=1$, $n^5 - n = 0$, which is divisible by 5.

Induction: Assume $k \in \mathbb{N}$, $(k+1)^5 - (k+1) = (k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1) - (k+1) = (k^2 - k) + 5k^4 + 10k^3 + 10k^2 + 5k$, which is divisible by 5.

So $n^5 - n$ is divisible by 5 $\square$.

2.30 (b) Base case: for $n=1$, we have 1 hole and more than 1 pigeons already in the hole. So it's true for $n=1$.

We have s pigeons ($s > n+1$) roost in $n+1$ holes, from hole 1 to hole $n+1$.

If hole $n+1$ is roosted by more than 1 pigeons, we are done

$\square$