GDP Convergence

Macro Measurement Project 2

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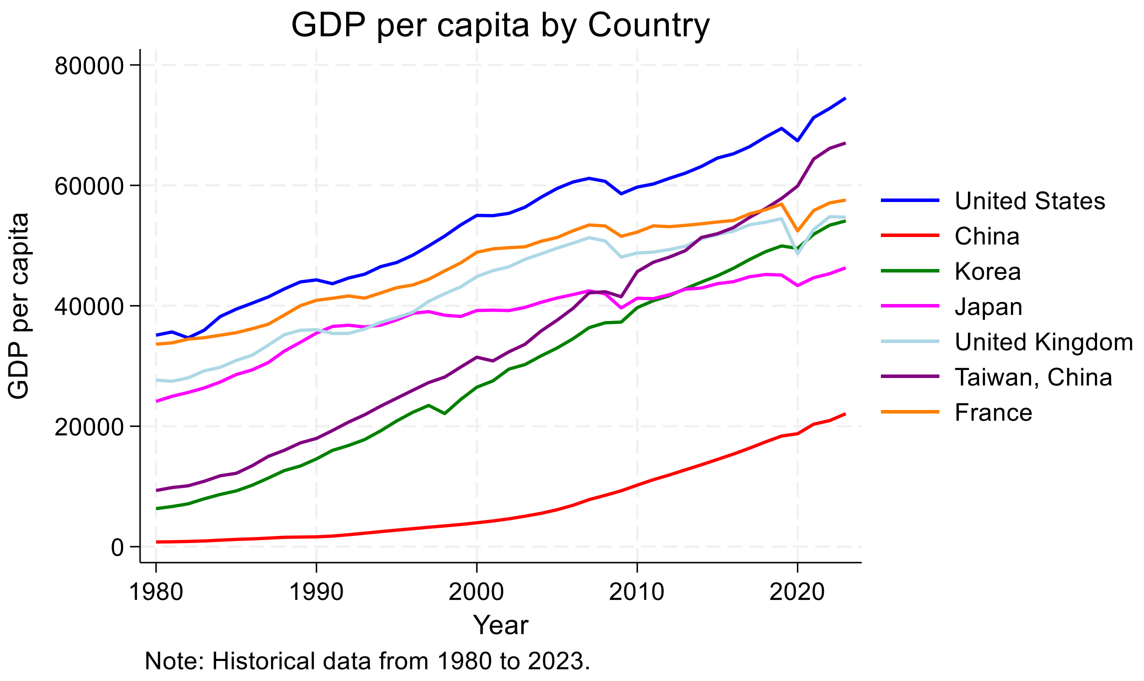
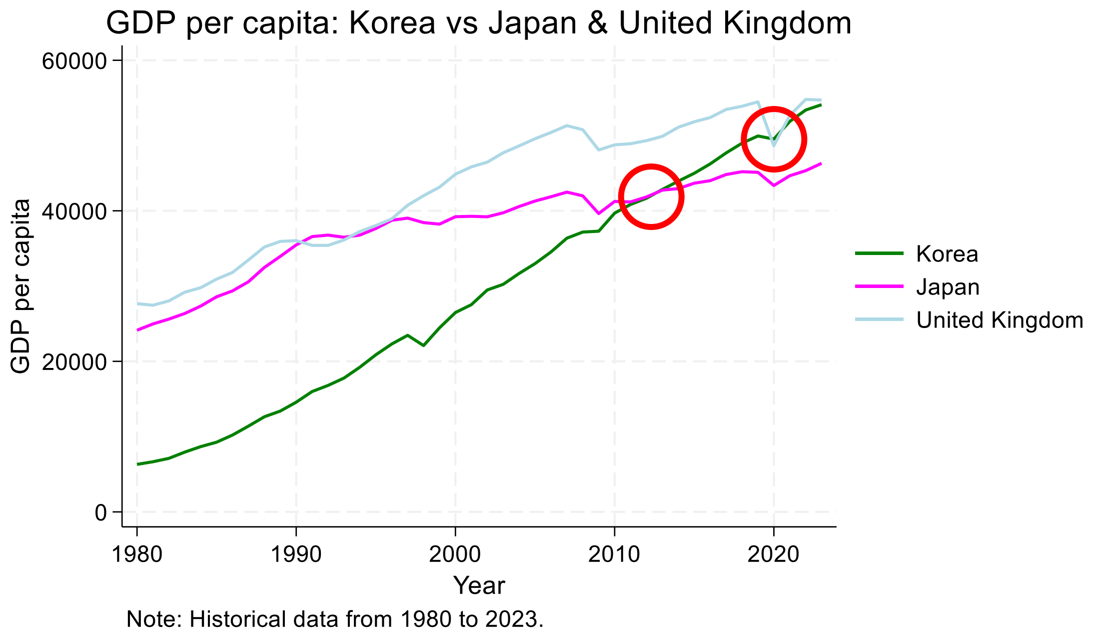
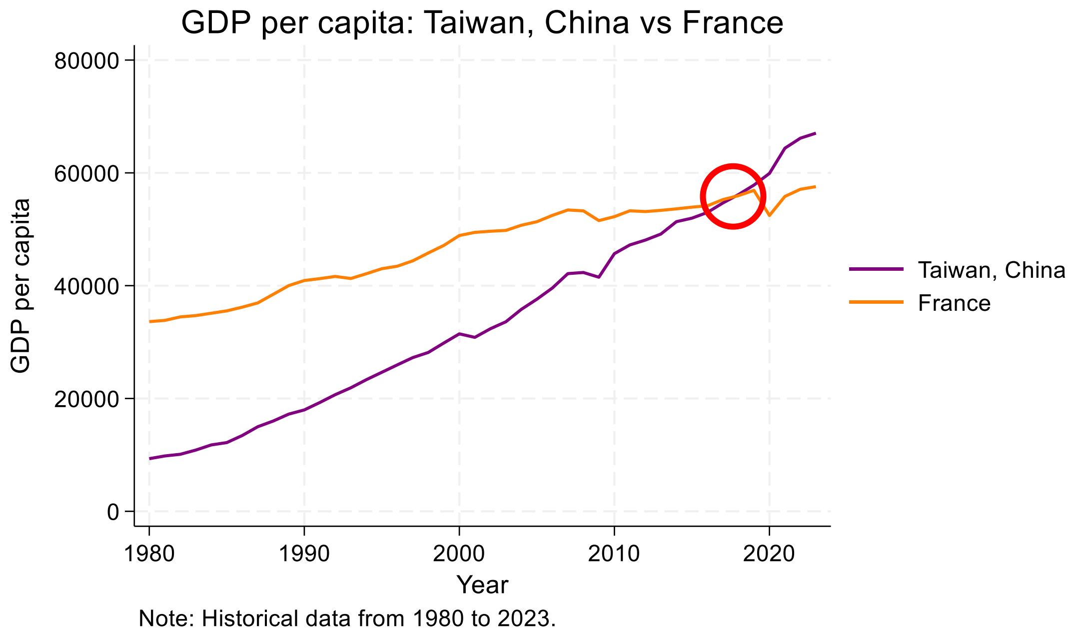
Background

- Research question:
  - Per capita GDP is a key indicator of residents' living standards and economic development.
  - China's per capita GDP has grown rapidly in decades; United States grows moderately but stably.
  - We want to ask: **will China's per capita GDP surpass United States' or achieve convergence? If so, which prediction model has the best predictive ability? What about the case between Japan and South Korea?**
- Data:
  - Source: per capita GDP from WEO (measured in purchasing power parity, 2017 international dollar)
  - Time period: 1980 – 2023
  - Sample countries: USA, China, South Korea, Japan, UK, Taiwan Province of China, France
- Contributions:
  - Use authoritative data & econometric models to explore per capita GDP convergence between countries.
  - **Provide a comparison among different prediction models and estimation windows** by checking whether a specific model successfully predict established facts or not.
  - Among Asian developed economies, Korea first surpassed Japan in per capita GDP (PPP-adjusted, 2017 international dollar) in 2013. This study will explore the subsequent evolution of their per capita GDP gap (e.g., whether it widens or leads to a crossover again).
- Potential Applications:
  - A reference of the robustness and predictive performance of AR(1) models for forecasting per capita GDP in the context of open economies.

Historical Data and Facts





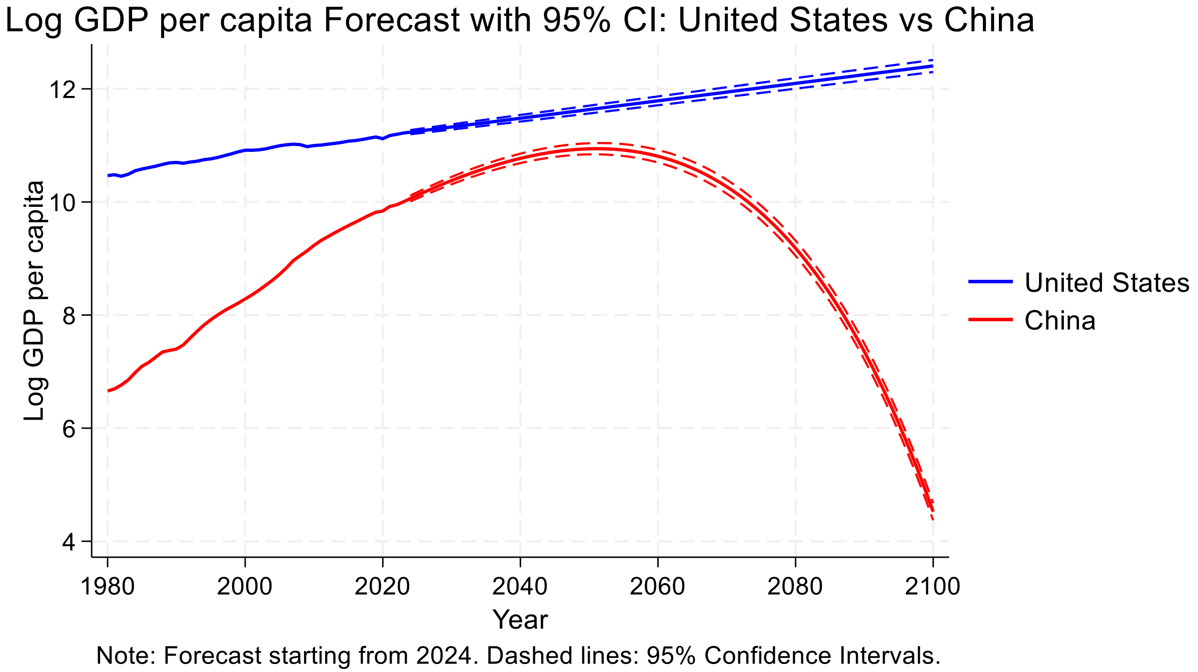


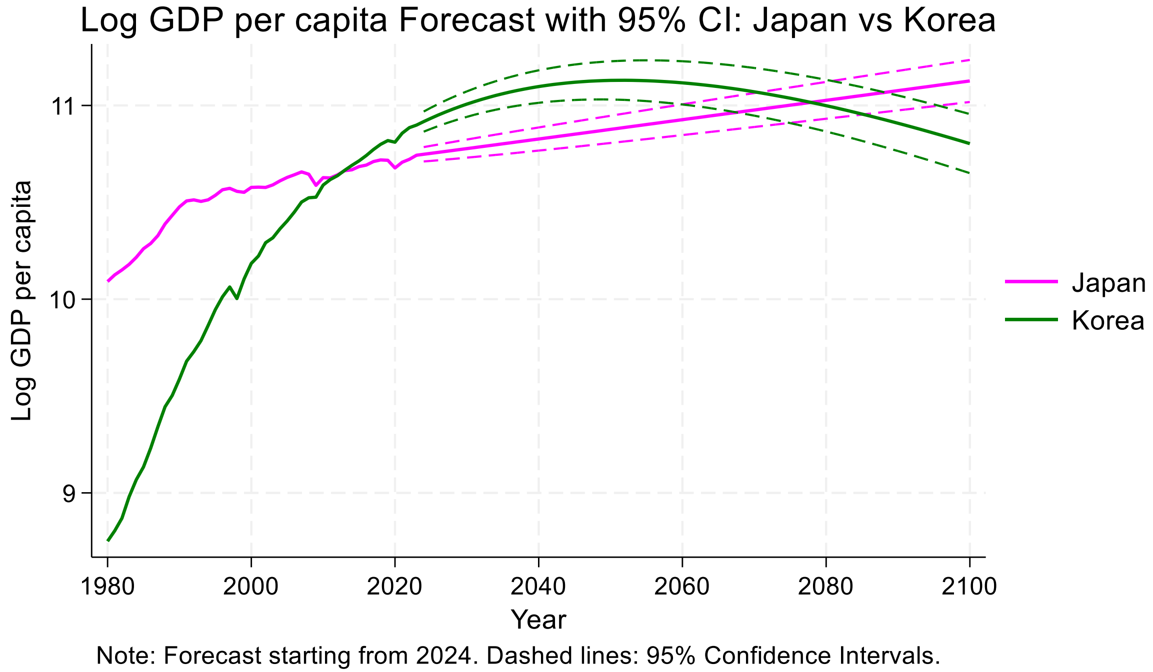
- Three notable crossovers in history:
  - Korea surpassed Japan in 2013
  - Korea surpassed UK in 2020
  - Taiwan surpassed France in 2018
- Whether Korea or Japan was the chaser depends on the estimation windows chosen.
- Use the facts to check models’ predictive ability.

Methodology

- Model specification: $\ln(y_t) = \alpha + \beta t + \gamma \ln(y_{t-1}) + u_t,\ u_t \sim \mathrm{N}(0, \sigma^2)$
- Estimation window: 1980 – 2023
- Predicting method:
  - Step 1: For every country, run the regression over the estimation window using OLS. Obtain $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\sigma}$.
  - Step 2: To predict $\widehat{\ln(y_{2024})}$, use t = 2024 and actual data $\ln(y_{2023})$. Then compute the 95% confidence interval. 95% CI = $\widehat{\ln(y_{2024})} \pm 1.96 \times \hat{\sigma}$.
  - Step 3: To predict $\widehat{\ln(y_{2025})}$, use t = 2025 and the predicted data $\widehat{\ln(y_{2024})}$. For the 95% CI, we prefer a gradually widening CI to one with equal width. Define a standard deviation expanding over time (a rule of thumb): $s.e._{2025} = \hat{\sigma} \cdot \sqrt{1 + (2025 - 2024) \times 0.1} = \sqrt{1.1} \cdot \hat{\sigma}$. 95% CI = $\widehat{\ln(y_{2025})} \pm 1.96 \times s.e._{2025}$.
  - Step 4: To predict $\widehat{\ln(y_{2026})}$, use t = 2026 and the predicted data $\widehat{\ln(y_{2025})}$, with a lager standard deviation than $s.e._{2025}$: $s.e._{2026} = \hat{\sigma} \cdot \sqrt{1 + (2026 - 2024) \times 0.1} = \sqrt{1.2} \cdot \hat{\sigma}$. 95% CI = $\widehat{\ln(y_{2026})} \pm 1.96 \times s.e._{2026}$.
  - Step 5: The same logic applies to predicting $\widehat{\ln(y_t)}$ until t = 2100. For any year t, define the standard deviation $s.e._t = \hat{\sigma} \cdot \sqrt{1 + (t - 2024) \times 0.1}$. 95% CI = $\widehat{\ln(y_t)} \pm 1.96 \times s.e._t$.
- Calculate the range of crossover year: $[\underline{t}, \overline{t}]$
  - From the first year $\underline{t}$ the CIs of the two countries intersect, to the last year $\overline{t}$ they intersect.
- Calculate the range of crossover level of GDP per capita: $[\underline{y}, \overline{y}]$
  - Apply linear interpolation to the 95% CI bounds. The range is defined by the lowest and highest values among all calculated intersection points.

Main Findings





- Model specification: $\ln(y_t) = \alpha + \beta t + \gamma \ln(y_{t-1}) + u_t$
- Estimation window: 1980 – 2023
- Findings:
  - China is predicted to **never exceed** United States in per capita GDP.
  - Japan is predicted to retake the lead in per capita GDP over South Korea around the year **2078**. Considering the 95% CI, this crossover is projected to occur within an interval of **2060 to 2095**, at a corresponding GDP per capita level ranging from **54,706 to 67,941** (2017 international dollar), in purchasing power parity.

Main Findings

Table 1: Regression Results for Log-log AR(1) Model with Linear Trend

	(1)	(2)	(3)	(4)
Variable	United States	China	Japan	Korea
Year	0.0020	-0.0031	0.0005	-0.0004
	(0.0013)	(0.0055)	(0.0006)	(0.0015)
Lagged Log GDP per capita	0.8678***	1.0305***	0.9818***	0.9723***
	(0.0737)	(0.0669)	(0.0386)	(0.0282)
Constant	-2.6268	6.1050	0.0659	1.1584
	(1.7451)	(10.5282)	(0.7622)	(2.6252)
Observations	43	43	43	43
R-squared	0.993	0.999	0.988	0.998

Notes: The dependent variable is Log GDP per capita. Standard errors are in parentheses.
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

- Explanation:
  - Mathematically, it is a **negative coefficient** of linear trend t, i.e., $Year$, that results in a decrease in per capita GDP for China and Korea after hitting a peak.
  - Empirically, it is likely that **COVID-19** caused China and Korea to deviate from their long-term per capita GDP growth paths. At least, they were more vulnerable to Covid-19 than United States and Japan.

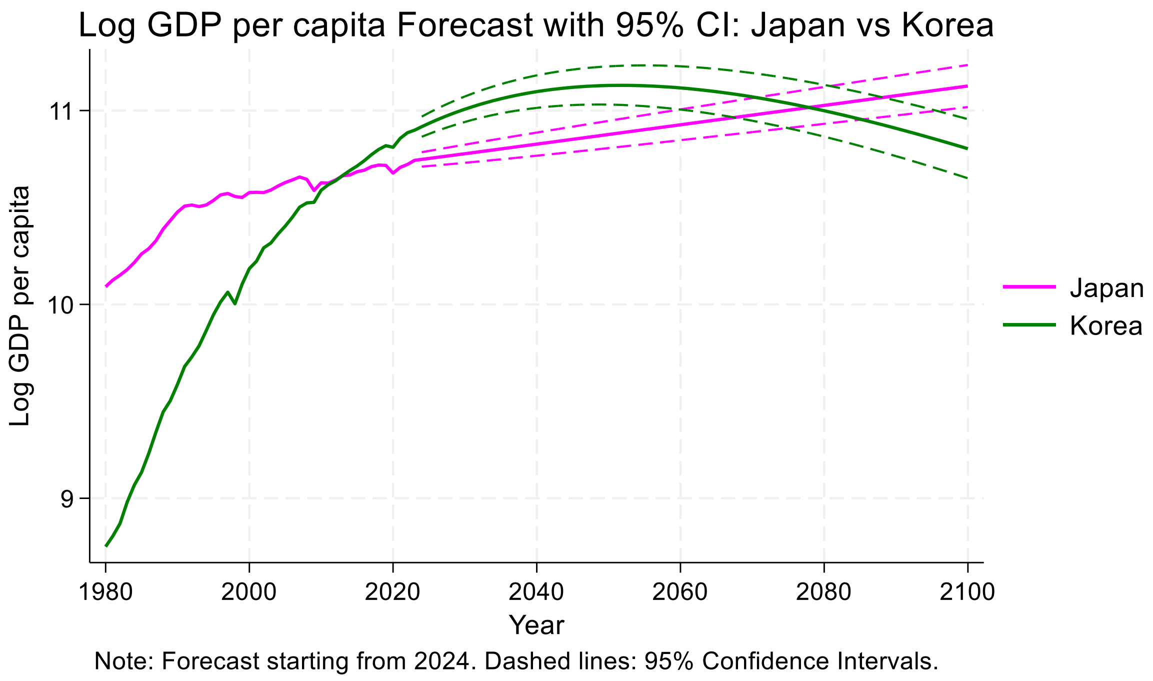
Sensitivity Analysis

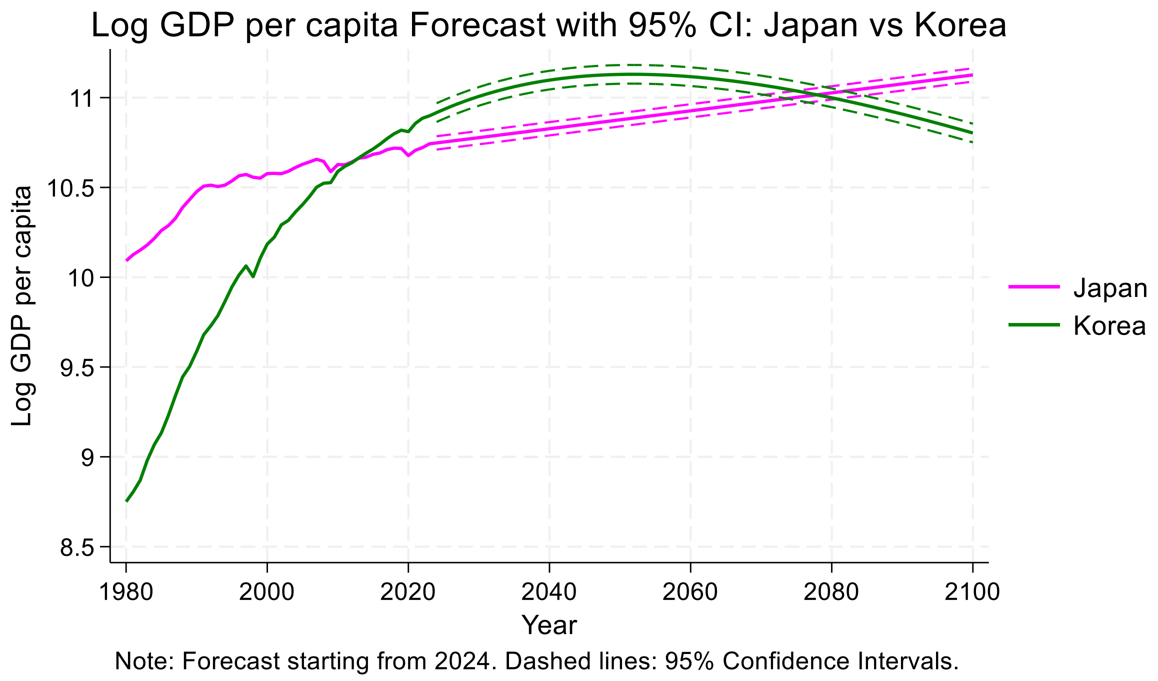
- Sensitivity analysis 1:
  - We use $s.e._t = \hat{\sigma} \cdot \sqrt{1 + (t - 2024) \times 0.1}$ as a rule of thumb to represent an expanding standard deviation and an increasingly widening 95% confidence band.
  - What if a constant standard deviation, i.e., $s.e._t = \hat{\sigma}$ for any year t, and thus a constant-width 95% confidence band? And what about the Delta Method?
- Sensitivity analysis 2:
  - We use a log-log AR(1) model and assume a linear trend.
  - What if a level-level model and a quadratic trend?
  - We test the following models:

$$\ln(y_t) = \alpha + \beta t + \gamma t^2 + \delta \ln(y_{t-1}) + u_t$$
$$y_t = \alpha + \beta t + \gamma y_{t-1} + u_t$$
$$y_t = \alpha + \beta t + \gamma t^2 + \delta y_{t-1} + u_t$$

- Sensitivity analysis 3:
  - We use the estimation window from 1980 to 2023.
  - What if 1980 – 2019 to exclude the impact of Covid-19?
  - What if 1980 – 2007 to exclude the impact of 2008 Financial Crisis?
  - What if 1980 – 1999 to exclude the impact of China's entrance to WTO?
- Additional question: which model has the best predictive ability?
  - We check whether a specific model successfully predict selected facts or not.

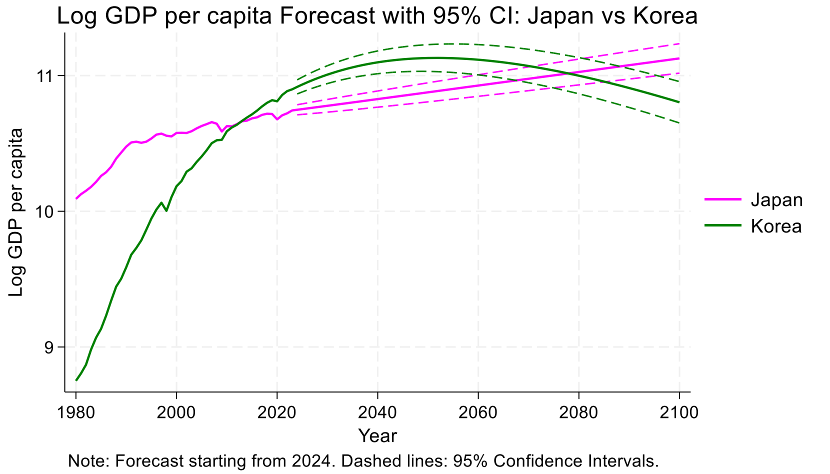
Sensitivity Analysis 1 – Constant-width CI

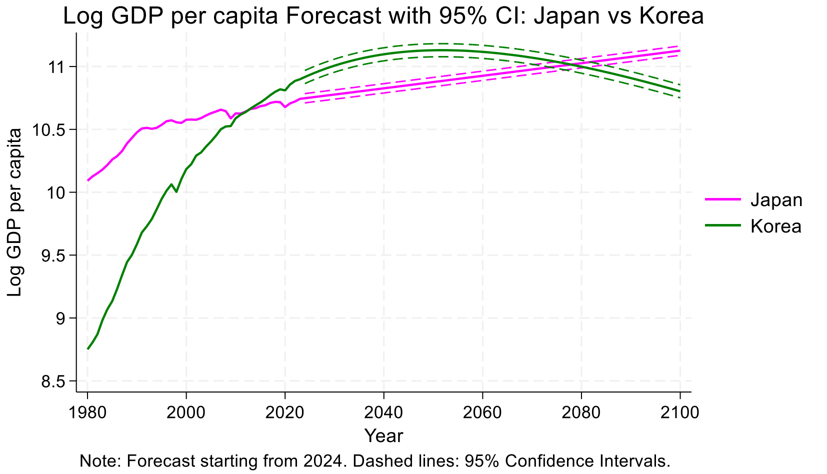


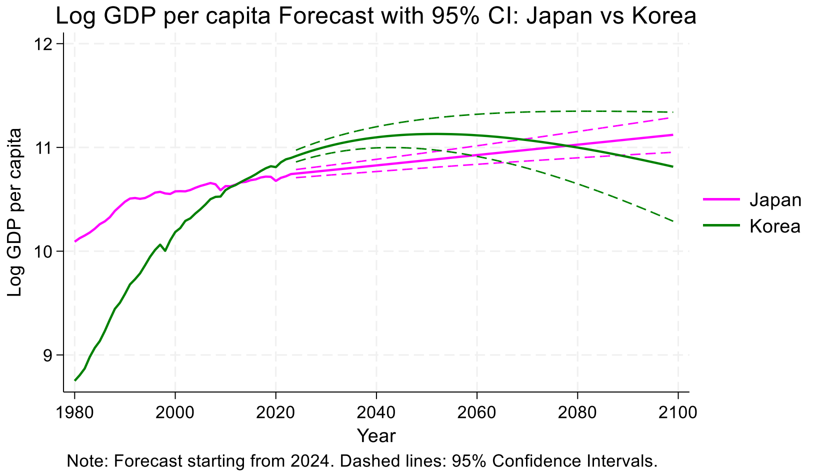


- Model specification: $\ln(y_t) = \alpha + \beta t + \gamma \ln(y_{t-1}) + u_t$
- Estimation window: 1980 – 2023
- Findings:
  - For the case in Japan and Korea, after changing to a constant standard deviation $s.e._t = \hat{\sigma}$, the **overlapping area becomes smaller**, with a **narrower** crossover time interval of **2071 to 2084**, compared to a previous 2060 to 2095, at a corresponding GDP per capita level ranging from **58,277 to 63,495**, (2017 international dollar), in purchasing power parity, compared to a previous range of 54,706 to 67,941.
  - We remain using $s.e._t = \hat{\sigma} \cdot \sqrt{1 + (t - 2024) \times 0.1}$ in our following work.

Sensitivity Analysis 1 – Delta Method



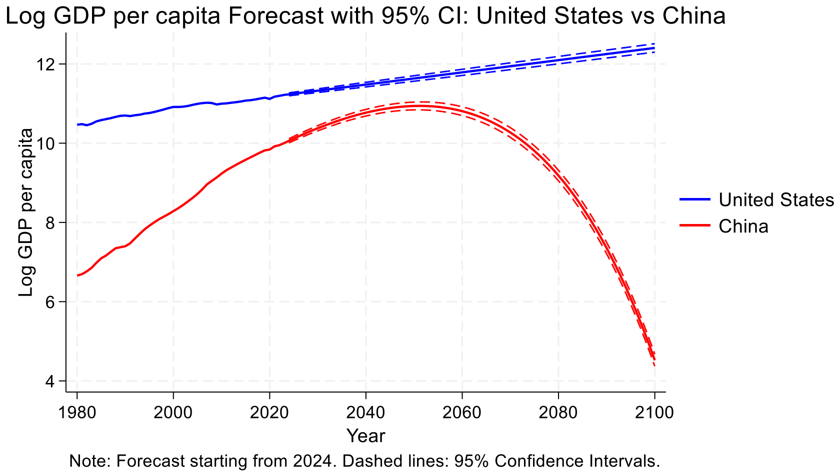


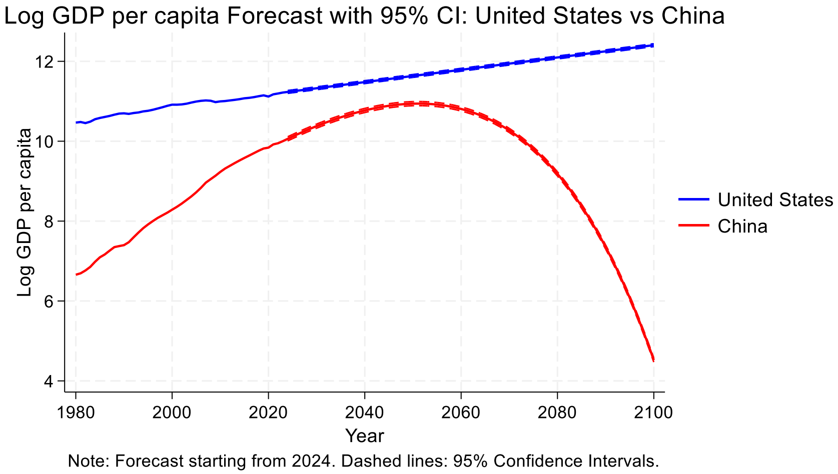


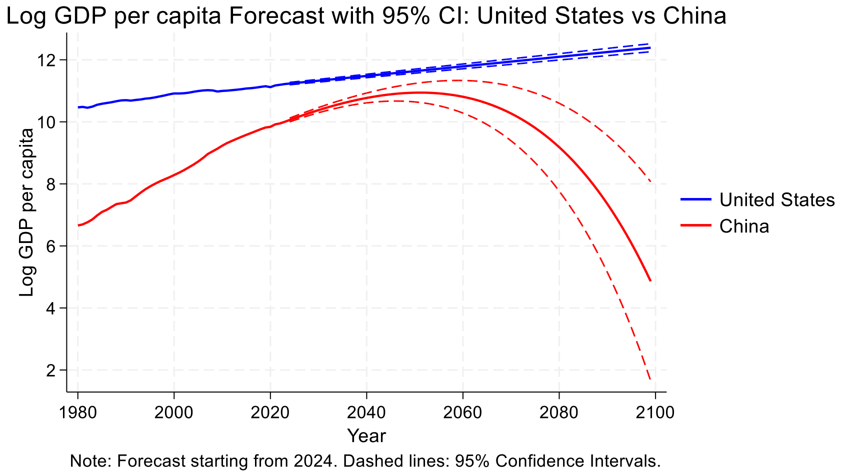
$$s.e._t = \hat{\sigma} \cdot \sqrt{1 + (t - 2024) \times 0.1}$$

$$s.e._t = \hat{\sigma}$$

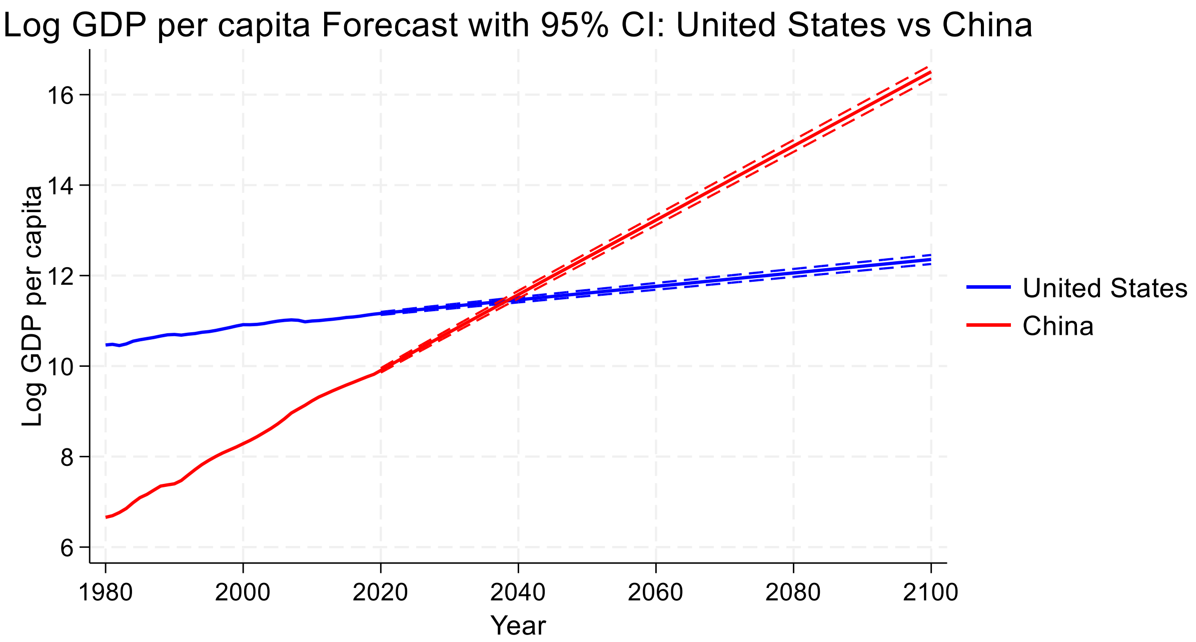
Delta Method



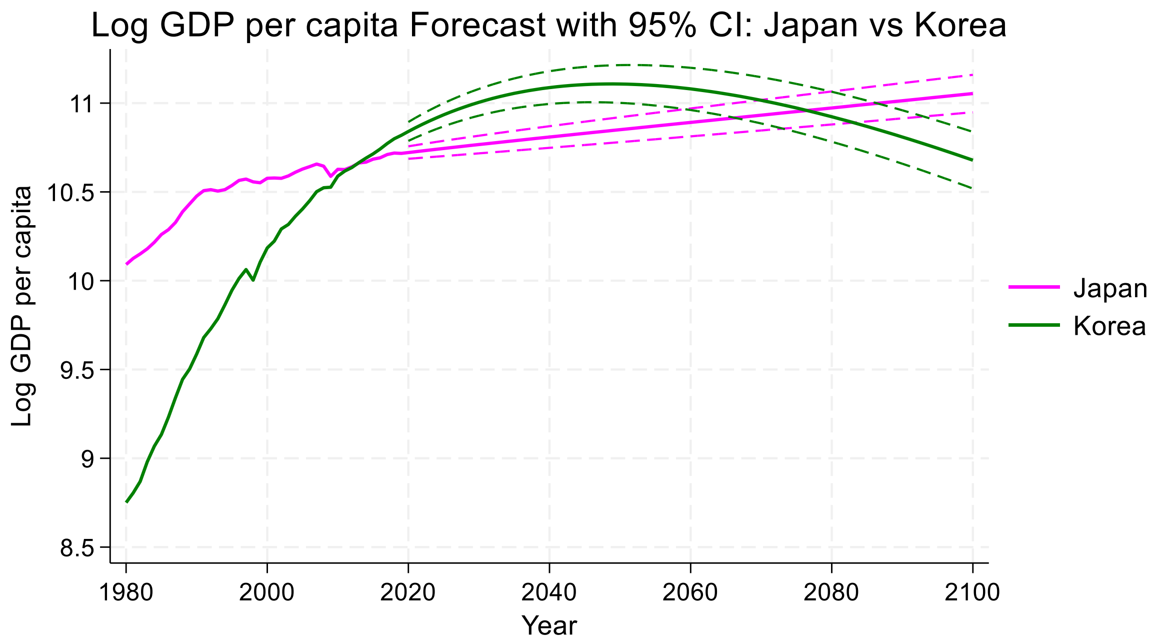




Sensitivity Analysis 2 & 3 – Example 1



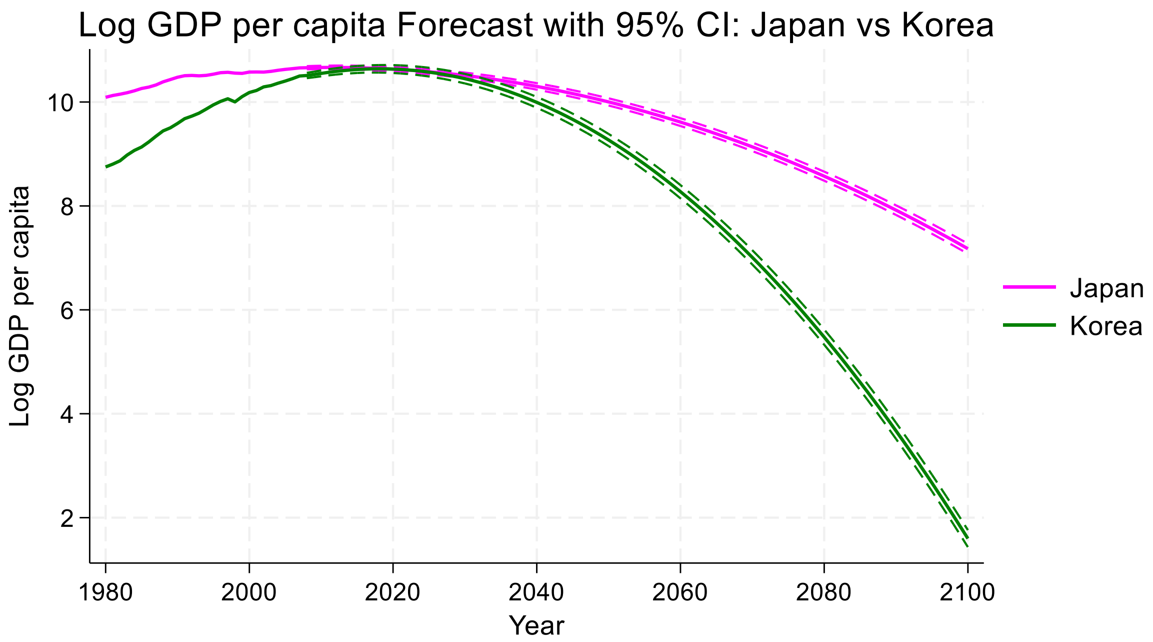
Note: Forecast starting from 2020. Dashed lines: 95% Confidence Intervals.



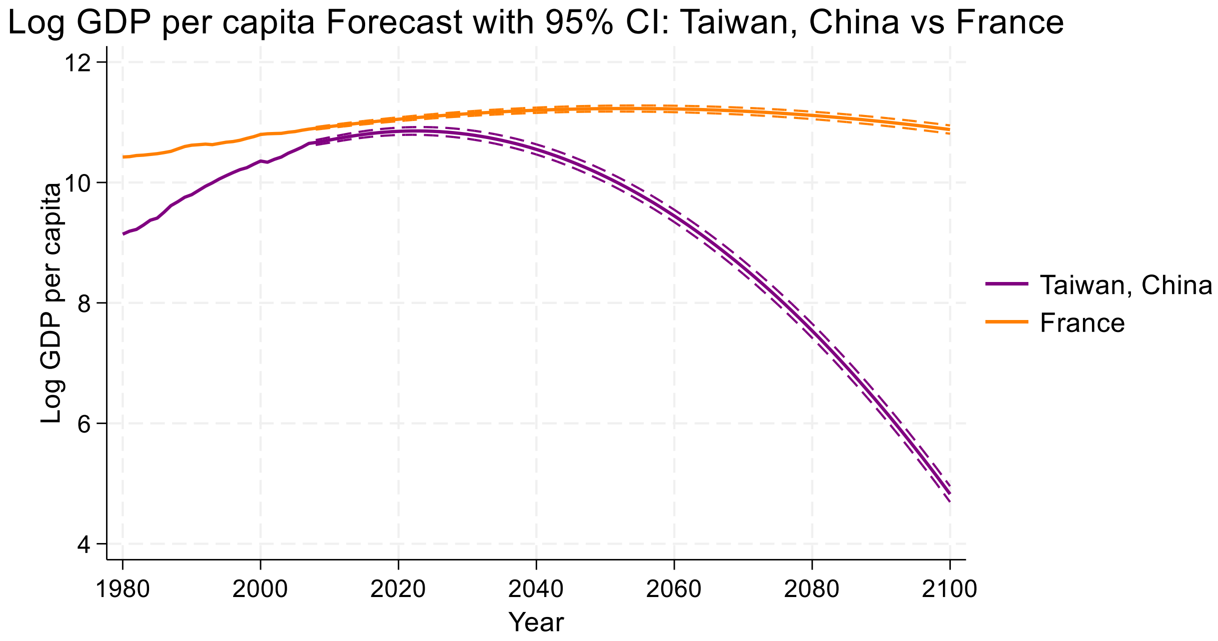
Note: Forecast starting from 2020. Dashed lines: 95% Confidence Intervals.

- Model specification: $\ln(y_t) = \alpha + \beta t + \gamma \ln(y_{t-1}) + u_t$
- Estimation window: 1980 – 2019
- Findings:
  - Using the **same model**, after **excluding the impact of Covid-19**, China's per capita GDP is predicted to exceed United States' around the year 2039; Japan's per capita GDP is predicted to exceed Korea's around the year 2077, compared to a previous 2078.
  - The log-log AR(1) model with linear trend is **sensitive** to the choice of estimation window for the United States and China case, but is relatively **robust** for the Japan and Korea case.

Sensitivity Analysis 2 & 3 – Example 2



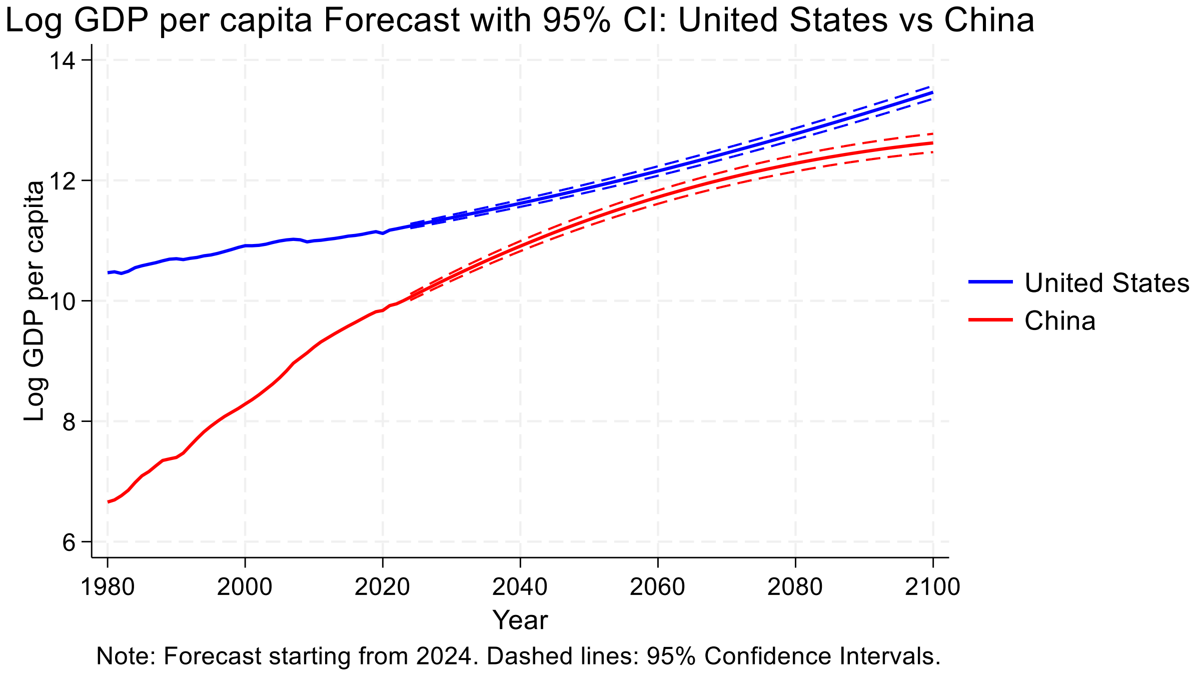
Note: Forecast starting from 2008. Dashed lines: 95% Confidence Intervals.

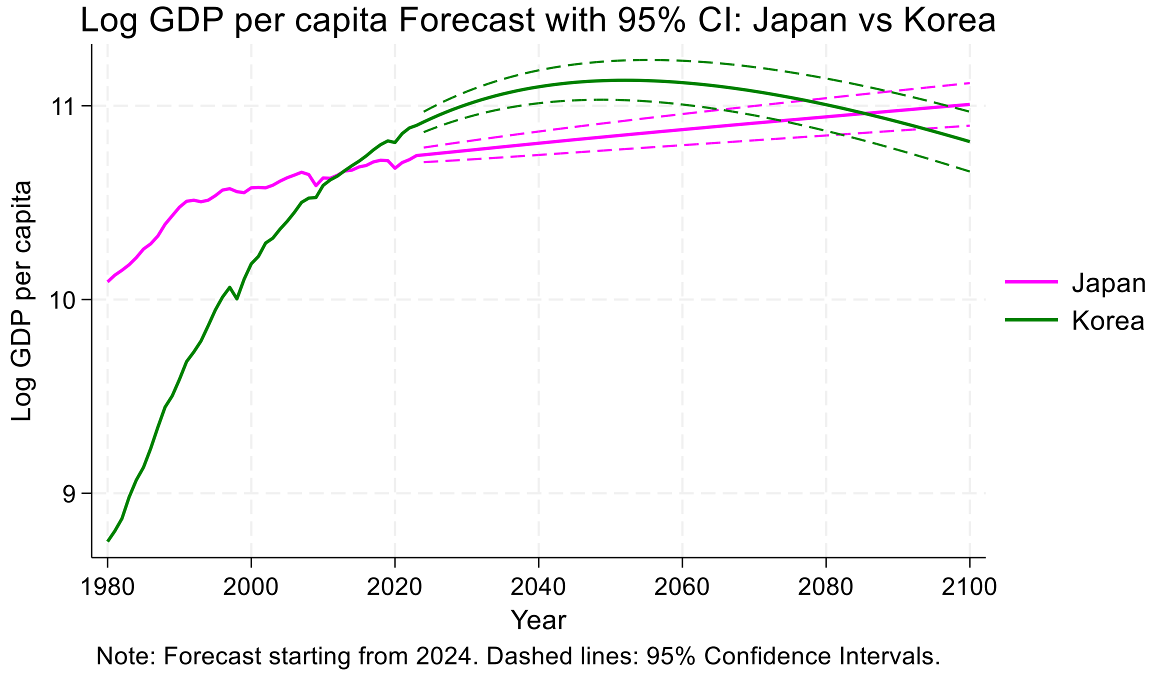


Note: Forecast starting from 2008. Dashed lines: 95% Confidence Intervals.

- Model specification: $\ln(y_t) = \alpha + \beta t + \gamma t^2 + \delta \ln(y_{t-1}) + u_t$
- Estimation window: 1980 – 2007
- Findings:
  - Under this estimation window, **Korea is the chaser** at the beginning of the forecast. Using a different log-log AR(1) model with **quadratic trend**, the crossover time range is 2012 – 2034. This combination of regression model and estimation window **seems to be credible**, because Korea did surpass Japan in 2013.
  - However, if we apply this model to the country pair of Taiwan and France, it **fails to predict** their known crossover in 2018.

Sensitivity Analysis 2 & 3 – Example 3





- Model specification: $\ln(y_t)^A = \alpha_1 + \beta_1 t + \gamma_1 \ln(y_{t-1})^A + \delta_1 \ln(y_{t-1})^B + u_t$

$$\ln(y_t)^B = \alpha_2 + \beta_2 t + \gamma_2 \ln(y_{t-1})^A + \delta_2 \ln(y_{t-1})^B + \varepsilon_t$$

- Estimation window: 1980 – 2023
- Findings:
  - The VAR model can avoid situations where one of the country’s GDP growth path goes all the way down, because of a positive correlation between two countries’ per capita GDP.
  - However, this “lift-up” power is limited, so it cannot guarantee a convergence.

Sensitivity Analysis – All Results

Model specification	Estimation window	United States vs China			Japan vs Korea			Does the crossover year range of this combination of model and estimation window correctly predict the facts?		
		The chaser before forecast	Crossover year (range)	Crossover level of GDP per capita (range)	The chaser before forecast	Crossover year (range)	Crossover level of GDP per capita (range)	Korea surpassed Japan in 2013	Taiwan surpassed Fance in 2018	Korea surpassed UK in 2020
Log-log + AR(1) + linear trend: $\ln(y_t) = \alpha + \beta t + \gamma \ln(y_{t-1}) + u_t$	1980 – 2023	China	No cross by 2100		Japan	2060 – 2095	54,706 – 67,941	N/A	N/A	N/A
	1980 – 2019	China	2037 – 2040	85,529 – 101,738	Japan	2060 – 2093	51,867 – 63,904	N/A	N/A	✓
	1980 – 2007	**China**	**2037 – 2041**	**108,278 – 137,392**	**Korea**	**2009 – 2013**	**41,643 – 45,906**	✓	✓	✓
	1980 – 1999	China	2039 – 2044	118,172 –162,384	Korea	2004 – 2005	32,046 – 35,977	✗	✗	✓
Log-log + AR(1) + quadratic trend: $\ln(y_t) = \alpha + \beta t + \gamma t^2 + \delta \ln(y_{t-1}) + u_t$	1980 – 2023	China	No cross by 2100		Japan	2029 – 2040	46,498 – 52,240	N/A	N/A	N/A
	1980 – 2019	China	2043 – 2052	66,348 – 75,913	Japan	2022 – 2036	44,976 – 50,223	N/A	N/A	✗
	1980 – 2007	China	2030 – 2032	79,131 – 91,592	Korea	2012 – 2034	41,411 – 44,301	✓	✗	✗
	1980 – 1999	China	2053 – 2072	90,005 – 110,494	Korea	2006 – 2009	27,892 – 32,066	✗	✗	✗
Level-level + AR(1) + linear trend: $y_t = \alpha + \beta t + \gamma y_{t-1} + u_t$	1980 – 2023	China	2070 – 2074	109,960 – 124,396	Japan	No cross by 2100		N/A	N/A	N/A
	1980 – 2019	China	2054 – 2056	96,369 – 105,311	Japan	No cross by 2100		N/A	N/A	✗
	1980 – 2007	China	2023	74,892 – 79,411	Korea	2012 – 2018	42,967 – 47,865	✓	✗	✗
	1980 – 1999	China	No cross by 2100		Korea	2006 – 2008	31,338 – 34,372	✗	✗	✗
Level-level + AR(1) + quadratic trend: $y_t = \alpha + \beta t + \gamma t^2 + \delta y_{t-1} + u_t$	1980 – 2023	China	2068 – 2073	106,274 – 120,454	Japan	No cross by 2100		N/A	N/A	N/A
	1980 – 2019	China	2055 – 2058	84,896 – 92,978	Japan	No cross by 2100		N/A	N/A	✗
	1980 – 2007	China	2018	71,378 – 75,359	Korea	2010 – 2012	41,519 – 44,032	✗	✓	✗
	1980 – 1999	China	No cross by 2100		Korea	2005 – 2006	31,787 – 34,451	✗	✗	✗

Sensitivity Analysis – All Results

<table>
<tr><th rowspan="2">Model specification</th><th rowspan="2">Estimation window</th><th colspan="3">United States vs China</th><th colspan="3">Japan vs Korea</th><th colspan="3">Does the crossover year range of this combination of model and estimation window correctly predict the facts?</th></tr>
<tr><th>The chaser before forecast</th><th>Crossover year (range)</th><th>Crossover level of GDP per capita (range)</th><th>The chaser before forecast</th><th>Crossover year (range)</th><th>Crossover level of GDP per capita (range)</th><th>Korea surpassed Japan in 2013</th><th>Taiwan surpassed Fance in 2018</th><th>Korea surpassed UK in 2020</th></tr>
<tr><td rowspan="4">Log-log + VAR(1) + linear trend:

$\ln(y_t)^A = \alpha_1 + \beta_1 t + \gamma_1 \ln(y_{t-1})^A + \delta_1 \ln(y_{t-1})^B + u_t$

$\ln(y_t)^B = \alpha_2 + \beta_2 t + \gamma_2 \ln(y_{t-1})^A + \delta_2 \ln(y_{t-1})^B + u_t$</td><td>1980 – 2023</td><td>China</td><td colspan="2">No cross by 2100</td><td>Japan</td><td>2066 – later than 2100</td><td>51,627 – more than 64,443</td><td>N/A</td><td>N/A</td><td>N/A</td></tr>
<tr><td>1980 – 2019</td><td>China</td><td>2038 – 2041</td><td>90,005 – 108,310</td><td>Japan</td><td>2057 – 2083</td><td>47,878 – 57,827</td><td>N/A</td><td>N/A</td><td>✗</td></tr>
<tr><td colspan="10">Haven’t done yet.</td></tr>
<tr><td colspan="10">Haven’t done yet.</td></tr>
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Conclusion & Takeaways

- Research Question:
  - Will China's per capita GDP surpass United States' or achieve convergence? If so, which model has the best predictive ability? What about the case between Japan and South Korea?
- Conclusion:
  - Whether there will be a per capita GDP convergence **depends on the assumption of models and estimation windows**. 13 out of 18 of our models predict a convergence between China and United States before 2100, and another 14 out of 18 of our models predict a convergence between Japan and Korea before 2100.
  - In terms of sensitivity, AR(1) models are **generally sensitive** to the assumption of estimation windows.
  - In terms of predictive ability or credibility, the models are **generally incredible**, because only a few of them can be supported by selected facts. Among them, the most credible model is **the log-log AR(1) model with linear trend, using the estimation window of 1980 to 2007**. It successfully predicts all the three crossover facts. However, the predicted time **range largely relies on the rule of thumb** we used to construct the 95% CI.
- Caveats/Limitations:
  - Need a more reasonable way to estimate the standard errors to construct the 95% CI.
  - We only use confidence intervals to project the range of **expected values** of dependent variable. If we want to predict the **individual values** of GDP per capita, it is more reasonable to use **prediction intervals**, with a **larger margin of error**, due to larger standard errors, than CI.
  - Fail to perform a unit root test first for the AR(1) models to avoid spurious regression.