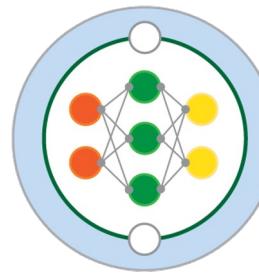


Learning Ocean Model Errors from Data Assimilation Increments

Tarun Verma, Feiyu Lu, Alistair Adcroft

01/10/2023

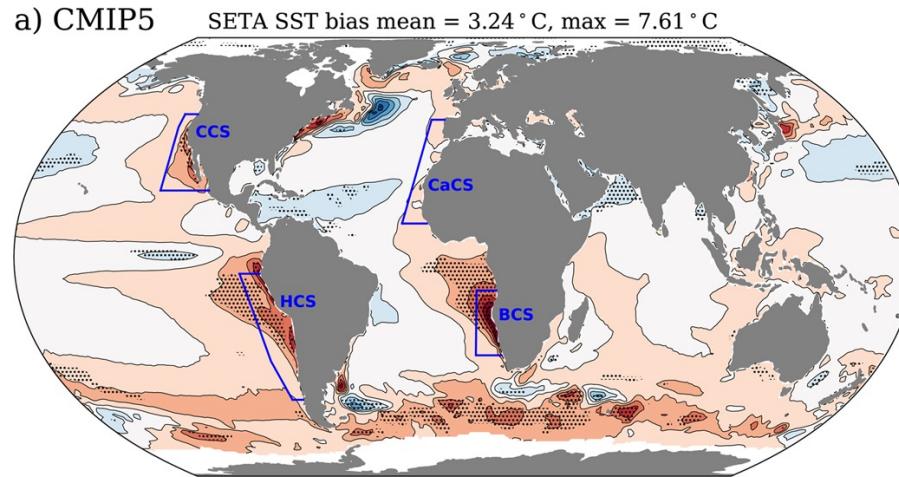




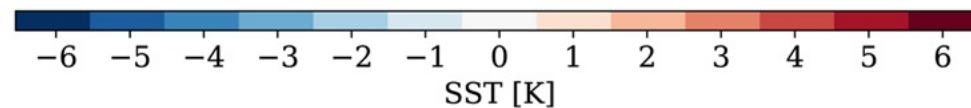
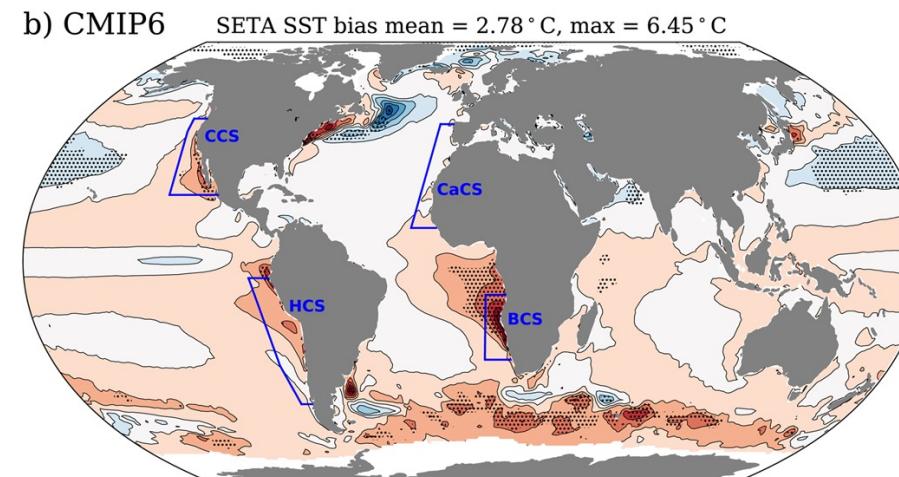
Bias: a persistent problem in climate modeling

Multi-model Mean SST Bias (Farneti et al. 2022)

CMIP5 (3.25° C)



CMIP6 (2.78° C)



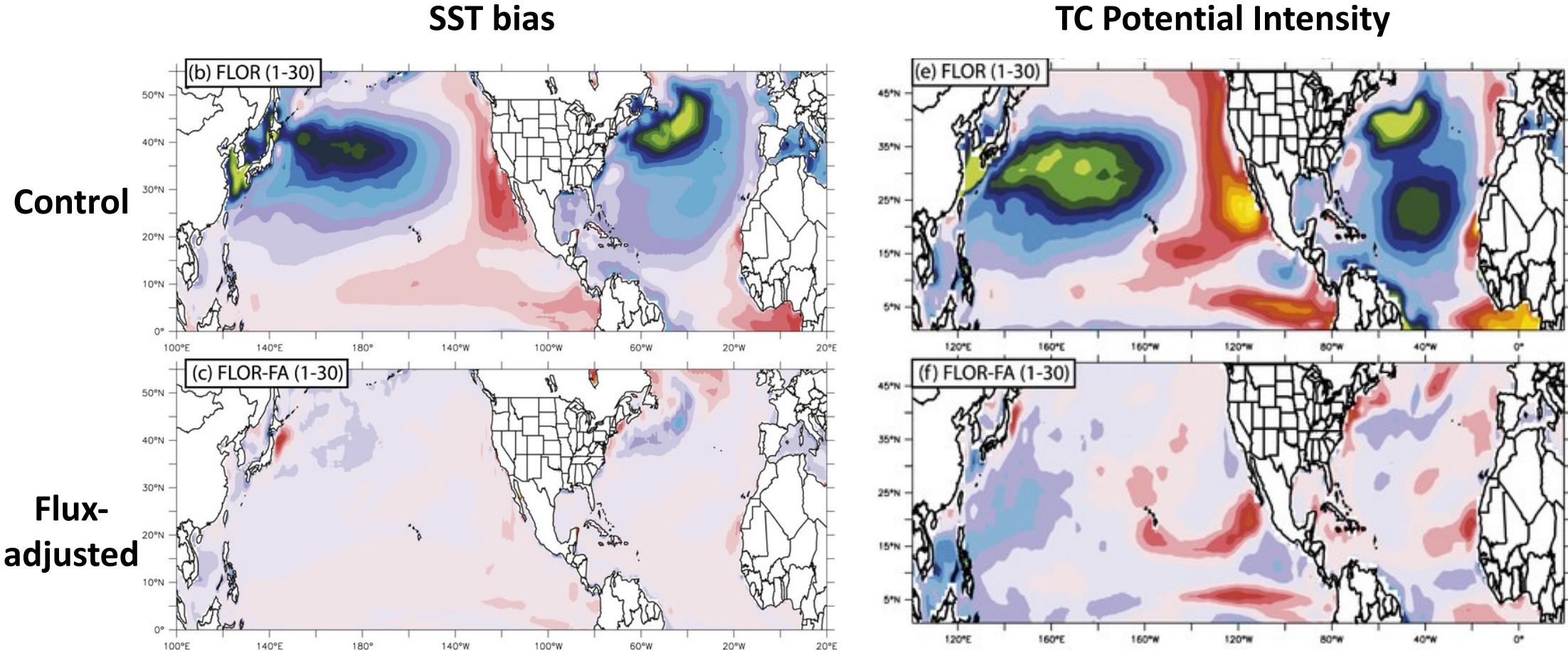
High-res MIP (1.78° C)

Ocean MIP2 (1.14° C)



SST bias affects many other important phenomena, e.g., Tropical Cyclones (TC)

Reduction in SST bias improves TC activity (Vecchi et al. 2014)

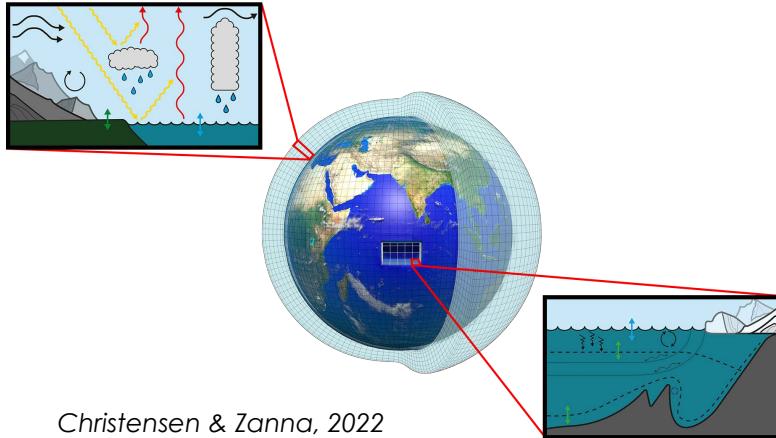




Part of the bias stems from fast physics errors

Model error

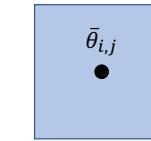
→ Poor or lacking representation of key processes



Christensen & Zanna, 2022

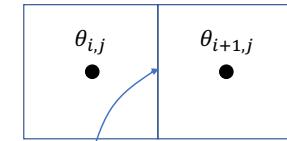
→ Numerics

a) Finite volume



$$V_{i,j} \frac{\partial \theta_{i,j}}{\partial t} = \oint \kappa \nabla \theta \cdot dA$$

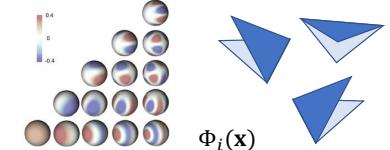
b) Finite difference



$$\frac{\partial \theta}{\partial x} \approx \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta x}$$

truncate the functional operators

c) Spectral or finite element



$$\theta(\mathbf{x}) = \sum_{i=1}^n A_i \Phi_i$$

$$\frac{\partial}{\partial t} \int W \theta \, dV = \int \kappa \nabla \theta \cdot \nabla W \, dV$$

truncate the functional space

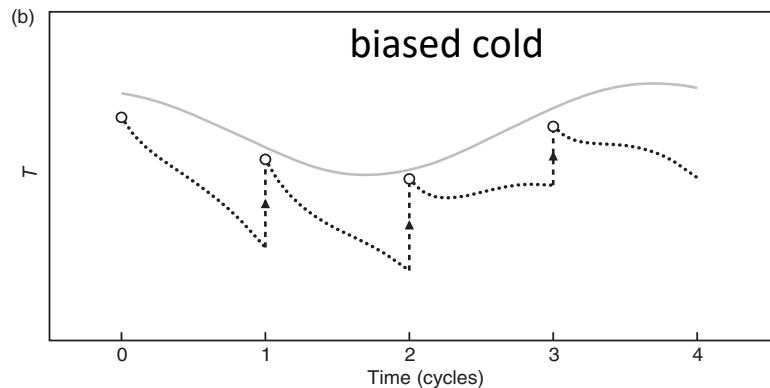
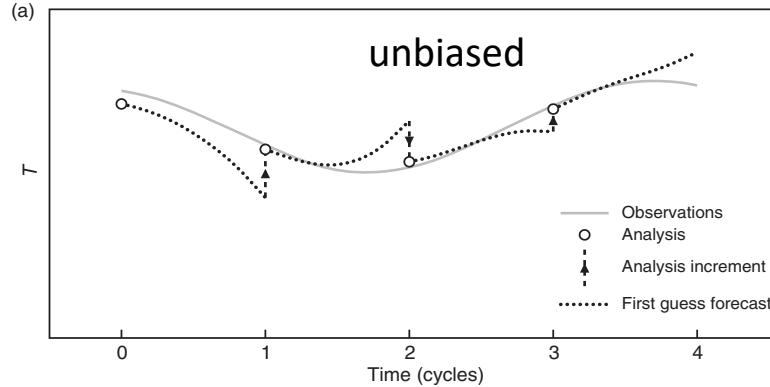
- Missing or inaccurate physics,
 - vertical mixing, sub-meso-mesoscale eddies etc.
- Numerics.

Here we propose to (machine) learn ocean model corrections at these fine-scales using data assimilation increments.



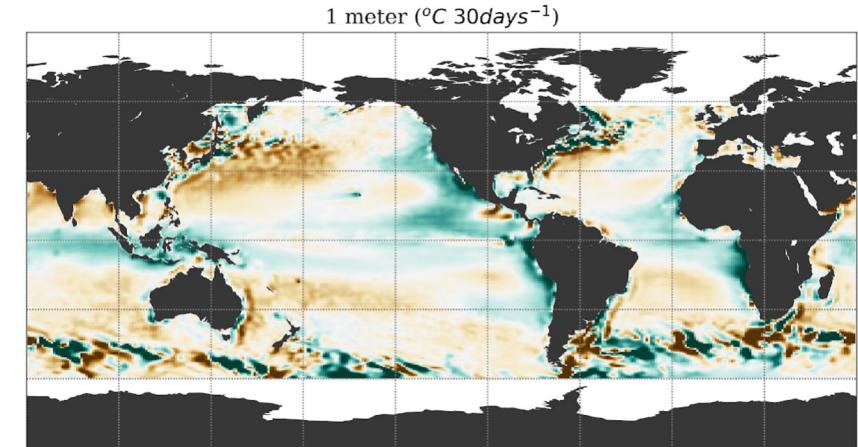
Data Assimilation (DA) Increments and Model Bias

Palmer and Weisheimer 2011

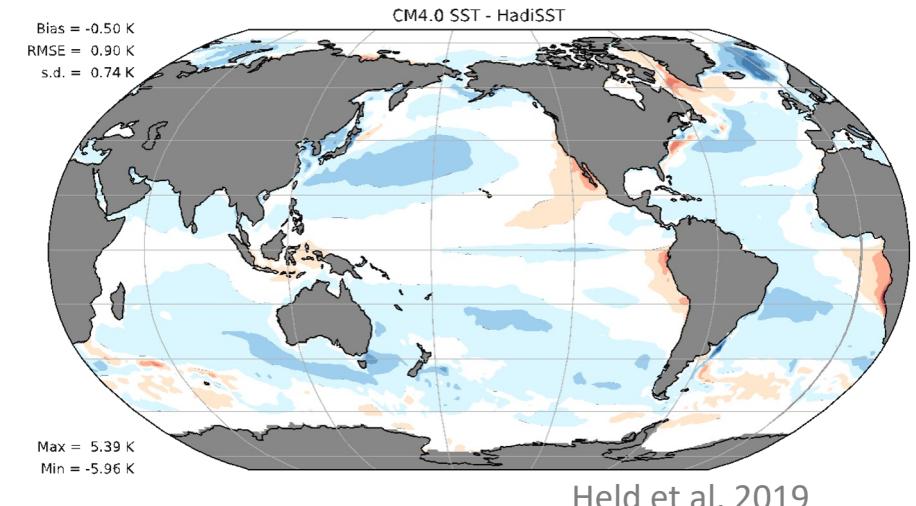


- a) DA corrects random errors.
 - b) DA corrects systematic errors.
- Learning corrections = Learning Increments

Mean Increments in GFDL SPEAR

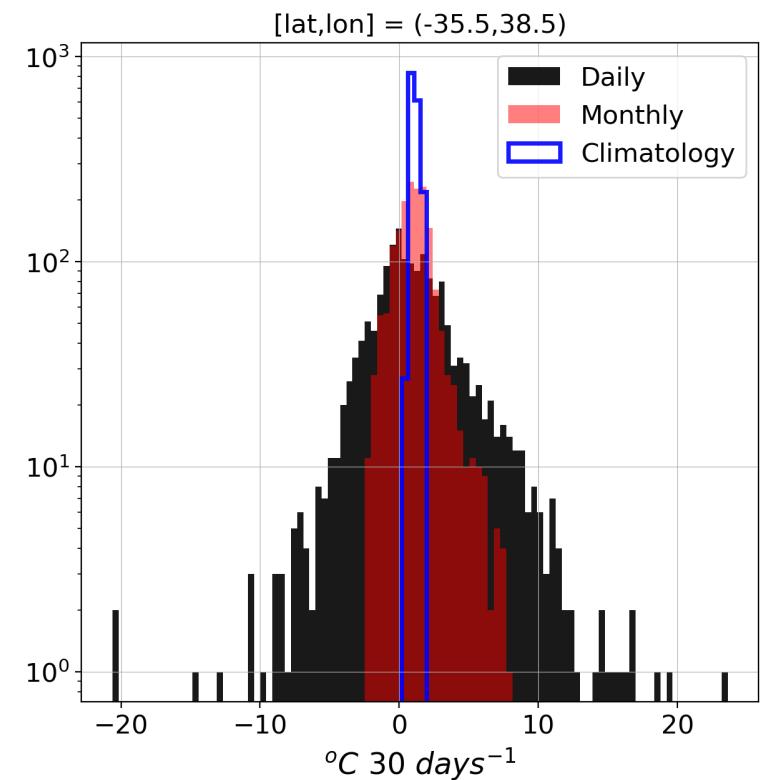
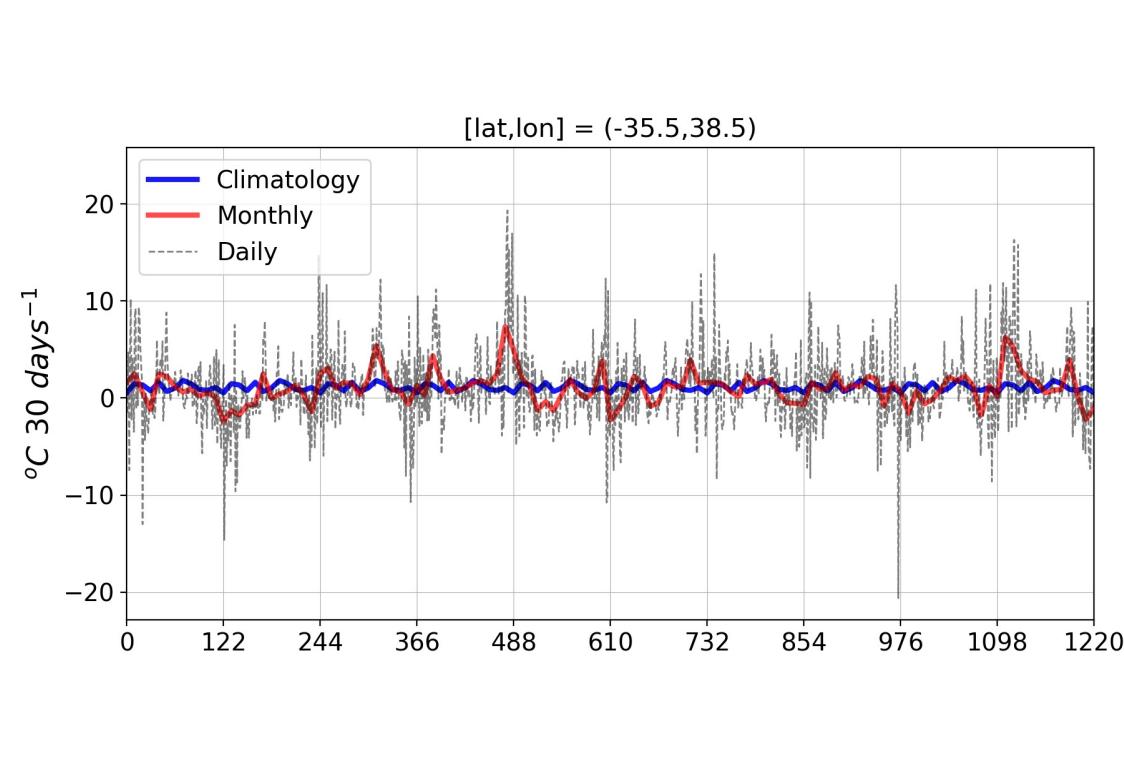


Mean SST Bias in CM4.0





Learn high frequency or low frequency increments?



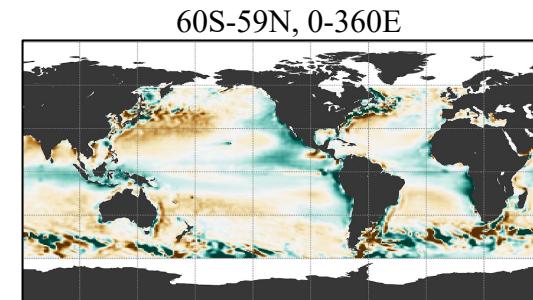
- $X_{t,s}$ is a daily DA increment with t and s indices representing time and space.
- $\langle X_{t,s} \rangle^{d,m,c}$ is a smoothing (or high frequency filtering) operation with daily, monthly or climatological timescales
- Loss function for the neural network can then be written as

$$E \left[\left(\hat{X}_{t,s} - \langle X_{t,s} \rangle^{d,m,c} \right)^2 \right]$$



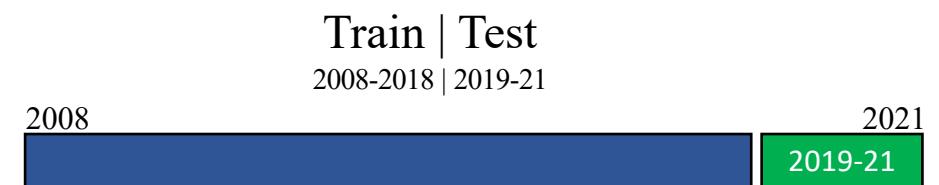
Use Neural Nets to learn nonlinear mapping from model state to DA increments

- ✓ **Goal:** local, state-dependent (~parameterization)
- ✓ **Inputs:** daily T , T_z , U_z , V_z , $hfdz$, τ_{ax} , τ_{ay} etc.
- ✓ **Output:**
 - 1) daily climatology of temperature increments
 - 2) raw daily temperature increments
- ✓ **Architecture:** Fully connected neural network
(2h32 to 5h256)



- ✓ near-global domain
- ✓ sub-sampled to $\sim 2^\circ$ horizontally
- ✓ coarsened to 19 levels in vertical
- ✓ sub-sampled every 3rd day

- ✓ Using **GFDL SPEAR DA system** which assimilates SST, ARGO observations
- ✓ 2004-2021



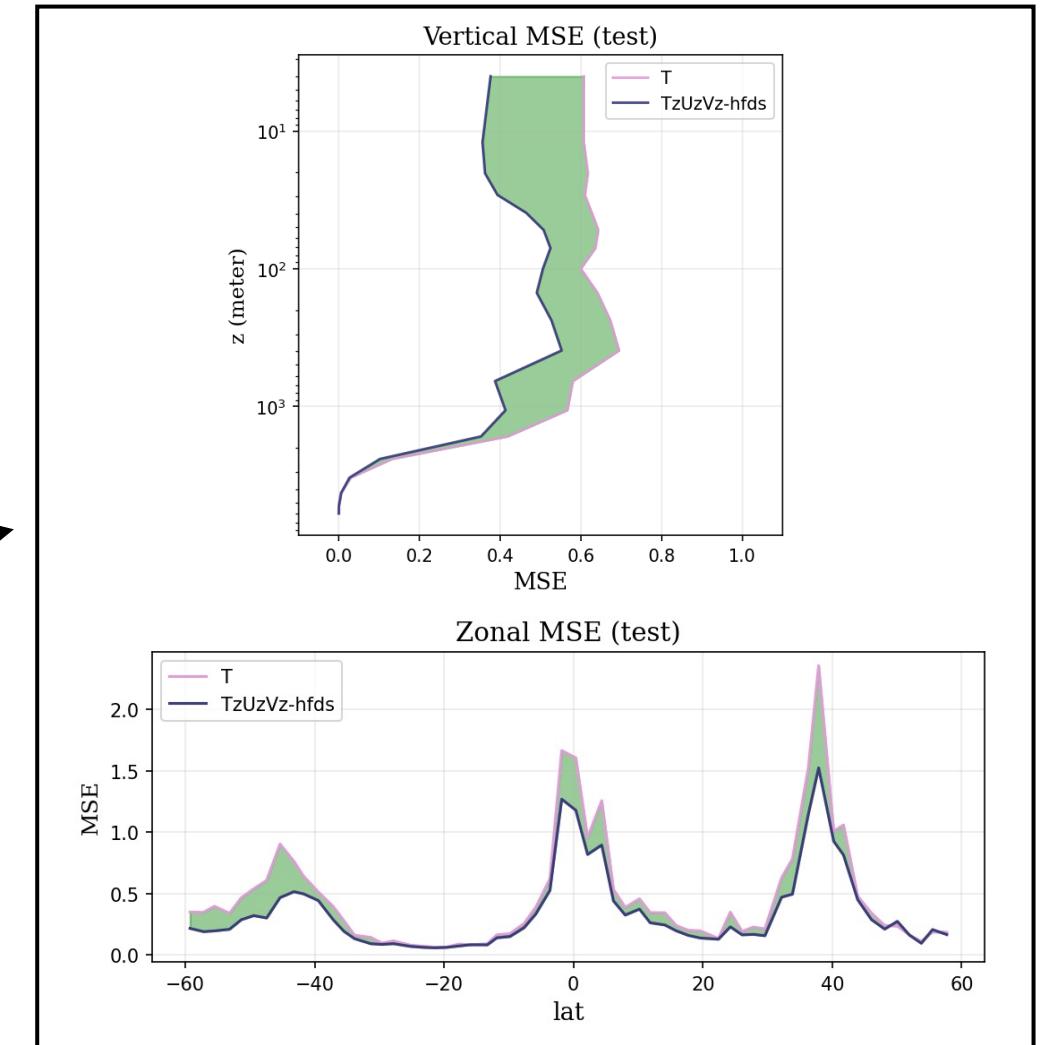
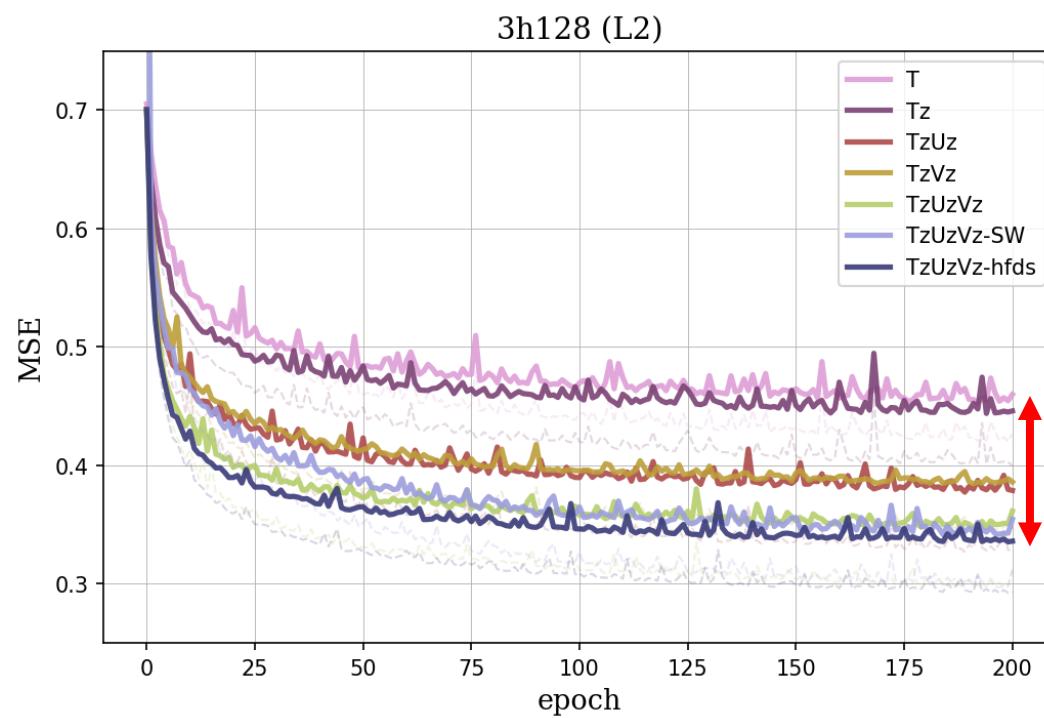


- 1) Learning daily climatology of temperature increments



Sensitivity to different input predictors

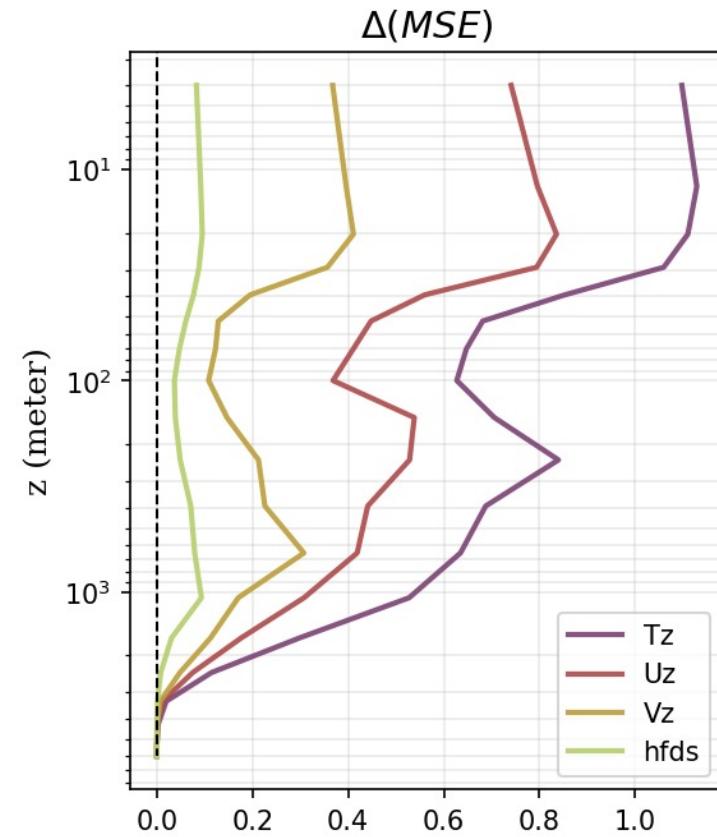
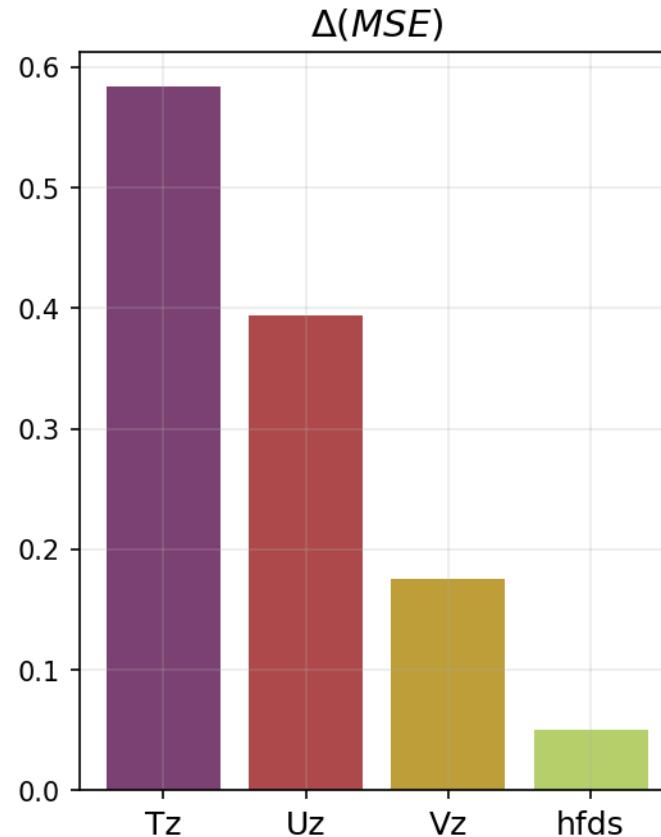
$[T]$, $[T_z]$, $[T_zU_z]$, $[T_zV_z]$, $[T_zU_zV_z]$, $[T_zU_zV_z\text{ sw}]$, $[T_zU_zV_z\text{ hfds}]$





Change in MSE due to randomizing input predictors

$$\overline{\delta T} = \widehat{\mathcal{F}}[T_z, U_z, V_z, hfds; \theta] ; \text{NN} = 3h128$$

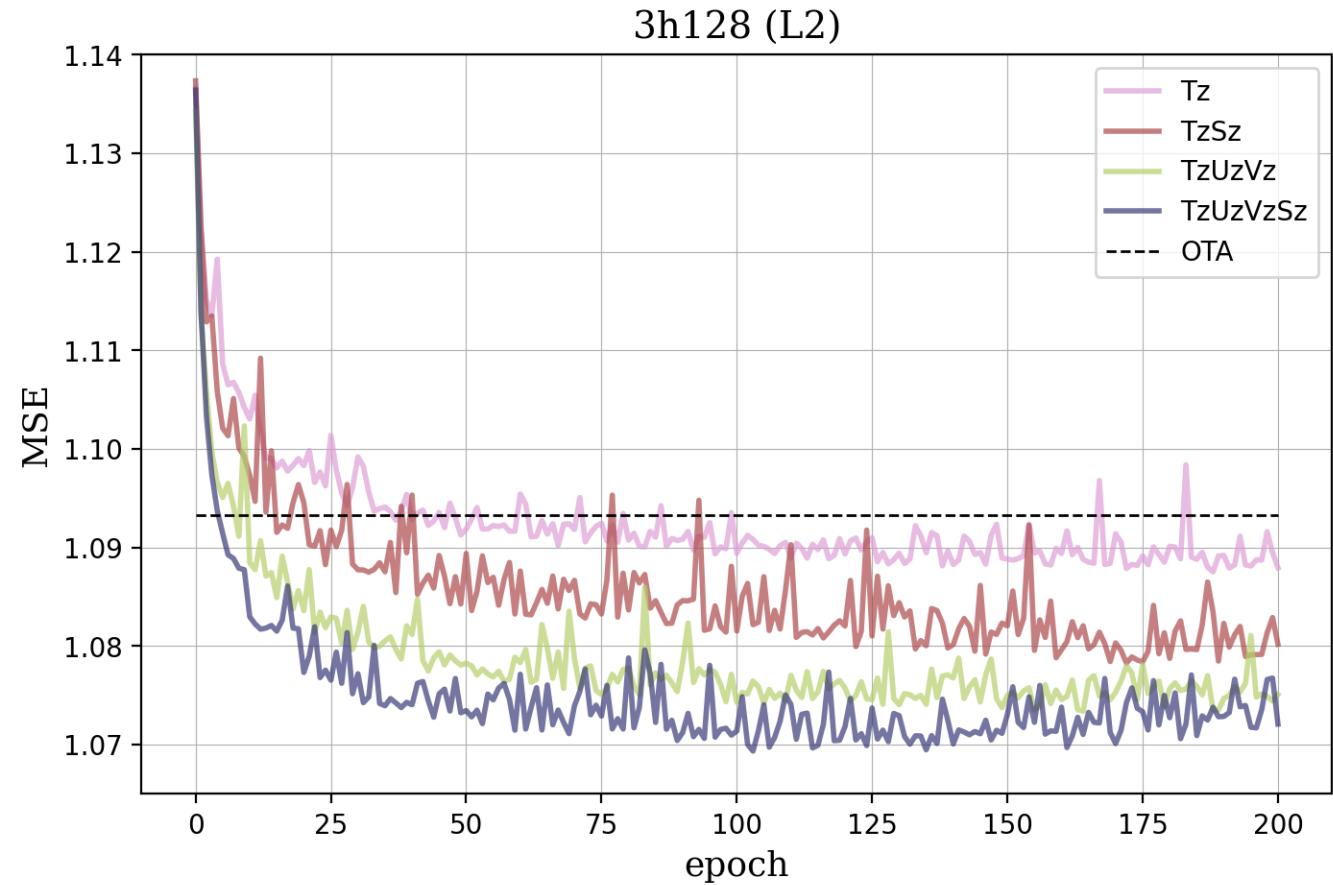




2) Learning raw daily temperature increments



Sensitivity to different input predictors

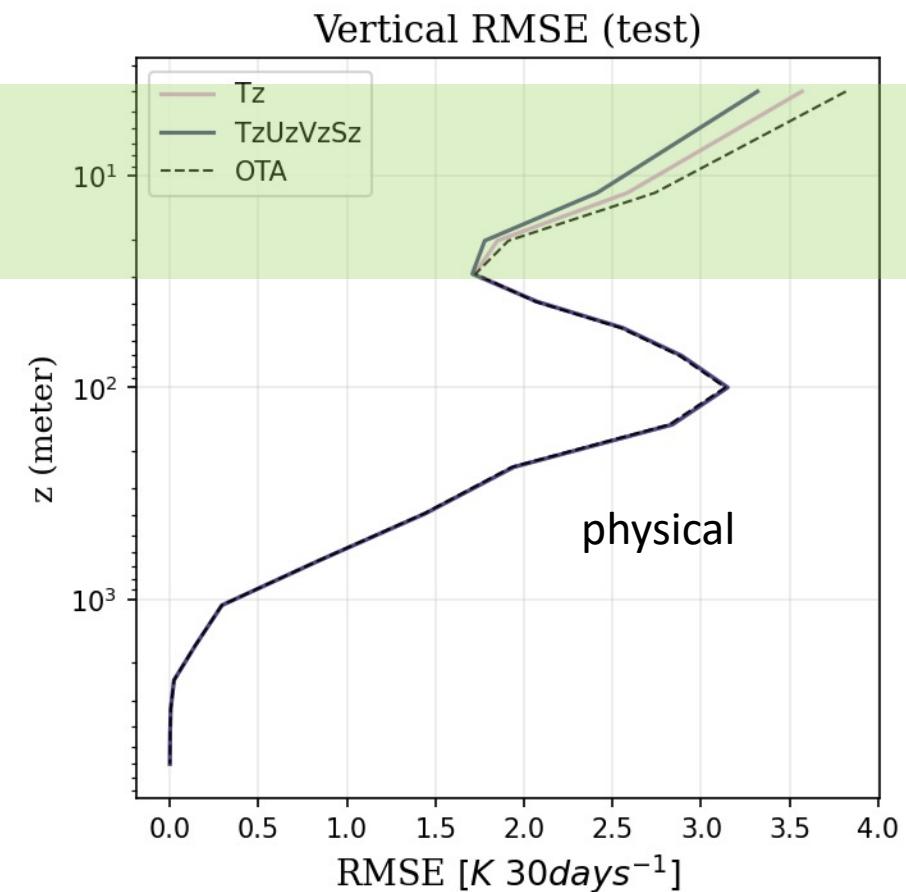
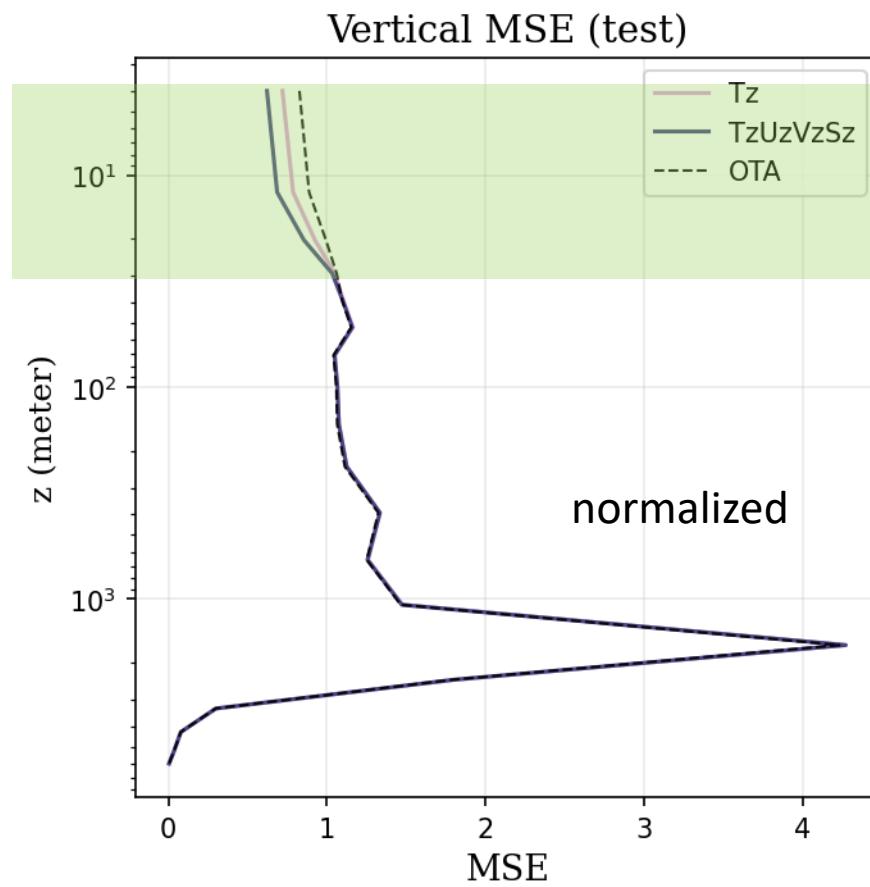


OTA = benchmark based on daily climatology of temperature increments



Vertical Structure of MSE/RMSE

OTA =
benchmark
based on daily
climatology of
temperature
increments

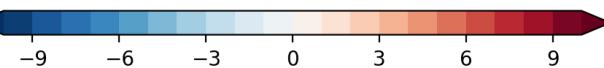
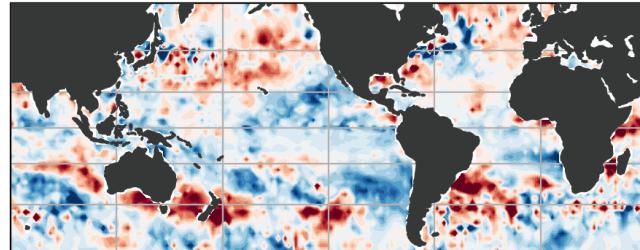


	OTA	Tz	TzUzVzSz
R² (0-30m) (%)	12.1	18.4	25.0

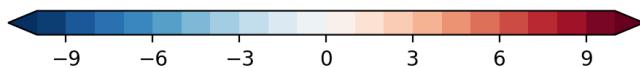
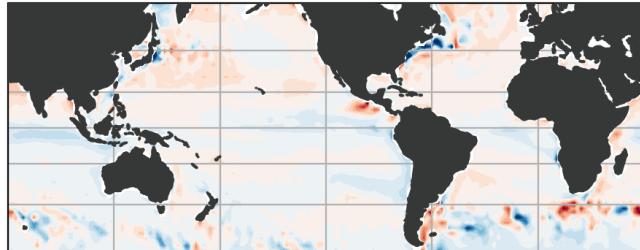


2019-01-01 Truth and Predictions (learning daily vs climatology)

Daily

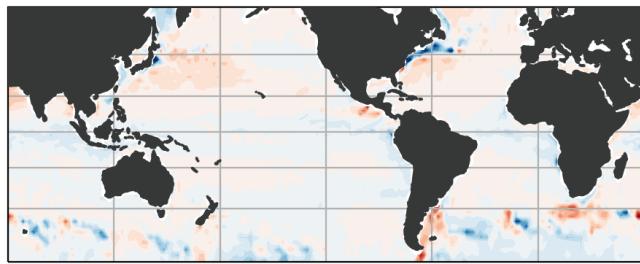
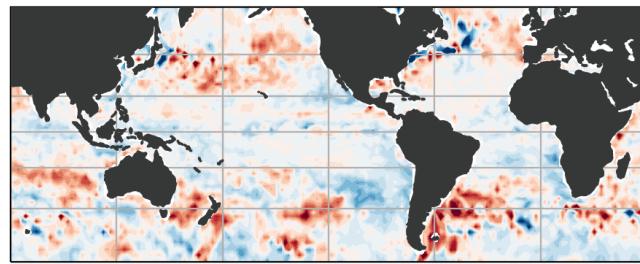


Climatology



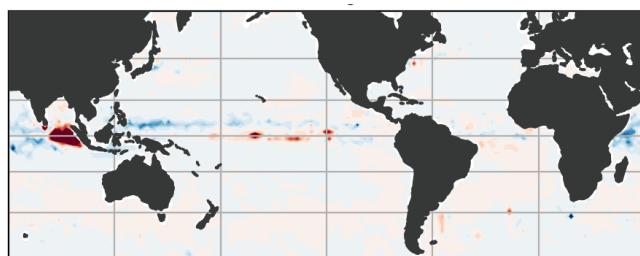
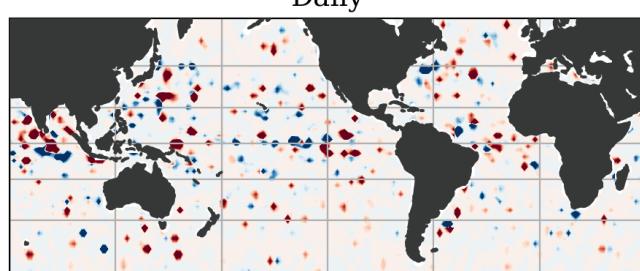
Truth

4 meter (K 30-days⁻¹)

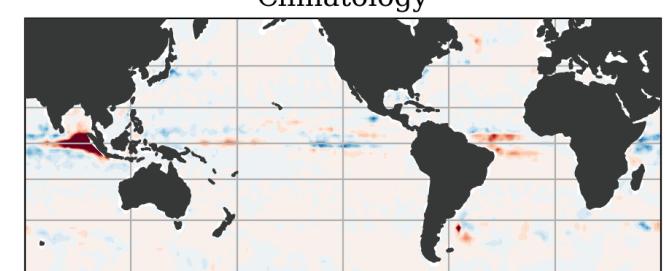


NN

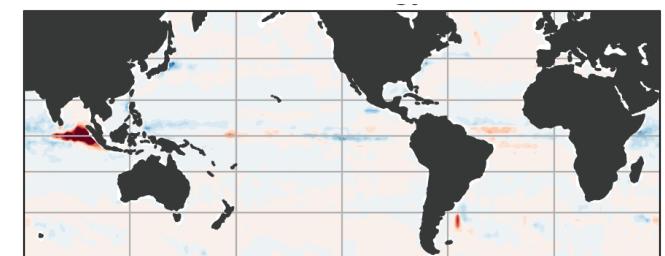
100 meter (K 90-days⁻¹)



Climatology

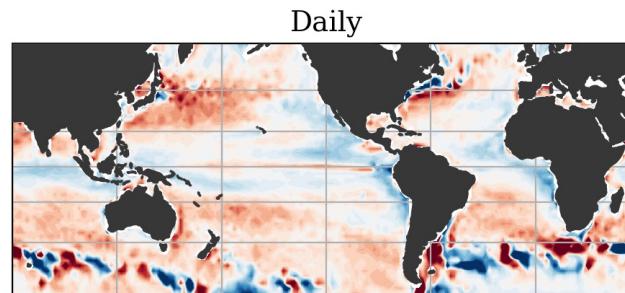
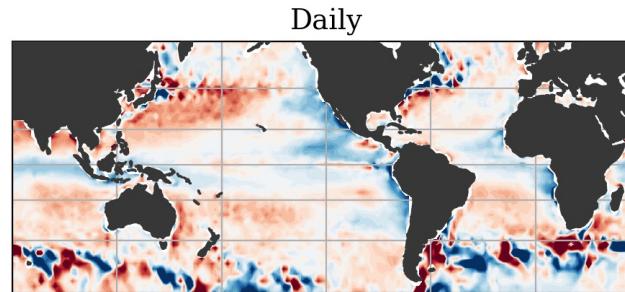


Truth

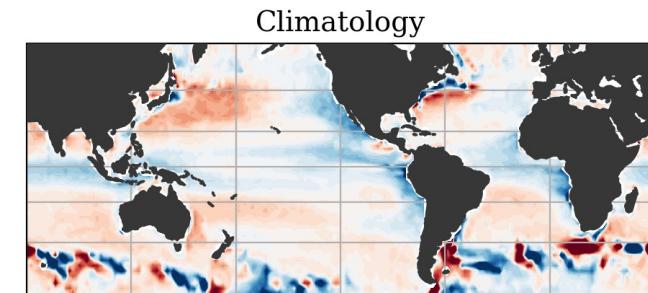
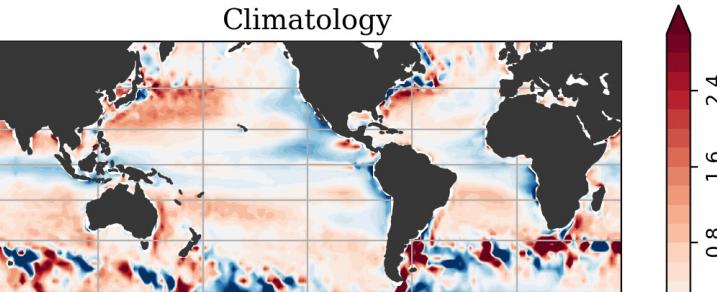


NN

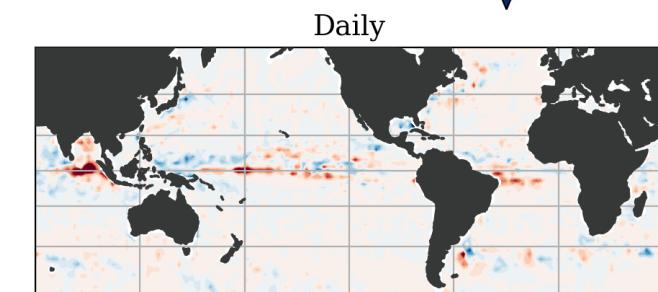
2019-2021 mean Truth and Predictions (learning daily vs climatology; K 30-days⁻¹)



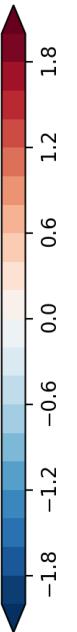
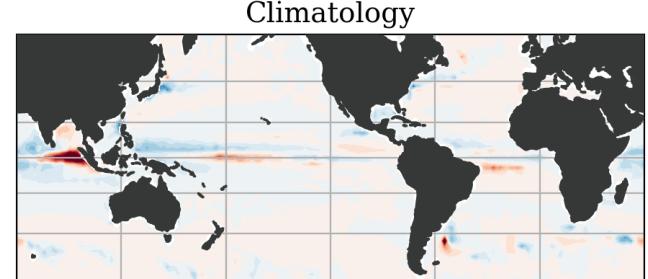
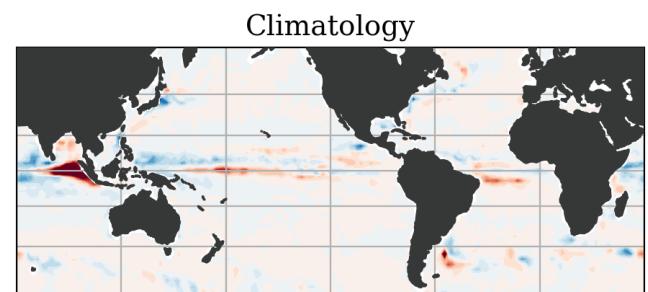
100 meter



Truth



NN





Conclusions

- ✓ DA increments can be used to learn systematic fast ocean model errors.
- ✓ Neural network shows significant offline skill in learning nonlinear relationships between model state and DA increments.
- ✓ The offline skill improves with the addition of dynamically relevant quantities.
- ✓ A network trained on raw daily temperature increments has better skill in the upper 30 meters which relaxes to the one trained on daily climatology of temperature increments below that.

Next...

- ✓ Expand to global domain, remove subsampling, coarsening
- ✓ Joint prediction of temperature and salinity increments.
- ✓ Interpretation and learn missing/inaccurate physics
- ✓ Online inference and evaluation