1.

а) $E(X) = 300$

по неравенству Маркова: $P(x > A) \le \frac{E(X)}{A}$ ответ: 0.75

$\Updownarrow$

$P(x > 400) \le 0.75$

б) $P(X \le A) + P(X > A) = 1$

$\Updownarrow$

$P(X > A) = \frac{E(x)}{A} \ge 1 - P(X \le A)$

$\Updownarrow$

$P(X \le A) \ge 1 - \frac{E(X)}{A} = 1 - \frac{300}{500} = 0.4$ ответ: 0.4

2.

$n=1600 \quad p=0.3 \quad \varepsilon=50$

по схеме Бернулли

$E(X)=np=480$

$D(X)=np(1-p)=336$

по неравенству Чебышева: $\forall \varepsilon>0 \quad P(|X-E(X)|\geq\varepsilon)\leq\frac{D(X)}{\varepsilon^2}$

$\therefore \; P(|X-480|<50)\geq 1-\frac{336}{50^2}=0.8656$

ответ: 0.8656

3. $D(X) = 1 = \sigma^2 \qquad X = \{9, 5, 7, 7, 4, 10\}$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{9+5+7+7+4+10}{6} = 7$$

$1 - \frac{\alpha}{2} = 0.995$ по таблице: $Z_\alpha = 2.58$

$$\Delta = \frac{\sigma}{\sqrt{n}} Z_\alpha = \frac{1}{\sqrt{6}} \cdot 2.58 \approx 1.05$$

Ответ: доверительный интервал $(\bar{x} - \Delta, \bar{x} + \Delta) = (5.95; 8.05)$

4.

$$X_i \sim N(\mu, \sigma^2)$$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$L = \prod_{i=1}^{n} p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2}$$

$$\Updownarrow$$

$$\ln L = \ln \frac{1}{(\sqrt{2\pi\sigma^2})^n} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2$$

$$(\ln L)'_\mu = \frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i-\mu) = 0 \Rightarrow \frac{1}{\sigma^2}\,\frac{\bar{X}-\mu}{n} = 0 \Rightarrow \hat{\mu} = \bar{X}$$

$$(\ln L)'_{\sigma^2} = \left[\left(-\frac{n}{2}\right)(\ln 2\pi + \ln\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2\right]' = 0$$

$$\Rightarrow -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0 \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$$

Ombem: $\hat{\mu} = \bar{X}$, $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$