

פרויקט בחינה למדע העיוני. חלק א'

הסתברות, חוק ק"ס:

(1) א

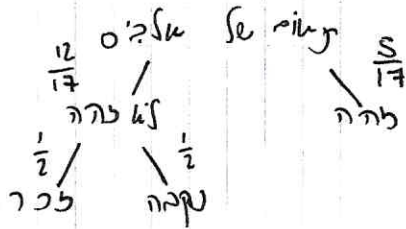
ידוע שלמלגה יש שני סוגים, והסתברותי לכל אחד היא:

$$\frac{12}{17} \cdot \frac{1}{2} + \frac{5}{17} \cdot \frac{1}{2} = \frac{17}{1700}$$

↓

הסתברות שלמה = $\frac{5}{17}$

הסתברות של לא למה = $\frac{12}{17}$



$$P(\text{נראה} | \text{מלגה}) = \frac{\frac{5}{17}}{\frac{5}{17} + \frac{12}{17} \cdot \frac{1}{2}} = \frac{\frac{5}{17}}{\frac{11}{17}} = \frac{5 \cdot 17}{11 \cdot 17} = \frac{5}{11}$$

(2) ב

$$P(\text{קופסה 1} | \text{שוקדים}) = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{3}{4}} = \frac{\frac{3}{8}}{\frac{3}{8} + \frac{1}{4}} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3 \cdot 8}{5 \cdot 8} = \frac{3}{5} = 0.6$$

(2) ג

$$P(\text{מבחן 1994} | \text{מבחן נכון}) = \frac{\frac{1}{2} \cdot 0.2 + \frac{1}{2} \cdot 0.2}{(\frac{1}{2} \cdot 0.14 + \frac{1}{2} \cdot 0.2) + (\frac{1}{2} \cdot 0.2 + \frac{1}{2} \cdot 0.1)} = \frac{\frac{3}{25} + \frac{1}{5}}{\frac{3}{25} + \frac{1}{5}} = \frac{\frac{5}{8}}{\frac{8}{8}} = \frac{5}{8} = 0.625$$

לא נראה
1 כיוון שיש
3 צדדים המכילים
מדענים אוכלוס

→ $P(\frac{H}{E}) = \frac{P(E|H) \cdot P(H)}{P(E)}$ $H = \text{חולה}$, $E = \text{קדירה חיונית}$ (נדרש למשק)

סבוי שחלה
והקדירה מפוא לא

$P(E|H) = \frac{99}{100}$ ← מה הסבוי שחלה בקדירה חיונית? (שן)

$P(H) = \frac{1}{10,000} = 10^{-4}$ ← מה הסבוי שהאדם יחלה?

$P(E) = \frac{9999}{10,000}$ ← כולל מר: 0.010099

$P(\text{חולה} | \text{חיונית}) = 0.99$

$P(\text{חולה} | \text{לא חיונית}) = 0.01$

$$P(\frac{H}{E}) = \frac{0.99 \cdot 10^{-4}}{0.010099} = \frac{1}{102}$$

$\frac{1}{200} = 5 \cdot 10^{-3} \leftarrow P(H)$, $\frac{99}{100} \leftarrow P(E|H)$ (7)

$$P(\frac{H}{E}) = \frac{0.99 \cdot 5 \cdot 10^{-3}}{0.0149} = \frac{99}{298} \approx 0.33$$

$0.99 \cdot \frac{1}{200} + 0.01 \cdot \frac{199}{200} = \frac{199}{10,000} = 0.0199 \leftarrow P(E)$

: הן קשה יותר לזכור את הנוסחה (170) (4)

(17) 0.125 - 100% = 0.875 (170) הן 1 הן 100%

Random Variables:

- ① Chances to get sum that can be divided by 3:

1+5, 5+1, 1+2, 2+1, 2+4, 4+2, 3+3, 3+6, 6+3, 4+5, 5+4, 6+6

12 options

There are 36 combinations when rolling 2 dices in a game.

Roi's chances to win is $\frac{12}{36} = \frac{1}{3}$
Hence, Roi's chances to lose are $\frac{2}{3}$

Expected Value: $(\frac{1}{3} \cdot 6\$) + (\frac{2}{3} \cdot (-3\$)) = 2\$ - 2\$ = 0\$$

②

The sum is 12	The sum is more than 12	The sum is less than 12
2+10, 3+9, 4+8, 5+7, 6+6	3+10, 4+9, 4+10, 5+8, 5+9, 5+10, 6+9, 6+10, 7+8, 7+9, 7+10, 8+7, 8+8, 8+9, 8+10, 9+6, 9+7, 9+8, 9+9, 10+5, 10+6, 10+7, 10+8, 10+9, 11+4, 11+5, 11+6, 11+7, 11+8, 12+3, 12+4, 12+5, 12+6, 12+7, 12+8, 12+9	1+6, 1+7, 1+8, 1+9, 1+10, 2+6, 2+7, 2+8, 2+9, 3+6, 3+7, 3+8, 4+6, 4+7, 5+6
$\frac{4}{20}$	$\frac{6}{20}$	$\frac{15}{20}$

Expected Value: $(\frac{6}{20} \cdot 5\$) + (\frac{4}{20} \cdot 0\$) + (\frac{15}{20} \cdot (-6\$)) = 1.5\$ + 0\$ - 4.5\$ = -3\$$

③

Mean: $\mu = n \cdot p = 8 \cdot 0.4 = 3.2$

The formula to calculate std is: $\sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} = \sqrt{\sum_{i=1}^N p_i (x_i - \mu)^2}$

$x_i = i$, number of males ($0 \leq i \leq 8$)

$p_i = \text{probability of } i \text{ males}$ All the options to choose 8-i females

For 0 man, $p_i = 0.01525$, std = 0.156

For 1 man, $p_i = 0.0837$, std = 0.418

For 2 man, $p_i = 0.20948$, std = 0.302

For 3 man, $p_i = 0.28417$, std = 0.011

For 4 man, $p_i = 0.2358$, std = 0.15

For 5 man, $p_i = 0.1225$, std = 0.397

For 6 man, $p_i = 0.0389$, std = 0.305

For 7 man, $p_i = 0.0069$, std = 0.0999

For 8 man, $p_i = 0.00052$, std = 0.012

$\Rightarrow \text{std} = \sqrt{\sum_{i=1}^8 p_i (x_i - 3.2)^2} = 1.36 \approx 1.4$

: in the normal distribution

(4)

$$\text{Mean} = \mu = 26$$

$$\text{Std} = \sigma = 2$$

$$\Rightarrow P(26 < X < 30) = P(z_1, z_2)$$

$$z = \frac{x - \mu}{\sigma}$$

$$P(26 < X < 30) = P\left(\frac{26-26}{2} < z < \frac{30-26}{2}\right) = P(0 < z < 2) = P(z < 2) - P(z < 0)$$

$$P(26 < X < 30) = 0.977 - 0.5 = 0.477 \approx 0.48$$

(5)

$P(X > 3)$ is the area under the graph from 3.

$$P(X > 3) = \frac{0.4 \cdot (5-3)}{2} = \frac{0.4 \cdot 2}{2} = 0.4$$

(6)

4 employees
 $\frac{2}{5}$ / $\frac{3}{5}$

don't have children have children

$$P\left(\frac{3}{4} \text{ have children}\right) = 4 \cdot \left(\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}\right) = 4 \cdot \frac{54}{625} = \frac{216}{625} = 0.3456$$

number
of
combinations

(7)

$$\text{Expected Value of } X: (0.1 \cdot (-10)) + (0.35 \cdot (-5)) + (0.1 \cdot 0) + (0.35 \cdot 5) + (0.1 \cdot 10) = 0$$