Supplementary Information for A Dynamic Dirichlet Process Mixture Model for the Partisan Realignment of Civil Rights Issues in the U.S. House of Representatives*

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A Summary Statistics

Table A.1: Summary Statistics of Four Types of Legislative Activities Related to Civil Rights in the U.S. House of Representatives (1933–1973)

Con.	Year	Roll Calls			Petitions			Speeches		Bill Sponsorship	
		No.	Dem.	Rep.	No.	Dem.	Rep.	Dem.	Rep.	Dem.	Rep.
73	1933-35	0			1	0.156 (0.363)	0.717 (0.453)	0.025 (0.156)	0.025 (0.157)		
74	1935-37	0			1	0.392 (0.489)	0.771 (0.422)	0.037 (0.189)	0.029 (0.167)		
75	1937-39	2	0.507 (0.5)	0.521 (0.501)	2	0.224 (0.417)	0.618 (0.487)	0.059 (0.236)	0.097 (0.297)		
76	1939-41	2	0.476 (0.5)	0.964 (0.186)	3	0.161 (0.368)	0.354 (0.479)	0.052 (0.222)	0.096 (0.295)		
77	1941-43	3	0.589 (0.492)	0.975 (0.157)	4	0.16 (0.366)	0.222 (0.416)	0.029 (0.169)	0.018 (0.132)		
78	1943-45	2	0.478 (0.5)	0.924 (0.265)	4	0.178 (0.383)	0.209 (0.407)	0.022 (0.147)	0.005 (0.068)		
79	1945-47	3	0.531 (0.499)	0.895 (0.307)	4	0.305 (0.46)	0.312 (0.464)	0.036 (0.188)	0.015 (0.123)		
80	1947-49	1	0.422 (0.495)	0.941 (0.235)	2	0.19 (0.393)	0.081 (0.273)	0.051 (0.22)	0.004 (0.063)	0.102 (0.606)	0.087 (0.399)
81	1949-51	4	0.567 (0.496)	0.83 (0.376)	3	0.234 (0.424)	0.076 (0.265)	0.053 (0.224)	0.023 (0.149)	0.132 (0.768)	0.08 (0.292)
82	1951-53	0			1	0.05 (0.218)	0.019 (0.138)	0.074 (0.263)	0.024 (0.154)	0.107 (0.536)	0.053 (0.33)
83	1953-55	0			2	0.272 (0.445)	0.045 (0.208)	0.073 (0.261)	0.041 (0.198)	0.178 (0.79)	0.032 (0.199)
84	1955-57	1	0.522 (0.501)	0.876 (0.33)	1	0.403 (0.491)	0.232 (0.423)	0.14 (0.348)	0.039 (0.195)	0.43 (1.625)	0.074 (0.358)
85	1957-59	2	0.55 (0.498)	0.896 (0.306)	2	0.154 (0.361)	0.071 (0.257)	0.142 (0.349)	0.078 (0.27)	0.446 (1.494)	0.118 (0.428)
86	1959-61	5	0.591 (0.492)	0.87 (0.337)	1	0.564 (0.497)	0.289 (0.455)	0.143 (0.351)	0.075 (0.265)	0.254 (1.022)	0.176 (0.792)
87	1961-63	1	0.655 (0.476)	0.855 (0.353)	0			0.095 (0.294)	0.023 (0.149)	0.374 (1.453)	0.158 (1.004)
88	1963-65	2	0.635 (0.482)	0.861 (0.347)	2	0.283 (0.451)	0.066 (0.249)	0.182 (0.386)	0.143 (0.351)	0.517 (1.642)	0.522 (1.169)
89	1965-67	2	0.761 (0.427)	0.793 (0.406)	0			0.225 (0.418)	0.161 (0.369)	0.225 (0.684)	0.629 (0.845)
90	1967-69	0			0			0.23 (0.422)	0.053 (0.224)	0.254 (0.953)	0.026 (0.191)
91	1969-71	0			1	0.46 (0.499)	0.11 (0.314)			0.352 (0.951)	0.14 (0.481)
92	1971-73	0			1	0.318 (0.467)	0.406 (0.492)			0.512 (1.813)	0.299 (0.993)

Note: This table shows summary statistics of four types of legislative activities related to civil rights, by Congress and by political parties. Standard deviations are shown in parentheses. Variables of roll calls are dummies indicating whether a member voted "yes" for a civil rights bill; variables of petitions are dummies for signing a discharge petition for advancing a civil right bill; variables of speeches are dummies indicating whether a member delivered at least one pro-civil rights speech during a certain Congress; bill sponsorship is a count variable measuring how many civil rights bills a member sponsored during a certain Congress.

B Markov Chain Monte Carlo Algorithm for the Empirical Application

In Section 3.4, we introduced the Markov Chain Monte Carlo (MCMC) algorithm for a general dynamic DP mixture model. In this section, we present a specific algorithm tailored to the empirical application. Most components of this algorithm are identical to those in the general algorithm. Here, we explain the parts that are specific to the application example.

As in Section 3.4, at the beginning of the algorithm, set an arbitrarily large number K to truncate the number of clusters. Then, initialize the starting values of $g^D[st]$ and $g^R[st]$ for $s = 1, 2, ..., 50$ and $t = 73, 74, ..., 92$. After initialization, each iteration of the Gibbs sampler proceeds as follows:

1. Update $\theta_k, \eta_k, \omega_k, \lambda_k$ for $k = 1, 2, ..., K$

   (a) The Posterior Distribution of θ_k

$$\theta_k \sim \mathsf{Beta}(\alpha_\theta + N_\theta^1, \beta_\theta + N_\theta^0)$$

$$N_\theta^1 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D}\sum_{j=1}^{J_t^V} V_{istj}^D \mathbb{I}(g^D[st] = 1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R}\sum_{j=1}^{J_t^V} V_{istj}^R \mathbb{I}(g^R[st] = 1)$$

$$N_\theta^0 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D}\sum_{j=1}^{J_t^V} (1-V_{istj}^D) \mathbb{I}(g^D[st] = 1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R}\sum_{j=1}^{J_t^V} (1-V_{istj}^R) \mathbb{I}(g^R[st] = 1)$$

   (b) The Posterior Distribution of η_k

$$\eta_k \sim \mathsf{Beta}(\alpha_\eta + N_\eta^1, \beta_\eta + N_\eta^0)$$

$$N_\eta^1 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D}\sum_{j=1}^{J_t^P} P_{istj}^D \mathbb{I}(g^D[st] = 1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R}\sum_{j=1}^{J_t^P} P_{istj}^R \mathbb{I}(g^R[st] = 1)$$

$$N_\theta^0 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D}\sum_{j=1}^{J_t^P} (1-P_{istj}^D) \mathbb{I}(g^D[st] = 1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R}\sum_{j=1}^{J_t^P} (1-P_{istj}^R) \mathbb{I}(g^R[st] = 1)$$

   (c) The Posterior Distribution of ω_k

$$\omega_k \sim \mathsf{Beta}(\alpha_\omega + N_\omega^1, \beta_\omega + N_\omega^0)$$

$$N_\omega^1 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D} S_{ist}^D \mathbb{I}(g^D[st]=1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R} S_{ist}^R \mathbb{I}(g^R[st]=1)$$

$$N_\omega^0 = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D} (1-S_{ist}^D) \mathbb{I}(g^D[st]=1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R} (1-S_{ist}^R) \mathbb{I}(g^R[st]=1)$$

(d) The Posterior Distribution of λ_k

$$\lambda_k \sim \mathsf{Gamma}(\alpha_\omega + C_\lambda, \beta_\omega + N_\lambda)$$

$$C_\lambda = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D} B_{ist}^D \mathbb{I}(g^D[st]=1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R} B_{ist}^R \mathbb{I}(g^R[st]=1)$$

$$N_\lambda = \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^D} \mathbb{I}(g^D[st]=1) + \sum_{t=73}^{92}\sum_{s=1}^{50}\sum_{i=1}^{I_{st}^R} \mathbb{I}(g^R[st]=1)$$

2. Sample the Stickiness Parameter p

   To sample p, we first introduce a series of dummy variables d_{st}^D and d_{st}^R for $t = 2, 3, ...T$ and $i = 1, 2, ..., N$ to indicate whether a unit directly stays in the same cluster as in the previous period.

   (a) Sample d_{st}^D and d_{st}^D

$$p(d_{st}^D = 1) = \begin{cases} 0 & \text{if } g^D[st] \neq g^D[s,t-1]; \\ \frac{p}{p+(1-p)q_k^t} & \text{if } g^D[st] = g^D[s,t-1] = k. \end{cases}$$

$$p(d_{st}^R = 1) = \begin{cases} 0 & \text{if } g^R[st] \neq g^R[s,t-1]; \\ \frac{p}{p+(1-p)q_k^t} & \text{if } g^R[st] = g^R[s,t-1] = k. \end{cases}$$

   (b) Sample p

$$p \sim \mathsf{Beta}(\alpha_p + N_1, \beta_p + N_2)$$

$$N_1 = \sum_{s=1}^{50}\sum_{t=72}^{93} d_{st}^D + \sum_{s=1}^{50}\sum_{t=72}^{93} d_{st}^R$$

$$N_2 = \sum_{s=1}^{50}\sum_{t=72}^{93} (1-d_{st}^D) + \sum_{s=1}^{50}\sum_{t=72}^{93} (1-d_{st}^R)$$

3. Update the Stick-breaking Weight π_k^t and q_k^t:

$$\pi_k^t \sim Beta(1 + n_k^{t-1} + n_k^t, \gamma + \sum_{l=k+1}^{K} n_l^{t-1} + \sum_{l=k+1}^{K} n_l^t)$$

$$n_k^{t-1} = \sum_{s=1}^{50} \mathbb{I}(g^D[s, t-1] = k) + \sum_{s=1}^{50} \mathbb{I}(g^R[s, t-1] = k)$$

$$n_k^t = \sum_{i=1}^{N} (1 - d_{st}^D)\mathbb{I}(g^D[st] = k) + \sum_{i=1}^{N} (1 - d_{st}^R)\mathbb{I}(g^R[st] = k)$$

$$q_k^t = \pi_k^t \prod_{l=1}^{k-1} (1 - \pi_l^t)$$

4. Update $g^D[st]$ and $g^R[st]$

   Here we introduce the sampling algorithm for $g^D[st]$. The algorithm for $g^R[st]$ follows the same pattern.

   Let $\mathbf{g}^D[t] \equiv (g^D[1t], g^D[2t], ..., g^D[50t])'$ and $\mathbf{q}^t \equiv (q_1^t, q_2^t, ..., q_K^t)'$. Let Y_{st}^D represent the collection of $(V_{istj}^D, P_{istj}^D, S_{ist}^D, B_{ist}^D)'$ for all $i = 1, 2, ..., I_{st}^D$ and $j = 1, 2, ..., J_t^V/J_t^P$; $\mathbf{Y}_t^D \equiv (Y_{1t}^D, Y_{2t}^D, ..., Y_{50t}^D)'$. Define $\Theta_k \equiv (\theta_k, \eta_k, \omega_k, \lambda_k)'$.

   We sample $\mathbf{g}^D$[92],...,$\mathbf{g}^D[t]$, ..., $\mathbf{g}^D$[73]in turn.

$$Pr(g^D[st] = k|\mathbf{g}^D[92], ..., \mathbf{g}^D[t+1], p, \mathbf{q}^{92}, ..., \mathbf{q}^{73}, \mathbf{Y}_{92}^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K)$$

$$\propto \underbrace{Pr(g^D[s, t+1]|p, g^D[st] = k, \mathbf{q}^{t+1})}_{\text{part 1}} \underbrace{Pr(g^D[st] = k|p, \mathbf{q}^t, ..., \mathbf{q}^{73}, \mathbf{Y}_t^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K)}_{\text{part 2}}$$

   part 1:

$$Pr(g^D[s, t+1] = l|p, g^D[st] = k, q_l^{t+1}) = (1-p)q_l^{t+1} + p\mathbb{I}(l = k)$$

   part 2:

$$Pr(g^D[st] = k|p, \mathbf{q}^t, ..., \mathbf{q}^{73}, \mathbf{Y}_t^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K))$$

$$\propto \underbrace{f(Y_{st}|g^D[st] = k, \mathbf{Y}_{t-1}^D, ..., \mathbf{Y}_1^D, \Theta_1, ..., \Theta_K)}_{\text{part } a} \underbrace{Pr(g^D[st] = k|p, \mathbf{q}^t, ..., \mathbf{q}^{73}, \mathbf{Y}_{t-1}^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K)}_{\text{part } b}$$

   part a:

$$f(Y_{st}^D|g^D[st] = k, \mathbf{Y}_{t-1}^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K) = f(Y_{st}^D|\Theta_{g^D[st]})$$

$$f(Y_{st}^D|\Theta_{g^D[st]}) = f_V f_P f_S f_B$$

$$f_V = \prod_{i=1}^{I_{st}^D} \prod_{j=1}^{J_t^V} (\theta_{g^D[st]}^{V_{istj}^D} (1 - \theta_{g^D[st]})^{(1-V_{istj}^D)})$$

$$f_P = \prod_{i=1}^{I_{st}^D} \prod_{j=1}^{J_t^P} (\eta_{g^D[st]}^{p_{istj}^D} (1 - \eta_{g^D[st]})^{(1-P_{istj}^D)})$$

$$f_S = \prod_{i=1}^{I_{st}^D} (\omega_{g^D[st]}^{S_{ist}^D} (1 - \omega_{g^D[st]})^{(1-S_{ist}^D)})$$

$$f_S = \prod_{i=1}^{I_{st}^D} \frac{\lambda_{g^D[st]}^{B_{ist}^D} e^{-\lambda_{g^D[st]}}}{B_{ist}^D!}$$

part b:

$$Pr(g^D[st] = k | p, \mathbf{q}^t, ..., \mathbf{q}^{73}, \mathbf{Y}_{t-1}^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K)$$

$$= \sum_l Pr(g^D[st] = k | p, q_k^t, g^D[s, t-1] = l) Pr(g^D[s, t-1] = l | p, \mathbf{q}^{t-1}, ..., \mathbf{q}^{73}, \mathbf{Y}_{t-1}^D, ..., \mathbf{Y}_{73}^D, \Theta_1, ..., \Theta_K)$$

C Simulation Study

In this section, we describe two simulations to highlight two kinds of group membership changes over time. In one simulation, we generate group memberships based on a structural break model; in the other simulation, we generate group memberships based on a gradual change model. We will show that the proposed method works well in both situations.

C.1 Data Generating Process

Recall the question of changing voting group we described in the motivating example. To simplify, we set up the question as the following: there are $T = 30$ parliamentary sessions, $N = 50$ representatives, and $M = 4$ issues to vote in each session. There are different voting groups in the parliament. For representative i in a voting group g, the probability of voting "yea" for issue j in session t follows a Bernoulli distribution, $\mathsf{Bernoulli}(\theta_{gjt})$. We use different ways to generate group memberships in simulation 1 and simulation 2.

Simulation 1: a structural break model. Two transition points at $t = 11$ and 21 separate the 30 parliamentary sessions into three periods. In the first period, there are 3 groups, with 20 representatives in group 1, 20 representatives in group 2, and 10 representatives in group 3; entering into the second period, 5 representatives in group 1 change to group 2 and 10 representatives in group 2 shift to group 1; in the last period, 5 representatives in group 1 change to group 2, 5 representatives in group 3 move to group 1, and the other 5 representatives in group 3 move to group 2.

To summarize, there are 3 groups in the first and second periods, and only 2 groups in the last period. For each group, we generate the parameter θ_{gjt} of the Bernoulli distribution that models the voting outcomes from a uniform distribution. For group 1, the four uniform distributions for the four voting issues are $\mathbf{Uniform}(0.8, 1)$, $\mathbf{Uniform}(0.7, 1)$, $\mathbf{Uniform}(0, 0.2)$ and $\mathbf{Uniform}(0, 0.3)$; for group 2, they are $\mathbf{Uniform}(0, 0.2)$, $\mathbf{Uniform}(0, 0.3)$, $\mathbf{Uniform}(0.8, 1)$ and $\mathbf{Uniform}(0.7, 1)$; for group 3, they are $\mathbf{Uniform}(0.7, 1)$, $\mathbf{Uniform}(0, 0.2)$, $\mathbf{Uniform}(0, 0.3)$ and $\mathbf{Uniform}(0.8, 1)$.

Simulation 2: a gradual change model. In the first 5 parliamentary sessions, there are 3 groups with 20, 20, and 10 representatives in each group. From $t = 6$ to $t = 25$, representatives in group 1 may change to group 2 with a probability of 0.5, and this change may happen at any time during this period; similarly, with a probability of 0.5, representatives in group 2 may shift to group 1 at a random time; for representatives in group 3, they will shift to group 1 with a probability of 0.5 and otherwise they will shift to group 2. The process to generate θ_{gjt} for each group g and each issue j in session t is the same as the process in simulation 1.

C.2 The Model

In section 3, we only describe a general version of the proposed method. Here, we introduce the detailed model we use to analyze the simulated data.

Let $g[it]$ represent the group of representative i in session t. Then,

$$g \sim \mathsf{IgCRP}(\gamma, \alpha_{\mathsf{p}}, \beta_{\mathsf{p}})$$

For V_{ijt}, the vote of issue j that representative i in sessions t casts, we assume it follows a Bernoulli distribution $\mathsf{Bernoulli}(\theta_{\mathsf{jk}})$ for $g[it] = k$. Unlike the data generating process, we assume that θ_{jk} does not change with t. As we will show, the model still works well under this assumption.

$$V_{ijt} \sim \mathsf{Bernoulli}(\theta_{\mathsf{j,g[it]}})$$

For $g[it] = 1, 2, ..., k, ...$, we assume:

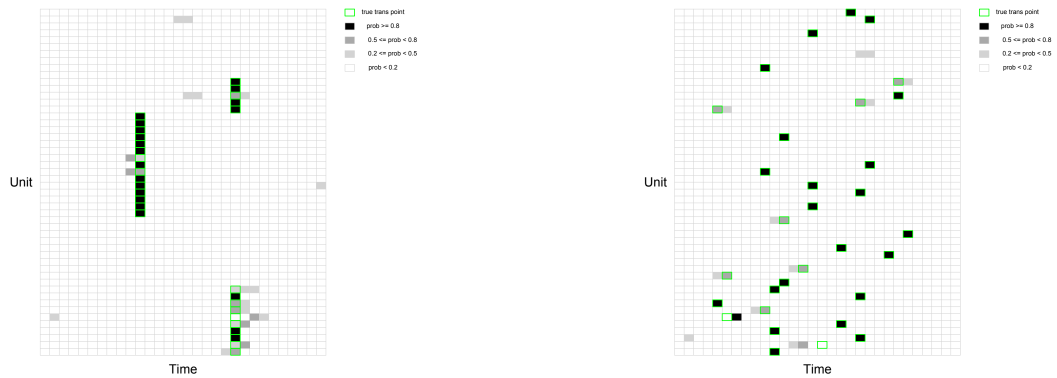
$$\theta_{jk} \sim \mathsf{Beta}(\alpha_{\theta,\beta_\theta}).$$

The MCMC algorithm for this model is a simplified version of the MCMC algorithm we use for the empirical application. Thus, we skip the detailed algorithm here.

C.3 Results

We first investigate whether the proposed method can detect the true transition points. For each unit, we calculate the probability that the unit in the current time and in the former time are in different groups. A probability approaching 1 indicates a transition point. As shown in Figure C.1, the proposed method detects almost all transition points, successfully recovering both the structural break model and the gradual change model.

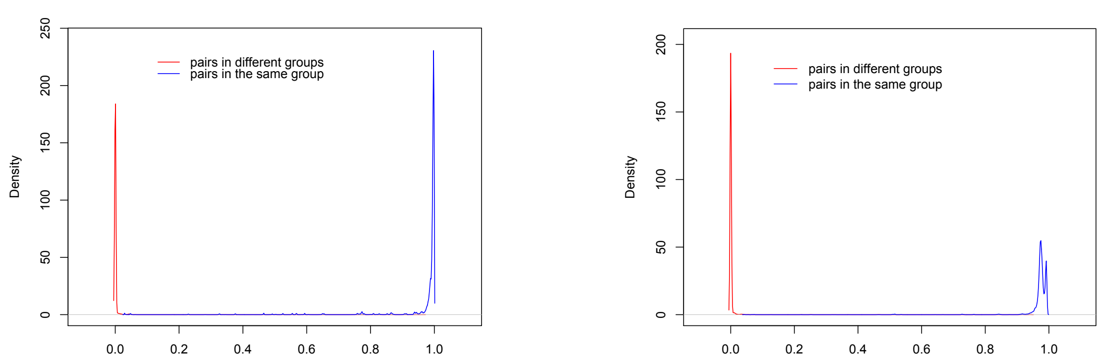
Besides checking whether the proposed method can detect the true transition points, we also investigate whether our method recovers true group memberships across units and over time together. For this purpose, we calculate the probability that two observations (they are either from different units,or in different time points, or both) are in the same group for all possible pairs. As we know the true group memberships in simulation studies, we separate pairs in different groups from pairs in the same group. For pairs in different groups, we expect the density of probabilities that two observations are in the same group to center around 0; for pairs in the same, we expect the density of probabilities to center around 1. As shown in Figure C.2, the proposed method discovers the true group memberships for both the structural break model and the gradual transition model.



(1) Recover Structural Break Model

(2) Recover Gradual Change Model

Figure C.1: Probabilities of Changing to a New Group. In the left figure, data is generated through a structural break model; in the right figure, data is generated through a gradual change model. A square represents the probability that the unit in the current time changes to a different group. The true transition points are circulated with green lines. This figure shows that no matter the data is generated through a structural break model or a gradual change model, the proposed method works well in identifying the transition point.



(1) Recover Structural Break Model

(2) Recover Gradual Change Model

Figure C.2: Densities of Probabilities that Two Observations are in the Same Groups. As we know the true memberships, we can separate pairs in the same groups from pairs in different groups and calculate the densities separately.