Model-Based and Data-Driven Hierarchical Control and Topology Co-Design for Robust Networked Systems

Shirantha Welikala, Zihao Song, Hai Lin, and Panos J. Antsaklis

Abstract— **In this paper, we consider a class of networked systems comprised of an interconnected set of linear subsystems along with disturbance inputs and performance outputs. For such networked systems, using dissipativity theory, we first propose a model-based hierarchical control design strategy to ensure the closed-loop networked system is dissipative from its disturbance inputs to performance outputs. This design process involves designing the local subsystem-level controllers and co-designing the distributed global controllers and the interconnection topology. A key assumption underlying this hierarchical control design process is the knowledge of subsystem dynamics, which may not hold for many real-world networked systems. Motivated by this, we also propose a data-driven hierarchical control design strategy for the considered class of networked systems, assuming only the availability of rich data trajectories obtained from the subsystems. The proposed data-driven design process accounts for disturbances in the data. Finally, the effectiveness of the proposed data-driven control design is evaluated for a networked system representative of a vehicular platoon.**

Index Terms—Data-Driven Control, Distributed Control, Topology Design, Dissipativity.

I. Introduction (From: Arxiv Paper of Conference paper)

Large-scale networked systems have gained a renewed attention over the past several years due to their broad applications in infrastructure networks [1]–[3], biological networks [4]–[6], vehicle platooning [7]–[9], electronic circuits [10], [11], mechanical networks [12], [13] and so on. Related to such networked systems, the main research thrusts have been focused on addressing problems such as analysis/verification [14]–[16], optimization [17]–[19], abstraction [20]–[22], controller synthesis [23]–[25] and network topology synthesis [14], [26]–[28]. Moreover, the literature on large-scale networked systems can also be categorized based on the nature of the constituent subsystems as: discrete [29], continuous [1], linear [23], piece-wise linear [19], non-linear [2], stochastic [20], switched [30] and hybrid [24].

With respect to the literature above, this paper aims to address the research problems: analysis/verification and network topology synthesis of large-scale networked systems comprised of non-linear subsystems (henceforth referred to as *analysis* and *synthesis* of non-linear networked systems). In particular, we are interested in *analyzing* the stability/dissipativity properties of a given non-linear networked system and, when necessary, *synthesizing* the interconnection topology of the non-linear networked system to guarantee/optimize such stability/dissipativity properties.

To achieve these goals, we only assume the knowledge of the involved subsystem dissipativity properties such as their passivity, passivity indices, L_2-stability and L_2-gain values. Note that identifying such dissipativity properties is far more convenient, efficient and reliable than having to identify entire dynamic models [2], [31]–[34]. Further, we formulate the interested non-linear networked system analysis and synthesis problems as linear matrix inequality (LMI) problems. Thus, they can be conveniently implemented and efficiently solved [35], [36]. Furthermore, we formulate these LMI problems so that they can be implemented in a decentralized and compositional manner. Consequently, new subsystems can be added/removed to/from a networked system conveniently without having to re-design the existing interconnections [1], [23]. Due to these unique qualities, the proposed solutions in this paper can be used for higher-level design of large-scale networked systems. To summarize, we provide dissipativity-based centralized and decentralized LMI techniques to analyze and synthesize non-linear networked systems.

This paper was inspired by the well-known work [2], [15] where dissipativity based centralized LMI techniques have been developed to analyze non-linear networked systems. Compared to [2], [15], we consider two special non-linear networked system configurations (NSCs) with multiple applications. Moreover, as mentioned earlier, we not only provide techniques for analysis but also address the corresponding synthesis and decentralization problems.

It should be noted that even though the analysis techniques proposed in [2], [15] have been derived in a compositional manner, they can only be evaluated in a centralized setting. To address this challenge, inspired by Sylvester's criterion [37], the work in [1], [23] propose a decentralized and compositional iterative process to analyze linear networked systems. In this paper, we adopt this idea to analyze non-linear networked systems in a decentralized and compositional manner.

A critical difference between this work and [1], [23] is that we only use dissipativity properties of the subsystems, whereas [1], [23] use subsystem dynamic models. Note, however, that this additional information and the linear dynamics assumed in [1], [23] also enable the synthesis of standard feedback controllers and observers for linear networked systems. In contrast, due to the said complexities that we consider, here we do not aim to synthesize controllers. Instead,

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we aim to synthesize appropriate interconnection topologies (in a centralized or decentralized and compositional manner) for non-linear networked systems when the analysis fails.

Nevertheless, there is a wealth of literature that focuses on the synthesis of controllers for networked systems. Some recent examples are as follows. Considering a linear networked system, a decentralized observer based controller is synthesized in [38]. For stochastic hybrid networked systems, to enforce a particular class of global specifications, a compositional framework is proposed in [24] to construct local control barrier functions. This solution is extended in [39] for discrete-time partially observable stochastic networked systems. For non-linear dynamical systems, a feed-forward control procedure is proposed in [25] using model reduction and bifurcation theory. A decentralized controller synthesis approach is proposed in [40] for discrete-time linear networked systems exploiting approximations of robust controlled invariant sets [41]. Compositionally constructed abstractions of networked systems are used to synthesize controllers for discrete-time stochastic [20], [30], [42], [43], continuous-time hybrid [44], [45] and nonlinear [21] networked systems.

As mentioned earlier, these controller synthesis techniques require additional information and assumptions regarding the involved subsystems and their interconnection topology. In contrast, in this paper, we only use subsystem dissipativity properties and focus on synthesizing the interconnection topology - which describes how each subsystem output is connected to the other subsystem inputs in the considered networked system (characterized by a block matrix called the “interconnection matrix”). Note that this approach is more reasonable when there are ready-made standard controllers (perhaps with tunable parameters that may change their dissipativity properties). For example, see the scenarios considered in [2], [14], [16], [27], [28], [33], [46].

However, literature focusing directly on synthesizing this interconnection matrix of networked systems are few and far between (especially for non-linear networked systems). In fact, the leading paper that motivated us to address this particular synthesis problem is [33] (see also [47]). In particular, [33], [47] consider a networked system comprised only of one or two subsystems and propose a set of non-linear inequalities to guide the synthesis of the corresponding interconnection matrix. In contrast, we consider a networked system with an arbitrary number of subsystems and provide LMI conditions to efficiently and conveniently synthesize the interconnection matrix (centrally or decentrally). Therefore, the proposed synthesis results in this paper can be seen as a generalization of [33], [47]. Note also that the techniques proposed in [33], [47] have influenced several recent works on problems such as designing switched controllers [48], networked controllers [49] and adaptive controllers [46]. Therefore, it is reasonable to expect that the proposed techniques in this paper may also be able to generalize the techniques proposed in [46], [48], [49].

In addition, the work in [14] has considered the interconnection topology synthesis problem limited to linear and positive networked systems. However, it requires the explicit knowledge of the involved linear positive subsystems. In [27], symmetries in the interconnection matrix are exploited to reduce the computational complexity of the networked system analysis. However, this approach only allows the interconnection matrix to be diagonally scaled from a predefined value (to recover symmetries). For stability analysis of large-scale interconnected systems, similar symmetry based techniques have been used in [28], [50]. However, they assume the interconnection matrix (hence the topology) as a given - rather than treating it as a decision variable. Taking a graph theoretic approach, necessary conditions for the stability of a class of non-linear networked systems are given in terms of the interconnection matrix in [16]. While this solution leads to identify several types of interconnection topologies that guarantee stability, it is only applicable to a small class of non-linear systems with known dynamics.

*a) **Contributions**:* With respect to the literature mentioned above, our contributions can be summarized as follows: (1) We consider several networked system configurations (NSCs) that can be used to model several exciting and widely used non-linear networked system configurations; (2) For the centralized analysis (stability/dissipativity) of such NSCs, we propose linear matrix inequality (LMI) based techniques; (3) For the centralized synthesis (interconnection topology) of such NSCs, we propose LMI based approaches; (4) We propose decentralized and compositional (i.e., resilient to subsystem additions and removals) counterparts for the proposed centralized analysis and synthesis approaches; and (5) We provide several numerical results to support our theoretical results.

*b) **Organization**:* This paper is organized as follows. In Section **??**, we provide several preliminary concepts and results related to dissipativity, network matrices and their positive definiteness analysis. Different networked system configurations and applicable centralized techniques to analyze their stability and dissipativity are discussed in Sec. **??**. In Sec. **??**, we propose centralized techniques to synthesize the interconnection matrices involved in the considered networked systems. Section **??** provide the decentralized counterparts of the proposed centralized analysis and centralized synthesis techniques. Finally, Sec. **??** discusses several numerical examples before concluding the paper in Sec. VI.

II. Preliminaries

A. Notations

$\mathbb{R}$ and $\mathbb{N}$ denote sets of real and natural numbers, respectively, and $\mathbb{N}^0 \triangleq \mathbb{N} \cup \{0\}$. For any $N \in \mathbb{N}$, we define $\mathbb{N}_N \triangleq \{1, 2, .., N\}$ and $\mathbb{N}_N^0 \triangleq \{0, 1, 2, .., N\}$. An $n \times m$ block matrix A is denoted as $A = [A_{ij}]_{i\in\mathbb{N}_n, j\in\mathbb{N}_m}$. $[A_{ij}]_{j\in\mathbb{N}_m}$ and $\text{diag}([A_{ii}]_{i\in\mathbb{N}_n})$ represent a block row matrix and a block diagonal matrix, respectively. $\mathbf{0}$ and $\mathbf{I}$ respectively denote zero and identity matrices (their dimensions will be clear from the context). A symmetric positive definite (semi-definite) matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $A > 0$ ($A \geq 0$). The symbol $\star$ represents conjugate matrices in symmetric matrices and conjugate blocks in symmetric block matrices (their meaning will be clear from the context). We also define $\mathcal{H}(A) \triangleq A + A^\top$, $\mathbf{1}_{\{\cdot\}}$ as the indicator function and $\text{e}_{ij} \triangleq \mathbf{1}_{\{i=j\}}$. The acronym RHS (LHS) means the right (left) hand side of an equation.

B. Some Useful Lemmas

Here we provide several useful lemmas regarding matrices and their positive definiteness.

Lemma 1: (Schur's complement [51]) For some matrices Θ, Φ, Γ (with appropriate dimensions),
$$\{\Gamma - \Phi^\top\Theta^{-1}\Phi \geq 0,\ \Theta > 0\} \iff \begin{bmatrix}\Theta & \Phi\\ \Phi^\top & \Gamma\end{bmatrix} \geq 0.$$

Lemma 2: (Congruence principle [51]) A matrix $W \geq 0$ if and only if $PWP^\top \geq 0$ where P is a full-rank matrix.

Corollary 1: For some block matrices $\Theta \triangleq [\Theta_{ij}]_{i,j\in\mathbb{N}_2}$, $\Phi \triangleq \begin{bmatrix}\Phi_1\\ \Phi_2^\top\end{bmatrix}$, and Γ (with appropriate dimensions)
$$\begin{bmatrix}\Theta_{11} & \Phi_1 & \Theta_{12}\\ \Phi_1^\top & \Gamma & \Phi_2\\ \Theta_{12}^\top & \Phi_2^\top & \Theta_{22}\end{bmatrix} \geq 0 \iff \begin{bmatrix}\Theta_{11} & \Theta_{12} & \Phi_1\\ \Theta_{12}^\top & \Theta_{22} & \Phi_2^\top\\ \Phi_1^\top & \Phi_2 & \Gamma\end{bmatrix} \geq 0$$
$$\iff \{\Gamma - \Phi^\top\Theta^{-1}\Phi \geq 0,\ \Theta > 0\}.$$

Proof: First, by applying Lm 2, we can interchange block rows/columns 2 and 3 via selecting P to be the corresponding permutation matrix. Then, the proof is complete by applying Lm. 1. ∎

Lemma 3: (Block-element-wise form) [52] Let $\Psi = [\Psi^{kl}]_{k,l\in\mathbb{N}_m}$ be an $m\times m$ block matrix where each $\Psi^{kl}, k, l \in \mathbb{N}_m$ is an $n \times n$ block matrix (with appropriate dimensions), and the block-element-wise (BEW) form of Ψ be defined as $\text{BEW}(\Psi) \triangleq [[\Psi_{ij}^{kl}]_{k,l\in\mathbb{N}_m}]_{i,j\in\mathbb{N}_n}$. Then,
$$\Psi \geq 0 \iff \text{BEW}(\Psi) \geq 0.$$

Lemma 4: (Matrix S-Lemma) Let M, N be symmetric matrices,
$$\mathcal{S}_N \triangleq \{Z \in \mathbb{R}^{n\times k} : \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix}^\top N \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix} \geq 0\},$$
and $\mathcal{S}_N^+$ be defined similarly to $\mathcal{S}_N$ except with a strict inequality. Assume that there exists some $\bar{Z} \in \mathcal{S}_N^+$. Then, if $\mathcal{S}_N$ is bounded, the implication (for any matrix Z)
$$\begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix}^\top N \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix} \geq 0 \implies \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix}^\top M \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix} > 0$$
holds if and only if there exists $\lambda \geq 0$ such that $M - \lambda N > 0$. If $\mathcal{S}_N$ is not bounded, the implication (for any matrix Z)
$$\begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix}^\top N \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix} \geq 0 \implies \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix}^\top M \begin{bmatrix}\mathbf{I}\\ Z\end{bmatrix} \geq 0$$
holds if and only if there exists $\lambda \geq 0$ such that $M - \lambda N \geq 0$.

C. Dissipativity

Consider the discrete-time dynamical system
$$\begin{aligned} x(t+1) &= f(x(t), u(t)),\\ y(t) &= h(x(t), u(t)), \end{aligned} \tag{1}$$
where the state $x(t) \in \mathbb{R}^{n_x}$, input $u(t) \in \mathbb{R}^{n_u}$, output $y(t) \in \mathbb{R}^{n_y}$ and $t \in \mathbb{N}^0$. Suppose under $u(t) = \mathbf{0}$, $x(t) = \mathbf{0}$ is an equilibrium point, i.e., $\mathbf{0} = f(\mathbf{0}, \mathbf{0})$, and $y(t) = h(\mathbf{0}, \mathbf{0}) = \mathbf{0}$.

Definition 1: The system (1) is *dissipative* (from input u to output y) under the *supply rate function* $s : \mathbb{R}^{n_u} \times \mathbb{R}^{n_y} \to \mathbb{R}$ if there exists a *storage function* $V : \mathbb{R}^{n_x} \to \mathbb{R}$ such that $V(0) = 0$, $V(x(t)) > 0$ and
$$\Delta V \triangleq V(x(t+1)) - V(x(t)) \leq s(u(t), y(t)),$$
for all $(u(t), t, x(0)) \in \mathbb{R}^{n_u} \times \mathbb{N}^0 \times \mathbb{R}^{n_x}$.

The above-defined dissipativity property can be specialized based on the used supply rate function $s(\cdot, \cdot)$. Following [53], we use a quadratic supply rate function determined by a coefficient matrix $\mathcal{X}$ to define a specialized dissipativity property named *$\mathcal{X}$-dissipativity* as follows.

Definition 2: The system (1) is *$\mathcal{X}$-dissipative* where $\mathcal{X} = \mathcal{X}^\top \triangleq [\mathcal{X}^{kl}]_{k,l\in\mathbb{N}_2} \in \mathbb{R}^{(n_u+n_y)\times(n_u+n_y)}$ if it is dissipative under the supply rate function:
$$s(u, y) \triangleq \begin{bmatrix}u\\ y\end{bmatrix}^\top \begin{bmatrix}\mathcal{X}^{11} & \mathcal{X}^{12}\\ \mathcal{X}^{21} & \mathcal{X}^{22}\end{bmatrix} \begin{bmatrix}u\\ y\end{bmatrix}.$$

The above-defined $\mathcal{X}$-dissipativity is identical to the conventional (Q, S, R)-dissipativity [54], and based on the used $\mathcal{X}$, it can represent several properties of interest as follows.

Remark 1: If the system (1) is $\mathcal{X}$-EID with:

1) $\mathcal{X} = \begin{bmatrix}\mathbf{0} & \frac{1}{2}\mathbf{I}\\ \frac{1}{2}\mathbf{I} & \mathbf{0}\end{bmatrix}$, then it is *passive*;
2) $\mathcal{X} = \begin{bmatrix}-\nu\mathbf{I} & \frac{1}{2}\mathbf{I}\\ \frac{1}{2}\mathbf{I} & -\rho\mathbf{I}\end{bmatrix}$, then it is IF-OFP($\nu, \rho$), i.e., input feedforward and output feedback passive with indices ν and ρ, respectively [32];
3) $\mathcal{X} = \begin{bmatrix}\gamma^2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix}$, then it is L2G($\gamma$), i.e., finite-gain L_2-*stable* with gain γ [32];
4) $\mathcal{X} = \begin{bmatrix}-a\mathbf{I} & \frac{a+b}{2b}\mathbf{I}\\ \frac{a+b}{2b}\mathbf{I} & -\frac{1}{b}\mathbf{I}\end{bmatrix}$ (or $\mathcal{X} = \begin{bmatrix}-ab\mathbf{I} & \frac{a+b}{2}\mathbf{I}\\ \frac{a+b}{2}\mathbf{I} & -\mathbf{I}\end{bmatrix}$) with $b > a$ (or $b > a$ and $b > 0$), then it is sector bounded where a and b are sector bound parameters [54].

D. Dissipativity of Linear Time-Invariant (LTI) Systems

If (1) is linear time-invariant (LTI), a necessary and sufficient condition for its $\mathcal{X}$-dissipativity can be found in the form of a linear matrix inequality (LMI) problem as follows.

Proposition 1: [55] The linear time-invariant (LTI) system
$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{2}$$

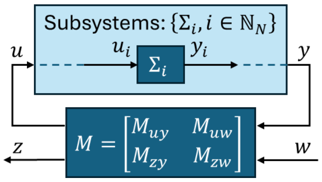

Fig. 1: A generic networked system $\Sigma : w \to z$.

is $\mathcal{X}$-dissipative (from input u to output y) if there exists a matrix $P \in \mathbb{R}^{n_x \times n_x}$ such that $P > 0$ and

$$\begin{bmatrix} P & PA & PB \\ \star & P + C^\top \mathcal{X}^{22} C & C^\top \mathcal{X}^{21} + C^\top \mathcal{X}^{22} D \\ \star & \star & \mathcal{X}^{11} + \mathcal{H}(\mathcal{X}^{12} D) + D^\top \mathcal{X}^{22} D \end{bmatrix} \geq 0.$$

Consider the LTI system (2) with a local controller $u(t) \triangleq Kx(t) + \bar{u}(t)$ and $D \triangleq \mathbf{0}$, and let $\tilde{u}(t) \triangleq B\bar{u}$ denote an external input (which may also include disturbances). The following corollary provides an LMI problem for designing this local controller (i.e., L) to enforce/optimize the corresponding closed-loop $\mathcal{X}$-dissipativity from $\tilde{u}(t)$ to $y(t)$.

***Corollary** 2:* [55] The closed-loop LTI system

$$\begin{aligned} x(t+1) &= (A + BK)x(t) + \tilde{u}(t), \\ y(t) &= Cx(t), \end{aligned} \tag{3}$$

? A B = I

is $\mathcal{X}$-EID with $\mathcal{X}^{22} < 0$ from external input $\tilde{u}(t)$ to output $y(t)$ if and only if there exists $P > 0$ and $\bar{K}$ such that

$$\begin{bmatrix} (-\mathcal{X}^{22})^{-1} & \mathbf{0} & CP & \mathbf{0} \\ \star & P & AP + B\bar{K} & \mathbf{I} \\ \star & \star & P & PC^\top \mathcal{X}^{21} \\ \star & \star & \star & \mathcal{X}^{11} \end{bmatrix} \geq 0,$$

n×m?

m×m

with $K = \bar{K}P^{-1}$.

E. Networked Systems Design

Consider the networked system Σ shown in Fig. 1 comprised of N independent discrete-time dynamic subsystems $\Sigma_i, i \in \mathbb{N}_N$ and a static interconnection matrix M that defines how the subsystem inputs and outputs, and the networked system's exogenous inputs $w \in \mathbb{R}^{n_w}$ and performance outputs $z(t) \in \mathbb{R}^{n_z}$ are interconnected.

Each subsystem $\Sigma_i, i \in \mathbb{N}_N$ follows the dynamics

$$\begin{aligned} x_i(t+1) &= f_i(x_i(t), u_i(t)), \\ y_i(t) &= h_i(x_i(t), u_i(t)), \end{aligned} \tag{4}$$

where the state $x_i(t) \in \mathbb{R}^{n_{xi}}$, input $u_i(t) \in \mathbb{R}^{n_{ui}}$, output $y_i(t) \in \mathbb{R}^{n_{yi}}$ and $t \in \mathbb{N}^0$. Further, under $u_i(t) = \mathbf{0}$, $x_i(t) = \mathbf{0}$ is an equilibrium point, i.e., $\mathbf{0} = f_i(\mathbf{0}, \mathbf{0})$, and $y_i(t) = h_i(\mathbf{0}, \mathbf{0}) = \mathbf{0}$. Furthermore, it is $\mathcal{X}_i$-dissipative (from u_i to y_i) where $\mathcal{X}_i = \mathcal{X}_i^\top \triangleq [\mathcal{X}_i^{kl}]_{k,l \in \mathbb{N}_2}$.

The interconnection matrix M is structured as given in the interconnection relationship:

$$\begin{bmatrix} u \\ z \end{bmatrix} = M \begin{bmatrix} y \\ w \end{bmatrix} \equiv \begin{bmatrix} M_{uy} & M_{uw} \\ M_{zy} & M_{zw} \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix}, \tag{5}$$

where $u \triangleq [u_i^\top]_{i \in \mathbb{N}_N}^\top \in \mathbb{R}^{n_u}$ and $y \triangleq [y_i^\top]_{i \in \mathbb{N}_N}^\top \in \mathbb{R}^{n_y}$ with $n_u \triangleq \sum_{i \in \mathbb{N}_N} n_{ui}$ and $n_y \triangleq \sum_{i \in \mathbb{N}_N} n_{yi}$.

As shown in [55], the interconnection matrix M (5) can be designed using an LMI problem to enforce/optimize the $\mathcal{Y}$-dissipativity of the networked system Σ (from w to z). However, this requires two mild assumptions as follows.

***Assumption** 1:* The desired $\mathcal{Y}$-dissipativity specification for the networked system Σ is such that $\mathcal{Y}^{22} < 0$.

***Remark** 2:* According to Rm. 1, As. 1 holds if we require the networked system Σ to be L2G(γ) (i.e., L_2-stable), or IF-OFP(ν, ρ) with $\rho > 0$ (i.e., output feedback passive). As such properties are desirable, As. 1 is mild.

***Assumption** 2:* Each subsystem $\Sigma_i, i \in \mathbb{N}_N$ in the networked system Σ is $\mathcal{X}_i$-EID with $\mathcal{X}_i^{11} > 0$.

***Remark** 3:* According to Rm. 1, As. 2 holds if each subsystem $\Sigma_i, i \in \mathbb{N}_N$ is L2G(γ_i) (i.e., L_2-stable), or IF-OFP(ν_i, ρ_i) with $\nu_i < 0$ (i.e., lacks input feedback passivity). If such conditions are not met, local controllers can be used (e.g., see Co. 2). Therefore, As. 2 is also mild.

***Proposition** 2:* [55] Under Assumptions 1 and 2, the networked system Σ is $\mathcal{Y}$-dissipative (from w to z) if the interconnection matrix M (5) is designed via the LMI problem:

$$\begin{aligned} &\text{Find: } L_{uy}, L_{uw}, M_{zy}, M_{zw}, \{p_i \in \mathbb{R} : i \in \mathbb{N}_N\} \\ &\text{Sub. to: } p_i > 0, \forall i \in \mathbb{N}_N, \text{ and } (8) \end{aligned} \tag{6}$$

where $\mathcal{X}_p^{kl} \triangleq \text{diag}([p_i \mathcal{X}_i^{kl}]_{i \in \mathbb{N}_N}), \forall k, l \in \mathbb{N}_2$, $\bar{\mathcal{X}}^{12} \triangleq \text{diag}([(\mathcal{X}_i^{11})^{-1} \mathcal{X}_i^{12}]_{i \in \mathbb{N}_N})$, $\bar{\mathcal{X}}^{21} \triangleq (\bar{\mathcal{X}}^{12})^\top$ with $M_{uy} \triangleq (\mathcal{X}_p^{11})^{-1} L_{uy}$ and $M_{uw} \triangleq (\mathcal{X}_p^{11})^{-1} L_{uw}$.

The above design technique for M can also be used when M is partially known. A specialized result for such a scenario is given in the following corollary.

***Corollary** 3:* Under Assumptions 1 and 2, the networked system Σ with $M_{uw} \triangleq \mathbf{I}, M_{zy} \triangleq \mathbf{I}$ and $M_{zw} \triangleq \mathbf{0}$, is $\mathcal{Y}$-dissipative (from w to z) if the remaining interconnection matrix block M_{uy} (5) is designed via the LMI problem:

$$\text{Find: } L_{uy}, \{p_i \in \mathbb{R} : i \in \mathbb{N}_N\}$$

$$\text{Sub. to: } p_i > 0, \forall i \in \mathbb{N}_N, \text{ and}$$

$$\begin{bmatrix} \mathcal{X}_p^{11} & \mathbf{0} & L_{uy} & \mathcal{X}_p^{11} \\ \star & -\mathcal{Y}^{22} & -\mathcal{Y}^{22} & \mathbf{0} \\ \star & \star & -\mathcal{H}(\bar{\mathcal{X}}^{21} L_{uy}) - \mathcal{X}_p^{22} & -\mathcal{X}_p^{21} + \mathcal{Y}^{21} \\ \star & \star & \star & \mathcal{Y}^{11} \end{bmatrix} \geq 0. \tag{7}$$

with $M_{uy} \triangleq (\mathcal{X}_p^{11})^{-1} L_{uy}$.

Proof: The proof follows substituting the known interconnection matrix blocks: $M_{uw} \triangleq \mathbf{I}$, $M_{zy} \triangleq \mathbf{I}$, and $M_{zw} \triangleq \mathbf{0}$ directly in (8), and then simplifying it using the relationships $M_{uw} \triangleq (\mathcal{X}_p^{11})^{-1} L_{uw}$ and $\bar{\mathcal{X}}^{21} \mathcal{X}_p^{11} = \mathcal{X}_p^{21}$. ■

III. Model-Based Hierarchical Design of Networked Systems

In this section, we consider a special case of networked systems discussed in Sec. II-E motivated by their occurrence across diverse applications, such as supply chains [55], vehicular platoons [56], and microgrids [57]. Figure 2 shows such a networked system, which is comprised of subsystems that follow discrete-time LTI dynamics - as opposed to generic non-linear dynamics (4), and hence referred to as a *linear networked system*. Most importantly, in such a linear networked system, each subsystem is considered to include a local controller that has to be designed so that the resulting closed-loop subsystem dissipativity property

$$\begin{bmatrix} \mathcal{X}_p^{11} & \mathbf{0} & L_{uy} & L_{uw} \\ \star & -\mathcal{Y}^{22} & -\mathcal{Y}^{22}M_{zy} & -\mathcal{Y}^{22}M_{zw} \\ \star & \star & -\mathcal{H}(\bar{\mathcal{X}}^{21}L_{uy}) - \mathcal{X}_p^{22} & -\bar{\mathcal{X}}^{21}L_{uw} + M_{zy}^\top \mathcal{Y}^{21} \\ \star & \star & \star & \mathcal{H}(\mathcal{Y}^{12}M_{zw}) + \mathcal{Y}^{11} \end{bmatrix} \geq 0 \tag{8}$$

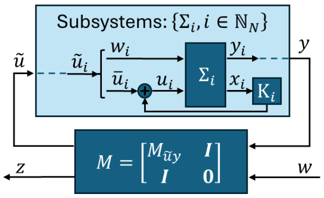

Fig. 2: A linear networked system $\Sigma : w \to z$.

(i.e., $\mathcal{X}_i$-dissipativity) is favorable for the design of the interconnection matrix.

A. Local Controller Design

Consider a linear networked system (as shown in Fig. 2) where each subsystem $\Sigma_i, i \in \mathbb{N}_N$ follows the dynamics

$$\begin{aligned} x_i(t+1) &= A_i x_i(t) + B_i u_i(t) + w_i(t), \\ y_i(t) &= C_i x_i(t). \end{aligned} \tag{9}$$

Data

In (9), $w_i(t) \in \mathbb{R}^{n_{wi}}$ represents the local disturbance component in $w \triangleq [w_i^\top]_{i\in\mathbb{N}_N}^\top \in \mathbb{R}^{n_w}$ (i.e., the exogenous input affecting the networked system) with $n_w \triangleq \sum_{i\in\mathbb{N}_N} n_{wi}$. At each subsystem $\Sigma_i, i \in \mathbb{N}_N$, we also include a local state-feedback controller of the form:

$$u_i(t) = K_i x_i(t) + \bar{u}_i(t), \tag{10}$$

where $\bar{u}_i(t)$ is a distributed global controller (to be designed later on, when designing the interconnection matrix). Applying (10) in (9), we get the closed-loop subsystem dynamics

This is input

$$\begin{aligned} x_i(t+1) &= (A_i + B_i K_i)x_i(t) + \tilde{u}_i(t), \\ y_i(t) &= C_i x_i(t). \end{aligned} \tag{11}$$

where $\tilde{u}_i(t) \triangleq B_i\bar{u}_i(t) + w_i(t)$. $B_i = I$

The following proposition details how the local controller K_i in (10) can be designed so that the closed-loop subsystem (11) is $\mathcal{X}_i$ dissipative - as required for the subsequent design of the interconnection matrix (e.g., via Prop. 2).

Proposition 3: At each subsystem $\Sigma_i, i \in \mathbb{N}_N$, the local controller K_i in (10) can be designed to enforce/optimize the closed-loop $\mathcal{X}_i$-dissipativity (of (11), from $\tilde{u}_i$ to y_i, under Assumptions 1 and 2) using the LMI problem:

Find: $\bar{K}_i, P_i, \mathcal{X}_i$

Sub. to: $P_i > 0$, and

nxn nxm

$$\begin{array}{c} r \\ n \\ n \\ m \end{array} \begin{bmatrix} (-\mathcal{X}_i^{22})^{-1} & \mathbf{0} & C_i P_i & \mathbf{0} \\ \star & P_i & A_i P_i + B_i \bar{K}_i & \mathbf{I} \\ \star & \star & P_i & P_i C_i^\top \mathcal{X}_i^{21} \\ \star & \star & \star & \mathcal{X}_i^{11} \end{bmatrix} \geq 0, \tag{12}$$

r n n m mxm

with $K_i = \bar{K}_i P_i^{-1}$.

Proof: Comparing (11) with (3), it is clear that Co. 2 is applicable, which leads to the given LMI problem (12). ■

Comparing (11) with (3), it is clear that the local controller K_i in (10) can be designed using Co. 2 to enforce/optimize the $\mathcal{X}_i$-dissipativity of the closed-loop subsystem (11) from $\tilde{u}_i$ to y_i (as required for the subsequent design of the interconnection matrix, e.g., using Prop. 2).

B. Distributed Global Controller Design

At each subsystem $\Sigma_i, i \in \mathbb{N}_N$, we consider a distributed global controller $\bar{u}_i(t)$ (see (10)) of the form:

$$\bar{u}_i(t) = \sum_{j\in\mathbb{N}_N} K_{ij} y_j(t) \iff \bar{u}(t) = K y(t) \tag{13}$$

where the equation in the RHS is from vectorizing the equation in the LHS, with the notations $\bar{u} \triangleq [\bar{u}_i^\top]_{i\in\mathbb{N}_N}^\top$, $y \triangleq [y_i^\top]_{i\in\mathbb{N}_N}^\top$ and $K \triangleq [K_{ij}]_{i,j\in\mathbb{N}_N}$. Note that, in (13), $K_{ij} = \mathbf{0}$ if the output information from the subsystem Σ_j is not required at the subsystem Σ_i (implying no communication link is necessary from Σ_j to Σ_i). Therefore, while designing the distributed global controller K (13), we can also optimize the underlying communication topology. Consequently, this distributed global controller design stage is referred to as a communication and control co-design stage [56].

For this co-design task, we can apply Prop. 2 exploiting the subsystem dissipativity properties enforced by the local controller design (12). However, we first have to identify the underlying interconnection matrix structure of the considered networked system. To this end, recall the external input affecting the closed-loop subsystems, i.e., $\tilde{u}_i(t)$ in (11), that can be vectorized and simplified (using (13)) respectively as

$$\begin{aligned} \tilde{u}_i(t) \triangleq B_i\bar{u}_i + w_i(t) \iff \tilde{u}(t) &= B\bar{u}(t) + w(t) \\ \iff \tilde{u}(t) &= BKy(t) + w(t), \end{aligned} \tag{14}$$

where $\tilde{u} \triangleq [\tilde{u}_i^\top]_{i\in\mathbb{N}_N}^\top$ and $B \triangleq \text{diag}([B_i]_{i\in\mathbb{N}_N})$. On the other hand, let us define $z_i(t) \triangleq y_i(t)$ as the local performance output component in $z \triangleq [z_i^\top]_{i\in\mathbb{N}_N}^\top$. Therefore, the performance output of the networked system is $z(t) \triangleq y(t)$. Combining this result with (14), we get the underlying interconnection matrix structure through the interconnection relationship

h = ml

$$\begin{bmatrix} \tilde{u} \\ z \end{bmatrix} = \begin{bmatrix} M_{\tilde{u}y} & M_{\tilde{u}w} \\ M_{zy} & M_{zw} \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix} \equiv \begin{bmatrix} BK & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix}. \tag{15}$$

Based on (15), it is clear that we can use Co. 3 (instead of Prop. 2) for this co-design task. The details are summarized in the following proposition.

Proposition 4: Under local controllers (10) (e.g., designed via (12)) that make each closed-loop subsystem $\Sigma_i, i \in \mathbb{N}_N$ (11) $\mathcal{X}_i$-dissipative (from $\tilde{u}_i$ to y_i, under As. 1), the distributed global controller and the underlying communication topology (jointly captured by K in (13)) can be co-designed to enforce/optimize the $\mathcal{Y}$-dissipativity (from w to z, under As. 2) of the closed-loop linear networked system using the LMI problem:

$$\min_{\substack{\bar{K},\mathcal{Y},\\ \{p_i\in\mathbb{R}:i\in\mathbb{N}_N\}}} \quad J \triangleq \|\bar{K}\|_1 - \phi([p_i]_{i\in\mathbb{N}_N}) + \psi(\mathcal{Y})$$

Sub. to: $p_i > 0, \forall i \in \mathbb{N}_N,$ and

$$\begin{bmatrix} \mathcal{X}_p^{11} & \mathbf{0} & \mathcal{X}^{11}B\bar{K} & \mathcal{X}_p^{11} \\ \star & -\mathcal{Y}^{22} & -\mathcal{Y}^{22} & \mathbf{0} \\ \star & \star & -\mathcal{H}(\mathcal{X}^{21}B\bar{K}) - \mathcal{X}_p^{22} & -\mathcal{X}_p^{21} + \mathcal{Y}^{21} \\ \star & \star & \star & \mathcal{Y}^{11} \end{bmatrix} \geq 0, \tag{16}$$

where $\mathcal{X}_p^{kl} \triangleq \text{diag}([p_i\mathcal{X}_i^{kl}]_{i\in\mathbb{N}_N})$, $\mathcal{X}^{kl} \triangleq \text{diag}([\mathcal{X}_i^{kl}]_{i\in\mathbb{N}_N})$, $\forall k,l \in \mathbb{N}_2$, $\mathcal{Y} \triangleq [\mathcal{Y}^{kl}]_{k,l\in\mathbb{N}_2}$, and $\bar{K} \triangleq [\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$ with $K \triangleq [p_i^{-1}\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$.

Proof: We start by applying Co. 3 with $M_{uy} = BK$ (15). According to Co. 3, $M_{uy} = (\mathcal{X}_p^{11})^{-1}L_{uy}$. Therefore, we have to replace the L_{uy} term appearing in the main LMI condition in Co. 3 using $L_{uy} = \mathcal{X}_p^{11}BK$. Here, since the RHS is bilinear in decision variables $\{p_i \in \mathbb{R} : i \in \mathbb{N}_N\}$ and K, we use a change of variables: $\bar{K} \triangleq [\bar{K}_{ij}]_{i,j\in\mathbb{N}_N} \equiv [p_iK_{ij}]_{i,j\in\mathbb{N}_N}$. Consequently, we get:

$$\begin{aligned} &L_{uy} = \mathcal{X}_p^{11}BK \\ &\iff L_{uy}^{ij} = p_i\mathcal{X}_i^{11}B_iK_{ij} = \mathcal{X}_i^{11}B_i\bar{K}_{ij}, \forall i,j \in \mathbb{N}_N \\ &\iff L_{uy} = \mathcal{X}^{11}B\bar{K}. \end{aligned}$$

Using the above derived substitution for L_{uy} with the fact

$$\begin{aligned} \bar{\mathcal{X}}^{21}\mathcal{X}^{11} &= \text{diag}([X_i^{21}(X_i^{11})^{-1}]_{i\in\mathbb{N}_N})\text{diag}([X_i^{11}]_{i\in\mathbb{N}_N}) \\ &= \text{diag}([X_i^{21}]_{i\in\mathbb{N}_N}) = \mathcal{X}^{21} \end{aligned}$$

we get the LMI condition in (16). ■

***Remark** 4:* Unlike in Co. 3, in Prop. 4, we propose to optimize an objective function J (16). The first term in J, i.e., $\|\bar{K}\|_1$ motivates the sparsity of the communication topology. The second term $\phi([p_i]_{i\in\mathbb{N}_N})$ motivates smaller controller gains. The third term $\psi(\mathcal{Y})$ optimizes the control performance of the networked system (e.g., optimizing the passivity indices or L2-gain, see Rm .1). Overall, the LMI problem (16) provides a distributed controller design that also optimizes the underlying communication topology.

C. Local Controller Design Revisit

The global co-design problem (16), particularly its feasibility and effectiveness, is clearly dependent on the subsystem dissipativity properties enforced by the local controller design problem (12). Therefore, we propose to influence the local controller design (12) to achieve favorable subsystem dissipativity properties that can potentially lead to feasible and effective global co-designs. The main idea is to identify a necessary condition for the LMI conditions in the global co-design problem (16), and then to decompose it into a form that can be included in the local controller design problems (12) at different subsystems.

Using Lm. 3, it is easy to see that the main LMI in (16) is equivalent to $[W_{ij}]_{i,j\in\mathbb{N}_N} \geq 0$, where W_{ij} is as defined in (21). Now, a set of necessary condition for $[W_{ij}]_{i,j\in\mathbb{N}_N} \geq 0$ can be identified as $\{W_{ii} \geq 0, \forall i \in \mathbb{N}_N\}$. Dividing these necessary conditions by the respective scalars in $\{p_i : i \in \mathbb{N}_N\}$ and defining $\bar{\mathcal{Y}}_{ii}^{kl} \triangleq p_i^{-1}\mathcal{Y}_{ii}^{kl}, \forall k,l \in \mathbb{N}_2, I \in \mathbb{N}_N$, we get the necessary conditions as $p_i^{-1}W_{ii} \geq 0, \forall i \in \mathbb{N}_N$, i.e.,

$$\begin{bmatrix} \mathcal{X}_i^{11} & \mathbf{0} & p_i^{-1}\mathcal{X}_i^{11}B_i\bar{K}_{ii} & \mathcal{X}_i^{11} \\ \star & -\bar{\mathcal{Y}}_{ii}^{22} & -\bar{\mathcal{Y}}_{ii}^{22} & \mathbf{0} \\ \star & \star & -\mathcal{H}(p_i^{-1}\mathcal{X}_i^{21}B_i\bar{K}_{ii}) - \mathcal{X}_i^{22} & -\mathcal{X}_i^{21} + \bar{\mathcal{Y}}_{ii}^{21} \\ \star & \star & \star & \bar{\mathcal{Y}}_{ii}^{11} \end{bmatrix} \geq 0, \quad \forall i \in \mathbb{N}_N.$$

To further simplify these necessary conditions, we make the following assumption regarding subsystem and networked system dissipativity properties.

***Assumption** 3:* The desired subsystem and networked system dissipativity properties, i.e., $\{\mathcal{X}_i : i \in \mathbb{N}_N\}$ and $\mathcal{Y}$ matrices, respectively, have inner blocks that are scalar matrices such that $\mathcal{X}_i^{kl} = x_i^{kl}\mathbf{I}$ and $\mathcal{Y}_{ii}^{kl} = y_{ii}^{kl}\mathbf{I}$, for any $k,l \in \mathbb{N}_2, i \in \mathbb{N}_N$.

It is worth noting that As. 3 holds for dissipativity properties given in Rm. 1. Using As. 3, Lm. 2 (with $P \triangleq \text{diag}(\begin{bmatrix}\mathbf{I} & p_i\mathbf{I} & \mathbf{I} & \mathbf{I}\end{bmatrix})$), and the change of variables $\tilde{K}_{ii}^{11} \triangleq p_i^{-1}x_i^{11}\bar{K}_{ii}$, $\bar{y}_{ii}^{kl} \triangleq p_i^{-1}y_{ii}^{kl}$ and $\tilde{y}_{ii}^{22} \triangleq p_iy_{ii}^{22}$ for any $k,l \in \mathbb{N}_2, \forall i \in \mathbb{N}_N$, we can simplify the identified necessary conditions to obtain

$$\begin{bmatrix} x_i^{11}\mathbf{I} & \mathbf{0} & B_i\tilde{K}_{ii}^{11} & x_i^{11}\mathbf{I} \\ \star & -\tilde{y}_{ii}^{22}\mathbf{I} & -y_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \star & -\mathcal{H}(B_i\tilde{K}_{ii}^{21}) - x_i^{22}\mathbf{I} & -x_i^{21}\mathbf{I} + \bar{y}_{ii}^{21}\mathbf{I} \\ \star & \star & \star & \bar{y}_{ii}^{11}\mathbf{I} \end{bmatrix} \geq 0, \quad y_{ii}^{22} < 0, \quad \forall i \in \mathbb{N}_N, \tag{17}$$

where $y_{ii}^{22} < 0$ is equivalent for $p_i > 0$, and each $\tilde{K}_{ii}^{21}, i \in \mathbb{N}_N$ should satisfy the constraint

$$\tilde{K}_{ii}^{21} = (x_i^{11})^{-1}x_i^{21}\tilde{K}_{ii}^{11}. \tag{18}$$

By relaxing the latter non-linear equality constraints and letting each $\tilde{K}_{ii}^{21}, i \in \mathbb{N}_N$ to be a free matrix, (17) becomes a set of necessary conditions for (16).

Note that each necessary condition in (17) is an LMI, enabling its inclusion in a corresponding local controller design problem. Particularly, the i-th necessary condition in (17), where $i \in \mathbb{N}_N$, is an LMI in variables: $\{x_i^{kl} : k,l \in \mathbb{N}_2\}$ (subsystem dissipativity properties) and $\{\tilde{K}_{ii}^{11}, \tilde{K}_{ii}^{21}, \bar{y}_{ii}^{11}, \bar{y}_{ii}^{12}, \bar{y}_{ii}^{21}, y_{ii}^{22}, \tilde{y}_{ii}^{22}\}$. Using the latter variables and the used change of variables relationships, while it appears that one can "recover" some of the global co-design variables (e.g., $p_i = \tilde{y}_{ii}^{22}(y_{ii}^{22})^{-1}$, $y_{ii}^{11} = p_i\bar{y}_{ii}^{11}$, $y_{ii}^{12} = p_i\bar{y}_{ii}^{12}$, $y_{ii}^{21} = p_i\bar{y}_{ii}^{21}$, and $\bar{K}_{ii} = p_i(x_i^{11})^{-1}\tilde{K}_{ii}^{11}$), such "recovered" variables are neither required nor expected to align perfectly with the subsequent corresponding global co-design variable values observed when (16) is solved. After all, (17) is only necessary (not sufficient) for (16).

***Remark** 5:* When each subsystem Σ_i has an input matrix B_i that contains a zero, then the matrix block $B_i\tilde{K}_{ii}^{11}$ in (17) has at least one zero in its main diagonal. Using this fact, we can re-apply Lm. 3 to identify a set of necessary conditions for (17) (and thus for (16)) as

$$\begin{bmatrix} \mathcal{X}_i^{11} & 0 & 0 & \mathcal{X}_i^{11} \\ \star & -\tilde{\mathcal{Y}}_{ii}^{22} & -\mathcal{Y}_{ii}^{22} & \mathbf{0} \\ \star & \star & -\mathcal{X}_i^{22} & -\mathcal{X}_i^{21}+\bar{\mathcal{Y}}_{ii}^{21} \\ \star & \star & \star & \bar{\mathcal{Y}}_{ii}^{11} \end{bmatrix} \geq 0, \tag{19}$$

$$\mathcal{Y}_{ii}^{22} < 0, \quad \forall i \in \mathbb{N}_N.$$

Finally, the following proposition combines the obtained necessary conditions (17) with the corresponding local LMI problems (12) to formulate a comprehensive local controller design problem that also accounts for the subsequent global co-design problem (16).

***Proposition** 5:* Under Assumptions 1-3, at each subsystem $\Sigma_i, i \in \mathbb{N}_N$, to enforce/optimize the closed-loop subsystem $\mathcal{X}_i$-dissipativity (from $\tilde{u}_i$ to y_i, where $\mathcal{X}_i \triangleq [\mathcal{X}_i^{kl}\mathbf{I}]_{k,l\in\mathbb{N}_2}$) so that it favors the subsequent global co-design stage (i.e., (16) in Prop. 4), the local controller K_i (10) can be designed using the LMI problem:

$$\begin{aligned} &\text{Find: } \tilde{K}_i, \bar{P}_i, \{\bar{\mathcal{X}}_i^{11}, \mathcal{X}_i^{12}, \mathcal{X}_i^{21}, \mathcal{X}_i^{22}\}, \\ &\qquad \{\tilde{K}_{ii}^{11}, \hat{K}_{ii}^{21}, \tilde{\mathcal{Y}}_{ii}^{11}, \bar{\mathcal{Y}}_{ii}^{12}, \bar{\mathcal{Y}}_{ii}^{21}, \hat{\mathcal{Y}}_{ii}^{22}, \tilde{\mathcal{Y}}_{ii}^{22}\} \\ &\text{Sub. to: } \bar{P}_i > 0,\ \mathcal{X}_i^{22} < 0,\ \hat{\mathcal{Y}}_{ii}^{22} < 0, \\ &\begin{bmatrix} \mathbf{I} & \mathbf{0} & C_i\bar{P}_i & \mathbf{0} \\ \star & \bar{P}_i & A_i\bar{P}_i + B_i\tilde{K}_i & (-\mathcal{X}_i^{22})\mathbf{I} \\ \star & \star & \bar{P}_i & \mathcal{X}_i^{21}\bar{P}_iC_i^\top \\ \star & \star & \star & \bar{\mathcal{X}}_i^{11}\mathbf{I} \end{bmatrix} \geq 0, \text{ and} \\ &\begin{bmatrix} \bar{\mathcal{X}}_i^{11}\mathbf{I} & \mathbf{0} & B_i\tilde{K}_{ii}^{11} & \bar{\mathcal{X}}_i^{11}\mathbf{I} \\ \star & -\tilde{\mathcal{Y}}_{ii}^{22}\mathbf{I} & -\hat{\mathcal{Y}}_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \star & -\mathcal{H}(B_i\hat{K}_{ii}^{21})+\mathbf{I} & -\mathcal{X}_i^{21}\mathbf{I}+\bar{\mathcal{Y}}_{ii}^{21}\mathbf{I} \\ \star & \star & \star & \tilde{\mathcal{Y}}_{ii}^{11}\mathbf{I} \end{bmatrix} \geq 0, \end{aligned} \tag{20}$$

with $K_i = \tilde{K}_i\bar{P}_i^{-1}$ and $\mathcal{X}_i^{11} = (-\mathcal{X}_i^{22})^{-1}\bar{\mathcal{X}}_i^{11}$.

Proof: Applying As. 3 in (12) we get

$$\begin{bmatrix} (-\mathcal{X}_i^{22})^{-1}\mathbf{I} & \mathbf{0} & C_iP_i & \mathbf{0} \\ \star & P_i & A_iP_i + B_i\bar{K}_i & \mathbf{I} \\ \star & \star & P_i & \mathcal{X}_i^{21}P_iC_i^\top \\ \star & \star & \star & \mathcal{X}_i^{11}\mathbf{I} \end{bmatrix} \geq 0.$$

By multiplying the LHS by $(-\mathcal{X}_i^{22})$ and applying the change of variables $\bar{P}_i \triangleq (-\mathcal{X}_i^{22})P_i$, $\tilde{K}_i \triangleq (-\mathcal{X}_i^{22})\bar{K}_i$, and $\bar{\mathcal{X}}_i^{11} \triangleq (-\mathcal{X}_i^{22})\mathcal{X}_i^{11}$, we get the first, second and fourth LMIs in (20).

From the previous discussion, we derived (17) as a necessary condition for (16). By multiplying the first LMI in (17) by $(-\mathcal{X}_i^{22})^{-1}$ and then applying Lm. 2 with $P = \text{diag}([(-\mathcal{X}_i^{22}) \quad \mathbf{I} \quad \mathbf{I} \quad (-\mathcal{X}_i^{22})])$, we get (22). Next, using the change of variables used above and $\tilde{\mathcal{Y}}_{ii}^{11} \triangleq (-\mathcal{X}_i^{22})\bar{\mathcal{Y}}_{ii}^{11}$, $\tilde{\mathcal{Y}}_{ii}^{22} \triangleq (-\mathcal{X}_i^{22})^{-1}\tilde{\mathcal{Y}}_{ii}^{22}$, $\hat{\mathcal{Y}}_{ii}^{22} \triangleq (-\mathcal{X}_i^{22})^{-1}\mathcal{Y}_{ii}^{22}$, $\hat{K}_{ii}^{21} \triangleq (-\mathcal{X}_i^{22})^{-1}\tilde{K}_{ii}^{21}$, we get the fifth LMI in (20). Finally, the second LMI in (17), i.e., $\mathcal{Y}_{ii}^{22} < 0$, via the used change of variables, is equivalent to $\hat{\mathcal{Y}}_{ii}^{22} < 0$, i.e., the third LMI in (20). This completes the proof. ■

***Remark** 6:* In the revised local controller design problem (20), one can include an objective function (besides just finding a feasible solution) to maximize the influence on the global co-design effectiveness. To see this, note that the global co-design problem variables can be "recovered" from (20) as: $p_i = (\hat{\mathcal{Y}}_{ii}^{22})^{-1}\tilde{\mathcal{Y}}_{ii}^{22}$, $\mathcal{Y}_{ii}^{11} = p_i(-\mathcal{X}_i^{22})^{-1}\tilde{\mathcal{Y}}_{ii}^{11}$, $\mathcal{Y}_{ii}^{12} = p_i\bar{\mathcal{Y}}_{ii}^{12}$, $\mathcal{Y}_{ii}^{21} = p_i\bar{\mathcal{Y}}_{ii}^{21}$, $\mathcal{Y}_{ii}^{22} = (-\mathcal{X}_i^{22})\hat{\mathcal{Y}}_{ii}^{22}$ and $\bar{K}_{ii} = p_i(\mathcal{X}_i^{11})^{-1}\tilde{K}_{ii}^{11}$. Therefore, to optimize the global co-design problem variables $p_i, \mathcal{Y}_{ii}^{11}, \mathcal{Y}_{ii}^{12}, \mathcal{Y}_{ii}^{21}, \mathcal{Y}_{ii}^{22}$ and $\bar{K}_{ii}$, one can optimize the local controller design variables $\tilde{\mathcal{Y}}_{ii}^{22}, \tilde{\mathcal{Y}}_{ii}^{11}, \bar{\mathcal{Y}}_{ii}^{12}, \bar{\mathcal{Y}}_{ii}^{21}, \hat{\mathcal{Y}}_{ii}^{22}$ and $\tilde{K}_{ii}^{11}$, respectively.

IV. Data-Driven Hierarchical Design of Networked Systems

In this section, for the considered class of linear networked systems (see Fig. 2), we provide a data-driven hierarchical control design framework. In particular, we exploit the model-based LMI formulations developed for local controller design (20) and global co-design (16) in Sec. III, but strategically replace the role of subsystem model parameters by modifying the overall design framework using data trajectories obtained from the subsystems.

Overall, the proposed approach is a direct data-driven control design strategy, where controllers (both local and global) are designed directly based on observed data trajectories, without estimating subsystem model parameters (like in indirect data-driven control design). Therefore, it is more practical and efficient compared to indirect data-driven control design approaches. Moreover, it leads to robust networked system designs that can also be extended conveniently to integrate online design, fault detection, and recovery mechanisms.

A. Data-Driven Local Controller Design

While each subsystem $\Sigma_i, i \in \mathbb{N}_N$ follows the dynamic model (9), here we assume that we do not know its exact model parameters $\Theta_i \equiv (A_i, B_i, C_i)$ in (9)), but have finite data trajectories $\Phi_i \triangleq \{(u_i(t), x_i(t), y_i(t)) : t \in \mathbb{N}_T^0\}$ at our disposal. For each subsystem $\Sigma_i, i \in \mathbb{N}_N$, let us define the data matrices (as $1 \times T$ block row matrices):

$$\begin{aligned} X_i &\triangleq [x_i(t)]_{t\in\mathbb{N}_{T-1}^0}, \quad \bar{X}_i \triangleq [x_i(t)]_{t\in\mathbb{N}_T}, \\ U_i &\triangleq [u_i(t)]_{t\in\mathbb{N}_{T-1}^0}, \quad Y_i \triangleq [y_i(t)]_{t\in\mathbb{N}_{T-1}^0}, \\ W_i &\triangleq [w_i(t)]_{t\in\mathbb{N}_{T-1}^0}. \end{aligned} \tag{23}$$

Note that, while we know the "system" data matrices $\bar{\Phi}_i \triangleq (X_i, \bar{X}_i, U_i, Y_i),\ i \in \mathbb{N}_N$, we do not know "disturbance" data matrices $W_i, i \in \mathbb{N}_N$. However, we make the following assumption about such disturbance data matrices.

***Assumption** 4:* For each subsystem $\Sigma_i, i \in \mathbb{N}_N$, its disturbance data matrix W_i (23) is bounded such that

$$\begin{bmatrix} \mathbf{I} \\ W_i^\top \end{bmatrix}^\top Q_{wi} \begin{bmatrix} \mathbf{I} \\ W_i^\top \end{bmatrix} \geq 0 \tag{24}$$

where $Q_{wi} \triangleq [Q_{wi}^{kl}]_{k,l\in\mathbb{N}_2}$ is symmetric and $Q_{wi}^{22} < 0$.

***Remark** 7:* The above As. 4 is a standard assumption used in data-driven control literature [**?**]. It represents an eliptical bound on disturbances, and hence covers simple spherical bounds on disturbances like $W_iW_i^\top \leq \gamma_i\mathbf{I}$ where $\gamma_i > 0$ is a constant. In some instances (e.g., with spherical bounds on disturbances), we can also treat each Q_{wi} as a design matrix, of which the corresponding elliptical boundary needs to be maximized, so as to increase the flexibility and the disturbance robustness of the data-driven design.

$$W_{ij} \triangleq \begin{bmatrix} p_i\mathcal{X}_i^{11}\mathrm{e}_{ij} & \mathbf{0} & \mathcal{X}_i^{11}B_i\bar{K}_{ij} & p_i\mathcal{X}_i^{11}\mathrm{e}_{ij} \\ \star & -\mathcal{Y}_{ij}^{22} & -\mathcal{Y}_{ij}^{22} & \mathbf{0} \\ \star & \star & -\mathcal{H}(\mathcal{X}_i^{21}B_i\bar{K}_{ij}) - p_i\mathcal{X}_i^{22}\mathrm{e}_{ij} & -p_i\mathcal{X}_i^{21}\mathrm{e}_{ij} + \mathcal{Y}_{ij}^{21} \\ \star & \star & \star & \mathcal{Y}_{ij}^{11} \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} (-x_i^{22})x_i^{11}\mathbf{I} & \mathbf{0} & B_i\tilde{K}_{ii}^{11} & (-x_i^{22})x_i^{11}\mathbf{I} \\ \star & -(-x_i^{22})^{-1}\tilde{y}_{ii}^{22}\mathbf{I} & -(-x_i^{22})^{-1}y_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \star & -\mathcal{H}(B_i(-x_i^{22})^{-1}\tilde{K}_{ii}^{21}) - (-x_i^{22})^{-1}x_i^{22}\mathbf{I} & -x_i^{21}\mathbf{I} + \bar{y}_{ii}^{21}\mathbf{I} \\ \star & \star & \star & (-x_i^{22})\bar{y}_{ii}^{11}\mathbf{I} \end{bmatrix} \tag{22}$$

We also make the following standard assumption that implies the persistence of excitation in the data Φ_i [?].

***Assumption** 5:* For each subsystem $\Sigma_i, i \in \mathbb{N}_N$,

$$\text{rank}\begin{bmatrix} U_i \\ X_i \end{bmatrix} = n_{ui} + n_{xi} \quad \text{(i.e., full row rank).}$$

For each subsystem $\Sigma_i, i \in \mathbb{N}_N$, as the data matrices (23) are resulting from the dynamics (9), they satisfy:

$$\begin{aligned} \bar{X}_i &= A_iX_i + B_iU_i + W_i, \\ Y_i &= C_iX_i. \end{aligned} \tag{25}$$

Consequently, the set of model parameters that is consistent under the observed data matrices $\bar{\Phi}_i$ for some disturbance data matrix W_i satisfying (24), can be defined as:

$$\Psi_i \triangleq \Big\{(A_i, B_i, C_i) : \text{(25) holds for some } W_i \text{ satisfying (24)}\Big\}. \tag{26}$$

Using (25), for any W_i that satisfies (24), we can obtain the following quadratic matrix inequality (QMI) constraint on A_i and B_i in Θ_i:

$$\begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix}^\top \underbrace{\begin{bmatrix} \mathbf{I}_n & \bar{X}_i \\ \mathbf{0}_n & -X_i \\ \mathbf{0}_{mn} & -U_i \end{bmatrix} Q_{wi} \begin{bmatrix} \mathbf{I}_n & \bar{X}_i \\ \mathbf{0}_n & -X_i \\ \mathbf{0}_{mn} & -U_i \end{bmatrix}^\top}_{\triangleq \bar{Q}_{wi} \Leftarrow (n+n+m)\times(n+n+m)} \begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix} \geq 0. \tag{27}$$

$n\times T$

$n\times T$

$(n+T)\times(n+T)$

$m\times T$

On the other hand, using (25), we can obtain the following constraint on C_i in Θ_i:

$$C_i = Y_i\check{K}_i \text{ where } \check{K}_i \in \Pi_i \triangleq \{\check{K}_i : X_i\check{K}_i = \mathbf{I}\}. \tag{28}$$

Using Assumptions 4 and 5, we have the following lemma.

***Lemma** 5:* Under Assumptions 4 and 5, we have

$$\Psi_i = \Gamma_i \triangleq \{(A_i, B_i, C_i) : \text{(27) and (28) holds}\},$$

and for any $\Theta_i \in \Psi_i$, (28) uniquely determines $C_i, i \in \mathbb{N}_N$.

Proof: First, we prove that $\Gamma_i \subseteq \Psi_i$ by showing that any $\Theta_i \in \Gamma_i \implies \Theta_i \in \Psi$. Consider a particular $\Theta_i \in \Gamma_i$, which implies that (27) and (28) hold for that Θ_i. Using this Θ_i and the observed data matrices $\bar{\Phi}_i$, we can define a disturbance matrix as $W_i(\Theta_i) \triangleq \bar{X}_i - A_iX_i - B_iU_i$. Clearly, this W_i satisfies

$$\begin{bmatrix} \mathbf{I} \\ W_i^\top \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \bar{X}_i \\ \mathbf{0} & -X_i \\ \mathbf{0} & -U_i \end{bmatrix}^\top \begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix},$$

and thus, it also satisfies (24) (via (27)). Moreover, by definition, for this W_i, Θ_i (particularly its A_i and B_i) satisfies the first equation in (25).

On the other hand, we also have $Y_i\check{K}_i = C_i$ from (28), where $\check{K}_i$ satisfies $X_i\check{K}_i = \mathbf{I}$ (as X_i is full row rank due to As. 5, solution space Π_i for $\check{K}_i$ is non-empty). Pre-multiplying both sides of the latter equation by C_i we get $C_iX_i\check{K}_i = C_i$. This, when subtracted from prior equation, we get $(Y_i - C_iX_i)\check{K}_i = \mathbf{0}$, which implies that C_i satisfies the second equation in (25). Therefore, we can conclude that any $\Theta_i \in \Gamma_i \implies \Theta_i \in \Psi$, in other words, $\Gamma_i \subseteq \Psi_i$.

Next, we prove that $\Gamma_i \supseteq \Psi_i$ by showing that any $\Theta_i \in \Psi \implies \Theta_i \in \Gamma_i$. Note that, if any $\Theta_i \in \Psi$, that Θ_i satisfies (25) for some W_i, where $W_i = \bar{X}_i - A_iX_i - B_iU_i$ and satisfies (24). Using these two facts about W_i, we can obtain (27), implying Θ_i (particularly its A_i and B_i) satisfies (27).

On the other hand, as Θ_i satisfies (25), we have $Y_i = C_iX_i$. Let us select $\check{K}_i$ such that $X_i\check{K}_i = \mathbf{I}$. Using these two equations, we get $C_iX_i\check{K}_i = C_i \iff Y_i\check{K}_i = C_i$. This implies that C_i satisfies (28). Therefore, we can conclude that any $\Theta_i \in \Psi \implies \Theta_i \in \Gamma_i$, in other words, $\Gamma_i \supseteq \Psi_i$.

Combining these results, we have $\Psi_i = \Gamma_i$. Finally, to show that C_i is uniquely determined by (28), assume the opposite: let $C_i^{(1)}$ and $C_i^{(2)}$ be two distinct solutions obtained from (28) for C_i, where $\check{K}_i^{(1)}$ and $\check{K}_i^{(2)}$ are their corresponding $\check{K}_i$ values in Π_i. Applying these definitions in (28), we can easily obtain $C_i^{(1)} - C_i^{(2)} = Y_i(\check{K}_i^{(1)} - \check{K}_i^{(2)})$ and $X_i(\check{K}_i^{(1)} - \check{K}_i^{(2)}) = \mathbf{0}$. Applying from (25), the prior equation becomes $C_i^{(1)} - C_i^{(2)} = C_iX_i(\check{K}_i^{(1)} - \check{K}_i^{(2)})$, which can be further simplified using the latter equation to get $C_i^{(1)} - C_i^{(2)} = \mathbf{0}$. This contradicts the earlier non-uniqueness assumption and thus completes the proof. ∎

***Remark** 8:* Data measurement noise can also be included in (25). To handle this, we first have to extend the boundedness assumption (24) to include such measurement noise. Next, the QMI (27) has to be extended to include all three of the parameters A_i, B_i, C_i in a unified QMI (without having C_i uniquely determined via (28)). Upon these extensions, a similar equivalence lemma as the above Lm. 5 can be established (like [?, Lm.1]). Continuing this, the subsequent results presented in this paper can also be generalized to include data measurement noise (exploiting dualization lemma [xxx], inspired by recent work [xxx]). However, in this paper, we do not provide this generalization to maintain brevity and simplicity without obscuring the fundamental details of the proposed data-driven design technique. A formal treatment of data measurement noise is the subject of ongoing research and will be presented as an extension of this paper.

Now, the goal is to design a local controller K_i (10) at

$n = m$ in $\mathcal{X}_i$-dissipativity

each subsystem $\Sigma_i, i \in \mathbb{N}_N$, to make the closed-loop subsystem (11) $\mathcal{X}_i$-dissipative under all possible model parameters $\Theta_i \in \Psi_i$ given the observed data matrices $\bar{\Phi}_i$. The required data-driven local controller design process is proven in the following proposition.

***Proposition** 6:* At each subsystem $\Sigma_i, i \in \mathbb{N}_N$, using the observed data matrices $\bar{\Phi}_i$ and under Assumptions 1, 2, 4, and 5, to enforce/optimize $\mathcal{X}_i$-dissipativity for the closed-loop subsystem dynamics (11) for any subsystem model parameters $\Theta_i \in \Psi_i$, the local controller K_i (10) can be designed using the LMI problem:

$$\begin{aligned} &\text{Find: } \bar{K}_i, \tilde{K}_i, \mathcal{X}_i, \lambda_i \\ &\text{Sub. to: } \lambda_i \geq 0, \quad \text{and} \quad \begin{bmatrix} Q_i & S_i \\ S_i^\top & R_i \end{bmatrix} \geq 0, \end{aligned} \tag{29}$$

where

$$Q_i \triangleq \begin{bmatrix} (-\mathcal{X}_i^{22})^{-1} & Y_i\tilde{K}_i & \mathbf{0} \\ \star & X_i\tilde{K}_i & \tilde{K}_i^\top Y_i^\top \mathcal{X}_i^{21} \\ \star & \star & \mathcal{X}_i^{11} \end{bmatrix},$$

$$S_i \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & X_i\tilde{K}_i & \bar{K}_i^\top \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$R_i \triangleq \begin{bmatrix} X_i\tilde{K}_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \lambda_i \begin{bmatrix} \mathbf{I} & \bar{X}_i \\ \mathbf{0} & -X_i \\ \mathbf{0} & -U_i \end{bmatrix} Q_{wi} \begin{bmatrix} \mathbf{I} & \bar{X}_i \\ \mathbf{0} & -X_i \\ \mathbf{0} & -U_i \end{bmatrix}^\top. \tag{30}$$

with $K_i = \bar{K}_i(X_i\tilde{K}_i)^{-1}$.

Proof: We start by applying Prop. 3, which require:

$$\begin{bmatrix} (-\mathcal{X}_i^{22})^{-1} & \mathbf{0} & C_iP_i & \mathbf{0} \\ \star & P_i & A_iP_i + B_i\bar{K}_i & \mathbf{I} \\ \star & \star & P_i & P_iC_i^\top \mathcal{X}_i^{21} \\ \star & \star & \star & \mathcal{X}_i^{11} \end{bmatrix} \geq 0.$$

This is where the error comes!

Using Co. 1, we can obtain an equivalent condition as

$$P_i - [\star]^\top Q_i^{-1} \begin{bmatrix} \mathbf{0} \\ P_iA_i^\top + \bar{K}_i^\top B_i^\top \\ \mathbf{I} \end{bmatrix} \geq 0$$

where

$$Q_i \triangleq \begin{bmatrix} (-\mathcal{X}_i^{22})^{-1} & C_iP_i & \mathbf{0} \\ \star & P_i & P_iC_i^\top \mathcal{X}_i^{21} \\ \star & \star & \mathcal{X}_i^{11} \end{bmatrix}$$

This condition can be restated as a QMI in A_i and B_i as

$$\begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix}^\top \bar{R}_i \begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix} \geq 0$$

where

$$\bar{R}_i \triangleq \begin{bmatrix} P_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - [\star]^\top Q_i^{-1} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & P_i & \bar{K}_i^\top \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

For this QMI to hold for any $\Theta_i \triangleq (A_i, B_i, C_i) \in \Psi_i$ (see also Lm. 5 and (27)), both necessary and sufficient conditions can be found using the matrix S-lemma (i.e., Lm. 4) as

$$\bar{R}_i - \lambda_i \bar{Q}_{wi} \geq 0,$$

for some $\lambda_i \geq 0$ where we recall

$$\bar{Q}_{wi} \triangleq \begin{bmatrix} \mathbf{I} & \bar{X}_i \\ \mathbf{0} & -X_i \\ \mathbf{0} & -U_i \end{bmatrix} Q_{wi} \begin{bmatrix} \mathbf{I} & \bar{X}_i \\ \mathbf{0} & -X_i \\ \mathbf{0} & -U_i \end{bmatrix}^\top.$$

Now, we apply Lm. 1 to obtain an equivalent condition as

$$\bar{R}_i - \lambda_i \bar{Q}_{wi} \geq 0 \iff \begin{bmatrix} Q_i & S_i \\ S_i^\top & R_i \end{bmatrix} \geq 0,$$

where Q_i is as defined before,

$$S_i \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & P_i & \bar{K}_i^\top \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \text{and} \quad R_i \triangleq \begin{bmatrix} P_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \lambda_i \bar{Q}_{wi}.$$

Note that this LMI contains C_i terms, particularly inside its block Q_i. As established in Lm. 5, such terms can be replaced using (28). Consequently, Q_i can be restated as

$$Q_i = \begin{bmatrix} (-\mathcal{X}_i^{22})^{-1} & Y_i\check{K}_iP_i & \mathbf{0} \\ \star & P_i & P_i\check{K}_i^\top Y_i^\top \mathcal{X}_i^{21} \\ \star & \star & \mathcal{X}_i^{11} \end{bmatrix},$$

where $\check{K}_i$ satisfies $X_i\check{K}_i = \mathbf{I}$. To remove the bilinear terms in $\check{K}_i$ and P_i, we next use the change of variables:

$$\tilde{K}_i \triangleq \check{K}_iP_i \text{ and } P_i = X_i\tilde{K}_i,$$

where the latter is due to $X_i\check{K}_i = \mathbf{I} \iff X_i\check{K}_iP_i = P_i$. Upon making these substitutions, we obtain the main LMI condition in (29) along with the same Q_i, S_i and R_i definitions given in (30).

Finally, as $K_i = \bar{K}_iP_i^{-1}$ according to Prop. 3, here, it becomes $K_i = \bar{K}_i(X_i\tilde{K}_i)^{-1}$ due to the introduced change of variables. This completes the proof. ∎

The above result can be interpreted as the data-driven version of Prop. 3. As the next step, we present the data-driven version of Prop. 5, which is aimed at designing the local controllers so that they are more favorable for the subsequent global co-design stage.

***Proposition** 7:* At each subsystem $\Sigma_i, i \in \mathbb{N}_N$, using the observed data matrices $\bar{\Phi}_i$ and under Assumptions 1-5, to enforce/optimize $\mathcal{X}_i$-dissipativity for the closed-loop subsystem dynamics (11) for any subsystem model parameters $\Theta_i \in \Psi_i$ so that it favors the subsequent global co-design stage, the local controller K_i (10) can be designed using the LMI problem:

$$\begin{aligned} &\text{Find: } \tilde{K}_i, \hat{K}_i, \lambda_{i1}, \lambda_{i2}, \{\bar{x}_i^{11}, x_i^{12}, x_i^{21}, x_i^{22}\}, \\ &\qquad \{\tilde{K}_{ii}^{11}, \tilde{K}_{ii}^{21}, \bar{y}_{ii}^{11}, \bar{y}_{ii}^{12}, \bar{y}_{ii}^{21}, \hat{y}_{ii}^{22}, \check{y}_{ii}^{22}\} \\ &\text{Sub. to: } x_i^{22} < 0, \ \hat{y}_{ii}^{22} < 0, \ \lambda_{i1} \geq 0, \ \lambda_{i2} \geq 0, \\ &\qquad \begin{bmatrix} Q_{i1} & S_{i1} \\ S_{i1}^\top & R_{i1} \end{bmatrix} \geq 0, \text{ and } \begin{bmatrix} Q_{i2} & S_{i2} \\ S_{i2}^\top & R_{i2} \end{bmatrix} \geq 0, \end{aligned} \tag{31}$$

$u \leftarrow m$ 5 same X_i -dissip.
$x \leftarrow n$
$y \leftarrow r$
$\tilde{K}_i \leftarrow m\times n$
$\hat{K}_i \leftarrow T\times n$
$\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix}$

where

$$Q_{i1} \triangleq \begin{bmatrix} \mathbf{I} & Y_i\hat{K}_i & \mathbf{0} \\ \star & X_i\hat{K}_i & x_i^{21}\hat{K}_i^\top Y_i^\top \\ \star & \star & \bar{x}_i^{11}\mathbf{I} \end{bmatrix},$$

$$S_{i1} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & X_i\hat{K}_i & \tilde{K}_i^\top \\ (-x_i^{22})\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$R_{i1} \triangleq \begin{bmatrix} X_i\hat{K}_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \lambda_{i1}\bar{Q}_{wi},$$

$\llcorner (n+n+m)\times(n+n+m)$

$$Q_{i2} \triangleq \begin{bmatrix} -\tilde{y}_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \tilde{y}_{ii}^{11}\mathbf{I} \end{bmatrix}, \tag{32}$$

$$S_{i2} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & -\tilde{y}_{ii}^{22}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \bar{x}_i^{11}\mathbf{I} & \mathbf{0} & \mathbf{0} & -x_i^{21}\mathbf{I} + \tilde{y}_{ii}^{21}\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$R_{i2} \triangleq \begin{bmatrix} \bar{x}_i^{11}\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \tilde{K}_{ii}^{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\tilde{K}_{ii}^{11})^\top & \mathbf{I} & \mathbf{0} & -\hat{K}_{ii}^{12} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\hat{K}_{ii}^{21} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \lambda_{i2}\begin{bmatrix} \bar{Q}_{wi} & \mathbf{0} \\ \mathbf{0} & \bar{Q}_{wi} \end{bmatrix},$$

with $K_i = \tilde{K}_i(X_i\hat{K}_i)^{-1}$ and $x_i^{11} = (-x_i^{22})^{-1}\bar{x}_i^{11}$.

$m\times n$ $\quad m\times n \quad n\times T \quad T\times n$

Proof: We start by applying Prop. 5, which requires two main LMI conditions, where the first main LMI condition is:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} & C_i\bar{P}_i & \mathbf{0} \\ \star & \bar{P}_i & A_i\bar{P}_i + B_i\tilde{K}_i & (-x_i^{22})\mathbf{I} \\ \star & \star & \bar{P}_i & x_i^{21}\bar{P}_iC_i^\top \\ \star & \star & \star & \bar{x}_i^{11}\mathbf{I} \end{bmatrix} \geq 0.$$

For this, by applying Co. 1, an equivalent condition can be obtained as

$$\bar{P}_i - [\star]^\top Q_{i1}^{-1}\begin{bmatrix} \mathbf{0} \\ \bar{P}_iA_i^\top + \tilde{K}_i^\top B_i^\top \\ (-x_i^{22})\mathbf{I} \end{bmatrix} \geq 0,$$

where

$$Q_{i1} \triangleq \begin{bmatrix} \mathbf{I} & C_i\bar{P}_i & \mathbf{0} \\ \star & \bar{P}_i & x_i^{21}\bar{P}_iC_i^\top \\ \star & \star & \bar{x}_i^{11}\mathbf{I} \end{bmatrix}.$$

This LMI condition can now be restated as a QMI in A_i and B_i as

$$\begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix}^\top \bar{R}_{i1}\begin{bmatrix} \mathbf{I} \\ A_i^\top \\ B_i^\top \end{bmatrix} \geq 0,$$

where

$$\bar{R}_{i1} \triangleq \begin{bmatrix} \bar{P}_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - [\star]^\top Q_{i1}^{-1}\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{P}_i & \tilde{K}_i^\top \\ (-x_i^{22})\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

For this QMI to hold for any $(A, B_i, C_i) \in \Psi_i$, matrix S-lemma (Lm. 4) gives a necessary and sufficient condition as $\bar{R}_{i1} - \lambda_{i1}\bar{Q}_{wi} \geq 0$, for some $\lambda_{i1} \geq 0$. Applying Lm. 1, we get an equivalent condition for this as

$$\begin{bmatrix} Q_{i1} & S_{i1} \\ S_{i1}^\top & R_{i1} \end{bmatrix} \geq 0,$$

where Q_{i1} is as defined before,

$$S_{i1} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{P}_i & \tilde{K}_i^\top \\ (-x_i^{22})\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \text{ and}$$

$$R_{i1} \triangleq \begin{bmatrix} \bar{P}_i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \lambda_{i1}\bar{Q}_{wi}.$$

The C_i terms in this LMI (particularly inside its block Q_{i1}) now can be equivalently replaced using (28), which leads to

$$Q_{i1} = \begin{bmatrix} \mathbf{I} & Y_i\check{K}_i\bar{P}_i & \mathbf{0} \\ \star & \bar{P}_i & x_i^{21}\bar{P}_i\check{K}_i^\top Y_i^\top \\ \star & \star & \bar{x}_i^{11}\mathbf{I} \end{bmatrix},$$

where $\check{K}_i$ satisfies $X_i\check{K}_i = \mathbf{I}$. Now, using the change of variables

$$\underset{T\times n}{\hat{K}_i} \triangleq \underset{T\times n}{\check{K}_i}\underset{n\times n}{\bar{P}_i} \text{ with } \bar{P}_i = X_i\hat{K}_i,$$

where the latter is due to $X_i\check{K}_i = \mathbf{I} \iff X_i\check{K}_i\bar{P}_i = \bar{P}_i$, we can obtain the first main LMI condition given in (31) along with the same Q_{i1}, S_{i1} and R_{i1} definitions given in (32).

Next, we consider the second main LMI condition required in Prop. 5, which takes the form

$m\times n \quad m\times n$

$$\begin{bmatrix} \bar{x}_i^{11}\mathbf{I} & \mathbf{0} & B_i\tilde{K}_{ii}^{11} & \bar{x}_i^{11}\mathbf{I} \\ \star & -\tilde{y}_{ii}^{22}\mathbf{I} & -\tilde{y}_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \star & -\mathcal{H}(B_i\hat{K}_{ii}^{21}) + \mathbf{I} & -x_i^{21}\mathbf{I} + \tilde{y}_{ii}^{21}\mathbf{I} \\ \star & \star & \star & \tilde{y}_{ii}^{11}\mathbf{I} \end{bmatrix} \geq 0.$$

First, using Lm. 2, we obtain an equivalent LMI condition for it as

$$\begin{bmatrix} -\tilde{y}_{ii}^{22}\mathbf{I} & \mathbf{0} & \mathbf{0} & -\tilde{y}_{ii}^{22}\mathbf{I} \\ \star & \tilde{y}_{ii}^{11}\mathbf{I} & \bar{x}_i^{11}\mathbf{I} & -x_i^{21}\mathbf{I} + \tilde{y}_{ii}^{21}\mathbf{I} \\ \star & \star & \bar{x}_i^{11}\mathbf{I} & B_i\tilde{K}_{ii}^{11} \\ \star & \star & \star & -\mathcal{H}(B_i\hat{K}_{ii}^{21}) + \mathbf{I} \end{bmatrix} \geq 0,$$

which, through applying Lm. 1, can be restated as

$$\begin{bmatrix} \bar{x}_i^{11}\mathbf{I} & B_i\tilde{K}_{ii}^{11} \\ \star & -\mathcal{H}(B_i\hat{K}_{ii}^{21}) + \mathbf{I} \end{bmatrix} - \bar{S}_{i2}^\top Q_{i2}^{-1}\bar{S}_{i2} \geq 0$$

where

$$Q_{i2} \triangleq \begin{bmatrix} -\tilde{y}_{ii}^{22}\mathbf{I} & \mathbf{0} \\ \star & \tilde{y}_{ii}^{11}\mathbf{I} \end{bmatrix} \text{ and } \bar{S}_{i2} \triangleq \begin{bmatrix} \mathbf{0} & -\tilde{y}_{ii}^{22}\mathbf{I} \\ \bar{x}_i^{11}\mathbf{I} & -x_i^{21}\mathbf{I} + \tilde{y}_{ii}^{21}\mathbf{I} \end{bmatrix}.$$

To write this condition as a QMI in A_i and B_i, we first define $\Theta_i^\top \triangleq [\mathbf{I} \quad A_i \quad B_i]$, using which we express its first term as

$n+n+m \quad n+n+m \quad (n+n+m)\times n$

$$\begin{bmatrix} \bar{x}_i^{11}\mathbf{I} & B_i\tilde{K}_{ii}^{11} \\ \star & -\mathcal{H}(B_i\hat{K}_{ii}^{21}) + \mathbf{I} \end{bmatrix} = [\star]^\top \underbrace{\begin{bmatrix} \bar{R}_{i2}^{11} & \bar{R}_{i2}^{12} \\ \star & \bar{R}_{i2}^{22} \end{bmatrix}}_{\triangleq \bar{R}_{i2}} \begin{bmatrix} \Theta_i & \mathbf{0} \\ \mathbf{0} & \Theta_i \end{bmatrix}$$

where

$$\bar{R}_{i2}^{11} \triangleq \begin{bmatrix} \bar{x}_i^{11}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \bar{R}_{i2}^{12} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \tilde{K}_{ii}^{11} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\bar{R}_{i2}^{22} \triangleq \begin{bmatrix} \mathbf{I} & \mathbf{0} & -\hat{K}_{ii}^{12} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\hat{K}_{ii}^{21} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

Next, we express $\bar{S}_{i2}$ (in its second term) as

$$\bar{S}_{i2} \triangleq \begin{bmatrix}\mathbf{0} & -\hat{y}_{ii}^{22}\mathbf{I}\\ \bar{x}_i^{11}\mathbf{I} & -x_i^{21}\mathbf{I}+\bar{y}_{ii}^{21}\mathbf{I}\end{bmatrix} = \underbrace{\begin{bmatrix}S_{i2}^{11} & S_{i2}^{12}\\ S_{i2}^{21} & S_{i2}^{22}\end{bmatrix}}_{\triangleq S_{i2}} \begin{bmatrix}\bar{\Theta}_i & \mathbf{0}\\ \mathbf{0} & \bar{\Theta}_i\end{bmatrix}$$

where

$$S_{i2}^{11} \triangleq \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad S_{i2}^{12} \triangleq \begin{bmatrix}-\hat{y}_{ii}^{22}\mathbf{I} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$
$$S_{i2}^{21} \triangleq \begin{bmatrix}\bar{x}_i^{11}\mathbf{I} & \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad S_{i2}^{22} \triangleq \begin{bmatrix}-x_i^{21}\mathbf{I}+\bar{y}_{ii}^{21}\mathbf{I} & \mathbf{0} & \mathbf{0}\end{bmatrix}.$$

Using these two representations, we obtain the QMI:

$$\begin{bmatrix}\bar{\Theta}_i & \mathbf{0}\\ \mathbf{0} & \bar{\Theta}_i\end{bmatrix}^\top \underbrace{(\bar{R}_{i2} - S_{i2}^\top Q_{i2}^{-1} S_{i2})}_{\triangleq \tilde{R}_{i2}} \begin{bmatrix}\bar{\Theta}_i & \mathbf{0}\\ \mathbf{0} & \bar{\Theta}_i\end{bmatrix} \geq 0.$$

On the other hand, from (27), we have the QMI:

$$\begin{bmatrix}\bar{\Theta}_i & \mathbf{0}\\ \mathbf{0} & \bar{\Theta}_i\end{bmatrix}^\top \underbrace{\begin{bmatrix}\bar{Q}_{wi} & \mathbf{0}\\ \mathbf{0} & \bar{Q}_{wi}\end{bmatrix}}_{\triangleq \tilde{Q}_{wi}} \begin{bmatrix}\bar{\Theta}_i & \mathbf{0}\\ \mathbf{0} & \bar{\Theta}_i\end{bmatrix} \geq 0.$$

For this latter QMI to imply the former QMI, both a necessary and sufficient condition can be found using the matrix S-lemma as $\tilde{R}_{i2} - \lambda_{i2}\tilde{Q}_{wi} \geq 0$, for some $\lambda_{i2} \geq 0$. Applying Lm. 1, we can obtain an equivalent condition for this as

$$\begin{bmatrix}Q_{i2} & S_{i2}\\ S_{i2}^\top & R_{i2}\end{bmatrix} \geq 0,$$

where $R_{i2} \triangleq \tilde{R}_{i2} - \lambda_{i2}\tilde{Q}_{wi}$, which is the second main LMI condition given in (31) along with the same Q_{i2}, S_{i2} and R_{i2} definitions given in (32).

Finally, as $K_i = \tilde{K}_i \bar{P}_i^{-1}$ according to Prop. 5, here, it becomes $K_i = \tilde{K}_i(X_i \hat{K}_i)^{-1}$ due to the introduced change of variables. This completes the proof. ■

B. Data-Driven Global Co-design

The global co-design problem formulated in Prop. 4 primarily uses the already enforced closed-loop subsystem dissipativity properties. However, as evident from the $B\bar{K}$ terms appearing in (16), the global co-design problem also also requires the subsystem input matrices $B \triangleq \text{diag}([B_i]_{i\in\mathbb{N}_N})$. Consequently, one cannot follow up data-driven local controller designs (i.e., Prop. 6 or 7) with the global co-design in Prop. 4. To address this issue, using the observed subsystem data matrices $\{\bar{\Phi}_i : i \in \mathbb{N}_N\}$ subject to As. 4, the following proposition presents a data-driven global co-design problem.

***Proposition** 8:* Using the designed local controllers (e.g., via (31)) that make each closed-loop subsystem dynamics $\mathcal{X}_i$-dissipative at each subsystem $\Sigma_i, i \in \mathbb{N}_N$, to enforce/optimize $\mathcal{Y}$-dissipativity of the closed-loop linear networked system for any subsystem model parameters $\Theta_i \in \Psi_i, \forall i \in \mathbb{N}_N$, the distributed global controller and the underlying communication topology (jointly captured by K in (13)) can be co-designed using the LMI problem:

$$\min_{\substack{\bar{K},\mathcal{Y},\lambda\\ \{p_i\in\mathbb{R}:i\in\mathbb{N}_N\}}} \quad J \triangleq \|\bar{K}\|_1 - \phi([p_i]_{i\in\mathbb{N}_N}) + \psi(\mathcal{Y})$$

N×N with 2×2 blocks.

$$\text{Sub. to: } p_i > 0, \forall i \in \mathbb{N}_N, \ \lambda \geq 0, \text{ and} \qquad (33)$$
$$\begin{bmatrix}Q & S\\ S^\top & R\end{bmatrix} \geq 0,$$

where

$$Q \triangleq \begin{bmatrix}\mathcal{Y}^{11} & \mathbf{0}\\ \star & -\mathcal{Y}^{22}\end{bmatrix}, \quad \mathcal{Y} \triangleq [\mathcal{Y}^{kl}]_{k,l\in\mathbb{N}_2},$$

Nn×Nn, γI, Nn×Nn., I, Dissipativity, off-diag → 0.

$$S \triangleq \begin{bmatrix}S_{11} & S_{12} & \mathbf{0}\\ S_{21} & S_{22} & \mathbf{0}\end{bmatrix}, \quad S_{11} \triangleq \begin{bmatrix}\mathcal{X}_p^{11}\bar{\mathcal{X}}^{11} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$
$$S_{12} \triangleq \begin{bmatrix}-\mathcal{X}_p^{12}+\mathcal{Y}^{12} & \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad S_{21} \triangleq \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$
$$S_{12} \triangleq \begin{bmatrix}-\mathcal{Y}^{22} & \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad \bar{\mathcal{X}}^{11} \triangleq (\mathcal{X}^{11})^{-1},$$
$$\mathcal{X}_p^{kl} \triangleq \text{diag}([p_i\mathcal{X}_i^{kl}]_{i\in\mathbb{N}_N}), \quad \forall k,l \in \mathbb{N}_2,$$
$$\mathcal{X}^{kl} \triangleq \text{diag}([\mathcal{X}_i^{kl}]_{i\in\mathbb{N}_N}), \quad \forall k,l \in \mathbb{N}_2,$$
$$R \triangleq \begin{bmatrix}\tilde{R}_{11} & \tilde{R}_{12} & \mathbf{0}\\ \tilde{R}_{21} & -\tilde{R}_{22} & -\tilde{R}_{21}\\ \mathbf{0} & -\tilde{R}_{12} & \mathbf{0}\end{bmatrix} - \lambda\begin{bmatrix}E\bar{Q}_wE^\top & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & E\bar{Q}_wE^\top\end{bmatrix}, \qquad (34)$$
$$\tilde{R}_{11} \triangleq \begin{bmatrix}\bar{\mathcal{X}}_p^{11} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad \tilde{R}_{12} \triangleq \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \bar{K} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$
$$\tilde{R}_{21} \triangleq (\tilde{R}_{12})^\top, \quad \tilde{R}_{22} \triangleq \begin{bmatrix}\mathcal{X}_p^{22} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},$$
$$E \triangleq \begin{bmatrix}\text{diag}(\begin{bmatrix}\mathbf{I}_{n_{xi}} & \mathbf{0}_{n_{xi}} & \mathbf{0}_{n_{xi}\times n_{ui}}\end{bmatrix} : i \in \mathbb{N}_N)\\ \text{diag}(\begin{bmatrix}\mathbf{0}_{n_{xi}} & \mathbf{I}_{n_{xi}} & \mathbf{0}_{n_{xi}\times n_{ui}}\end{bmatrix} : i \in \mathbb{N}_N)\\ \text{diag}(\bar{E}_{3i}\begin{bmatrix}\mathbf{0}_{n_{xi}} & \mathbf{0}_{n_{xi}} & \bar{E}_{3i}^\top\end{bmatrix} : i \in \mathbb{N}_N)\end{bmatrix},$$
$$\bar{E}_{3i} \triangleq \begin{bmatrix}\mathbf{I}_{n_{ui}} & \mathbf{0}_{n_{ui}\times(n_{xi}-n_{ui})}\end{bmatrix}, \ \forall i \in \mathbb{N}_N,$$
$$\bar{Q}_w \triangleq \text{diag}([\bar{Q}_{wi}]_{i\in\mathbb{N}_N}), \quad \bar{\mathcal{X}}_p^{11} \triangleq \bar{\mathcal{X}}^{11}\mathcal{X}_p^{11}\bar{\mathcal{X}}^{11},$$

and $\bar{K} \triangleq [\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$ with $K \triangleq [p_i^{-1}\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$.

Proof: We start by applying Prop. 4, which requires:

$$\begin{bmatrix}\mathcal{X}_p^{11} & \mathbf{0} & \mathcal{X}^{11}B\bar{K} & \mathcal{X}_p^{11}\\ \star & -\mathcal{Y}^{22} & -\mathcal{Y}^{22} & \mathbf{0}\\ \star & \star & -\mathcal{H}(\mathcal{X}^{21}B\bar{K}) - \mathcal{X}_p^{22} & -\mathcal{X}_p^{21}+\mathcal{Y}^{21}\\ \star & \star & \star & \mathcal{Y}^{11}\end{bmatrix} \geq 0.$$

Using Lm. 2, we get the equivalent LMI condition

$$\begin{bmatrix}\bar{\mathcal{X}}_p^{11} & \mathbf{0} & B\bar{K} & \bar{\mathcal{X}}^{11}\mathcal{X}_p^{11}\\ \star & -\mathcal{Y}^{22} & -\mathcal{Y}^{22} & \mathbf{0}\\ \star & \star & -\mathcal{H}(\mathcal{X}^{21}B\bar{K}) - \mathcal{X}_p^{22} & -\mathcal{X}_p^{21}+\mathcal{Y}^{21}\\ \star & \star & \star & \mathcal{Y}^{11}\end{bmatrix} \geq 0,$$

where we have used the notations $\bar{\mathcal{X}}^{11} \triangleq (\mathcal{X}^{11})^{-1}$, and $\bar{\mathcal{X}}_p^{11} \triangleq \bar{\mathcal{X}}^{11}\mathcal{X}_p^{11}\bar{\mathcal{X}}^{11}$ from (34). Using Lm. 2, we restate the above LMI condition as

all Nn×Nn blocks.

$$\begin{bmatrix}\mathcal{Y}^{11} & \mathbf{0} & \mathcal{X}_p^{11}\bar{\mathcal{X}}^{11} & -\mathcal{X}_p^{12}+\mathcal{Y}^{12}\\ \star & -\mathcal{Y}^{22} & \mathbf{0} & -\mathcal{Y}^{22}\\ \star & \star & \bar{\mathcal{X}}_p^{11} & B\bar{K}\\ \star & \star & \star & -\mathcal{H}(\mathcal{X}^{21}B\bar{K}) - \mathcal{X}_p^{22}\end{bmatrix} \geq 0,$$

and then via Lm. 1, we obtain the equivalent condition

$$\bar{R} - \bar{S}^\top Q^{-1}\bar{S} \geq 0,$$

where

N×N with n×m, N×N block each m×n, All blocks. Nn × Nn

$$\bar{R} \triangleq \begin{bmatrix}\bar{\mathcal{X}}_p^{11} & B\bar{K}\\ \star & -\mathcal{H}(\mathcal{X}^{21}B\bar{K}) - \mathcal{X}_p^{22}\end{bmatrix},$$

$$Q \triangleq \begin{bmatrix}\mathcal{Y}^{11} & \mathbf{0}\\ \star & -\mathcal{Y}^{22}\end{bmatrix}, \text{ and } \bar{S} \triangleq \begin{bmatrix}\mathcal{X}_p^{11}\bar{\mathcal{X}}^{11} & -\mathcal{X}_p^{12}+\mathcal{Y}^{12}\\ \mathbf{0} & -\mathcal{Y}^{22}\end{bmatrix}.$$

All Nn×Nn blocks.

nN nN nN nN nN

Defining $\tilde{\Theta}^\top \triangleq \begin{bmatrix}\mathbf{I} & A & B\end{bmatrix}$ and $\hat{\Theta}^\top \triangleq \mathcal{X}^{21}\tilde{\Theta}^\top$, the $\bar{R}$ and $\bar{S}$ terms in this equivalent condition can be expressed as

(n+n+m)N (n+n+m)N

2nN x 2nN

$$\bar{R} = \begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix}^\top \underbrace{\begin{bmatrix}\tilde{R}_{11} & \tilde{R}_{12} & \mathbf{0}\\ \tilde{R}_{21} & -\tilde{R}_{22} & -\tilde{R}_{21}\\ \mathbf{0} & -\tilde{R}_{12} & \mathbf{0}\end{bmatrix}}_{\triangleq \tilde{R}} \begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix}, \quad \text{and}$$

3x1 block matrix (n+n+m)N x nN; nN x nN, nN x nN, mN x nN

all blocks (n+n+m)N x (n+n+m)N

(n+n+m)N (n+n+m)N (n+n+m)N

$$\bar{S} = \underbrace{\begin{bmatrix}S_{11} & S_{12} & \mathbf{0}\\ S_{21} & S_{22} & \mathbf{0}\end{bmatrix}}_{\triangleq S} \begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix},$$

2nN x 2nN

(n+n+m)N x nN

respectively, where all the inner elements of $\tilde{R}$ and S take the forms given in (34). Consequently, the obtained equivalent condition can be restated as a QMI:

3x3 block, each blk (n+n+m)N x (n+n+m)N

$$\begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix}^\top \underbrace{\left(\tilde{R} - S^\top Q^{-1} S\right)}_{\triangleq \check{R}} \begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix} \geq 0. \tag{35}$$

2nN x 2nN

To obtain a similarly structured QMI from (27), let us first denote $\bar{\Theta}_i^\top \triangleq \begin{bmatrix}\mathbf{I} & A_i & B_i\end{bmatrix}$, $\forall i \in \mathbb{N}_N$ and $\bar{\Theta} \triangleq \text{diag}([\bar{\Theta}_i]_{i\in\mathbb{N}_N})$. Using these notations, we can restate the local QMIs (27) as: $\bar{\Theta}_i^\top \bar{Q}_{wi}\bar{\Theta}_i \geq 0, \forall i \in \mathbb{N}_N$, which can be collectively represented as

(n+n+m)N x nN

nN x nN

$$\bar{\Theta}^\top \bar{Q}_w \bar{\Theta} \geq 0,$$

(n+n+m)N x (n+n+m)N

where $\bar{Q}_w \triangleq \text{diag}([\bar{Q}_{wi}]_{i\in\mathbb{N}_N})$ (as defined in (34)).

(n+n+m)N x nN

(n+n+m)N

Next, we identify the permutation matrix E that maps $\bar{\Theta} \to \tilde{\Theta}$ such that $\tilde{\Theta} = E\bar{\Theta}$. To this end, we consider E to be a block column matrix with three blocks, where each block $E_k \triangleq \text{diag}([E_{ki}]_{i\in\mathbb{N}_N})$, $k \in \mathbb{N}_3$ (is itself block diagonal) and satisfies $\mathbf{I} = E_1\bar{\Theta}$, $A^\top = E_2\bar{\Theta}$ and $B^\top = E_3\bar{\Theta}$, respectively. As $\bar{\Theta}$ is block diagonal, we require: $\mathbf{I}_{n_{xi}} = E_{1i}\bar{\Theta}_i$, $A_i^\top = E_{2i}\bar{\Theta}_i$ and $B_i^\top = E_{3i}\bar{\Theta}_i$ for any $i \in \mathbb{N}_N$. Using this, we can identify E_{1i}, E_{2i}, E_{3i} as

nN x nN
nN x nN
mN x nN

$$\bar{\Theta}_i = \begin{bmatrix}I\\ A_i^\top\\ B_i^\top\end{bmatrix}$$

$$E_{1i} \triangleq \begin{bmatrix}\mathbf{I}_{n_{xi}} & \mathbf{0}_{n_{xi}} & \mathbf{0}_{n_{xi}\times n_{ui}}\end{bmatrix},$$
$$E_{2i} \triangleq \begin{bmatrix}\mathbf{0}_{n_{xi}} & \mathbf{I}_{n_{xi}} & \mathbf{0}_{n_{xi}\times n_{ui}}\end{bmatrix},$$
$$E_{3i} \triangleq \bar{E}_{3i}\begin{bmatrix}\mathbf{0}_{n_{xi}} & \mathbf{0}_{n_{xi}} & \bar{E}_{3i}^\top\end{bmatrix},$$

where $\bar{E}_{3i} \triangleq \begin{bmatrix}\mathbf{I}_{n_{ui}} & \mathbf{0}_{n_{ui}\times(n_{xi}-n_{ui})}\end{bmatrix}$. This proves the permutation matrix E definition given in (34). Since E is a permutation matrix, $E^{-1} = E^\top$, and thus, $\bar{\Theta} = E^\top\tilde{\Theta}$.

Consequently, we can express the vectorized QMI obtained earlier from (27), i.e., $\bar{\Theta}^\top\bar{Q}_w\bar{\Theta} > 0$, as: $\tilde{\Theta}^\top E\bar{Q}_w E^\top\tilde{\Theta} > 0$. Using Lm. 2 with $P = \mathcal{X}^{12}$, this QMI also implies $\hat{\Theta}^\top E\bar{Q}_w E^\top\hat{\Theta} > 0$. Now, combining these two QMIs, we get a unified QMI as

end res 2nN x 2nN

$$\begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix}^\top \underbrace{\begin{bmatrix}E\bar{Q}_w E^\top & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & E\bar{Q}_w E^\top\end{bmatrix}}_{\triangleq \tilde{Q}_w} \begin{bmatrix}\tilde{\Theta} & \mathbf{0}\\ \mathbf{0} & \tilde{\Theta}\\ \mathbf{0} & \hat{\Theta}\end{bmatrix} \geq 0. \tag{36}$$

each blk (n+n+m)N x (n+n+m)N

For the implication (36) $\implies$ (35) to hold, both a necessary and sufficient condition can be found using the matrix S-lemma as $\check{R} - \lambda\tilde{Q}_w \geq 0$, for some $\lambda \geq 0$. Applying Lm. 1, we can obtain an equivalent condition for this as

2nN x 2nN

2nN x 3(n+n+m)N

$$\begin{bmatrix}Q & S\\ S^\top & R\end{bmatrix} \geq 0,$$

3x3 with (n+n+m)N x (n+n+m)N

where $R \triangleq \tilde{R} - \lambda\tilde{Q}_w$, which is the main LMI condition given in (33) along with the same Q, S and R definitions given in (34).

Finally, note that the identified condition above is still an LMI in design variables $\bar{K}$ and $\{p_i : i \in \mathbb{N}_N\}$. Therefore, according to Prop. 4, we can recover the design variable K via enforcing $\bar{K} \triangleq [\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$ and evaluating $K \triangleq [p_i^{-1}\bar{K}_{ij}]_{i,j\in\mathbb{N}_N}$. This completes the proof. ■

V. Application to DC Microgrid Control

To demonstrate the effectiveness of the proposed model-based and data-driven co-design techniques, we consider the DC Microgrid (DCMG) control problem, where the goal is to design local and distributed global controllers to simultaneously achieve voltage regulation and balanced current sharing with respect to rated voltages and currents.

*1) **DCMG Model**:* In particular, we consider a DCMG with N distributed generators (DGs) $\{\Sigma_i^{DG} : i \in \mathbb{N}_N\}$ and L lines $\{\Sigma_l^{Line} : l \in \mathbb{N}_L\}$. An example DCMG with 6 DGs and 7 Lines is shown in Fig. 3. The electrical schematic diagrams of a DG Σ_i^{DG} and a connected line Σ_l^{Line} in a DCMG are as shown in Fig. 4. We use the convention where electrical loads are considered to be ZI-loads (each with a constant impedance (Z) and a constant current (I) component) connected locally at each DG. The total load current $I_{Li}(t)$ at $\Sigma_i^{DG}, i \in \mathbb{N}_N$ can be expressed as

$$I_{Li}(t) \triangleq I_{Li}^Z(t) + I_{Li}^I(t) \equiv \frac{1}{R_{Li}} V_i(t) + \bar{I}_{Li},$$

where R_{Li} is the load's constant resistance and $\bar{I}_{Li}$ is the load's constant current.

Applying Kirchhoff's current and voltage laws, we can obtain the state space representation for Σ_i^{DG} as

$$\begin{aligned}\underbrace{\begin{bmatrix} \dot{V}_i(t) \\ \dot{I}_{ti}(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} -\frac{1}{C_i R_{Li}} & \frac{1}{C_i} \\ -\frac{1}{L_i} & -\frac{R_i}{L_i} \end{bmatrix}}_{\triangleq A_i} \underbrace{\begin{bmatrix} V_i(t) \\ I_{ti}(t) \end{bmatrix}}_{\triangleq x_i(t)} + \underbrace{\begin{bmatrix} -\frac{1}{C_i}\bar{I}_{Li} \\ 0 \end{bmatrix}}_{\triangleq \theta_i} \\ + \underbrace{\begin{bmatrix} -\frac{1}{C_i} & 0 \\ 0 & \frac{1}{L_i} \end{bmatrix}}_{\triangleq B_i} \underbrace{\begin{bmatrix} I_i(t) \\ V_{ti}(t) \end{bmatrix}}_{\triangleq u_i(t)} + \underbrace{\begin{bmatrix} -\frac{1}{C_i} & 0 \\ 0 & \frac{1}{L_i} \end{bmatrix}}_{\triangleq B_i} w_i(t), \end{aligned} \quad (37)$$

where L_i, R_i, C_i are the DG circuit elements (see Fig. 4). The DG state $x_i(t)$ is comprised of the components: $V_i(t)$, which denotes the voltage at the coupling point, and $I_{ti}(t)$, which denotes the internal current. The input $u_i(t)$ driving the DG dynamics is comprised of the components: $I_i(t)$ that represents the total current injected to the DCMG, and $V_{ti}(t)$ that represents the voltage command applied to the voltage source converter (VSC). The disturbance vector $w_i(t)$ affecting the DG dynamics is comprised of the components that represent the disturbances in $I_i(t)$ and $V_{ti}(t)$, respectively. These disturbance components are assumed to be zero-mean and bounded. Using the A_i, B_i and θ_i notations introduced in (37), the dynamics of Σ_i^{DG} can be expressed concisely as

$$\dot{x}_i(t) = A_i x_i(t) + \theta_i + B_i u_i(t) + B_i w_i(t). \quad (38)$$

*2) **DCMG Input**:* Note that the input $u_i(t)$ of Σ_i^{DG} contains an uncontrollable component $I_i(t)$ along with a controllable component $V_{ti}(t)$. In particular, $I_i(t)$ depends on the lines and the neighboring DGs in the physical topology of the DCMG, and takes the form

$$\begin{aligned} I_i(t) &= \sum_{j \in \mathcal{E}_i^P} \frac{1}{R_{ij}} (V_i(t) - V_j(t)) = \sum_{j \in \mathbb{N}_N} \bar{R}_{ij} V_j(t) \\ &= \sum_{j \in \mathbb{N}_N} \bar{R}_{ij} D_j^\top x_j(t), \end{aligned} \quad (39)$$

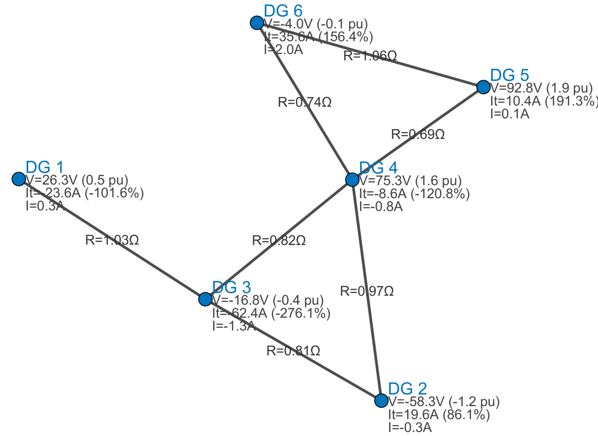

Fig. 3: An example DCMG with 6 DGs and 7 Lines (initial configuration).

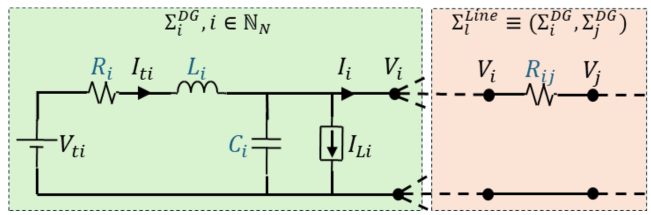

Fig. 4: Electrical schematic diagram of a DG Σ_i^{DG} and a connected Line Σ_l^{Line} in a DCMG.

where $\mathcal{E}_i^P$ represents the set of physically connected neighboring DGs to Σ_i^{DG}, R_{ij} represents the resistance of the line between Σ_i^{DG} and Σ_j^{DG}, and we denote

$$\bar{R}_{ij} \triangleq \begin{cases} 0, & j \notin \mathcal{E}_i^{DG} \cup \{i\}, \\ -\frac{1}{R_{ij}}, & j \in \mathcal{E}_i^{DG}, \\ \sum_{k \in \mathcal{E}_i^P} \frac{1}{R_{ik}}, & j = i, \end{cases}$$

$$D_i \triangleq \begin{bmatrix} 1 & 0 \end{bmatrix}^\top, \qquad \forall i \in \mathbb{N}_N.$$

On the other hand, $V_{ti}(t)$ is determined using a hierarchical controller involving: a steady-state controller (denoted V_{Sti}), a local feedback controller (denoted $V_{Lti}(t)$), and a global distributed feedback controller (denoted $V_{Gti}(t)$) implemented over the communication topology of the DCMG. In particular, $V_{ti}(t)$ takes the form

$$\begin{aligned} V_{ti}(t) &= V_{Sti} + V_{Lti}(t) + V_{Gti}(t) \\ &= V_{Sti} + K_i(x_i(t) - x_{Ei}) + \sum_{j \in \mathbb{N}_N} K_{ij}(x_j(t) - x_{Ej}), \end{aligned} \quad (40)$$

where the exact forms of V_{Sti} (steady-state input), x_{Ei} (DG state at the equilibrium), and controller gains $K_i, [K_{ij}]_{j \in \mathbb{N}_N}$ will be designed in the sequel. Now, combining (39) and (40), we can express the input $u_i(t)$ of Σ_i^{DG} as

$$\begin{aligned} u_i(t) &= \bar{D}_i V_{Sti} + \begin{bmatrix} 0 \\ K_i(x_i(t) - x_{Ei}) \end{bmatrix} \\ &+ \sum_{j \in \mathbb{N}_N} \begin{bmatrix} \bar{R}_{ij} D_j^\top x_j(t) \\ K_{ij}(x_j(t) - x_{Ej}) \end{bmatrix}, \end{aligned} \quad (41)$$

where $\bar{D}_i \triangleq \begin{bmatrix} 0 & 1 \end{bmatrix}^\top, \forall i \in \mathbb{N}_N$.

*3) **Equilibrium Point Design**:* For each DG $\Sigma_i, i \in \mathbb{N}_N$, we denote the rated state as $x_{Rat,i} \triangleq \begin{bmatrix} V_{Rat,i} & I_{Rat,ti} \end{bmatrix}^\top$, the rated input as $u_{Rat,i} \triangleq \begin{bmatrix} I_{Rat,i} & V_{Rat,ti} \end{bmatrix}^\top$, the equilibrium state as $x_{Ei} \triangleq \begin{bmatrix} V_{Ei} & I_{Eti} \end{bmatrix}^\top$, and the equilibrium input as $u_{Ei} \triangleq \begin{bmatrix} I_{Ei} & V_{Eti} \end{bmatrix}^\top$. Note that, while $x_{Rat,i}$ and $u_{Rat,i}$ are known, both x_{Ei} and u_{Ei} are to be designed. In particular, the design specifications for x_{Ei} and u_{Ei} are assumed to be convex constraints of the form:

$$x_{Ei} \in \mathcal{U}_{xi}(x_{Rat,i}), \quad u_{Ei} \in \mathcal{U}_{ui}(u_{Rat,i}).$$

For example, to achieve proportional current sharing among the DGs and bound deviations from the rated voltage level (both at the equilibrium), we can constrain x_{Ei} such that:

$$x_{Ei} = \begin{bmatrix} \epsilon_{Vi} & 0 \\ 0 & \epsilon_{Iti} \end{bmatrix} x_{Rat,i}, \quad \vert\epsilon_{Vi} - 1\vert \leq \delta_{Vi}, \quad \epsilon_{Iti} = \delta_I, \tag{42}$$

for some (or prespecified) $\delta_{Vi} \in [0,1],\ \forall i \in \mathbb{N}_N$ and $\delta_I \in [0,1]$. Similarly, to limit the total current injected to/from DGs and bound the voltage command applied to the VSC (at the equilibrium), we can constrain u_{Ei} so that

$$u_{Ei} = \begin{bmatrix} \epsilon_{Ii} & 0 \\ 0 & \epsilon_{Vti} \end{bmatrix} u_{Rat,i}, \quad \vert\epsilon_{Ii}\vert \leq \delta_{Ii}, \quad \vert\epsilon_{Vti} - 1\vert \leq \delta_{Vti}, \tag{43}$$

for some (or prespecified) $\delta_{Ii}, \delta_{Vti} \in [0,1],\ \forall i \in \mathbb{N}_N$.

Besides such design specifications, x_{Ei} and u_{Ei} must satisfy the equilibrium conditions (from (38) and (41)):

$$\begin{aligned} \mathbf{0} &= A_i x_{Ei} + \theta_i + B_i u_{Ei}, \\ u_{Ei} &= \bar{D}_i V_{Sti} + \sum_{j \in \mathbb{N}_N} \begin{bmatrix} \bar{R}_{ij} D_j^\top x_{Ej} \\ 0 \end{bmatrix}. \end{aligned} \tag{44}$$

Vectorizing these equilibrium conditions we can obtain:

$$\begin{aligned} \mathbf{0} &= A x_E + \theta + B u_E, \\ u_E &= \bar{D} V_{St} + D^\top \bar{R} D^\top x_E. \end{aligned} \tag{45}$$

where we have defined the vectors: $x_E \triangleq [x_{Ei}^\top]_{i\in\mathbb{N}_N}^\top$, $u_E \triangleq [u_{Ei}^\top]_{i\in\mathbb{N}_N}^\top$, $\theta \triangleq [\theta_i^\top]_{i\in\mathbb{N}_N}^\top$, $V_{St} \triangleq [V_{Sti}]_{i\in\mathbb{N}_N}^\top$, and the matrices: $A \triangleq \text{diag}([A_i]_{i\in\mathbb{N}_N})$, $B \triangleq \text{diag}([B_i]_{i\in\mathbb{N}_N})$, $D \triangleq \text{diag}([D_i]_{i\in\mathbb{N}_N})$, $\bar{D} \triangleq \text{diag}([\bar{D}_i]_{i\in\mathbb{N}_N})$, $\bar{R} \triangleq [\bar{R}_{ij}]_{i,j\in\mathbb{N}_N}$.

In our implementation, the equilibrium point (x_E, u_E) is designed such that constraints (42)-(45) are satisfied. Note that, through this equilibrium point design, we also indirectly obtain the required steady state control inputs V_{St} (for (41)) as the second component of u_{Ei} is $V_{Sti}, \forall i \in \mathbb{N}_N$ (see (44)).

*4) **Error Dynamics**:* By defining error variables $\tilde{x}_i(t) \triangleq x_i(t) - x_{Ei}$ and $\tilde{u}_i(t) \triangleq u_i(t) - u_{Ei}$ and applying them in (38) and (41), we obtain the error dynamics of DG Σ_i^{DG} as

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= A_i \tilde{x}_i(t) + B_i \tilde{u}_i(t) + \tilde{w}_i(t), \\ \tilde{y}_i(t) &= C_i \tilde{x}_i(t), \\ \tilde{u}_i(t) &= \bar{K}_i \tilde{x}_i(t) + \sum_{j \in \mathbb{N}_N} \bar{K}_{ij} \tilde{y}_j(t). \end{aligned} \tag{46}$$

where we have also used the equilibrium conditions (44) and the new notations $\tilde{w}_i(t) \triangleq B_i w_i(t)$, $C_i \triangleq \mathbf{I}$,

$$\bar{K}_i \triangleq \begin{bmatrix} 0 \\ K_i \end{bmatrix}, \quad \text{and} \quad \bar{K}_{ij} \triangleq \begin{bmatrix} \bar{R}_{ij} D_j^\top \\ K_{ij} \end{bmatrix}.$$

Clearly, this DG error dynamics, and consequently, the overall closed-loop DCMG error dynamics, are identical to the networked system model used this paper. Therefore, we can apply both the model-based and data-driven hierarchical control and topology co-design techniques developed in this paper (in Sections III and IV, respectively) to design the local feedback controllers, global distributed controllers and the communication topology for the considered DCMG system, particularly to evaluate the DG control inputs (40).

However, it is worth noting the additional design constraints on the effective local controller gains $\bar{K}_i, i \in \mathbb{N}_N$ and the effective global distributed controller gains $\bar{K}_{ij}, i,j \in \mathbb{N}_N$, due to the physical interconnections (lines) in (46), that can be expressed as:

$$\begin{aligned} &D_i^\top \bar{K}_i = 0,\ \forall i \in \mathbb{N}_N \text{ and} \\ &D_i^\top \bar{K}_{ij} = \bar{R}_{ij} D_j^\top, \forall i,j \in \mathbb{N}_N \iff D^\top \bar{K} = \bar{R} D^\top. \end{aligned} \tag{47}$$

Nevertheless, including these additional constraints in the design procedures given in Props. 3- 8 is quite straightforward. For example, when using Prop. 3, since the effective local controller $\bar{K}_i$ is to be determined via some LMI variables $\tilde{K}_i$ and P_i as $\bar{K}_i = \tilde{K}_i P_i^{-1}$, the required additional design constraint in (47) can be included in this LMI problem (12) as $D_i^\top \tilde{K}_i = \mathbf{0}$. This is because:

$$D_i^\top \bar{K}_i = \mathbf{0} \iff D_i^\top \bar{K}_i P_i = \mathbf{0} \iff D_i^\top \tilde{K}_i = \mathbf{0}.$$

Similarly, when using Prop. 8, since the effective global distributed controller $\bar{K}$ is to be determined via some LMI variables $\tilde{K} = [\tilde{K}_{ij}]_{i,j\in\mathbb{N}_N}$ and $\{P_i : i \in \mathbb{N}_N\}$ as $\bar{K} = [p_i^{-1}\tilde{K}_{ij}]_{i,j\in\mathbb{N}_N}$, the required additional design constraint in (47) can be included in this LMI problem as $D^\top \tilde{K} = P\bar{R}D^\top$ where $P \triangleq \text{diag}([p_i]_{i\in\mathbb{N}_N})$. This is because:

$$\begin{aligned} D^\top \bar{K} = \bar{R} D^\top &\iff D^\top [p_i^{-1}\tilde{K}_{ij}]_{i,j\in\mathbb{N}_N} = \bar{R} D^\top \\ &\iff D_i^\top p_i^{-1} \tilde{K}_{ij} = \bar{R}_{ij} D_j^\top, \forall i,j \in \mathbb{N}_N \\ &\iff D_i^\top \tilde{K}_{ij} = p_i \bar{R}_{ij} D_j^\top, \forall i,j \in \mathbb{N}_N \\ &\iff D^\top \tilde{K} = P\bar{R}D^\top. \end{aligned}$$

*5) **Model-Based Co-Design Results**:*

VI. Conclusion

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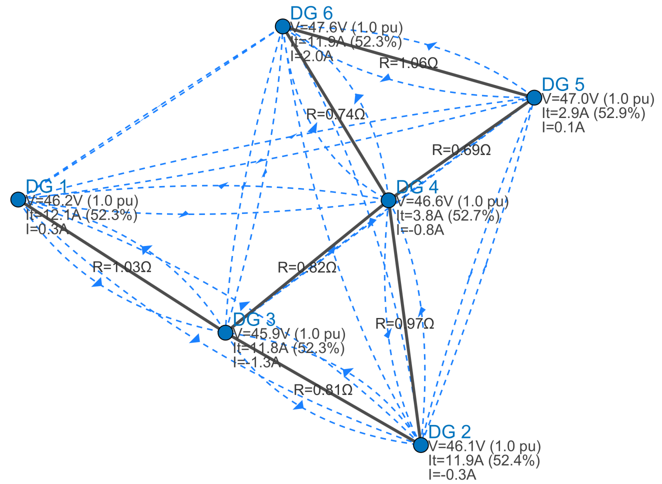
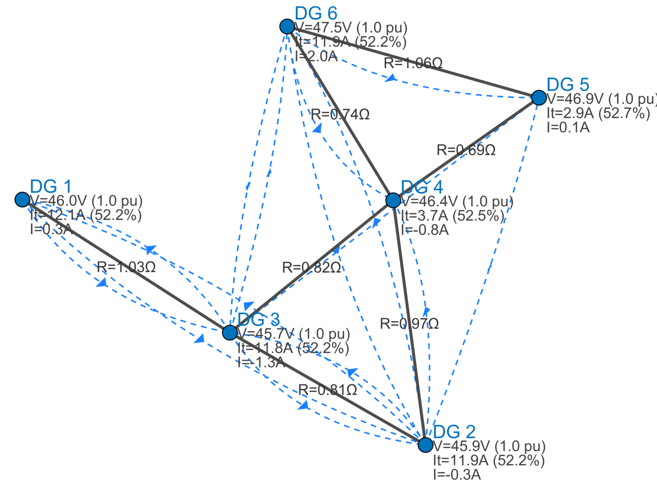
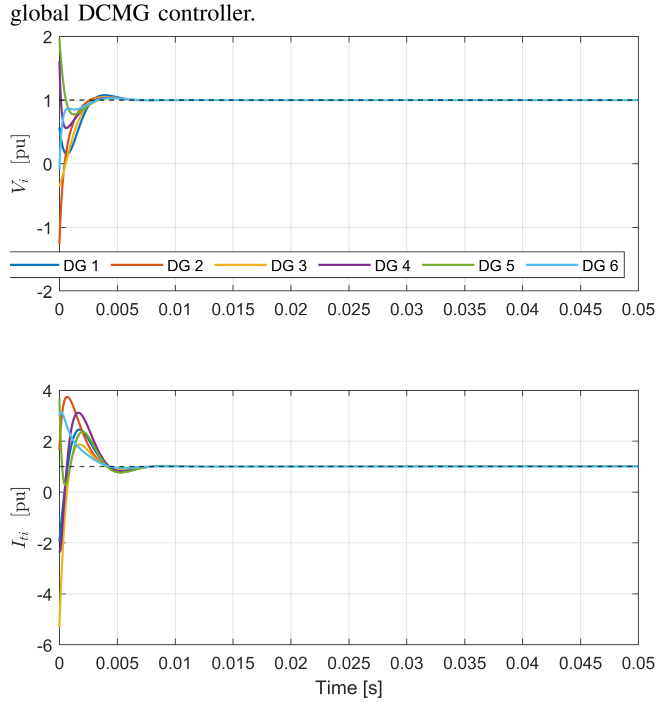


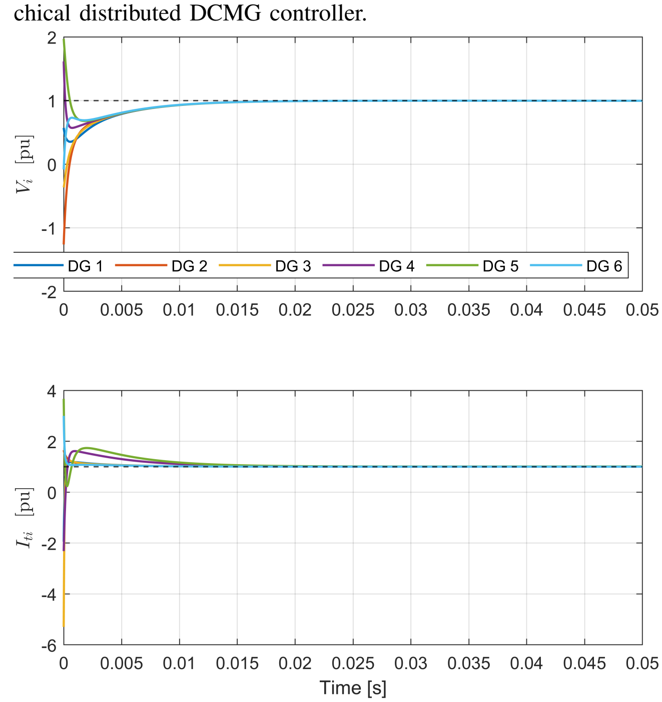
(a) Communication topology (fully-connected) and the final DCMG configuration observed under a model-based stabilizing global DCMG controller.



(b) Communication topology and the final DCMG configuration observed under the proposed model-based dissipativating hierarchical distributed DCMG controller.



(c) DG state trajectories observed under a model-based stabilizing global DCMG controller.



(d) DG state trajectories observed under the proposed model-based dissipativating hierarchical distributed DCMG controller.

Fig. 5: Numerical results comparison between stabilizing global control vs. dissipativating hierarchical distributed control for the DCMG voltage regulation and balanced current sharing tasks.

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